King Abdullah University of Science and Technology
Computer, Electrical and Mathematical Science and Engineering (CEMSE) Division

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King Abdullah University of
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Master of Science Thesis Defense

Feedback Reduction in Wireless Multiuser Networks

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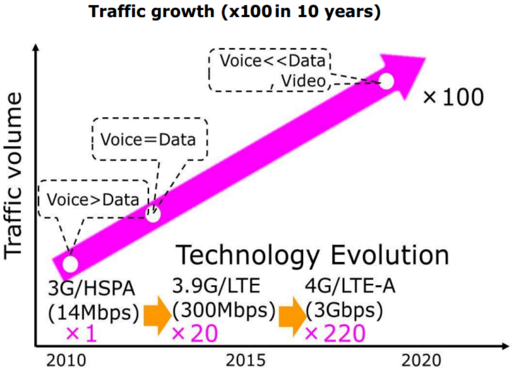


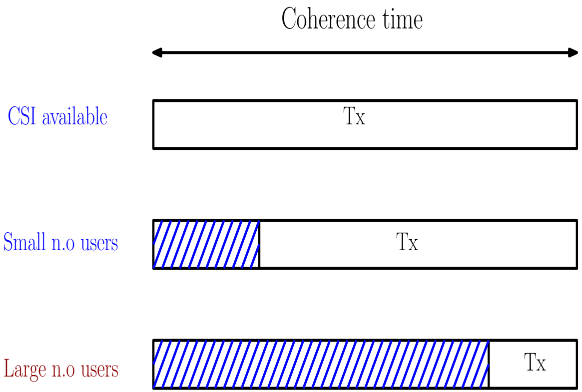
Figure 0.1: Explosive growth of traffic. Source: Ericsson Mobility Report, June 2013.

- To cope with the increasing demand, new solutions can be used such as
  - Higher-order modulation.
  - MIMO.
- Exploit randomness in the channel (MUD).

idea: Assign network resources to strong users.

Challenge channel state information (CSI).

Feedback is a key issue !



Challenge How to reduce feedback ?

Research Objectives

The aim of this work is to design efficient scheduling algorithms that minimize the feedback overhead of opportunistic schedulers without degrading the network performance.

Basic Assumptions

- $N << R$
- No direct link.
- i.i.d Rayleigh.
- Half duplex relays.
- DF or AF.

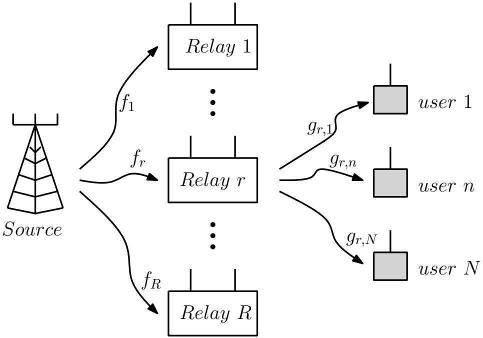
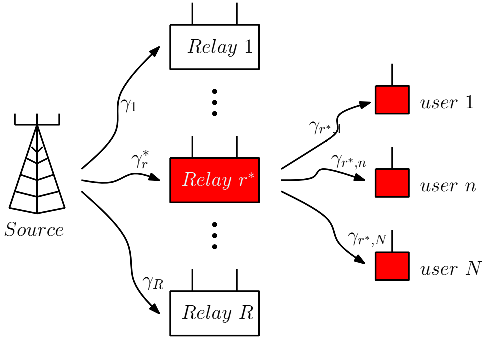


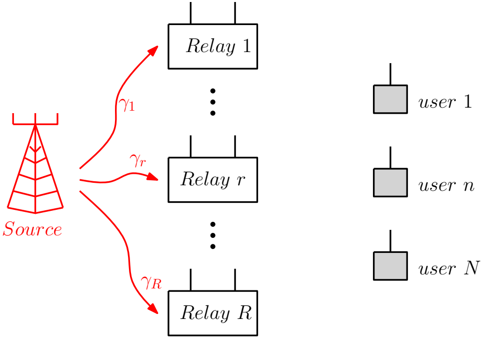
Figure 0.2: Multicast Relay Network Model

