ĐẠI HỌC QUỐC GIA HÀ NỘI
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CHAPTER 2: SPECTRAL ANALYSIS

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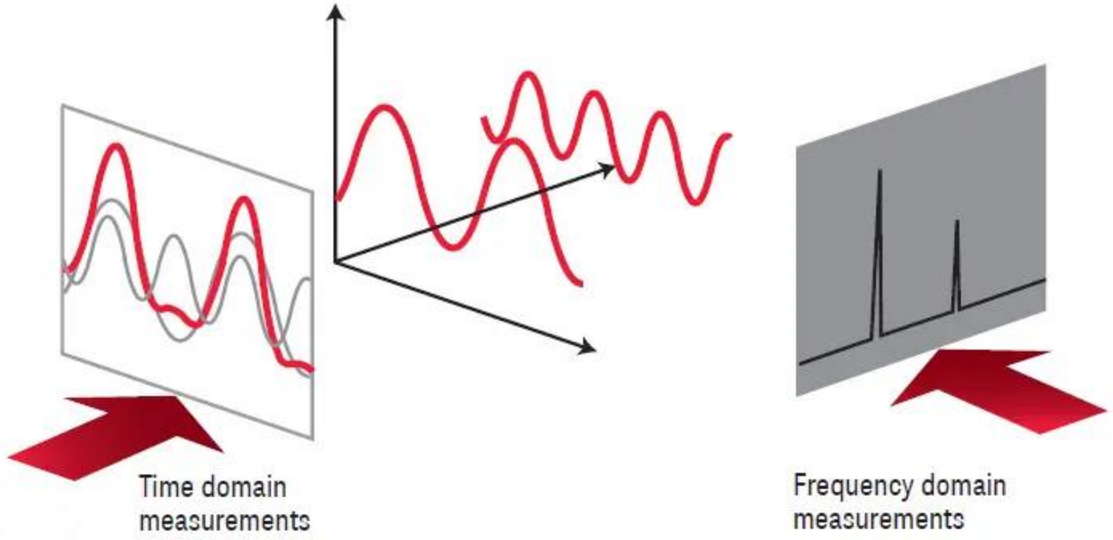
Spectral analysis

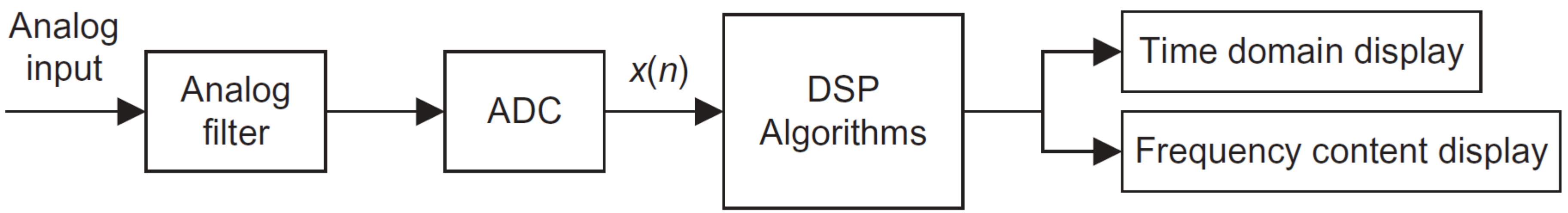
To study how different frequency components contribute to the overall signal, providing insight into its structure, patterns, and behavior.

- ✓ Frequency Domain Representation
- ✓ Fourier Transform (FT)
- ✓ Power Spectrum
- ✓ Windowing
- ✓ Short-Time Fourier Transform (STFT)
- ✓ Wavelet Transform



