t – test

Small Sample Case

By

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t-test (For small sample, n<30)

Test of significance of single mean

Setting up hypothesis

Null hypothesis(H_o):μ=μ_o ie population mean has specified value μ_o . **OR** there is no significance difference between sample mean and population mean.

Alternative hypothesis (H_1):μ≠μ_o ie population mean has not specified value μ_o **OR** there is significance difference between sample mean and population mean. (two tailed test)

H_1:μ>$\mu_o$ie population mean is greater than specified value μ_o .(right tail)

H_1:μ<$\mu_o$ie population mean less than specified value μ_o .(left tailed)

Test statistic: Under null hypothesis(H_o) ,the test statistics is

$$\mathbf{t} = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}} \quad \text{(When sample standard deviation is not known)}$$

If sample standard deviation is known, then

$$\mathbf{t} = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}}$$

where, μ= population mean, $\bar{X}$=sample mean, S=modified sample s.d. , s=sample s.d, n= sample size

S^2= unbiased estimate of the population variance = $\frac{1}{n-1}\sum(X-\bar{X})^2$ (Actual mean deviation method)

$$= \frac{1}{n-1}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] \quad \textbf{(Direct Method)}$$

s^2 = biased estimate of the population variance = $\frac{1}{n}\sum(X-\bar{X})^2$(Actual mean deviation method)

$$= \frac{1}{n}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] \quad \textbf{((Direct Method)}$$

$\sum(X-\bar{X})^2$ = Sum of square of deviation from mean

Contd…

Level of significance(α): Use 5% level of significance unless otherwise stated.

Degree of freedom= n-1

Tabulated Value $(\boldsymbol{t_{\alpha,n-1}})$= Obtained from Student's t- table

Decision:

If calculated value of t ≤ tabulated value of t $(\mathbf{i.e.\,,\, t_{Cal} \leq t_{tab}})$

We accept null hypothesis H_O, otherwise we reject H_O (i. e. accept H_1)

Note: *Degree of freedom means number of items which are chosen freely . OR It is total number of observations minus one*.

Numerical Example:

A random sample of size 16 showed a mean of 52 with a standard 4. Test the hypothesis that the mean of the population is 50.

Solution:

Sample size(n) =16

Sample mean ($\bar{X}$) = 52

Sample standard deviation (s) = 4

Population mean (μ) = 50

Setting up hypothesis

Null hypothesis(H_0):$\mu = 50$ i.e. The population mean is 50.

Alternative hypothesis(H_1):$\mu \neq 50$ i.e. The population mean is not 50.

Contd…

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}} \qquad \because \left[\frac{S}{\sqrt{n}} = \frac{s}{\sqrt{n-1}}\right]$$

$$= \frac{52-50}{\frac{4}{\sqrt{16-1}}} = \mathbf{1.936}$$

Level of significance(α) = 0.05

Degree of freedom: **n-1** = 16-1 = 15

Tabulated value $(t_{\alpha,\, n-1}) = t_{0.05,\, 15} = 2.131$ **(Two - tailed test)**

Decision: Since $t_{Cal} = 1.936 < t_{0.05,\, 15}$ **ie** $t_{tab} = 2.131$, we accept null hypothesis H_0.

Hence, we conclude that population mean is 50

Student’s t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	1.363	1.796	[illegible]	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	[illegible]	2.65	3.012	3.852	4.221
14	1.345	1.761	[illegible]	2.624	2.977	3.787	4.14
15	[illegible]	[illegible]	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	[illegible]	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Numerical Example:

An automobile tyre manufacturer claims that the average life of a particular grade of tyre is more than 20000 km when used under normal conditions. A random sample of 16 tyres was tested and a mean and standard deviation are found to be 22000 km an 5000 km respectively. Assuming the life of the tyres in km to be normally distributed, decide whether the manufacturer's claim is valid or not at 1% level of significance.

Solution:

Sample size (n) =16

Sample mean ($\bar{X}$) = 22000 km

Sample standard deviation (s) = 5000 km

Population mean (μ) = 20000 km

Setting up hypothesis

Null hypothesis(H_0): μ = 20000 km i.e. Average life time of the tyre is 20000 km.

Alternative hypothesis(H_1): μ > 20000 km i.e. Average life time of the tyre is more than 20000 km.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}} \quad \because \left[\frac{S}{\sqrt{n}} = \frac{s}{\sqrt{n-1}}\right]$$

$$= \frac{22000-20000}{\frac{5000}{\sqrt{16-1}}} = 1.549$$

Contd…

Level of significance(α) = 0.01

Degree of freedom: n-1 = 16-1 = 15

Tabulated value ($t_{\alpha, n-1}$) = $t_{0.01, 15}$ = 2.602 **(one - tailed test)**

Decision: Since $\mathbf{t_{Cal} = 1.549 < t_{0.01, 15}}$ **ie** $\mathbf{t_{tab} = 2.602}$, we accept null hypothesis H_0.

Hence, we conclude that average life time of particular brand of the tyre is 20000 km.

Student’s t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	[illegible]	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	[illegible]	63.656	318.289	636.578
2	1.886	2.92	4.303	[illegible]	9.925	22.328	31.6
3	1.638	2.353	3.182	[illegible]	5.841	10.214	12.924
4	1.533	2.132	2.776	[illegible]	4.604	7.173	8.61
5	1.476	2.015	2.571	[illegible]	4.032	5.894	6.869
6	1.44	1.943	2.447	[illegible]	3.707	5.208	5.959
7	1.415	1.895	2.365	[illegible]	3.499	4.785	5.408
8	1.397	1.86	2.306	[illegible]	3.355	4.501	5.041
9	1.383	1.833	2.262	[illegible]	3.25	4.297	4.781
10	1.372	1.812	2.228	[illegible]	3.169	4.144	4.587
11	1.363	1.796	2.201	[illegible]	3.106	4.025	4.437
12	1.356	1.782	2.179	[illegible]	3.055	3.93	4.318
13	1.35	1.771	2.16	[illegible]	3.012	3.852	4.221
14	1.345	1.761	2.145	[illegible]	2.977	3.787	4.14
15	[illegible]	[illegible]	[illegible]	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	[illegible]	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Numerical Example:

A test of breaking strength of six ropes manufactured by a company showed a 7,750 kg and standard deviation of 145 kg where the manufacturer's claim is that mean breaking strength is at least 8000 kg. Will you support the manufacturer's claim?