Basic Assumptions

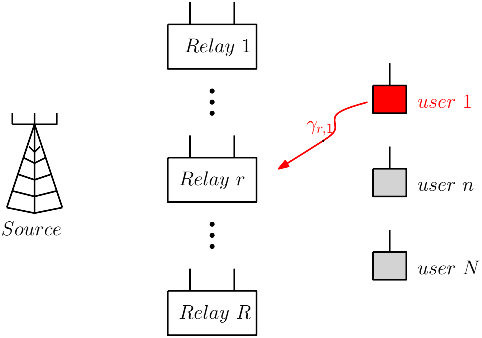
- $N << R$
- No direct link.
- i.i.d Rayleigh.
- Half duplex relays.
- DF or AF.



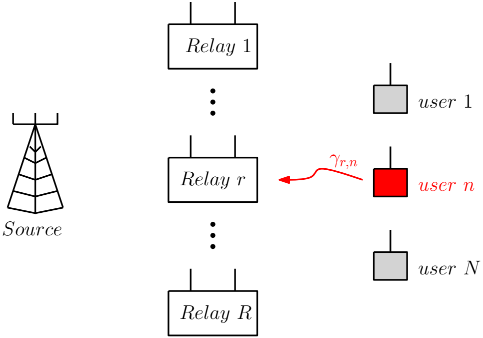
Equivalent SNR Estimation:



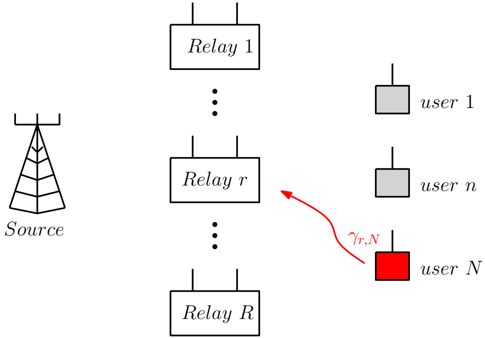
Equivalent SNR Estimation:



Equivalent SNR Estimation:



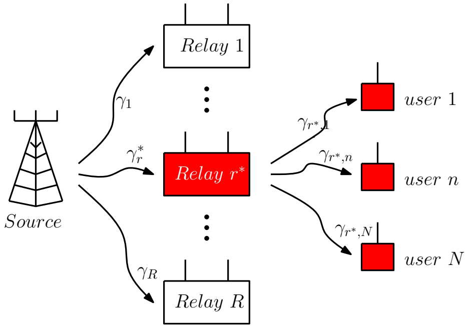
Equivalent SNR Estimation:



We need $N+1$ time instants for equivalent SNR estimation.

Equivalent SNR:

- DF: $\gamma_r^e = \min\left(\gamma_r, \min_{n\in\{1,\ldots,N\}}(\gamma_{r,n})\right)$.
- AF: $\gamma_r^e = \min_{n\in\left\{1,\ldots,N\right\}} \frac{\gamma_r\gamma_{r,n}}{\gamma_r+\gamma_{r,n}+1}$



Challenge How to determine r^* and $\gamma_{r^*}^e$?

Solution: Feedback

	Full CSI	Feedback
Overhead	R	$M < R$
Rate	$\mathcal{R}_{DF,AF}$	$< \mathcal{R}_{DF,AF}$

When $R \to \infty$

$$\gamma^*_{DF} = \mu^{-1}\left(\xi + \log R\right) \quad (1)$$

$$\gamma^*_{AF} = \left(\xi + \log\sqrt{\frac{\pi v\left(\mu + v\right)^{-1}}{2}} + \log R + \log\sqrt{\log R}\right)\left(\mu + v\right)^{-1} \quad (2)$$

$$\mathcal{R}_{DF} = \frac{1}{2}\log\left(1 + \gamma_{DF}\right) \quad (3)$$

$$\mathcal{R}_{AF} = \frac{1}{2}\log\left(1 + \gamma_{AF}\right) \quad (4)$$

$$\mathcal{R}_{DF} \approx \mathcal{R}_{AF} + \log\frac{\mu + v}{\mu} \quad (5)$$