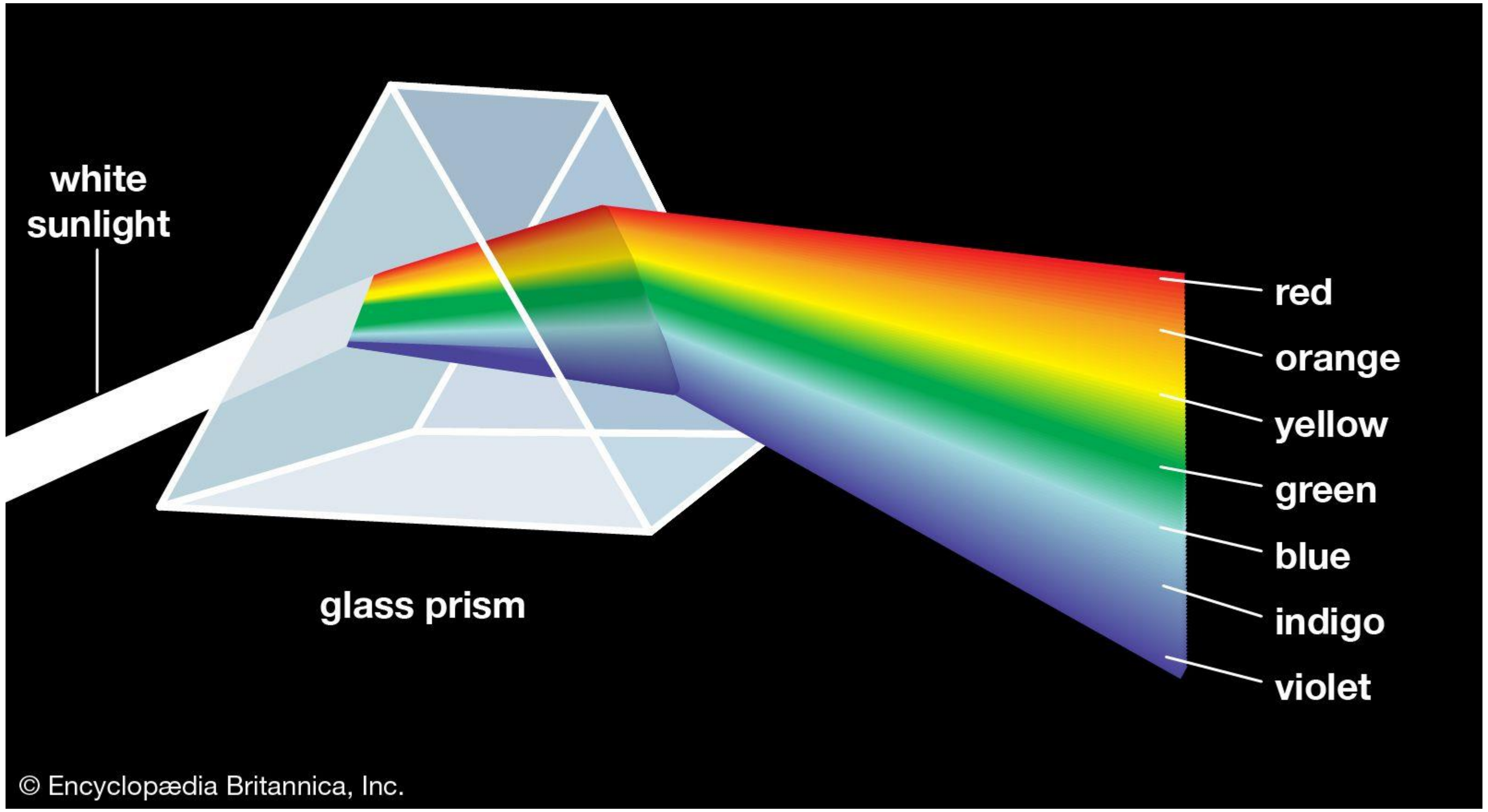
Frequency Domain Representation

- Time-domain analysis focuses on how a signal changes over time
- Frequency-domain analysis examines how much of the signal lies within each given frequency band.
- Spectral analysis involves converting a time-domain signal into its frequency-domain representation using tools like the Fourier Transform.



Frequency Domain Representation

- Fourier Series is used to represent periodic signals in the frequency domain.
- Fourier Transform is used for non-periodic signals in the frequency domain.

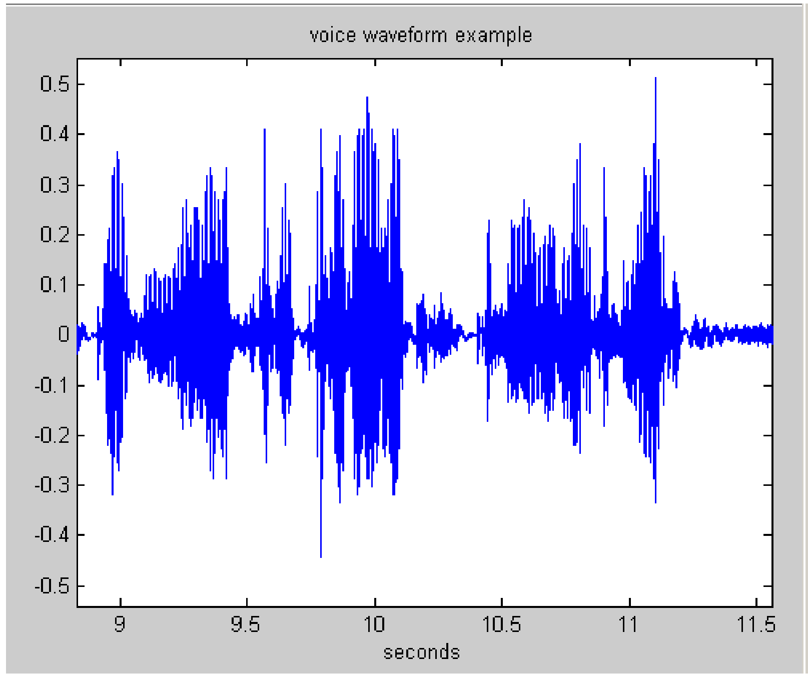


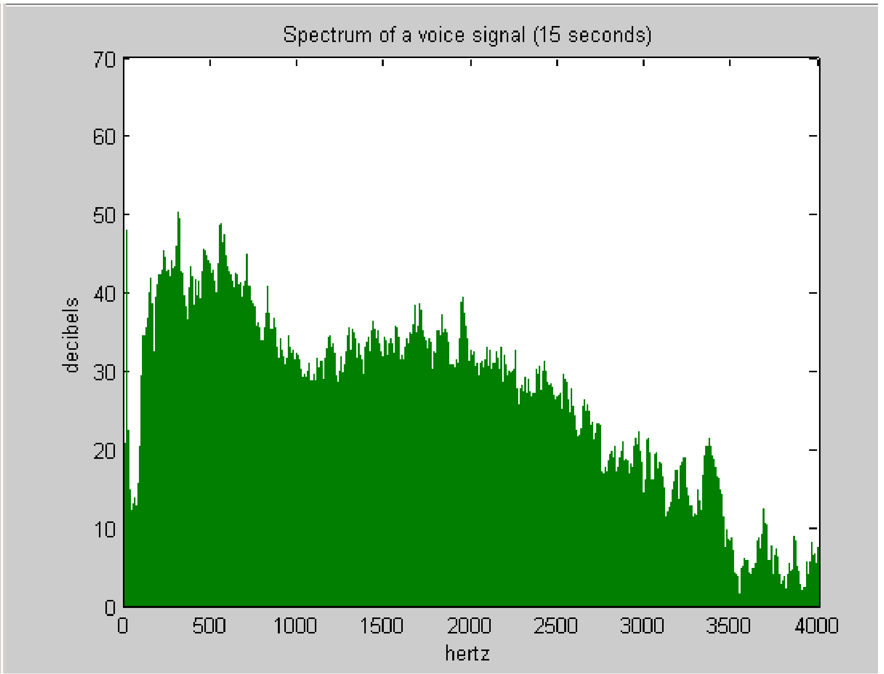
Fourier Transform (FT)

- Fourier Transform is the mathematical tool used to convert a signal from the time domain to the frequency domain.
- For discrete signals, the Discrete Fourier Transform (DFT) is commonly used
- For efficient computation, the Fast Fourier Transform (FFT) is employed

Power Spectrum

- The power spectrum shows how the power of a signal is distributed over frequency.
- The Power Spectral Density (PSD) is particularly useful for identifying dominant frequencies and understanding the energy distribution in different frequency bands.