Solution:

Sample size(n) =6

Sample mean ($\bar{X}$) = 7750 kg

Sample standard deviation (s) = 145 kg

Population mean (μ) = 8000 kg

Setting up hypothesis

Null hypothesis(H_0): $\mu \geq$ 8000 kg i.e. The mean breaking strength of the rope is at least 8000 kg.

Alternative hypothesis(H_1): $\mu <$ 8000 kg i.e. The mean breaking strength of the rope is less than 8000 kg.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}} \quad \because \left[\frac{S}{\sqrt{n}} = \frac{s}{\sqrt{n-1}}\right]$$

$$= \frac{7750-8000}{\frac{145}{\sqrt{6-1}}} = -3.86$$

$\therefore$ **$|t| = |-3.86| = 3.86$**

Level of significance(α) = 0.05

Degree of freedom: n-1 = 6-1 = 5

Tabulated value ($t_{\alpha, n-1}$) = $t_{0.05, 5}$ = 2.015 **(one - tailed test)**

Decision: Since **t_{Cal} = 3.86 > $t_{0.05, 5}$ ie t_{tab} =2.015** , we reject null hypothesis H_0.

Hence, we conclude that the mean breaking strength of the rope is less than 8000 kg.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	[illegible]	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	[illegible]	12.706	31.821	63.656	318.289	636.578
2	1.886	[illegible]	4.303	6.965	9.925	22.328	31.6
3	1.638	[illegible]	3.182	4.541	5.841	10.214	12.924
4	1.533	[illegible]	2.776	3.747	4.604	7.173	8.61
5	[illegible]	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	[illegible]	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
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30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Numerical Example:

Ministry of Tourism and Civil Aviation has claimed that average length of stay of tourist in Nepal is 13 days . To test his claim a researcher asked 9 tourists about their length of stay in Nepal and their length of stay in days were , 10 , 15, 11, 5 , 7 , 4, 8 , 14, and 11. On the basis of this sample result can we conclude that the average length of stay is 13 days?

Solution:

Given, Sample size (n) = 9

Population mean i.e. average length of stay of tourist in Nepal (μ)= 13 days

Let X be the length of stay of the tourist.

Calculation of sample mean($\bar{X}$) and sample standard deviation (S)

Now,

$$\bar{X}=\frac{\sum X}{n}=\frac{85}{9}=9.44 \text{ days}$$

$$S^2=\frac{1}{n-1}\left[\sum X^2-\frac{(\sum X)^2}{n}\right]=\frac{1}{9-1}\left[917-\frac{85^2}{9}\right]$$

$$=\frac{1}{8}\times 144.3722=14.3722$$

$$\therefore \text{S}=\sqrt{S^2}=\sqrt{14.3722}=3.7911$$

X	X^2
10	100
15	225
11	121
5	25
7	49
4	16
8	64
14	196
11	121
$\sum X=85$	$\sum X^2=917$

Setting up hypothesis

Null hypothesis(H_0): $\mu = 13$ days i.e. The average length of stay of tourist is 13 days.

Alternative hypothesis(H_1): $\mu \neq 13$ days i.e. The average length of stay of tourist is not 13 days.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\sqrt{\frac{s^2}{n}}} = \frac{9.4-13}{\sqrt{\frac{14.3722}{9}}} = -2.817$$

$\therefore |t| = |-2.817| = 2.817$

Level of significance(α) = 0.05

Degree of freedom: n-1 = 9-1 = 8

Tabulated value ($t_{\alpha, n-1}$) = $t_{0.05, 8}$ = 2.306 **(Two - tailed test)**

Decision: Since t_{Cal} **= 2.817 >** $t_{0.05, 8}$ **ie** t_{tab} **=2.306**, we reject null hypothesis H_0.

Hence, we conclude that the average length of stay of tourist is not 13 days.

Student’s t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
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4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	[illegible]	[illegible]	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
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19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
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26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Numerical Example:

A ABC television claims that its advertise time less than 6 minutes per hour. A random sample of 10 hours of this TV shows the following advertise times (rounded to the nearest minutes): 7, 3, 4, 6, 10, 5, 6, 4, 3, 8.

Is this evidence sufficient to support the TV's claim at 5% level of significance?

Solution:

Given, Sample size (n) = 10

Population mean (μ)= 6

Let X be the length of stay of the tourist.

Calculation of sample mean($\bar{X}$) and sample standard deviation (S)

Now,

$$\bar{X} = \frac{\sum X}{n} = \frac{56}{10} = 5.6$$

$$S^2 = \frac{1}{n-1}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] = \frac{1}{10-1}\left[360 - \frac{56^2}{10}\right] = \frac{1}{9} \times 46.4$$

$$= 5.16$$

$$\therefore S = \sqrt{S^2} = \sqrt{5.16} = 2.27$$

X	X^2
7	49
3	9
4	16
6	36
10	100
5	25
6	36
4	16
3	9
8	64
$\sum X = 56$	$\sum X^2 = 360$

Setting up hypothesis

Null hypothesis(H_0): $\mu = 6$ i.e. The average advertise time of ABC TV is 6 minutes.

Alternative hypothesis(H_1): $\mu < 6$ i.e. The average advertise time of ABC TV is less than 6 minutes.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\sqrt{\frac{s^2}{n}}} = \frac{5.6-6}{\sqrt{\frac{5.16}{10}}} = -0.557$$

$$\therefore |t| = |-0.557| = 0.557$$

Level of significance(α) = 0.05

Degree of freedom: n-1 = 10-1 =9

Tabulated value ($t_{\alpha,\, n-1}$) = 1.833 **(One- tailed test)**

Decision: Since $t_{Cal} = 0.557 < t_{0.05,\, 9}$ **ie** $t_{tab} = 1.833$, we accept null hypothesis H_0.