Feedback Model

- Thresholding: compare x_i to ζ
- Assign feedback codes to each relay.
- All relays feedback simultaneously

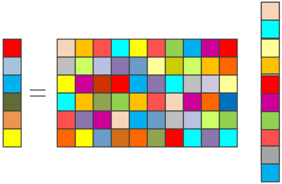
$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,R} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,R} \\ \vdots & \vdots & \vdots & \vdots \\ a_{M,1} & a_{M,2} & \cdots & a_{M,R} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_R \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}}_{\mathbf{w}}$$

or equivalently

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w} \tag{6}$$

Challenge How to resolve collision ?

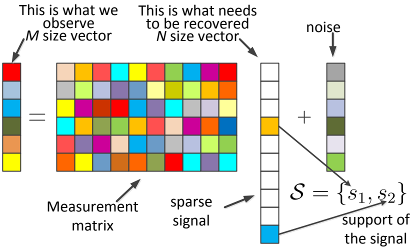
Background on Compressed Sensing (CS)



$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

$\mathbf{y} \in \mathbb{R}^{M \times 1}$, $\mathbf{A} \in \mathbb{R}^{M \times R}$ and $\mathbf{x} \in \mathbb{R}^{R \times 1}$
To be able to recover $\mathbf{x}$, we need to satisfy : $M \geq R$

$$\hat{\mathbf{x}}_{LS} = \left(\mathbf{A}^H \mathbf{A}\right)^{-1} \mathbf{A}^H \mathbf{y} \tag{7}$$



$$\mathbf{y} = \mathbf{A} \mathbf{x} + \mathbf{n}$$
$$\mathbf{y} = \mathbf{A}_{\mathcal{S}} \mathbf{x}_{\mathcal{S}} + \mathbf{n}$$

Figure 0.2: CS Linear Model

Performance guarantees (LASSO)

$$\mathcal{P}_{CS}(S) \geq 1 - 2R^{-1}\left(\frac{1}{\sqrt{2\pi \log R}} + \frac{S}{R}\right) \qquad (8)$$

Feedback recovery

- Recover the support $\mathcal{S} \rightarrow \mathbf{A}_{\mathcal{S}}$
- Recover the SNRs

Least Squares

$$\begin{aligned}\hat{\mathbf{x}}_{\mathcal{S}} &= \underbrace{(\mathbf{A}_{\mathcal{S}}^T \mathbf{A}_{\mathcal{S}})^{-1} \mathbf{A}_{\mathcal{S}}^T}_{\mathbf{A}_{\mathcal{S}}^{\dagger}} \mathbf{y} \\ &= \mathbf{x}_{\mathcal{S}} + \mathbf{e}\end{aligned} \tag{9}$$

where $\mathbf{e} = \mathbf{A}_{\mathcal{S}}^{\dagger} \mathbf{w}$

Problematic If the estimated SNR $>$ the actual one !

Solution: Back-off

- Back-off the est. SNR by Δ

$$SNR = SNR_{act} + e - \Delta \tag{10}$$

- Back-off Efficiency:

$$\begin{aligned} \eta &= \Pr\left(SNR \leq SNR_{act}\right) \\ &= 1 - Q\left(\frac{\Delta}{\sigma_e}\right) \end{aligned} \tag{11}$$

Performance How well does this perform ?

Performance Analysis

Feedback Air-Time

$$L = CS \log R + N + 1(mini - slot) \tag{12}$$

Rate

$$\mathcal{R} \geqslant \frac{1}{2}\log_2(1+\gamma-\Delta)\left(1-Q\left(\frac{\Delta}{\sigma_e}\right)\right)\sum_{r=1}^{R}\binom{R}{r}\left(1-F_{\gamma_r^e}(\zeta)\right)^r \times F_{\gamma_r^e}(\zeta)^{R-r}\left(1-2R^{-1}\left(\frac{1}{\sqrt{2\pi\log R}}+\frac{r}{R}\right)\right)(bit/sec/Hz) \tag{13}$$

Optimization Which Δ to use ?

