Quantum Teleportation of Superposition State for Squeezed States

Xin-Hua Cai
Department of Physics, Changde Normal College, Changde 415000, Hunan Province, China and Department of Physics, Hunan Normal University, Changsha 410081, China

Le-Man Kuang†
Department of Physics, Hunan Normal University, Changsha 410081, China

This paper proposes a scheme for teleporting an arbitrary coherent superposition state of two equal-amplitude and opposite-phase squeezed vacuum states (SVS) via a symmetric 50/50 beam splitter and photodetectors. It is shown that the quantum teleportation scheme has the successful probability 1/4. Maximally entangled SVS's are used as quantum channel for realizing the teleportation scheme. It is shown that if an initial quantum channel is in a pure but not maximally entangled SVS, the quantum channel may be distilled to a maximally entangled SVS through entanglement concentration.

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Quantum teleportation is a process by which a quantum state of a system is transported from one location to another. The state of the system is destroyed in one place and a perfect replica is created at a distant site by making measurements on the system and classical communication of the results to the remote location. The original proposals for quantum teleportation [1, 2] focused on teleporting quantum states of a system with a finite-dimensional (discrete variable) state space, such as the two polarizations of a photon or the discrete levels of an atoms. Discrete-variable teleportation has been demonstrated experimentally in optical systems [3] and liquid-state nuclear magnetic resonance systems [4].

Recently, quantum teleportation has been extended to continuous variables corresponding to states of infinite-dimensional (continuous-variable) systems [5, 6] such as optical fields or the motion of massive particle. In particular following the theoretical proposal of Ref. [6], continuous-variable teleportation has been realized for coherent states of a light field [7] by using entangled two-mode squeezed optical beams produced by parametric down-conversion in a sub-threshold optical parametric oscillator. Although coherent states are continuous and nonorthogonal states, they are very close to classical states. A real challenge for quantum teleportation is to teleport truly nonclassical states like squeezed states, quantum superposition states and entangled states. van Enk and Hirota [8] proposed a scheme to faithfully teleport a superposition state of two coherent states with equal amplitudes and opposite-phases though a quantum channel described by a maximally entangled coherent state with one ebit of entanglement.

In this paper we propose a scheme to teleport a superposition state of two SVS's with equal amplitudes and opposite phases using a beam splitter and photodetectors.

In the teleportation scheme one needs certain types of entangled states. We consider the following entangled squeezed states defined by (ESS's)

$$|\Phi\rangle_\pm = (|\xi\rangle_1|\xi\rangle_2 \pm |-\xi\rangle_1|-\xi\rangle_2)/\sqrt{N_\pm}, \tag{1}$$

$$|\Psi\rangle_\pm = (|\xi\rangle_1|-\xi\rangle_2 \pm |-\xi\rangle_1|\xi\rangle_2)/\sqrt{N_\pm}, \tag{2}$$

where both $|\xi\rangle$ and $|-\xi\rangle$ are the single-mode squeezed vacuum states

$$|\xi\rangle_1 = \hat{S}_1(\xi)|0\rangle, \qquad |\xi\rangle_2 = \hat{S}_2(\xi)|0\rangle, \tag{3}$$

with the single-mode squeezing operators

$$\hat{S}_1(\xi) = \exp\left(-\frac{\xi}{2}\hat{a}_1^{\dagger 2} + \frac{\xi^*}{2}\hat{a}_1^2\right), \tag{4}$$

$$\hat{S}_2(\xi) = \exp\left(-\frac{\xi}{2}\hat{a}_2^{\dagger 2} + \frac{\xi^*}{2}\hat{a}_2^2\right). \tag{5}$$

In Eqs. (1) and (2) the normalization factors are given by

$$N_\pm = 2(1 \pm k_\xi^2), \tag{6}$$

with $k_\xi = \langle\xi|-\xi\rangle = \sqrt{\frac{sech^2 r}{1+tanh^2 r}}$ being the overlap of the two squeezed vacuum states $|\xi\rangle$ and $|-\xi\rangle$.