Hence, we conclude that the average advertise time of ABC TV is 6 minutes. It means the claim of ABC TV is invalid.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	[illegible]	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	[illegible]	12.706	31.821	63.656	318.289	636.578
2	1.886	[illegible]	4.303	6.965	9.925	22.328	31.6
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8	1.397	[illegible]	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	[illegible]	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Question: A random variable of size 20 from a normal population showed a mean 42 and sum of squares of deviation from the mean is 720. Is population mean 44? Test at 5% level of significance.

Solution:

Sample size(n) =20

Sample mean ($\bar{X}$) = 42

Sum of square of deviation from mean $\sum(X-\bar{X})^2$ =720

Population mean (μ) = 44

Now, $\mathbf{S^2} = \frac{\mathbf{1}}{\boldsymbol{n}-\mathbf{1}}\sum(\boldsymbol{X}-\bar{\boldsymbol{X}})^{\mathbf{2}} = \frac{1}{20-1} \times 720 = \frac{1}{19} \times 720 = 37.894$

$\therefore$ S= $\sqrt{S^2}$ =$\sqrt{37.894}$ = 6.12

Setting up hypothesis

Null hypothesis(H_0): $\mu = 44$ i.e. The population mean is 44.

Alternative hypothesis(H_1): $\mu \neq 44$ i.e. The population mean is not 44.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}} = \frac{42-44}{\frac{6.12}{\sqrt{20}}} = -1.46$$

$$\therefore |t_{cal}| = |-1.46| = 1.46$$

Level of significance(α) = 0.05

Degree of freedom: n-1 = 20 -1 = 19

Tabulated value ($t_{\alpha, n-1}$) = $t_{0.05, 19}$ = 2.093 **(Two - tailed test)**

Decision: Since $t_{Cal} = 1.46 < t_{0.05, 19}$ **ie** $t_{tab} = 2.093$, we accept null hypothesis H_0.

Hence, we conclude that population mean is 44.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	1.363	1.796	[illegible]	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	[illegible]	2.65	3.012	3.852	4.221
14	1.345	1.761	[illegible]	2.624	2.977	3.787	4.14
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19	[illegible]	[illegible]	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	[illegible]	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
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60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Contd….

Example: A random variable of size 20 from a normal population showed a mean 42 and sum of squares of deviation from the mean is equal to 720. Test hypothesis the population mean is 44.

Solution:

Sample size(n) =20

Sample mean $(\bar{X})$ = 42

Sum of square of deviation from mean $\sum(X-\bar{X})^2$ =720

Population mean (μ) = 44

Now, $S^2 = \frac{1}{n-1}\sum(X-\bar{X})^2$

$= \frac{1}{20-1} \times 720 = \frac{1}{19} \times 720 = 37.894$

$\therefore$ S= $\sqrt{S^2}$ =$\sqrt{37.894}$ = 6.12

Setting up hypothesis

Null hypothesis(H_0): $\mu = 44$ i.e. The population mean is 44.

Alternative hypothesis(H_1): $\mu \neq 44$ i.e. The population mean is not 44.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{42 - 44}{\frac{6.12}{\sqrt{20}}} = -1.46$$

$$\therefore |t_{cal}| = |-1.46| = 1.46$$

Level of significance(α) = 0.05

Degree of freedom: n-1 = 20 -1 = 19

Tabulated value ($t_{\alpha, n-1}$) = $t_{0.05, 19}$ = 2.093 **(Two - tailed test)**

Decision: Since $t_{Cal} = 1.46 < t_{0.05, 19}$ **ie** $t_{tab} = 2.093$, we accept null hypothesis H_0.`

Hence, we conclude that population mean is 44.

Confidence interval estimate for population mean (μ) for small sample case (n<30)

C.I. for $\boldsymbol{\mu}$ = $\overline{\boldsymbol{X}} \pm t_{\alpha,n-1} \times \frac{\boldsymbol{S}}{\sqrt{\boldsymbol{n}}}$ (when sample standard deviation is not given)

= $\overline{\boldsymbol{X}} \pm t_{\alpha,n-1} \times \frac{\boldsymbol{s}}{\sqrt{\boldsymbol{n-1}}}$ (when sample standard deviation is given)

Where, S^2= unbiased estimate of the population variance $= \frac{1}{n-1}\sum(X-\bar{X})^2$

$$= \frac{1}{n-1}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right]$$

s^2 = biased estimate of the population variance $= \frac{1}{n}\sum(X-\bar{X})^2$

$$= \frac{1}{n}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right]$$

Numerical Example:

A race of car driver tested his car for time 0 to 60 mph, and in 20 tests obtained an average of 4.85 seconds with a standard deviation of 1.47 seconds. Calculate a 95% confidence interval estimate for the time 0 to 60 mph.

Solution:

Sample size (n) = 20

Sample mean ($\bar{X}$) = 4.85 secs

Sample standard deviation (s) = 1.47 secs.

Confidence level (1-α) =95% =0.95

$\alpha = 0.05$

Degree of freedom: n-1 = 20-1=19

Tabulated value ($t_{\alpha,\, n-1}$) = $t_{0.05,19}$ = 2.093

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	1.363	1.796	[illegible]	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	[illegible]	2.65	3.012	3.852	4.221
14	1.345	1.761	[illegible]	2.624	2.977	3.787	4.14
15	1.341	1.753	[illegible]	2.602	2.947	3.733	4.073
16	1.337	1.746	[illegible]	2.583	2.921	3.686	4.015
17	1.333	1.74	[illegible]	2.567	2.898	3.646	3.965
18	1.33	1.734	[illegible]	2.552	2.878	3.61	3.922
19	[illegible]	[illegible]	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	[illegible]	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
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26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Contd….