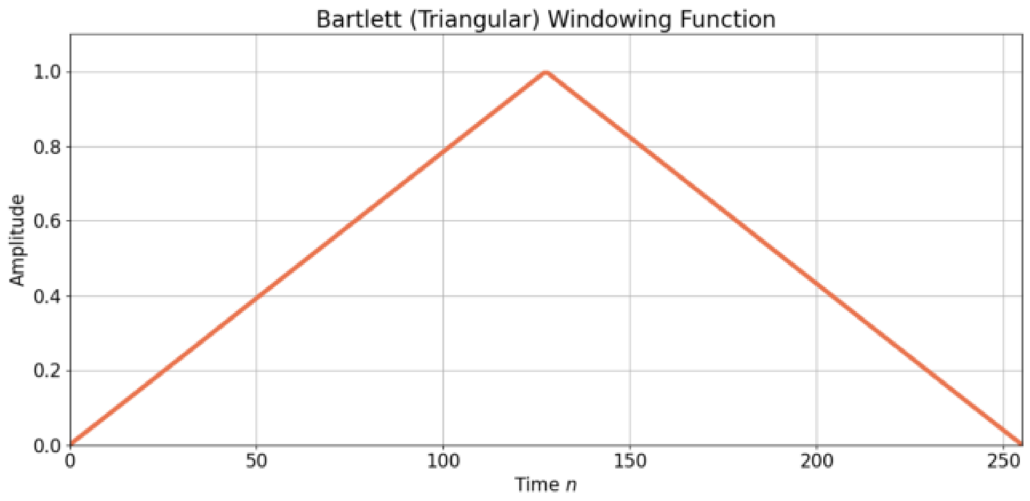


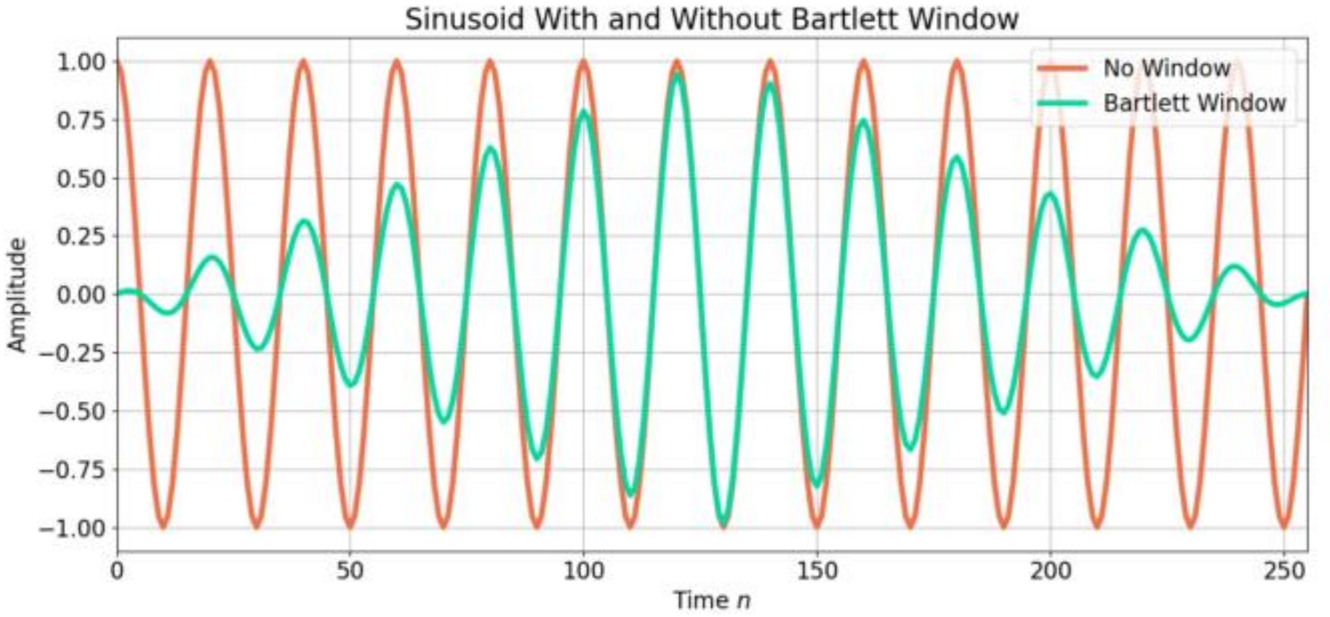


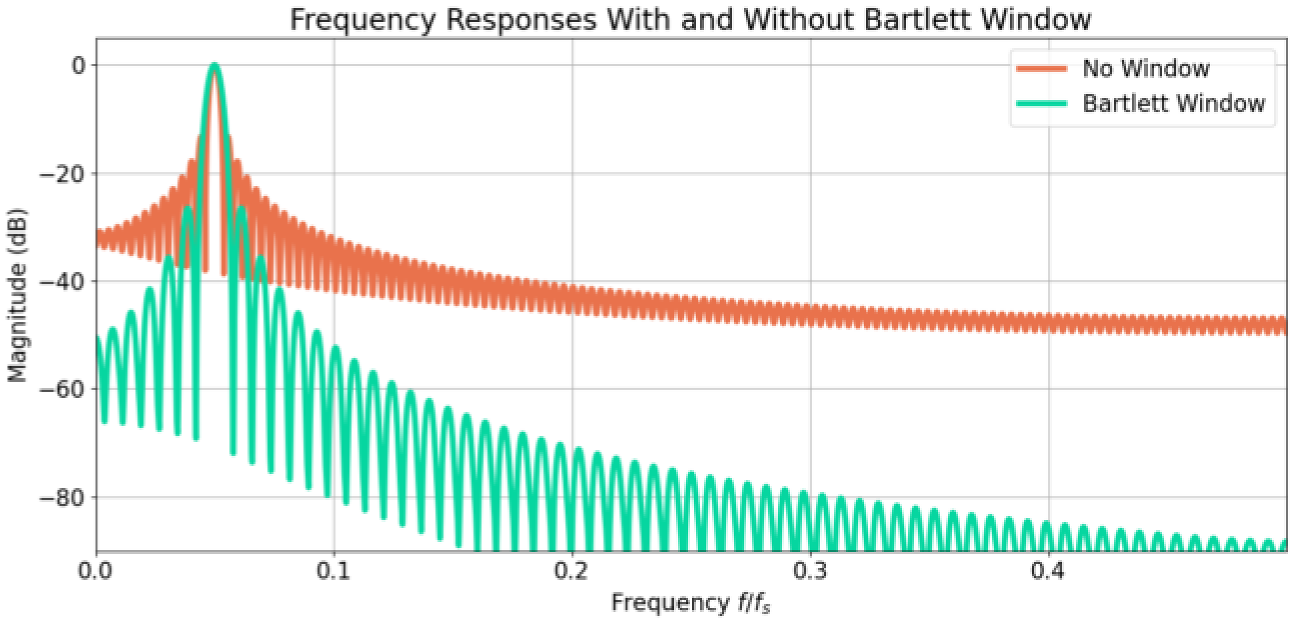
Windowing

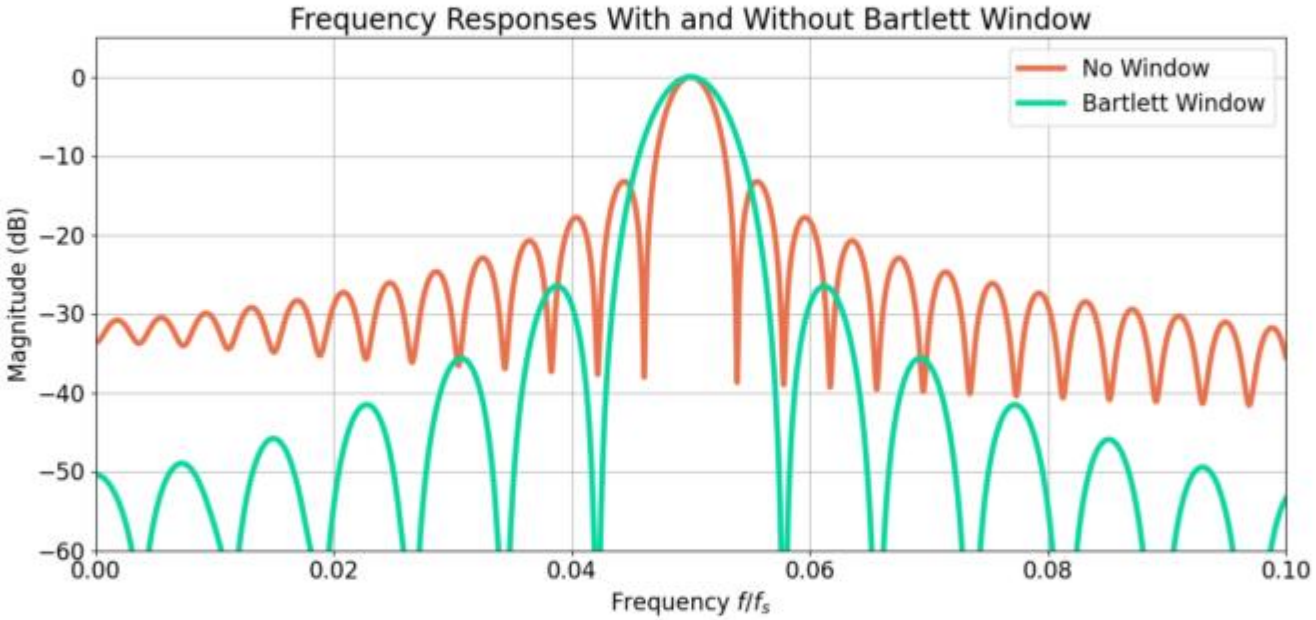
- Spectral leakage occurs when signal energy spreads across multiple frequencies, which can distort the true frequency content.
- When performing spectral analysis on non-periodic or finite-length signals, windowing techniques are applied to reduce spectral leakage. Windowing involves multiplying the signal with a window function before performing the Fourier Transform.

Windowing


Bartlett (Triangular) Windowing Function
Amplitude
Time n
0.0
0.2
0.4
0.6
0.8
1.0
0
50
100
150
200
250

