International IT University
Faculty of Computer technologies and cyber security
Department: MCM

Report

In the discipline «Numerical Analysis»

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Task 1: 1D Advection Equation

1. We have equation:

$$\frac{dN}{dt} = \alpha N - \beta N$$

- N(t) is the population at time t
- α is the birth rate coefficient
- β is the death rate coefficient
- t > 0 represents time evolution

2. The equation can be rewritten as:

$$\frac{dN}{dt} = (\alpha - \beta)N$$

So the Euler method for this is:

$$N_{n+1} = N_n + \Delta t \cdot f(N_n)$$

For the equation:

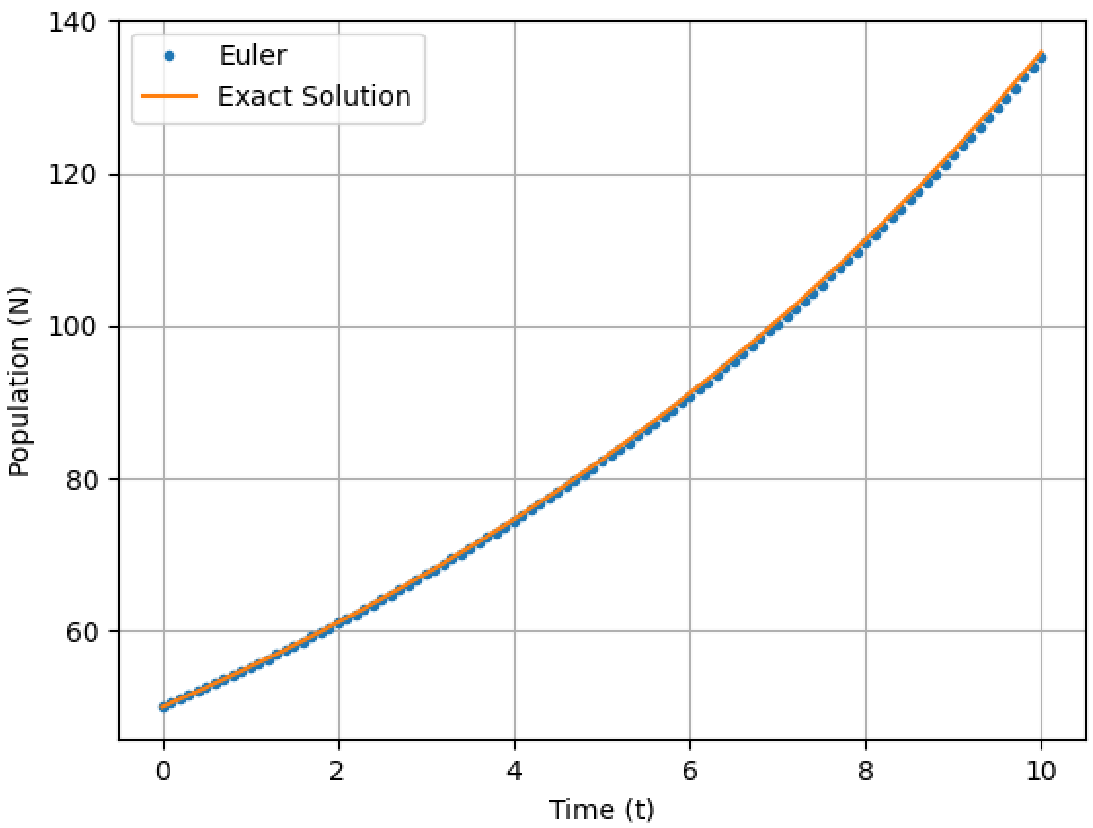
$$N_{n+1} = N_n + \Delta t \cdot (\alpha - \beta)N_n$$

$$N_{n+1} = N_n(1 + \Delta t \cdot (\alpha - \beta))$$

3. Code part:

```
import numpy as np
import matplotlib.pyplot as plt

alpha = 0.2
beta = 0.1
t0, t_final = 0, 10
dt = 0.1
N0 = 50

t_values = np.arange(t0, t_final + dt, dt)
N_values = np.zeros_like(t_values)
N_values[0] = N0

for i in range(1, len(t_values)):
    N_values[i] = N_values[i-1] + dt * (alpha - beta) * N_values[i-1]

N_exact = N0 * np.exp((alpha - beta) * t_values)

plt.plot(t_values, N_values, '.', label="Euler")
plt.plot(t_values, N_exact, '-', label="Exact Solution")
plt.xlabel("Time (t)")
plt.ylabel("Population (N)")
plt.legend()
plt.grid()
plt.show()
```

4. Graph:



Conclusion:

The graphs for different time points show that the population grows exponentially over time, following the expected trend. The numerical solution obtained using the explicit Euler method closely follows the exact analytical solution, but slight deviations appear due to numerical errors.