## The Evolution Operator of The Three Atoms Tavis-Cummings Model

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## Abstract

The explicit form of evolution operator of the three atoms Tavis-Cummings Model is given.

The Tavis-Cummings Models have been widely studied in the field of quantum optic and quantum computer. In this brief letter, The analytical form of the evolution operators of T-C model with three atoms is given. Compared with the result in the paper[1], our result is more convenient for further application.

Under the rotating-wave approximation and resonant condition, the Hamiltonian of the Tavis-Cummings model with three atoms in the interaction picture is

$$\hat{H}_{I} = \hbar \gamma \sum_{i=1}^{3} (\hat{a}\hat{\sigma}_{i}^{+} + \hat{a}^{\dagger}\hat{\sigma}_{i}^{-})$$
 (1)

We denote the ground and exited states for the atom by  $|g\rangle$  and  $|e\rangle$ . The evolution operator  $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$  in the atomic basis  $|eee\rangle, |eeg\rangle, |ege\rangle, |gee\rangle, |geg\rangle, |geg\rangle, |ggg\rangle,$  is found to be

$$\begin{pmatrix}
\hat{U}_{11} & \cdots & \hat{U}_{18} \\
\vdots & \ddots & \vdots \\
\hat{U}_{81} & \cdots & \hat{U}_{88}
\end{pmatrix}$$
(2)

where

$$\hat{U}_{11} = \frac{(7 + 2\hat{N} + \hat{\Omega})\cos(\hat{\Theta}_{1}\gamma t) + (-7 - 2\hat{N} + \hat{\Omega})\cos(\hat{\Theta}_{2}\gamma t)}{2\hat{\Omega}} \equiv u_{11}(\hat{N})$$

$$\hat{U}_{22} = \hat{a}^{\dagger} \frac{\left(-1 - 2\hat{N} + \hat{\Omega}\right)\cos(\hat{\Theta}_{1}\gamma t) + \left(1 + 2\hat{N} + \hat{\Omega}\right)\cos(\hat{\Theta}_{2}\gamma t) + 4\hat{\Omega}\cos(\sqrt{2 + \hat{N}}\gamma t)}{6\hat{\Omega}(\hat{N} + 1)} \hat{a}$$

$$\equiv \hat{a}^{\dagger} \frac{u_{22}(\hat{N})}{\hat{N} + 1} \hat{a}$$

$$\hat{U}_{88} = \hat{a}^{\dagger 3} \frac{\left(1 + 2\hat{N} + \hat{\Omega}\right)\cos(\hat{\Theta}_{1}\gamma t) + \left(-1 - 2\hat{N} + \hat{\Omega}\right)\cos(\hat{\Theta}_{2}\gamma t)}{2\hat{\Omega}(\hat{N} + 1)(\hat{N} + 2)(\hat{N} + 3)} \hat{a}^{3}$$

$$\begin{split} & \equiv \hat{a}^{\dagger 3} \frac{u_{88}(\hat{N})}{(\hat{N}+1)(\hat{N}+2)(\hat{N}+3)} \hat{a}^{3} \\ \hat{U}_{12} & = \frac{\hat{\Theta}_{1} \left(7+2\hat{N}+\hat{\Omega}\right) \sin(\hat{\Theta}_{1}\gamma t)+\hat{\Theta}_{2} \left(-7-2\hat{N}+\hat{\Omega}\right) \sin(\hat{\Theta}_{2}\gamma t)}{6i(1+\hat{N})\hat{\Omega}} \hat{a} \equiv \frac{u_{12}(\hat{N})}{\sqrt{\hat{N}+1}} \hat{a} \\ \hat{U}_{15} & = \frac{-\cos(\hat{\Theta}_{1}\gamma t)+\cos(\hat{\Theta}_{2}\gamma t)}{\hat{\Omega}} \hat{a}^{2} \equiv \frac{u_{15}(\hat{N})}{\sqrt{(\hat{N}+1)(\hat{N}+2)}} \hat{a}^{2} \\ \hat{U}_{25} & = \hat{a}^{\dagger} \frac{-\hat{\Theta}_{1} \left(2+\hat{N}\right) \sin(\hat{\Theta}_{1}\gamma t)+\hat{\Theta}_{2} \left(2+\hat{N}\right) \sin(\hat{\Theta}_{2}\gamma t)+\sqrt{2+\hat{N}}\hat{\Omega} \sin(\sqrt{2+\hat{N}}\gamma t)}{3i(1+\hat{N})(2+\hat{N})\hat{\Omega}} \hat{a}^{2} \\ & \equiv \hat{a}^{\dagger} \frac{u_{25}(\hat{N})}{(\hat{N}+1)\sqrt{\hat{N}+2}} \hat{a}^{2} \\ \hat{U}_{58} & = \hat{a}^{\dagger 2} \frac{\hat{\Theta}_{1} \left(1+2\hat{N}+\hat{\Omega}\right) \sin(\hat{\Theta}_{1}\gamma t)+\hat{\Theta}_{2} \left(-1-2\hat{N}+\hat{\Omega}\right) \sin(\hat{\Theta}_{2}\gamma t)}{6i(1+\hat{N})(2+\hat{N})(3+\hat{N})\hat{\Omega}} \hat{a}^{3} \\ & \equiv \hat{a}^{\dagger 2} \frac{u_{58}(\hat{N})}{(\hat{N}+1)(\hat{N}+2)\sqrt{\hat{N}+3}} \hat{a}^{3} \\ \hat{U}_{18} & = \frac{i\hat{\Theta}_{1}\hat{\Theta}_{2} \left(\hat{\Theta}_{2}\sin(\hat{\Theta}_{1}\gamma t)-\hat{\Theta}_{1}\sin(\hat{\Theta}_{2}\gamma t)\right)}{3(\hat{N}+1)(\hat{N}+3)\hat{\Omega}} \hat{a}^{3} \equiv \frac{u_{18}(\hat{N})}{\sqrt{(\hat{N}+1)(\hat{N}+2)(\hat{N}+3)}} \hat{a}^{3} \end{split}$$