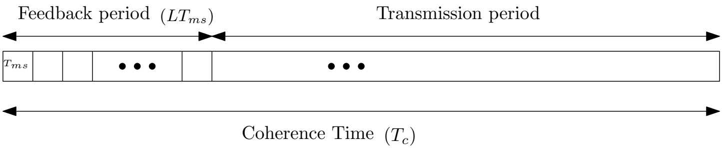
Optimize Δ

$$\left(\frac{1+\gamma-\Delta}{\sqrt{2\pi}\sigma_e}\right)\exp\left(-\frac{\Delta^2}{2\sigma_e^2}\right)\log\left(1+\gamma-\Delta\right)+Q\left(\frac{\Delta}{\sigma_e}\right)=1 \qquad (14)$$

Throughput:

$$\begin{aligned}\bar{\mathcal{R}} &= \mathcal{R}\frac{(T_c - LT_{ms})}{T_c} \\ &= \mathcal{R}\left(1 - L\tau\right)\end{aligned} \qquad (15)$$

$\tau = \frac{T_{ms}}{T_c}$.



Numerical Results

$\bar{\gamma}_r = \bar{\gamma}_d = 20$ dB, $N = 5$ users, and $C = 2.3$.

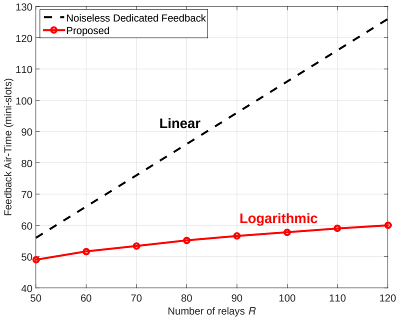


Figure 0.3: Feedback air-time versus the number of relays R.

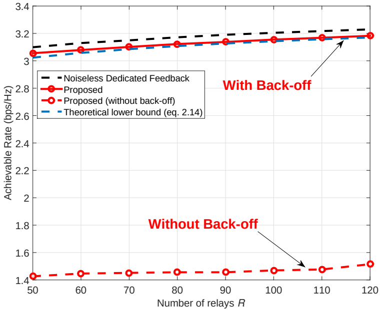


Figure 0.4: Rate versus the number of relays R.

Numerical Results

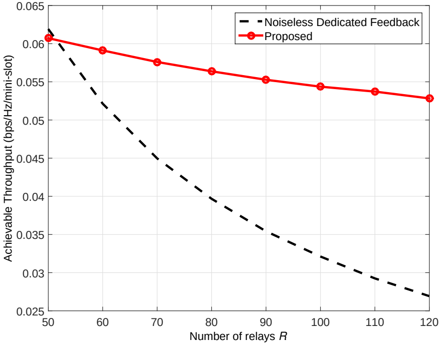


Figure 0.5: Achievable throughput versus the number of relays R.

Relay-aided Multiuser Networks

System Model

- The BS-relay link is modeled as Nakagami fading
- The channels, $(\gamma_n)_{n=1,2,\ldots,N}$ are i.i.d. Rayleigh.
- FD downlink.
- FD or HD uplink.

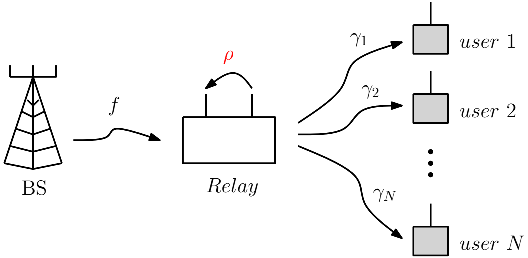


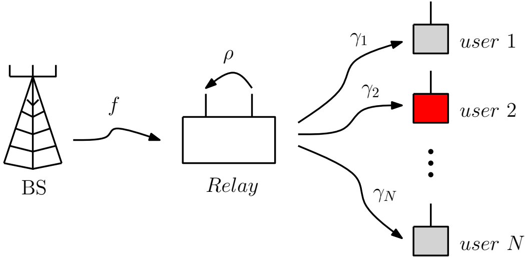
Figure 0.6: Network Model

Selection Rule

To maximize the downlink rate,

$$n^* = \arg \max_{1 \le n \le N} \gamma_n, \quad (16)$$

where $\gamma_n = \frac{P_r |g_n|^2}{N_0}$.