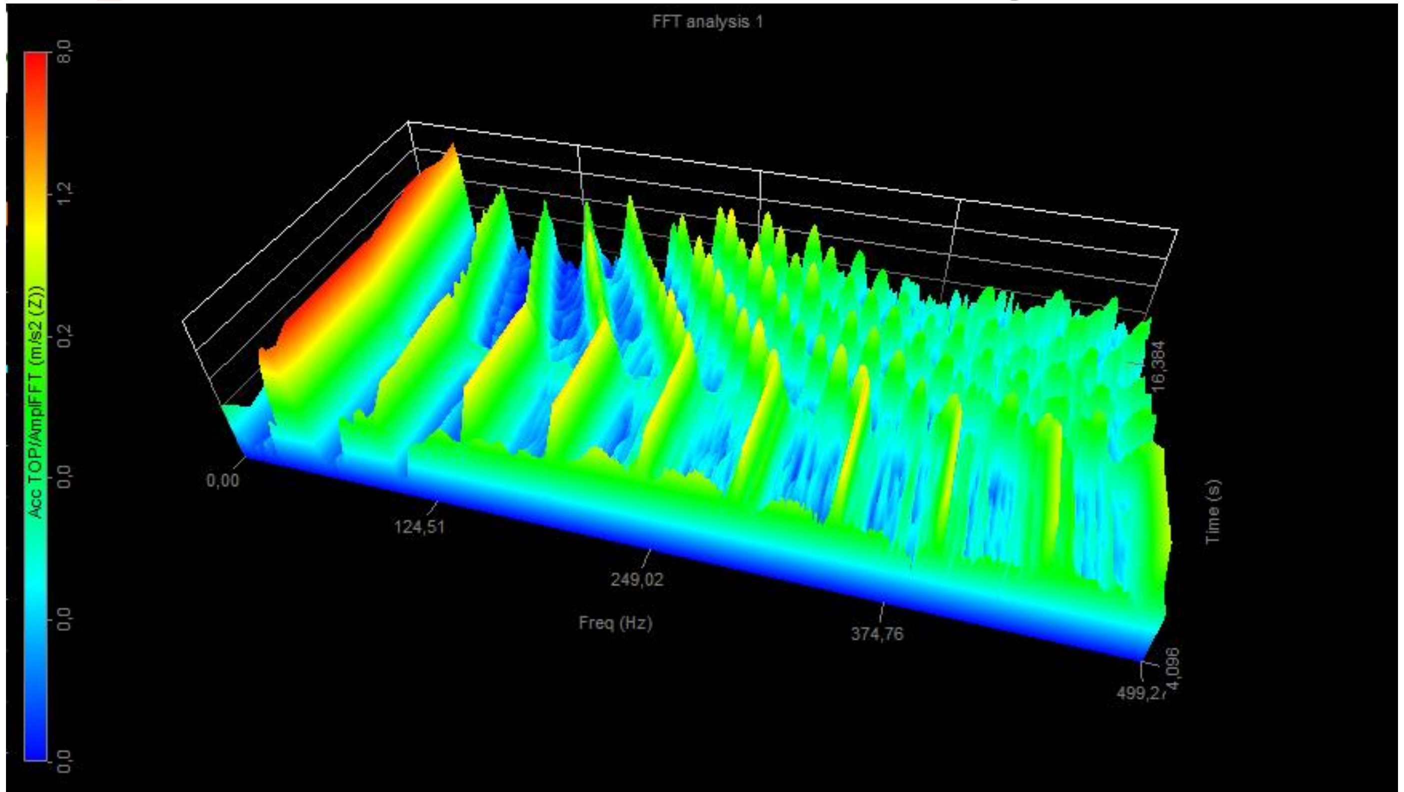

Sinusoid With and Without Bartlett Window
No Window
Bartlett Window
Amplitude
Time n
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
50
100
150
200
250


Frequency Responses With and Without Bartlett Window
No Window
Bartlett Window
Magnitude (dB)
Frequency f/f_s
0
−20
−40
−60
−80
0.0
0.1
0.2
0.3
0.4


Frequency Responses With and Without Bartlett Window
No Window
Bartlett Window
Magnitude (dB)
Frequency f/f_s
0
−10
−20
−30
−40
−50
−60
0.00
0.02
0.04
0.06
0.08
0.10

Short-Time Fourier Transform (STFT)

- The STFT is used for time-varying signals where the frequency content changes over time. It divides the signal into small time segments (windows) and applies the Fourier Transform to each segment, giving a time-frequency representation of the signal.



Wavelet Transform

- Unlike the Fourier Transform, which uses sinusoidal basis functions, the Wavelet Transform uses wavelets that are localized in both time and frequency. This allows better resolution of signals with non-stationary frequency content.
- Wavelet analysis is particularly useful in analyzing transient signals and for applications such as compression and noise reduction.

Short-time Fourier Transform (STFT)

Time-Frequency Analysis

- ✓ Showing how the signal's frequency content evolves over time
- ✓ This is crucial for non-stationary signals (e.g., speech, music, biological signals)

Short-time Fourier Transform (STFT)

Sliding Window Approach

- ✓ The STFT works by breaking a long signal into short, overlapping segments (called windows). Each window is assumed to be approximately stationary, and the Fourier Transform is applied to each segment.
- ✓ The window slides across the entire signal, and the transform is calculated for each windowed segment, generating a two-dimensional function of both time and frequency.

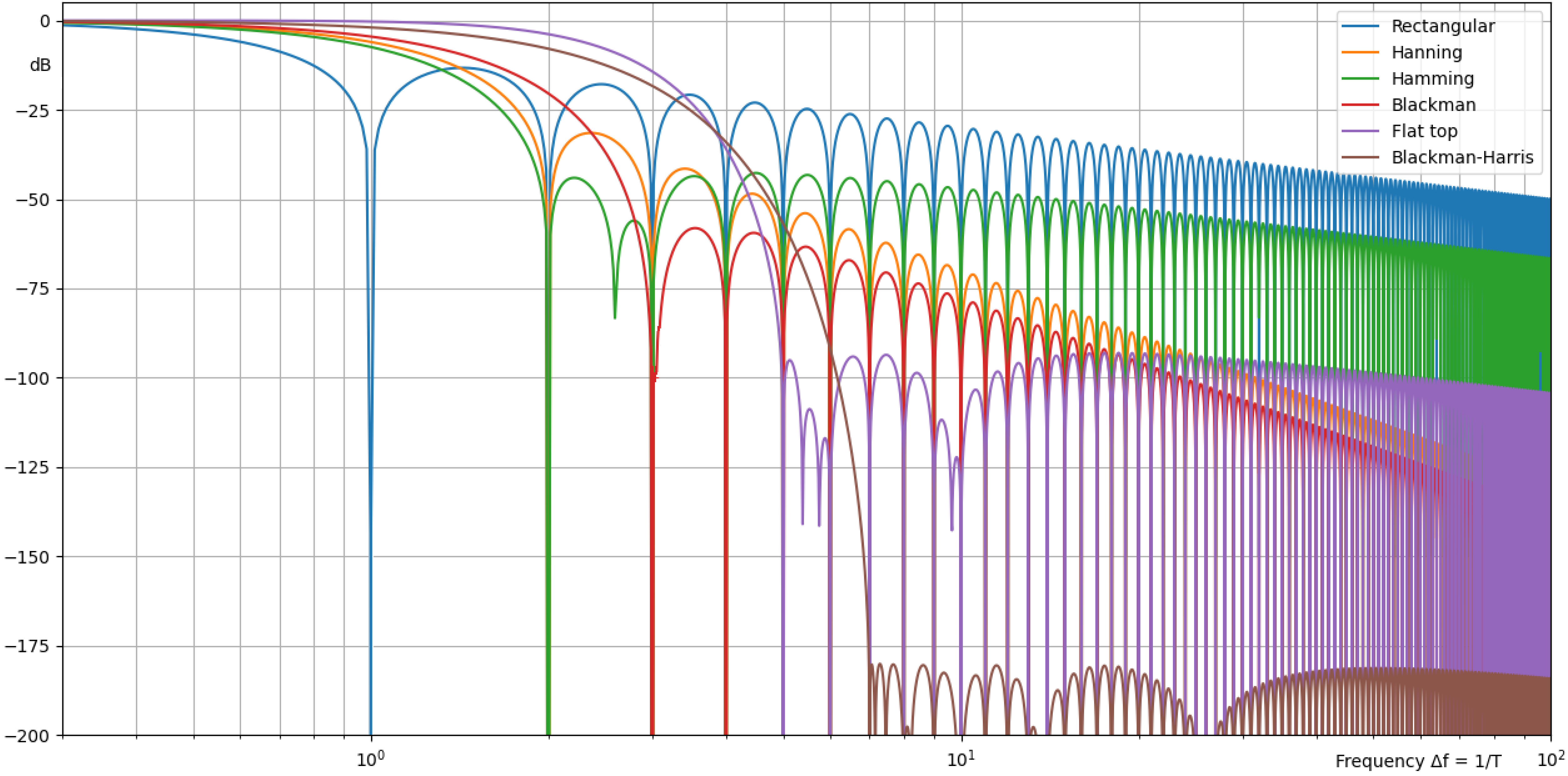
Short-time Fourier Transform (STFT)