95% confidence interval estimate for population mean (µ)

C. I for µ = $\bar{X} \pm t_{\alpha,\,n-1} \times \frac{s}{\sqrt{n-1}} = 4.85 \pm t_{0.05,19} \times \frac{1.47}{\sqrt{20-1}}$

$= 4.85 \pm 2.093 \times \frac{1.47}{\sqrt{19}} = 4.85 \pm 2.093 \times 0.33724$

$= 4.85 \pm 0.70584$

Lower limit = 4.85 – 0.70584 = 4.14416

Upper limit = 4.85 + 0.70584 = 5.55584

Numerical Example:

The weights in pounds at birth of sample of 15 babies born in a hospital are given below:

6	7	7	7	8	6	5	7
8	**8**	**8**	**5**	**6**	**9**	**8**	

- Construct 95% confidence interval estimate for the mean weight at birth of all such babies in the population.
- Construct 99% confidence interval estimate for the mean weight at birth of all such babies in the population.
- Discuss the effect of confidence interval estimate when you change the level of confidence.

Solution:

Here, sample size (n) = 15

Calculation of sample mean($\bar{X}$) and sample standard deviation (S)

Let X be the weight of babies at birth (in pounds)

X	6	7	7	7	8	6	5	7	8	8	8	5	6	9	6	$\sum X=105$
X^2	36	49	49	49	64	36	25	49	64	64	64	25	36	81	36	$\sum X^2=755$

Now,

Sample mean (($\bar{X}$) $= \frac{\sum X}{n} = \frac{105}{15}$ =7 pounds

$$S^2 = \frac{1}{n-1}\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] = \frac{1}{15-1}\left[755 - \frac{105^2}{15}\right] = \frac{1}{14} \times 20$$

=1.428

$\therefore$ S= $\sqrt{S^2}$ =$\sqrt{1.428}$ = 1.194

a)For 95% confidence interval estimate:

Confidence level (1- α) = 95% =0.95

Level of significance (α) = 5% = 0.05

Degree of freedom (n-1) = 15-1 =14

Tabulated value ($t_{\alpha,\, n-1}$) = $t_{0.05,\, 15-1}$= $t_{0.05,\, 14}$ =2.145

95% confidence interval estimate for mean weight at birth of all babies in the population is

$$\textbf{C. I for } \mu = \bar{X} \pm t_{\alpha,n-1} \times \frac{s}{\sqrt{n}} = 7 \pm t_{0.05,\,14} \times \frac{1.194}{\sqrt{15}}$$

$$= 7 \pm 2.145 \times \frac{1.194}{\sqrt{15}} = 7 \pm 2.145 \times 0.309$$

$$= 7 \pm 0.663$$

Lower limit = 7 – 0.663 = 6.337

Upper limit = 7 + 0.663 = 7.663

Hence , we conclude that , with 95% confidence that the mean weight at the birth of babies in the hospital is 6.337 and 7.663 pounds on the basis of sample of 15 babies born in that hospital.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	1.363	1.796	[illegible]	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	[illegible]	2.65	3.012	3.852	4.221
14	[illegible]	[illegible]	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	[illegible]	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
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22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
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27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

b)For 99% confidence interval estimate:

Confidence level $(1-\alpha) = 99\% = 0.99$, Level of significance $(\alpha) = 5\% = 0.05$

Degree of freedom $(n-1) = 15-1 = 14$

Tabulated value $(t_{\alpha,\, n-1}) = t_{0.01,\, 15-1} = t_{0.01,\, 14} = 2.997$

99% confidence interval estimate for mean weight at birth of all babies in the population is

$$\textbf{C. I for } \mu = \bar{X} \pm t_{\alpha,\, n-1} \times \frac{s}{\sqrt{n}} = 7 \pm 2.997 \times \frac{1.194}{\sqrt{15}}$$

$$= 7 \pm 2.997 \times 0.309 \quad = 7 \pm 0.920$$

Lower limit = 7 – 0.920 = 6.080

Upper limit = 7 + 0.920 = 7.920

Hence, we conclude that, with 99% confidence that the mean weight at the birth of babies in the hospital is 6.080 and 7.920 pounds on the basis of sample of 15 babies born in that hospital.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	[illegible]	318.289	636.578
2	1.886	2.92	4.303	6.965	[illegible]	22.328	31.6
3	1.638	2.353	3.182	4.541	[illegible]	10.214	12.924
4	1.533	2.132	2.776	3.747	[illegible]	7.173	8.61
5	1.476	2.015	2.571	3.365	[illegible]	5.894	6.869
6	1.44	1.943	2.447	3.143	[illegible]	5.208	5.959
7	1.415	1.895	2.365	2.998	[illegible]	4.785	5.408
8	1.397	1.86	2.306	2.896	[illegible]	4.501	5.041
9	1.383	1.833	2.262	2.821	[illegible]	4.297	4.781
10	1.372	1.812	2.228	2.764	[illegible]	4.144	4.587
11	1.363	1.796	2.201	2.718	[illegible]	4.025	4.437
12	1.356	1.782	2.179	2.681	[illegible]	3.93	4.318
13	1.35	1.771	2.16	2.65	[illegible]	3.852	4.221
14	[illegible]	[illegible]	[illegible]	[illegible]	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	[illegible]	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
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27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Contd…

c)Here,

95% confidence interval estimate (6.337,7.663) i.e. 1.326

99% confidence interval estimate (6.080, 7.920) i.e 1.84

Hence, increase in confidence level also increase in the confidence interval estimate.

Test of significance difference between two population means (Double Sample Case):

Setting up hypothesis

Null hypothesis(H_0): $\mu_1 = \mu_2$ **or** $\mu_1 - \mu_2 = 0$ i.e. There is no significant difference between two population means.**OR**.Two population means mean are equal.