Challenge Feedback

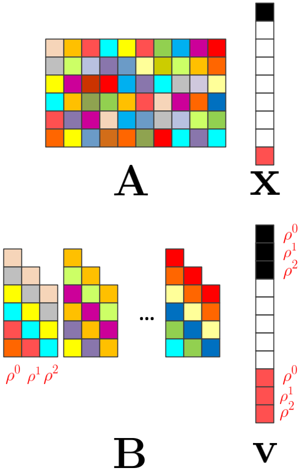
FD Feedback vs. HD Feedback

- **A** is the fback. code matrix.
- **x** is the fback. vector (sparse)

	FD	HD
RSI	$\rho \neq 0$	$\rho = 0$
Meas. matrix	$\mathbf{B}$	$\mathbf{A}$
Noise	$\mathbf{\Sigma}_z$	$\alpha\mathbf{I}$
Fback. vector	$\mathbf{v}$	$\mathbf{x}$
Struct.	block sparse	sparse
$\mathbf{y}$	$\mathbf{Bv}+\mathbf{z}$	$\mathbf{Ax}+\mathbf{z}$
N.o Measu.	M	2M

Tradeoff Accuracy vs. n.o. measurements.

Example



Question How much meas. do we really need ?

Important Result on Block-Sparse Recovery

- Robust recovery for a sparse vect. with size N and sparsity S: $M = \mathcal{O}\left(S \log N/S\right)$.
- Similarly to recover a block sparse vect. with block size J and sparsity S: $M = \mathcal{O}\left(JS \log N/S\right)$.

Structure in the sparse signal

Baraniuk *et al* (2010), showed that robustness guarantees can be achieved with

$$M = \mathcal{O}\left(JS + S \log N/S\right) \tag{17}$$

$\rightarrow$ substantial improvement over $\mathcal{O}\left(JS \log N/S\right)$

Feedback recovery

- Recover the support $\mathcal{I} \rightarrow \mathbf{B}_{\mathcal{I}}$
- Recover the SNRs

BLUE

$$\begin{aligned}\hat{\mathbf{x}}_{\mathcal{S}} &= \underbrace{(\mathbf{B}_{\mathcal{I}}^{t}\mathbf{\Sigma}_{z}^{-1}\mathbf{B}_{\mathcal{I}})^{-1}\mathbf{B}_{\mathcal{I}}^{t}}_{\mathbf{B}_{\mathcal{I}}^{\dagger}}\mathbf{y} \\ &= \mathbf{v}_{\mathcal{S}} + \mathbf{e}_{BLUE}\end{aligned} \tag{18}$$

where $\mathbf{e}_{BLUE} = \mathbf{B}_{\mathcal{I}}^{\dagger}\mathbf{z}$

- $\mathbf{e}_{BLUE}$ is Gaussian with zero mean and covariance $\left(\mathbf{B}_{\mathcal{I}}^t \mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1} \mathbf{B}_{\mathcal{I}}\right)^{-1}$
- $\sigma^2_{e,BLUE} = \frac{1}{JS} tr \left(\mathbf{B}_{\mathcal{I}}^t \mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1} \mathbf{B}_{\mathcal{I}}\right)^{-1}$

Lemma

Assume that $M \to +\infty$ with S and J fixed. Assume that $\limsup_M \left\|\mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1}\right\| < +\infty$. Then,

$$\left(\mathbf{B}_{\mathcal{I}}^t \mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1} \mathbf{B}_{\mathcal{I}}\right)^{-1} - \frac{M \mathbf{I}_{JS}}{tr\left(\mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1}\right)} \xrightarrow[M \to +\infty]{a.s.} \mathbf{O}_{JS}. \tag{19}$$