Window Functions

- ✓ A window function is applied to each segment of the signal before computing the Fourier Transform. This is necessary to limit the segment in time and avoid edge effects.
- ✓ Common window functions include the Hanning, Hamming, and Rectangular windows. Each window has different properties, and the choice of window affects the resolution in time and frequency.

Short-time Fourier Transform (STFT)

Window Functions



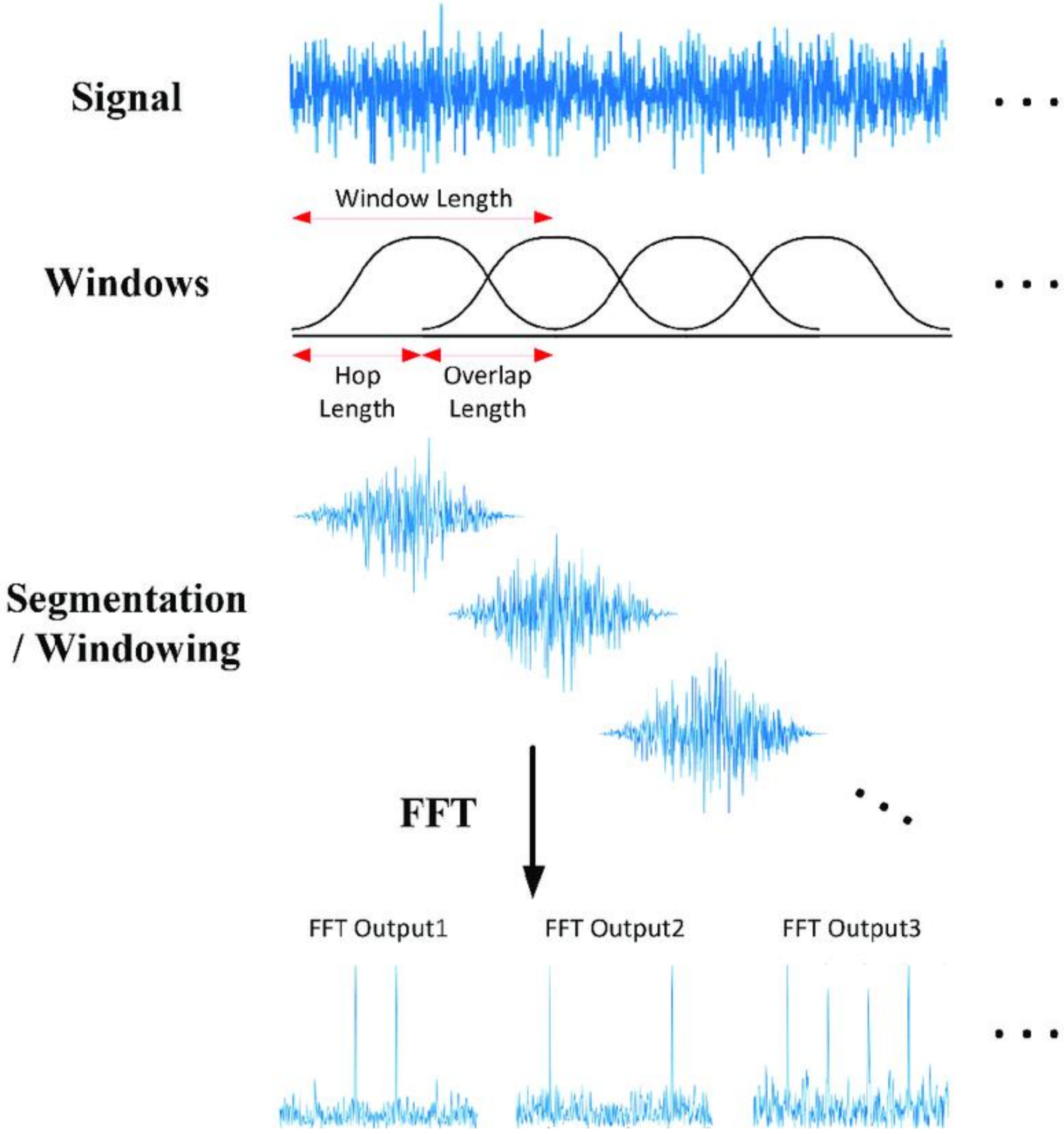
Short-time Fourier Transform (STFT)

Time-Frequency Trade-off

- ✓ The STFT involves a trade-off between time resolution and frequency resolution. A narrow window in time provides good time resolution but poor frequency resolution, while a wide window provides better frequency resolution at the cost of time resolution.
- ✓ The choice of window size depends on the nature of the signal and what features need to be analyzed.