Alternative hypothesis (H_1): $\mu_1 \neq \mu_2$ **or** $\mu_1 - \mu_2 \neq 0$ i.e.There is significant difference between two population means. **OR** Two population means are not equal. **(Two-tailed test)**

H_1: $\mu_1 > \mu_2$ i.e. First population mean is significantly greater than second population mean. **(Right tailed test)**

H_1 : $\mu_1 < \mu_2$ i.e. First population mean is significantly less than second population mean**. (Left tailed test)**

Test Statistic: Under H_0, the test statistic is

$$\mathbf{t} = \frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(\bar{X}_1-\bar{X}_2)-0}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Where, $\bar{X}_1 = \frac{\sum X_1}{n_1}$ = mean of the first sample, $\bar{X}_2 = \frac{\sum X_2}{n_2}$ = mean of the second sample

$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$ (When the sample variance (or standard deviations)

i.e. biased estimates are given) =**Pooled Variance**

$= \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ (When the unbiased estimate of the population variances

are known)

Note: 1. $S^2 = \frac{1}{n_1+n_2-2}\left[\sum X_1^2 - \frac{(\sum X_1)^2}{n_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{n_2}\right]$ **(Direct Method)**

2. $S^2 = \frac{1}{n_1+n_2-2}\left[\sum(X_1-\bar{X}_1)^2 + \sum(X_2-\bar{X}_2)^2\right]$ **(Actual mean deviation method)**

Level of significance(α) = 0.05 (Unless otherwise stated)

Degree of freedom = $\boldsymbol{n_1 + n_2 - 2}$

Tabulated Value = $\boldsymbol{t_{\alpha, n_1+n_2-2}}$ = Obtained from t-table.

Decision: If $\mathbf{t_{Cal}} > \boldsymbol{t_{\alpha, n_1+n_2-2}}$ **i.e** $\mathbf{t_{tab}}$, we reject H_0.

If $\mathbf{t_{Cal}} \leq \boldsymbol{t_{\alpha, n_1+n_2-2}}$ **i.e** $\mathbf{t_{tab}}$, we accept H_0 .

Example: Strength test carried out on s ample of two yarns spurn to the same count gave the following results:

Characteristics	Sample size	Sample mean	Sample Variance
Yarn A	4	52	42
Yarn B	9	42	56

The strength is expressed in kg. Test the significance of mean strengths of the sources which source of yarn is preferable?

Solution:

Here,

Characteristics	Sample Size	Sample mean	Sample variance
Yarn A	n_1=4	$\bar{X}_1$=52	s_1^2=42
Yarn B	n_2= 9	$\bar{X}_2$= 42	s_2^2 =56

Null hypothesis (H_0): $\boldsymbol{\mu_1 = \mu_2}$ i.e. Average strength of yarn A and yarn B is equal.

Alternative hypothesis (H_1): $\boldsymbol{\mu_1 \neq \mu_2}$ i.e. Average strength of yarn A and yarn B is not equal.

Test Statistic: Under $H_{0,}$ the test statistic is

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Where, $S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{4\times 42+9\times 56}{4+9-2} = 61$

$$t = \frac{52-42}{\sqrt{61\left(\frac{1}{4}+\frac{1}{9}\right)}} = 2.13$$

Level of significance: (α) = 0.05

Degree of freedom: $n_1 + n_2 - 2$ = 4+9-2 = 11

Tabulated value: (t_{α, n_1+n_2-2}) = $t_{0.05,11}$=2.201 (Two – tailed test)

Decision: Since, $\mathbf{t_{Cal} = 2.13 < t_{0.05,11}}$ **i.e $\mathbf{t_{tab}}$ =2.201**,we accept H_0.

Hence, we conclude that the average strength of Yarn A and Yarn B is equal.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	[illegible]	[illegible]	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Example:

A consumer research organization routinely selects several car models each year and evaluates their fuel efficiency. In this year's study of two similar subcompact models from two different automakers, the average gas mileage for 12 cars of brand A was 27.2 miles per gallon, and the standard deviation was 3.8 mpg. The 9 brand of B cars that were tested averaged 32.1 mpg, and the standard deviation was 4.3 mpg. At α =0.01, should it conclude that brand A cars have lower average gas mileage than do brand B cars?

Solution:

	Brand A	**Brand B**
Sample size (number of cars)	n_1=12	n_2= 9
Sample mean (average gas mileage)	$\bar{X}_1$= 27.2 mpg	$\bar{X}_2$= 32.1 mpg
Sample s.d.	s_1=3.8 mpg	s_2= 4.3 mpg

Setting up hypothesis

Null hypothesis (H_0): $\boldsymbol{\mu_1 = \mu_2}$ i.e. There is no significant difference between average mileage in Brand A and Brand B cars.

Alternative hypothesis (H_1): $\boldsymbol{\mu_1 < \mu_2}$ i.e. The Brand A cars have lower average mileage than that of Brand B.

Test Statistic: Under H_0, the test statistic is

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Where, $S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{12\times 3.8^2 + 9\times 4.3^2}{12+9-2} = 17.88$

$$t = \frac{27.2 - 32.1}{\sqrt{17.88\left(\frac{1}{12}+\frac{1}{9}\right)}} = \frac{-4.9}{\sqrt{3.477}} = -2.63$$

$\therefore |t_{cal}| = |-2.63| = 2.63$

Level of significance: (α) = 0.01

Degree of freedom: $n_1 + n_2 - 2$ = 12+9-2 = 19

Tabulated value: (t_{α, n_1+n_2-2}) = $t_{0.01,19}$=2.539 **(One – tailed test)**

Decision: Since, **t_{Cal} = 2.63 > $t_{0.05,19}$ i.e. t_{tab} =2.539** , we reject H_0.

Hence, we conclude that the brand A cars have lower average gas mileage than that of brand B cars.