Therefore,

$$\sigma^2_{e,BLUE} = \frac{M}{tr\left(\mathbf{\Sigma}_{\hat{\mathbf{z}}}^{-1}\right)} \tag{20}$$

Performance Analysis

Feedback Load

- HD Feedback

$$L_{\mathrm{HD}} = 2CS \log N/S \tag{21}$$

- FD Feedback

$$L_{\mathrm{FD}} = C\left(JS + S \log JN/S\right), \tag{22}$$

Rate

$$\begin{aligned} \mathcal{R} &\leq \mathbb{E}\left[\log\left(1+\gamma_{eq}-\Delta\right)\left(1-\mathcal{P}_o\right)\left(1-Q\left(\frac{\Delta}{\sigma_e}\right)\right)\right] \\ &\leq \log\left(1+\mathbb{E}[\gamma_{eq}]-\Delta\right)\left(1-\mathcal{P}_o\right)\left(1-Q\left(\frac{\Delta}{\sigma_e}\right)\right). \end{aligned} \tag{23}$$

Numerical Results

$b^2 = 20$ dB, $\sigma^2 = 0$ dB, $\sigma_g^2 = 5$ dB, and $\mathcal{P}_0 = 10^{-2}$.

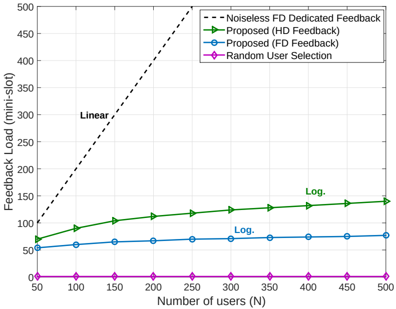


Figure 0.7: Feedback Load versus the number of users (N)

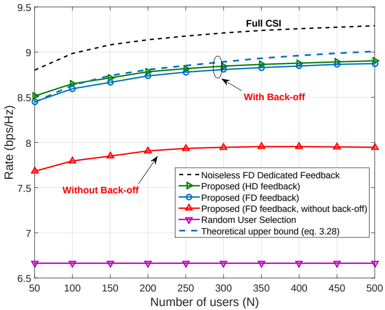


Figure 0.8: Average Rate versus the number of users (N)

Numerical Results

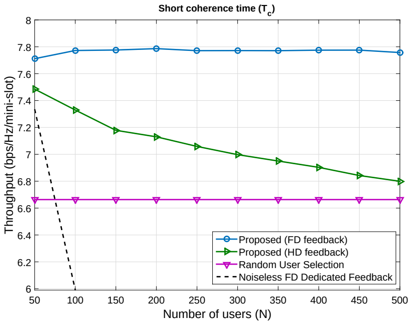


Figure 0.9: Average Throughput versus the number of users (N), $\tau = 1/600$

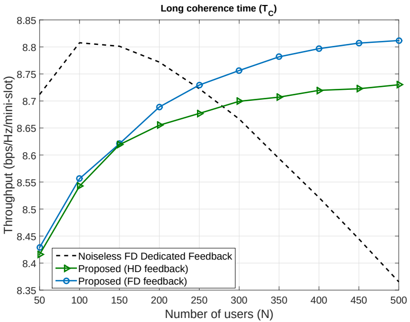


Figure 0.10: Average Throughput versus the number of users (N), $\tau = 10^{-4}$

User Selection in NW-MIMO

System Model

- M BSs with L antennas each and K users ($K \geq LM$).
- $\mathbf{h}_k = [h_{k,1}, h_{k,2}, ..., h_{k,LM}]$ channel from the BSs to the kth user.
- $\mathcal{K} = \{\pi(1), \pi(2), ..., \pi(|\mathcal{K}|)\}$