It has been shown that the ESS's in (1) and (2) can be produced via beam splitters and squeezed-vacuum-state phase-shift operations [9]. Entanglement in these ESS's can be measured by the partial entropy of each subsystem. In order to calculate the partial entropy, we express the ESS's in terms of two orthogonal Hilbert space vectors $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ as $|\xi\rangle = |0\rangle, |-\xi\rangle = M|1\rangle + k_\xi|0\rangle)$ with $M = \sqrt{1-k_\xi^2}$. Under these basic vectors, the ESS's (1), (2) can be rewriten as

$$\begin{aligned}|\Phi'\rangle_\pm = &[(1 \pm k_\xi^2)|0\rangle_1|0\rangle_2 \pm M^2|1\rangle_1|1\rangle_2 \\ &\pm Mk_\xi|1\rangle_1|0\rangle_2 \pm k_\xi M|0\rangle_1|1\rangle_2]/\sqrt{N_\pm},\end{aligned} \tag{7}$$

$$|\Psi'\rangle_{\pm} = [(k_\xi \pm k_\xi)|0\rangle_1|0\rangle_2 + M|0\rangle_1|1\rangle_2 \pm M|1\rangle_1|0\rangle_2]/\sqrt{N_\pm}. \tag{8}$$

Therefore, we transfer the continuous-variable state expressions (1) and (2) to the discrete-variable forms (7) and (8). From (7) and (8) it is easy to get the partial entropy of the subsystem 1 or 2

$$E_-(\Phi) = E_-(\Psi) = 1, \tag{9}$$

$$\begin{aligned} E_+(\Phi) &= E_+(\Psi) \\ &= -\frac{(1+k_\xi)^2}{2(1+k_\xi^2)}\log_2\frac{(1+k_\xi)^2}{2(1+k_\xi^2)} \\ &\quad -\frac{(1-k_\xi)^2}{2(1+k_\xi^2)}\log_2\frac{(1-k_\xi)^2}{2(1+k_\xi^2)}, \end{aligned} \tag{10}$$

which indicate that entangled SVS's $|\Phi\rangle_-$ and $|\Psi\rangle_-$ are maximally entangled squeezed-vacuum states which can be used as quantum channels for continuous-variable quantum teleportation.

Let us briefly review the action of a beam splitter on squeezed-vacuum states before we present our teleportation scheme. The lossless symmetric 50/50 beam splitter is described by a unitary transformation

$$\hat{U}_{1,2} = exp[i\frac{\pi}{4}(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1)] \tag{11}$$

where $\hat{a}_1$ and $\hat{a}_2$ are the annihilation operators for the two light beams entering the two input ports, respectively. Let $\hat{A}_1$ and $\hat{A}_2$ denote the annihilation operators for the two light beams leaving the two output ports for the 50/50 beam splitter, the photon annihilation and creation operators of the output modes $\hat{A}_i$ and $\hat{A}_i^+$, respectively, can then be obtained using the well-known input-output relations

$$\hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + i\hat{a}_2), \hat{A}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 + i\hat{a}_1). \tag{12}$$

Assume that before interacting with the beam splitter the two input light beams are in squeezed vacuum states

$$|\Psi_{in}\rangle = \hat{S}_1(\xi)\hat{S}_2(\xi)|0,0\rangle, \tag{13}$$

where $\hat{S}_1(\xi)$ and $\hat{S}_2(\xi)$ are given by Eqs. (4) and (5), respectively. After interacting with the beam splitter, the state vector of the output light beams becomes

$$\begin{aligned} |\Psi_{out}\rangle &= \exp[-\frac{1}{4}(\xi_1-\xi_2)\hat{A}_1^{\dagger 2} + \frac{1}{4}(\xi_1^*-\xi_2^*)\hat{A}_1^2 \\ &+ \frac{1}{4}(\xi_1-\xi_2)\hat{A}_2^{\dagger 2} - \frac{1}{4}(\xi_1^*-\xi_2^*)\hat{A}_2^2 \\ &- \frac{i}{2}(\xi_1+\xi_2)\hat{A}_1^\dagger\hat{A}_2^\dagger - \frac{i}{2}(\xi_1^*+\xi_2^*)\hat{A}_1\hat{A}_2]|0,0\rangle. \end{aligned} \tag{14}$$

For later use, let us set $\xi_i = r_i e^{i\varphi_i}$ with r_i being the squeezing amplitude and φ_i the squeezing angle, respectively, and consider the following two cases which play a key role in our teleportation scheme.