Example:

For a random sample of 10 pigs fed on diet A, the increase in weight(in lbs) in a certain period were 10, 17, 13, 12, 9, 8, 14, 15, 6 and 16. For another random sample of 12 pigs fed on diet B , the increase in weight in the same period were 14, 18, 8, 21, 23, 10,17, 12, 22, 15, 7 and 13. Test whether diet A and B differ significantly as regard their effect on increase in weight.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	[illegible]	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	[illegible]	63.656	318.289	636.578
2	1.886	2.92	4.303	[illegible]	9.925	22.328	31.6
3	1.638	2.353	3.182	[illegible]	5.841	10.214	12.924
4	1.533	2.132	2.776	[illegible]	4.604	7.173	8.61
5	1.476	2.015	2.571	[illegible]	4.032	5.894	6.869
6	1.44	1.943	2.447	[illegible]	3.707	5.208	5.959
7	1.415	1.895	2.365	[illegible]	3.499	4.785	5.408
8	1.397	1.86	2.306	[illegible]	3.355	4.501	5.041
9	1.383	1.833	2.262	[illegible]	3.25	4.297	4.781
10	1.372	1.812	2.228	[illegible]	3.169	4.144	4.587
11	1.363	1.796	2.201	[illegible]	3.106	4.025	4.437
12	1.356	1.782	2.179	[illegible]	3.055	3.93	4.318
13	1.35	1.771	2.16	[illegible]	3.012	3.852	4.221
14	1.345	1.761	2.145	[illegible]	2.977	3.787	4.14
15	1.341	1.753	2.131	[illegible]	2.947	3.733	4.073
16	1.337	1.746	2.12	[illegible]	2.921	3.686	4.015
17	1.333	1.74	2.11	[illegible]	2.898	3.646	3.965
18	1.33	1.734	2.101	[illegible]	2.878	3.61	3.922
19	[illegible]	[illegible]	[illegible]	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	[illegible]	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Solution:

Here, $n_1 = 10$ and $n_2 = 12$

Calculation of $\bar{X}_1, \bar{X}_2$ and S^2

Diet A		Diet B	
X_1	X_1^2	X_2	X_2^2
10	100	14	196
17	289	18	324
13	169	8	64
12	144	21	441
9	81	23	529
8	64	10	100
14	196	17	289
15	225	12	144
6	36	22	484
16	256	15	225
		7	49
		13	169
$\sum X_1=120$	$\sum X_1^2=1560$	$\sum X_2=180$	$\sum X_2^2= 3014$

$$\bar{X}_1 = \frac{\sum X_1}{n_1} = \frac{120}{10} = 12$$

$$\bar{X}_2 = \frac{\sum X_2}{n_2} = \frac{180}{12} = 15$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\sum X_1^2 - \frac{(\sum X_1)^2}{n_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{n_2}\right]$$

$$= \frac{1}{10+12-2}\left[1560 - \frac{(120)^2}{10} + 3014 - \frac{(180)^2}{12}\right] = \frac{1}{20} \times 437 = \mathbf{21.7}$$

Solution:

Setting up hypothesis

Null hypothesis(H_0): $\mu_1 = \mu_2$ i.e. There is no significant difference between average effect on increase in weight due to diets A and B.

Alternative hypothesis (H_1): $\mu_1 \neq \mu_2$ i.e. There is significant difference between average effect on increase in weight due to diets A and B.

Test Statistic: Under H_0, the test statistic is

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$t = \frac{12-15}{\sqrt{21.7\left(\frac{1}{10}+\frac{1}{12}\right)}} = -1.50$$

$\therefore |t_{cal}| = |-1.50| = 1.50$

Level of significance: (α) = 0.05

Degree of freedom: $n_1 + n_2 - 2$ = 10+12-2 = 20

Contd….

Level of significance: (α) = 0.05

Degree of freedom: $n_1 + n_2 - 2$ = 10+12-2 = 20

Tabulated value: (t_{α, n_1+n_2-2}) = $t_{0.05,20}$=2.086 (**Two – tailed test**)

Decision: Since, $t_{Cal} = 1.50 < t_{0.05,20}$ **i.e.** $t_{tab} = 2.086$, we accept H_0.

Hence, we conclude that there is no significant difference between average effect on increase in weight due to diets A and B.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	[illegible]	31.821	63.656	318.289	636.578
2	1.886	2.92	[illegible]	6.965	9.925	22.328	31.6
3	1.638	2.353	[illegible]	4.541	5.841	10.214	12.924
4	1.533	2.132	[illegible]	3.747	4.604	7.173	8.61
5	1.476	2.015	[illegible]	3.365	4.032	5.894	6.869
6	1.44	1.943	[illegible]	3.143	3.707	5.208	5.959
7	1.415	1.895	[illegible]	2.998	3.499	4.785	5.408
8	1.397	1.86	[illegible]	2.896	3.355	4.501	5.041
9	1.383	1.833	[illegible]	2.821	3.25	4.297	4.781
10	1.372	1.812	[illegible]	2.764	3.169	4.144	4.587
11	1.363	1.796	[illegible]	2.718	3.106	4.025	4.437
12	1.356	1.782	[illegible]	2.681	3.055	3.93	4.318
13	1.35	1.771	[illegible]	2.65	3.012	3.852	4.221
14	1.345	1.761	[illegible]	2.624	2.977	3.787	4.14
15	1.341	1.753	[illegible]	2.602	2.947	3.733	4.073
16	1.337	1.746	[illegible]	2.583	2.921	3.686	4.015
17	1.333	1.74	[illegible]	2.567	2.898	3.646	3.965
18	1.33	1.734	[illegible]	2.552	2.878	3.61	3.922
19	1.328	1.729	[illegible]	2.539	2.861	3.579	3.883
20	[illegible]	[illegible]	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	[illegible]	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Numerical Example

Two different types of drugs D_1 and D_2 were administered on certain patients for increasing weight at interval of one week time period. From the following observation, can we conclude that the second drug is more effective in increasing weight, use 1% level of significance?

D_1	**8**	**12**	**13**	**9**	**3**	**8**	**10**	**9**
D_2	**10**	**8**	**12**	**15**	**6**	**11**	**12**	**12**

Solution:

Here, $n_1 = 8$ and $n_2 = 8$

Calculation of $\bar{X}_1, \bar{X}_2$ and S^2

$$\bar{X}_1 = \frac{\sum X_1}{n_1} = \frac{72}{8} = 9$$

$$\bar{X}_2 = \frac{\sum X_2}{n_2} = \frac{86}{8} = 10.75$$

D_1		D_2	
X_1	X_1^2	X_2	X_2^2
8	64	10	100
12	144	8	64
13	169	12	144
9	81	15	225
3	9	6	36
8	64	11	121
10	100	12	144
9	81	12	144
$\sum X_1=72$	$\sum X_1^2 = 712$	$\sum X_2=86$	$\sum X_2^2 = 978$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\sum X_1^2 - \frac{(\sum X_1)^2}{n_1} + \sum X_2^2 - \frac{(\sum X_2)^2}{n_2}\right]$$

$$= \frac{1}{8+8-2}\left[712 - \frac{(72)^2}{8} + 978 - \frac{(86)^2}{8}\right] \quad = \frac{1}{14} \times 117.5 = \mathbf{8.393}$$

Setting up hypothesis

Null hypothesis(H_0): $\boldsymbol{\mu_1 = \mu_2}$ i.e. There is no significant difference between two drugs D_1 and D_2. OR Both drugs are equally effective.