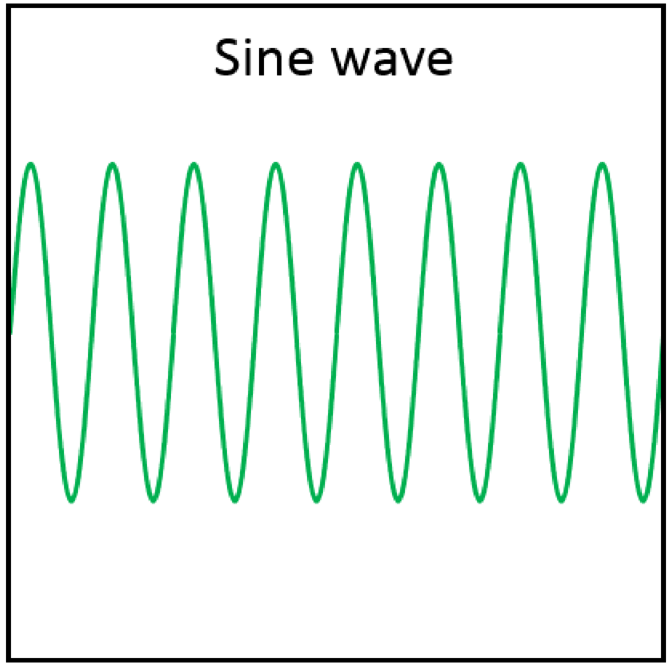
Short-time Fourier Transform (STFT)

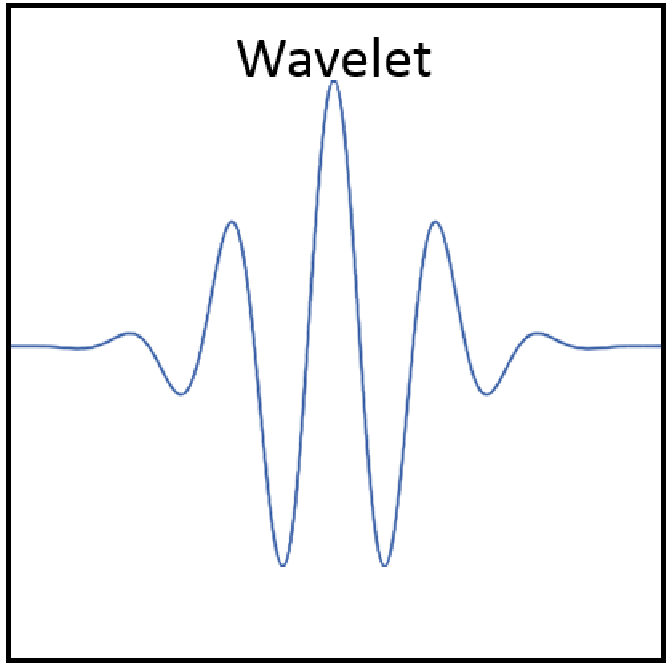
1. Signal Segmentation
2. Fourier Transform on Each Segment
3. Repetition
4. Spectrogram



Wavelet Transform

- The Wavelet Transform is a mathematical technique used for analyzing and processing signals, often providing time-frequency representation.
- Allows for both time and frequency localization, making it useful for analyzing signals that have non-stationary or transient characteristics.





Wavelet Transform

Continuous Wavelet Transform (CWT)

- ✓ In CWT, a signal is decomposed into scaled and translated versions of a chosen wavelet function.
- ✓ A wavelet is a small wave, oscillatory but with finite duration.
- ✓ The CWT provides a time-scale representation, which is useful for identifying structures in signals across different scales or resolutions.

$$CWT(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right) dt$$

Wavelet Transform

Discrete Wavelet Transform (DWT)

- ✓ DWT is a sampled version of the CWT, often used in practical applications due to its efficiency.
- ✓ It decomposes the signal into a set of orthogonal basis functions (wavelets).
- ✓ The signal is analyzed by passing it through a series of filters (low-pass and high-pass) at different scales.

Wavelet Transform

Discrete Wavelet Transform (DWT)

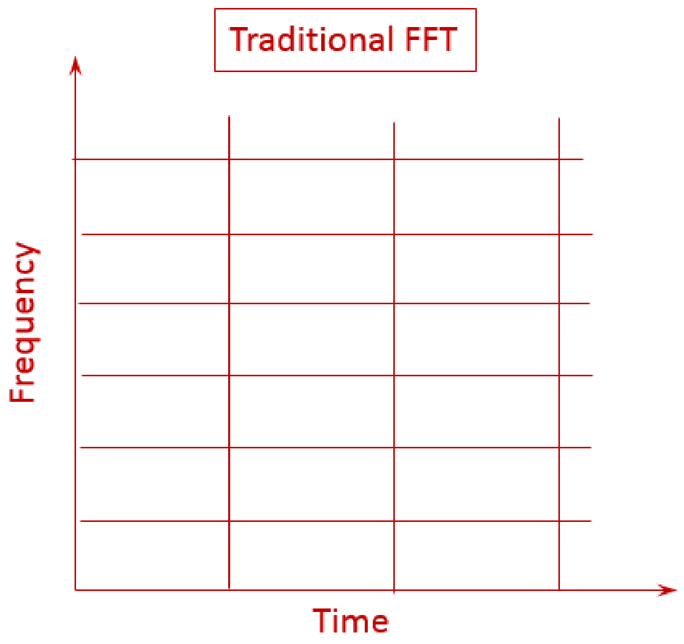
- ✓ In DWT, the signal is recursively decomposed into approximation coefficients (low-frequency components) and detail coefficients (high-frequency components).
- ✓ This process can be visualized as a multi-resolution analysis (MRA), where the signal is broken down at different resolutions or scales.
- ✓ The DWT is widely used in image compression (e.g., JPEG 2000) and denoising applications.