Case 1. The two input light beams have the same squeezing amplitudes and phases, i.e; $r_1 = r_2 = r$, and $\varphi_1 = \varphi_2$. In this case, from Eq.(14) it is easy to know that the state of the output light beams is simply a two-mode squeezed state

$$|\Psi_{out}\rangle = \exp[-re^{i(\varphi+\frac{\pi}{2})}\hat{A}_1^\dagger\hat{A}_2^\dagger + re^{-i(\varphi+\frac{\pi}{2})}\hat{A}_1\hat{A}_2]|0,0\rangle. \tag{15}$$

Case 2. The two input light beams have the same squeezing amplitudes but a phase difference π, i.e., $r_1 = r_2 = r$ and $\varphi_2 - \varphi_1 = \pi$. In this case, the output state of the beam splitter becomes a direct product of two single-mode squeezed vacuum states

$$\begin{aligned} |\Psi_{out}\rangle &= \exp(-\frac{1}{2}re^{i\varphi}\hat{A}_1^{\dagger 2} + \frac{1}{2}re^{-i\varphi}\hat{A}_1^2) \\ &\times \exp(-\frac{1}{2}re^{i(\varphi+\pi)}\hat{A}_2^{\dagger 2} + \frac{1}{2}re^{-i(\varphi+\pi)}\hat{A}_2^2)|0,0\rangle. \end{aligned} \tag{16}$$

As is known, it is possible to generate a superposition state of the two SVS's $|\xi\rangle$ and $|-\xi\rangle$ from a squeezed vacuum states $|\xi\rangle$ propagating through a nonlinear medium [10]. Now let us assume that by using the quantum channel described by the maximally entangled state $|\Phi\rangle_-$, Alice wants to teleport Bob a coherent superposition state of the two squeezed vacuum states

$$|\Psi\rangle_0 = (C_+|\xi\rangle_0 + C_-|-\xi\rangle_0)/\sqrt{N_\Psi}, \tag{17}$$

where C_+ and C_- are any complex numbers, N_Ψ normalization factor given by

$$N_\Psi = |C_+|^2 + |C_-|^2 + 2k_\xi Re(C_+C_-^*). \tag{18}$$

Then the initial state of the whole system consisting of system 0, 1, and 2 is given by

$$\begin{aligned} |\Phi\rangle_{012} &= |\Psi\rangle_0|\Phi\rangle_- \\ &= \frac{1}{\sqrt{N_\Psi N_-}}(C_+|\xi\rangle_0|\xi\rangle_1|\xi\rangle_2 - C_+|\xi\rangle_0|-\xi\rangle_1|-\xi\rangle_2 \\ &+ C_-|-\xi\rangle_0|\xi\rangle_1|\xi\rangle_2 - C_-|-\xi\rangle_0|-\xi\rangle_1|-\xi\rangle_2). \end{aligned} \tag{19}$$

Note that at this time the system 0 and 1 at the sender's side and system 2 at the receiver's side. We apply the beam splitter transformation $\hat{U}_{01}$ to the initial state (19), the state after the transformation becomes

$$\begin{aligned} |\Phi'\rangle_{012} = \frac{1}{\sqrt{N_\Psi N_-}}[&C_+\hat{S}_{0,1}(re^{i(\varphi+\frac{\pi}{2})})|0,0\rangle_{01}\otimes|\xi\rangle_2 \\ &-C_+\hat{S}_0(re^{i\varphi})\hat{S}_1(-re^{i\varphi})|0,0\rangle_{01}\otimes|-\xi\rangle_2 \\ &+C_-\hat{S}_0(-re^{i\varphi})\hat{S}_1(re^{i\varphi})|0,0\rangle_{01}\otimes|\xi\rangle_2 \\ &-C_-\hat{S}_{0,1}(re^{i(\varphi-\frac{\pi}{2})})|0,0\rangle_{01}\otimes|-\xi\rangle_2], \end{aligned} \tag{20}$$

where the two-mode squeezed operator is given by

$$\hat{S}_{0,1}(\xi) = \exp(-\xi\hat{A}_0^+\hat{A}_1^+ + \xi^*\hat{A}_1\hat{A}_0). \tag{21}$$