Alternative hypothesis (H_1): $\boldsymbol{\mu_1 < \mu_2}$ i.e. Second drug(D_2) is more effective than first drug (D_1).

Test Statistic: Under H_0, the test statistic is

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$t = \frac{9 - 10.75}{\sqrt{8.393\left(\frac{1}{8}+\frac{1}{8}\right)}} = \frac{-1.75}{\sqrt{2.09825}} = -1.208$$

$$\therefore |t_{cal}| = |-1.208| = 1.208$$

Contd….

Level of significance: (α) = 0.05

Degree of freedom: $n_1 + n_2 - 2$ = 8+8-2 = 14

Tabulated value: (t_{α, n_1+n_2-2}) = $t_{0.05,14}$ = 1.761 **(One – tailed test)**

Decision: Since, $\mathbf{t_{Cal} = 1.208 < t_{0.05,14}}$ **i.e.** $\mathbf{t_{tab} = 1.761}$, we accept H_0.

Hence, we conclude that there is no significant difference between two drugs D_1 and D_2. It means both drugs are equally effective.

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	[illegible]	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	[illegible]	12.706	31.821	63.656	318.289	636.578
2	1.886	[illegible]	4.303	6.965	9.925	22.328	31.6
3	1.638	[illegible]	3.182	4.541	5.841	10.214	12.924
4	1.533	[illegible]	2.776	3.747	4.604	7.173	8.61
5	1.476	[illegible]	2.571	3.365	4.032	5.894	6.869
6	1.44	[illegible]	2.447	3.143	3.707	5.208	5.959
7	1.415	[illegible]	2.365	2.998	3.499	4.785	5.408
8	1.397	[illegible]	2.306	2.896	3.355	4.501	5.041
9	1.383	[illegible]	2.262	2.821	3.25	4.297	4.781
10	1.372	[illegible]	2.228	2.764	3.169	4.144	4.587
11	1.363	[illegible]	2.201	2.718	3.106	4.025	4.437
12	1.356	[illegible]	2.179	2.681	3.055	3.93	4.318
13	1.35	[illegible]	2.16	2.65	3.012	3.852	4.221
14	[illegible]	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	[illegible]	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Paired t- test:

(For dependent case, pre/post, before/after, with/without etc)

Setting up hypothesis:

Null hypothesis(H_0): $\boldsymbol{\mu_x = \mu_y}$ **or** $\boldsymbol{\mu_x - \mu_y = 0}$ i.e. There is no significant difference between before treatment and after treatment. **OR**. Treatment is not effective.

Alternative hypothesis (H_1): $\boldsymbol{\mu_x \neq \mu_y}$ **or** $\boldsymbol{\mu_x - \mu_y \neq 0}$ i.e. There is significant difference between before treatment and after treatment.**(Two-tailed test)**

H_1: $\boldsymbol{\mu_x > \mu_y}$ i.e. There is positive(negative) impact in the observation after treatment. OR Treatment is effective **(Right tailed test)**

H_1 : $\boldsymbol{\mu_x < \mu_y}$ i.e. There is negative(positive) impact in the observation after treatment. OR Treatment is effective **(Left tailed test)**

Test statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{\bar{d}}{\sqrt{\frac{S^2}{n}}} \text{ (When sample standard deviation is not given)}$$

$$= \frac{\bar{d}}{\frac{s}{\sqrt{n-1}}} \text{ (When sample standard deviation is given)}$$

Where, $\bar{d}$ = mean of the difference = $\frac{\sum d}{n}$,d =X- Y or Y-X=difference between two observations (X= Before, Y= After)

$$S^2 = \frac{1}{n-1}\sum\left(d-\bar{d}\right)^2 \text{ (Actual mean deviation method)}$$

$$= \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right] \text{ (Direct Method)}$$

Level of significance(α): Use 5% level of significance unless otherwise stated.

Degree of freedom= n-1

Tabulated Value ($t_{\alpha,n-1}$)= Obtained from Student's t- table

Decision: If calculated value of t $\leq$ tabulated value of t **(i.e. , $t_{Cal} \leq t_{tab}$)** We accept null hypothesis H_O, otherwise we reject H_O (i. e. accept H_1)

Note: ***Generally one tailed test is performed in paired test t-test when the effectiveness is to be tested. Two tailed test is applied when the significance difference is to be calculated. Paired t-test is applied is always applied for the cases like before and after, pre/post with and without etc. for matched cases.***

Numerical Example: Sales of new Electronic item in six stores before and after special program are observed as follows

Store	1	2	3	4	5	6
Sales before campaign	50	30	31	48	55	42
Sales after campaign	52	29	30	52	56	45

Can you judge the promotional program a success? ($\alpha = 0.01$)

Solution:

Store	S3ales before Campaign(X)	Sales after Campaign (Y)	d = Y -X	d^2
1	50	52	2	4
2	30	29	-1	1
3	31	30	-1	1
4	48	52	4	16
5	55	56	1	1
6	42	45	3	9
Total			$\sum d$= 8	$\sum d^2$=32

Here, n= 6

$$\bar{d} = \frac{\sum d}{n} = \frac{8}{6} = 1.33$$

$$S^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$$

$$= \frac{1}{6-1}\left[32 - \frac{(8)^2}{6}\right]$$

$$= \frac{1}{5} \times 21.333 = \mathbf{4.267}$$

$$\therefore S = \sqrt{S^2} = \sqrt{4.267} = 2.065$$

Setting up hypothesis:

Null hypothesis(H_0): $\mu_x = \mu_y$ i.e. Special promotional program is not effective (successful)

Alternative hypothesis (H_1): $\mu_x < \mu_y$ i.e. Special promotional program is effective

Test statistic: Under H_0, the test statistic is,

$$\mathbf{t} = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{1.33}{\frac{2.065}{\sqrt{6}}} = 1.581$$

$\therefore$ **t_{cal} = 1.581**

Level of significance(α) = 0.01 =1%

Degree of freedom(n-1) = 6-1 =5

Tabulated value ($t_{\alpha,n-1}$) = $t_{0.01,5}$ =3.365**(One-tailed test)**

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	[illegible]	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	[illegible]	63.656	318.289	636.578
2	1.886	2.92	4.303	[illegible]	9.925	22.328	31.6
3	1.638	2.353	3.182	[illegible]	5.841	10.214	12.924
4	1.533	2.132	2.776	[illegible]	4.604	7.173	8.61
5				3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	[illegible]	3.707	5.208	5.959

Decision:

Since, $\mathbf{t_{Cal} = 1.581 < t_{tab} = 3.365}$, we accept null hypothesis H_0.

Hence, we conclude that special promotional campaign is not effective (successful).

Numerical Example:

A special coaching class on mathematics subject in a group of 10 students yield following changes in score,

8 , 10 ,-2 , 0 , -5 , -1, 9, 12, 6 , 5

Test at 5% level of significance whether the coaching class was effective.

Solution:

Here, n= 10

$$\bar{\mathbf{d}} = \frac{\sum \mathbf{d}}{\mathbf{n}} = \frac{42}{10} = 4.2$$

$$S^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$$

$$= \frac{1}{10-1}\left[480 - \frac{(42)^2}{10}\right]$$

$$= \frac{1}{9} \times 303.6 = 33.73$$

$$\therefore S = \sqrt{S^2} = \sqrt{33.73} = 5.808$$

d= X-Y	d^2
8	64
10	100
-2	4
0	0
-5	25
-1	1
9	81
12	144
6	36
5	25
$\sum d$= 42	$\sum d^2$= 480

Setting up hypothesis:

Null hypothesis(H_0): $\boldsymbol{\mu_x = \mu_y}$ i.e. Coaching class is not effective.

Alternative hypothesis (H_1): $\boldsymbol{\mu_x < \mu_y}$ i.e. Coaching class is effective.

Test Statistic: Under H_0, the test statistic is,

$$\mathbf{t} = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{4.2}{\frac{5.508}{\sqrt{10}}} = 2.241$$

Level of significance(α)= 0.05 =5%

Degree of freedom(n-1) = 10-1 =9

Tabulated value ($t_{\alpha,n-1}$) = $\boldsymbol{t_{0.05,9}}$ **= 1.833 (One-tailed test)**

Student's t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	[illegible]	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	[illegible]	12.706	31.821	63.656	318.289	636.578
2	1.886	[illegible]	4.303	6.965	9.925	22.328	31.6
3	1.638	[illegible]	3.182	4.541	5.841	10.214	12.924
4	1.533	[illegible]	2.776	3.747	4.604	7.173	8.61
5	1.476	[illegible]	2.571	3.365	4.032	5.894	6.869
6	1.44	[illegible]	2.447	3.143	3.707	5.208	5.959
7	1.415	[illegible]	2.365	2.998	3.499	4.785	5.408
8	1.397	[illegible]	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	[illegible]	2.228	2.764	3.169	4.144	4.587
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Decision: Since, $\mathbf{t_{Cal} = 2.241 > \mathit{t}_{0.05,9}}$ **i.e.** $\mathbf{t_{tab} = 1.833}$, we accept H_0.

Hence, we conclude that Coaching class is effective.

Numerical Example:

A certain stimulus administered to 12 patients resulted average increase in blood pressure by 2.58 with standard deviation 3.09. Can it be concluded that the stimulus will, in general, be accompanied by an increase in blood pressure?

Solution:

Here, sample size(n) =12

Resulted average increase blood pressure($\bar{d}$) =2.58

Sample Standard deviation(s) =3.09

Setting up hypothesis:

Null hypothesis(H_0): $\mu_x = \mu_y$ i.e. Stimulus is not effective to increase blood pressure.

Alternative hypothesis (H_1): $\mu_x < \mu_y$ i.e. Stimulus is effective to increase blood pressure.

Test Statistic: Under H_0, the test statistic is,

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n-1}}} = \frac{2.58}{\frac{3.09}{\sqrt{12-1}}} = 2.77$$

Level of significance(α) = 0.05 =5%

Degree of freedom(n-1) = 12-1 =11

Tabulated value ($t_{\alpha,n-1}$) = $t_{0.05,11}$ = 1.796 **(One-tailed test)**

Student’s t- Table

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	[illegible]	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	[illegible]	12.706	31.821	63.656	318.289	636.578
2	1.886	[illegible]	4.303	6.965	9.925	22.328	31.6
3	1.638	[illegible]	3.182	4.541	5.841	10.214	12.924
4	1.533	[illegible]	2.776	3.747	4.604	7.173	8.61
5	1.476	[illegible]	2.571	3.365	4.032	5.894	6.869
6	1.44	[illegible]	2.447	3.143	3.707	5.208	5.959
7	1.415	[illegible]	2.365	2.998	3.499	4.785	5.408
8	1.397	[illegible]	2.306	2.896	3.355	4.501	5.041
9	1.383	[illegible]	2.262	2.821	3.25	4.297	4.781
10	1.372	[illegible]	2.228	2.764	3.169	4.144	4.587
11	[illegible]	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	[illegible]	2.179	2.681	3.055	3.93	4.318

Decision: Since, $\mathbf{t_{Cal} = 2.77 > \mathit{t}_{0.05,11}}$ **i.e.** $\mathbf{t_{tab} = 1.796}$, we reject H_0.

Hence, we conclude that stimulus is effective to increase blood pressure.

Stay Home , Stay Safe

Thank You