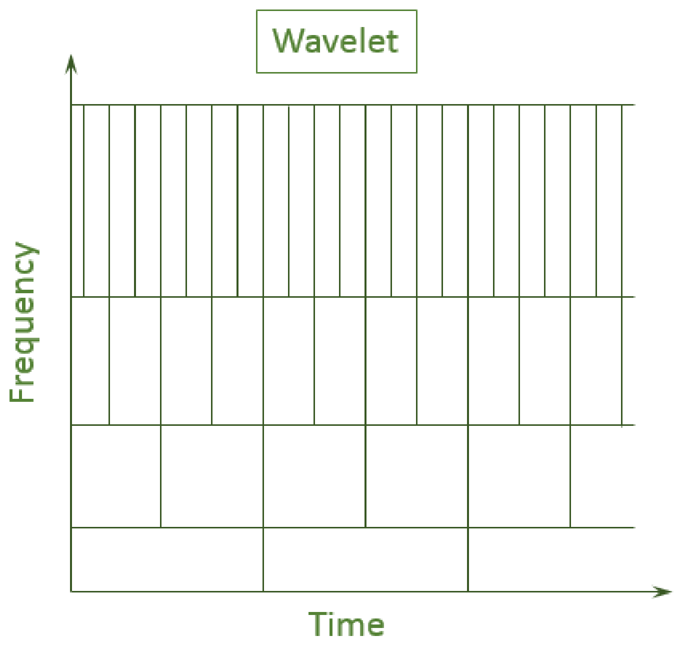
Wavelet Transform

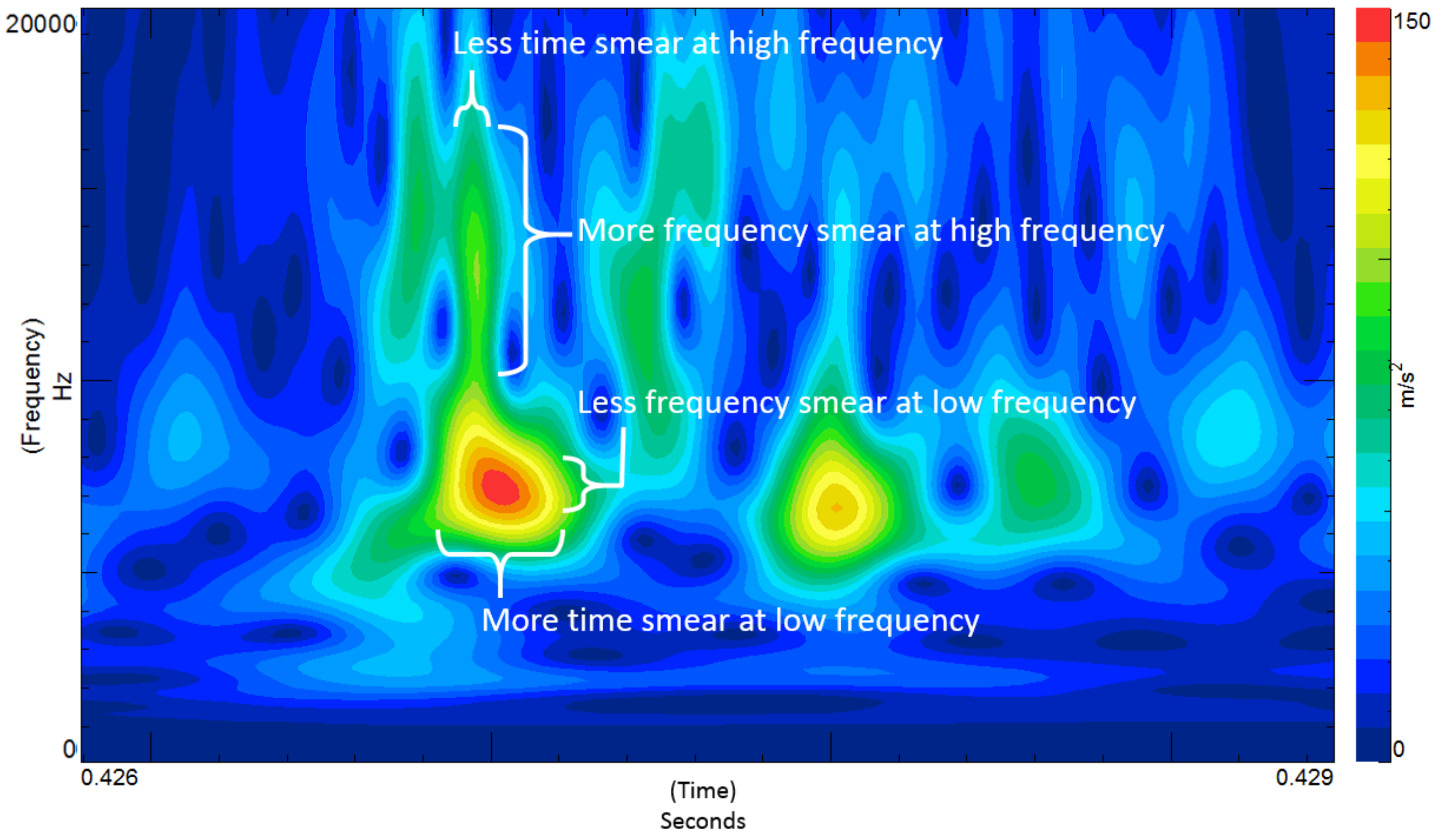
Wavelet Families

- ✓ Haar Wavelet: The simplest, used for its simplicity but not very smooth.
- ✓ Daubechies Wavelets: More advanced and widely used in signal and image processing due to their smoothness and compact support.
- ✓ Meyer Wavelets: These are smooth and have no sharp discontinuities, useful for certain applications.
- ✓ Morlet Wavelet: Often used in time-frequency analysis because of its close relation to sinusoids.

Wavelet Transform vs. Fourier Transform







Power Spectral Density Estimation

- Power Spectral Density (PSD) Estimation refers to the process of estimating the distribution of power across different frequency components of a signal.
- It quantifies how the power of a time-domain signal is distributed with frequency and is an important tool in signal processing, particularly for analyzing random processes or signals in domains such as communications, audio, and control systems.

Power Spectral Density Estimation

Definition of Power Spectral Density (PSD)

✓ The Power Spectral Density provides a measure of the power contained in a signal as a function of frequency. For a continuous-time signal x(t), the PSD describes the signal's power distribution in the frequency domain and is typically expressed in units of power per Hertz (W/Hz).

Power Spectral Density Estimation

Non-Parametric Methods: Non-parametric methods do not assume any underlying model for the data. These methods directly estimate the PSD from the signal data.

- ✓ Periodogram
- ✓ Modified Periodogram (Windowed)
- ✓ Welch's Method
- ✓ Multitaper Method

Power Spectral Density Estimation

Parametric Methods: Parametric methods assume a model for the signal (e.g., autoregressive models) and use that model to estimate the PSD. These methods typically require fewer data points and provide smoother estimates.

✓ Autoregressive (AR) Method

✓ Moving Average (MA) and ARMA Methods