Subsequently, Alice performs photon number measurements on system 0 and 1 on her side. From Eq.(20) it can be seen that for system 0 and 1 the second and third term on the RHS are single-mode SVS's which contain only even-number photon states in their number-state expansions, while the first and fourth terms are two-mode squeezed states in which each mode has the same photon numbers with both even and odd photons in the photon number-state representation.

After Alice measures an odd number of photons in either mode of systems 0 and 1, the state of the whole system collapses into the following state

$$\begin{aligned} |\Phi''\rangle_{012} &= \frac{1}{\sqrt{N_\Psi N_-}} sechr[e^{i(\varphi-\frac{\pi}{2})}\tanh r]^{2n+1} \\ &\times |2n+1,2n+1\rangle_{01}(C_+|\xi\rangle_2 + C_-|-\xi\rangle_2) \end{aligned} \tag{22}$$

which indicates that Alice transmits the coherent superposition of the two squeezed states $|\Psi\rangle_0$ given in Eq.(17) to Bob. The probability of finding an odd number of photons from the output state of the beam splitter (20) is given by

$$P(2n+1) = \frac{1}{N_-} sech^2 r(\tanh r)^{2(2n+1)}, \tag{23}$$

which is independent of the state to be teleported, hence we obtain the perfect teleportation for the odd number of photons. Then the successful probability of the teleportation scheme is given by

$$P = \sum_{n=0}^{\infty} P(2n+1). \tag{24}$$

These summation can be performed easily and the result is found to be independent of the squeezing amplitude r,

$$P = \frac{1}{4}. \tag{25}$$

In above teleportation scheme, a highly entangled state for the quantum channel plays a key role. If the initially prepared quantum channel is in a pure but not maximally ESS, the quantum channel may be distilled to a maximally entangled squeezed state before using it for quantum teleportation through entanglement concentration[11, 12]. Here we show that for an ESS the entanglement concentration may be simply realized by using a beam splitter and photodetectors.

Suppose an ensemble of a partially entangled pure squeezed state

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{N_\eta}}(\cos\eta|\xi\rangle_1|-\xi\rangle_2 - \sin\eta|-\xi\rangle_1|\xi\rangle_2), \tag{26}$$

from which we want to distill a subensemble of a maximally entangled squeezed state. In Eq.(26) N_η is a normalized factor given by

$$N_\eta = 1 - \sin(2\eta)\frac{sech^2 r}{1+\tanh^2 r}, \tag{27}$$

where the phase factor η, $0 < \eta < \pi/2$, determines the degree of entanglement for the state (26). It is obvious that the partially entangled pure squeezed state $|\Phi\rangle_{12}$ includes two ESS's $|\Phi\rangle_-$ and $|\Psi\rangle_-$ in Eq.(1) and (2) as two particular cases.

After sharing a quantum channel between Alice and Bob $|\Phi\rangle_{12}$ Alice prepares a pair of particles which are of the same entangled squeezed state $|\Phi\rangle_{34}$. The initial state of the whole system consisting of the four subsystems is then given by

$$|\Psi\rangle_{1234} = |\Phi\rangle_{12} \otimes |\Phi\rangle_{34}. \tag{28}$$

For convenience, now we assume mode 1 at the Bob's side and modes 2,3,4 at the Alice's side, then state (28) can be explicitly written as

$$\begin{aligned} |\Psi\rangle_{1234} &= \frac{1}{N_\eta}[\cos^2\eta|\xi\rangle_1 \otimes |-\xi\rangle_2 \otimes |\xi\rangle_3 \otimes |-\xi\rangle_4 \\ &- \frac{1}{2}\sin(2\eta)|\xi\rangle_1 \otimes |-\xi\rangle_2 \otimes |-\xi\rangle_3 \otimes |\xi\rangle_4 \\ &- \frac{1}{2}\sin(2\eta)|-\xi\rangle_1 \otimes |\xi\rangle_2 \otimes |\xi\rangle_3 \otimes |-\xi\rangle_4 \\ &+ \sin^2\eta|-\xi\rangle_1 \otimes |\xi\rangle_2 \otimes |-\xi\rangle_3 \otimes |\xi\rangle_4]. \end{aligned} \tag{29}$$