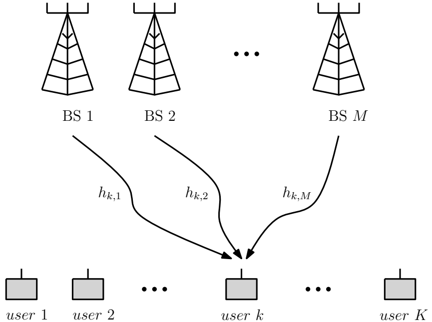
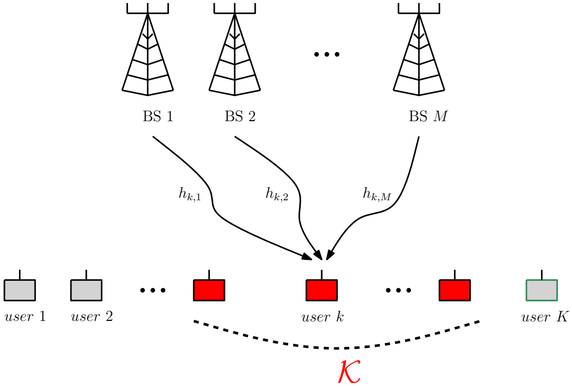


Figure 0.11: Network Model

User Selection



Challenge What's the optimal set $\mathcal{K}$?

Optimal set $\mathcal{K}$

$$\mathbf{W}(\mathcal{K}) = \mathbf{H}(\mathcal{K})^H \left(\mathbf{H}(\mathcal{K})\,\mathbf{H}(\mathcal{K})^H\right)^{-1} \tag{24}$$

$$R_{ZFBF}(\mathcal{K}) = \max_{P_i : \sum_{i\in\mathcal{K}} \gamma_i^{-1} P_i \le P} \sum_{i\in\mathcal{K}} \log(1 + P_i), \tag{25}$$

$$\gamma_i = \frac{1}{||w_i||^2}$$

$$R_{ZFBF} = \max_{\mathcal{K}\subset\{1,\cdots,K\} : |\mathcal{K}| \le M} R_{ZFBF}(\mathcal{K}) \tag{26}$$

Problem Computationally unfeasible for large K !

Semi-orthogonal user selection (SUS)

idea: choose users to be nearly orthogonals

1. $$\pi(1) = \underset{k \in \{1,2,\ldots,K\}}{\operatorname{argmax}} \left\|\mathbf{h}_k\right\|^2 \tag{27}$$

2. $$\pi(i+1) = \underset{k \in \mathcal{A}_i}{\operatorname{argmax}} \left\|\mathbf{h}_k\right\|^2, \tag{28}$$

   where $\mathcal{A}_i = \{1 \leq k \leq K : \frac{\left|\mathbf{h}_k \mathbf{h}_{\pi(j)}^H\right|}{\left\|\mathbf{h}_k\right\| \left\|\mathbf{h}_{\pi(j)}\right\|} \leq \epsilon, 1 \leq j \leq i\}$ and $i \leq M-1$

3. repeat until $|\mathcal{K}| = M$ or $\mathcal{A}_i = \emptyset$

Problem Full CSI needed or Fb. load $\sim \mathcal{O}(K)$

CS Approach

1. CDI and CQI for each user (training).
2. Sparsify users (thresholding).
3. Two fback info (CQI and ch. gain).
4. Assign two Gaussian codes for each user.

For the ith user selection,

if $\|\mathbf{h}_k\|^2 \geq \gamma_i$ **then**

transmit $(a_{m,2k-1}\mathit{pilot} + a_{m,2k}\mathit{CQI})$,

for $m = 1, 2, ..., J$

else

be silent

lth BS receives

$$\mathbf{y}_l = \mathbf{A}\left(\mathbf{g}_l \circ \mathbf{X}\right) + \mathbf{z}_l \qquad (29)$$

Feedback Recovery

$$\begin{bmatrix} h_{1,l} \\ h_{1,l} \\ h_{2,l} \\ h_{2,l} \\ \vdots \\ h_{K,l} \\ h_{K,l} \end{bmatrix} \circ \begin{bmatrix} pilot \\ CQI_1 \\ pilot \\ CQI_2 \\ \vdots \\ pilot \\ CQI_K \end{bmatrix} \tag{30}$$

