International IT University
Faculty of Computer technologies and cyber security
Department: MCM

Report

In the discipline «Numerical Analysis»

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Task 1: 1D Advection Equation

1. We have formula:

$$\frac{\partial U}{\partial t} + a\frac{\partial U}{\partial x} = 0,$$

- U (x, t) — function, we're looking for
- a — transfer coefficient
- x ∈ [0, L] — the area of space
- t > 0 — time

2. Approximation of the time derivative (step forward)

$$\frac{\partial U}{\partial x} \approx \frac{U_{i+1}^{n} - U_i^n}{\Delta x}$$

Approximation of the spatial derivative (step back)

$$\frac{\partial U}{\partial x} \approx \frac{U_i^n - U_{i-1}^n}{\Delta x}$$

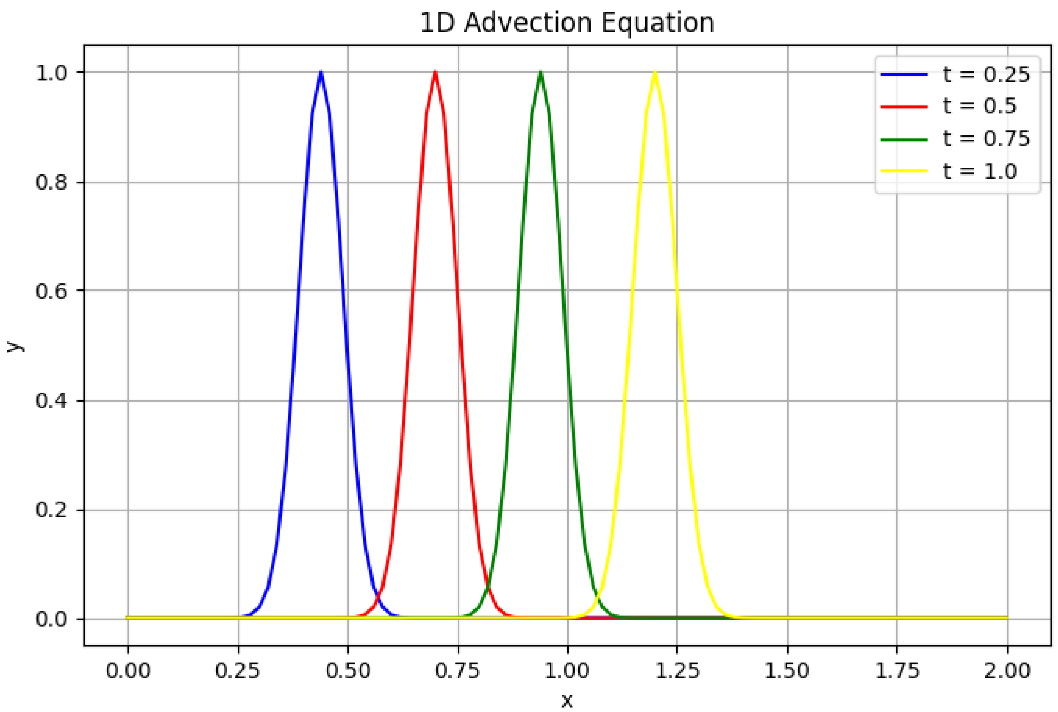
Now we substitute these into the basic formula:

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + a\frac{U_i^n - U_{i-1}^n}{\Delta x} = 0$$

3. Code part:

```python
import numpy as np
import matplotlib.pyplot as plt

t0, tn = 0, 1
x0, xn = 0, 2
N = 100
delta_x = (xn - x0) / N

delta_t = 0.02
N_t = int((tn - t0) / delta_t)

a = 1
C = a * delta_t / delta_x

X = np.linspace(x0, xn, N + 1)
T = np.arange(t0, tn + delta_t, delta_t)


def f(x):
    return np.exp(-200 * (x - 0.2) ** 2)


U = np.zeros((N_t + 1, N + 1))
U[0, :] = f(X)

for i in range(1, N_t + 1):
    for j in range(1, N + 1):
        U[i, j] = U[i - 1, j] - C * (U[i - 1, j] - U[i - 1, j - 1])


plot_times = [0.25, 0.5, 0.75, 1.0]
plot_indices = [int(t / delta_t) for t in plot_times]
colors = ['blue', 'red', 'green', 'yellow']


plt.figure(figsize=(8, 5))
for i, idx in enumerate(plot_indices):
    plt.plot(X, U[idx], color=colors[i], label=f"t = {plot_times[i]}")

plt.xlabel("x")
plt.ylabel("y")
plt.title("1D Advection Equation")
plt.legend()
plt.grid()
plt.show()
```

Graph:



Conclusion:

The graphs for different time points (t=0.25,0.5,0.75,1.0) show that the waveform remains approximately the same, but there is a slight blur.

Due to numerical diffusion, the wave becomes less sharp over time. This is due to the peculiarities of the method.