Subsequently, Alice let modes 2 and modes 3 enter the input ports of a beam splitter. After interacting with the beam splitter, the state of the whole system becomes

$$\begin{aligned} |\Psi'\rangle_{1234} &= \frac{1}{N_\eta}[\cos^2\eta|\xi\rangle_1 \otimes |-\xi\rangle_2 \otimes |\xi\rangle_3 \otimes |-\xi\rangle_4 \\ &- \frac{1}{2}\sin(2\eta)|\xi\rangle_1 \otimes \hat{S}_{23}(re^{i(\varphi-\frac{\pi}{2})})|0,0\rangle_{23} \\ &\otimes|\xi\rangle_4 \\ &- \frac{1}{2}\sin(2\eta)|-\xi\rangle_1 \otimes \hat{S}_{23}(re^{i(\varphi+\frac{\pi}{2})})|0,0\rangle_{23} \\ &\otimes|-\xi\rangle_4 \\ &+ \sin^2\eta|-\xi\rangle_1 \otimes |\xi\rangle_2 \otimes |-\xi\rangle_3 \otimes |\xi\rangle_4], \end{aligned} \tag{30}$$

where the two-mode squeezed operator is given by (21).

Now Alice performs photon number measurements for mode 2 and 3, when the results of measurements are odd number of photon, from Eq.(30) it can be seen that modes 1 and 4 collapse to a maximally ESS

$$|\Psi\rangle_{14} = \frac{1}{\sqrt{N_-}}(|\xi\rangle_1 \otimes |\xi\rangle_4 - |-\xi\rangle_1 \otimes |-\xi\rangle_4). \tag{31}$$

From Eq.(30), we can obtain the probability of finding $2n+1$ photons in modes 2 and 3 to be

$$P(2n+1) = \frac{N_-}{4N_\eta^2}\sin^2(2\eta) sech^2 r(\tanh r)^{2(2n+1)}, \tag{32}$$

which leads the probability of obtaining the maximally

ESS (31) to be

$$\begin{aligned} P &= \sum_{n=0}^{\infty} P(2n+1) \\ &= \frac{1}{4}\sin^2(2\eta)\frac{N_- \tanh^2 r}{N_\eta^2(1+\tanh^2 r)}, \end{aligned} \tag{33}$$

which implies that no matter how small the initial entanglement is, it is possible to distill some maximally entangled coherent channels from partially entangled pure channels.

In summary, we have present a scheme to optically teleport an arbitrary coherent superposition state of two equal-amplitude and opposite-phase SVS's via a symmetric 50/50 beam splitter and photodetectors with the successful probability 1/4. In particular, a perfect teleportation of a single-mode SVS can be obtained when one of the two coefficients in the coherent superposition state of two SVS's vanishes. In our scheme, maximally entangled SVS's are used as quantum channel for realizing the quantum teleportation scheme. We have also shown how to distill a maximally entangled SVS from an initial quantum channel in a pure but not maximally entangled SVS through entanglement concentration.

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†Corresponding author.

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] L. Davidovich, N. Zagury, M. Brune, J. M. Raimond, and S. Harroche, Phys. Rev. A **50**, R895 (1994).
[3] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature **390** 575 (1997).
[4] M. A. Nielsen, E. Knill, and R. Laflamme, Nature **396** 52 (1998).
[5] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
[6] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[7] A. Furusawa, J. L. Srensen, S. L. Braunstein, C. A.Fuchs, H. J. Kimble, and E. S. Polzik, Science **282** 706 (1998).
[8] S. J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).
[9] L. Zhou and L. M. Kuang, in preparation.
[10] B. C. Sanders, Phys. Rev. A **39**, 4284 (1989).
[11] C. H. Bennett, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).
[12] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).