- recover $CQI_{\pi(i)} \rightarrow h_{\pi(i),l}$
- Transmit $h^*_{\pi(i),l} / \left\| \mathbf{h}_{\pi(i)} \right\|$
- user j receives

$$c_j = \sum_{l=1}^{LM} \frac{h_{j,l} h^*_{\pi(i),l}}{\|\mathbf{h}_j\| \left\| \mathbf{h}_{\pi(i)} \right\|} = \frac{\mathbf{h}_j \mathbf{h}^H_{\pi(i)}}{\|\mathbf{h}_j\| \left\| \mathbf{h}_{\pi(i)} \right\|} \tag{31}$$

- compare c_j with ϵ.

Numerical Results

$M = 6$ BSs, $\epsilon = 0.25$

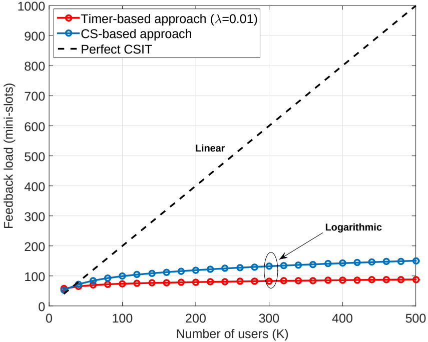


Figure 0.12: Feedback load versus the number of users K.

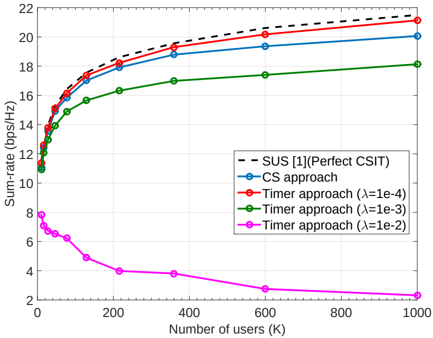


Figure 0.13: Sum-rate versus the number of users K.

- A compressed sensing based feedback algorithms has been established for different scenarios
- The feedback load grows logarithmically with the number of users/relays.
- The proposed feedback algorithms permit a substantial reduction in the feedback load with tolerable performance hit
- The proposed algorithms offer a practical framework that can be implemented in practice for various scenarios.

Publications

- (C1) K. Elkhalil, M. E. Eltayeb, H. Shibli, H. R. Bahrami and T. Y. Al-Naffouri "Opportunistic relay selection in multicast relay networks using compressive sens- ing ", Accepted in IEEE Global Communications Conference (GLOBECOM), Austin, Texas, USA, Dec 2014.
- (J1) M. E. Eltayeb, K. Elkhalil, H. R. Bahrami and T. Y. Al-Naffouri "Opportunis- tic Relay Selection with Limited Feedback", Major revision in IEEE Transactions on Communications.
- (J2) K. Elkhalil, M. E. Eltayeb, A. Kammoun, H. R. Bahrami and T. Y. Al-Naffouri "On the Feedback Reduction of Relay Aided Multiuser Networks using Compressive Sensing", Submitted to IEEE Transactions on Vehicular Technology.
- (C2) K. Elkhalil, M. E. Eltayeb, H. Dahrouj and T. Y. Al-Naffouri "Distributed User Selection in Network MIMO Systems with Limited Feedback", Submitted to IEEE 82nd Vehicular Technology Conference , 2015.
- (C3) M. E. Eltayeb, K. Elkhalil, A. A. Masud and T. Y. Al-Naffouri, "Relay Selection with Limited and Noisy Feedback", Submitted to IEEE 82nd Vehicular Technology Conference , 2015.
- (J3) K. Elkhalil, A. Kammoun and T. Y. Al-Naffouri, "Analytical derivation of the Inverse Moments for one-sided correlated Gram matrices with Applications", To be submitted to IEEE Transactions on Signal Processing.

Thank you for your attention