and

$$\begin{split} \hat{U}_{33} &= \hat{U}_{44} = \hat{U}_{22}; \\ \hat{U}_{55} &= \hat{U}_{66} = \hat{U}_{77} = \hat{a}^{\dagger 2} \frac{u_{22}(\hat{N}) - \frac{1}{\hat{\Omega}}(\cos(\hat{\Theta}_{1}\gamma t) - \cos(\hat{\Theta}_{2}\gamma t))}{(\hat{N}+1)(\hat{N}+2)} \hat{a}^{2} \\ \hat{U}_{23} &= \hat{U}_{24} = \hat{U}_{32} = \hat{U}_{34} = \hat{U}_{42} = \hat{U}_{43} = \hat{a}^{\dagger} \frac{u_{22}(\hat{N}) - \cos(\sqrt{\hat{N}+2\gamma t})}{\hat{N}+1} \hat{a} \\ \hat{U}_{56} &= \hat{U}_{57} = \hat{U}_{65} = \hat{U}_{67} = \hat{U}_{75} = \hat{U}_{76} = \hat{a}^{\dagger 2} \frac{u_{22}(\hat{N}) - \frac{1}{\hat{\Omega}}(\cos(\hat{\Theta}_{1}\gamma t) - \cos(\hat{\Theta}_{2}\gamma t)) - \cos(\sqrt{2+\hat{N}\gamma t})}{(\hat{N}+1)(\hat{N}+2)} \hat{a}^{2} \\ \hat{U}_{13} &= \hat{U}_{14} = \hat{U}_{12}; \qquad \hat{U}_{21} = \hat{U}_{31} = \hat{U}_{41} = \hat{a}^{\dagger} \frac{u_{12}(\hat{N})}{\sqrt{\hat{N}+1}} \\ \hat{U}_{15} &= \hat{U}_{16} = \hat{U}_{17}; \qquad \hat{U}_{51} = \hat{U}_{61} = \hat{U}_{71} = \hat{a}^{\dagger 2} \frac{u_{15}(\hat{N})}{\sqrt{(\hat{N}+1)(\hat{N}+2)}} \\ \hat{U}_{28} &= \hat{U}_{38} = \hat{U}_{48} = \hat{a}^{\dagger} \frac{u_{15}(\hat{N})}{(\hat{N}+1)\sqrt{(\hat{N}+1)(\hat{N}+2)}} \hat{a}^{3}; \\ \hat{U}_{82} &= \hat{U}_{83} = \hat{U}_{84} = \hat{a}^{\dagger 3} \frac{u_{15}(\hat{N})}{(\hat{N}+1)\sqrt{(\hat{N}+1)(\hat{N}+2)}} \hat{a} \\ \hat{U}_{25} &= \hat{U}_{26} = \hat{U}_{37} = \hat{U}_{46} = \hat{U}_{47}; \end{split}$$

$$\hat{U}_{52} = \hat{U}_{53} = \hat{U}_{62} = \hat{U}_{64} = \hat{U}_{73} = \hat{U}_{74} = \hat{a}^{\dagger 2} \frac{u_{25}(\hat{N})}{(\hat{N}+1)\sqrt{\hat{N}+2}} \hat{a}$$

$$\hat{U}_{27} = \hat{U}_{36} = \hat{U}_{45} = \hat{a}^{\dagger} \frac{u_{25}(\hat{N}) + \sin(\sqrt{2+\hat{N}\gamma t})}{(1+\hat{N})\sqrt{2+\hat{N}}} \hat{a}^{2};$$

$$\hat{U}_{54} = \hat{U}_{63} = \hat{U}_{72} = \hat{a}^{\dagger 2} \frac{u_{25}(\hat{N}) + \sin(\sqrt{2+\hat{N}\gamma t})}{(1+\hat{N})\sqrt{2+\hat{N}}} \hat{a}$$

$$\hat{U}_{68} = \hat{U}_{78} = \hat{U}_{58}; \qquad \hat{U}_{85} = \hat{U}_{86} = \hat{U}_{87} = \hat{a}^{\dagger 3} \frac{u_{58}(\hat{N})}{(\hat{N}+1)(\hat{N}+2)\sqrt{\hat{N}+3}} \hat{a}^{2}$$

$$\hat{U}_{81} = \hat{a}^{\dagger 3} \frac{u_{18}(\hat{N})}{\sqrt{(\hat{N}+1)(\hat{N}+2)(\hat{N}+3)}}$$

where  $\hat{\Omega}, \hat{\Theta_1}, \hat{\Theta_2}$  are defined by

$$\hat{\Theta}_1^2 = 5(\hat{N}+2) - \hat{\Omega}$$
  $\hat{\Theta}_2^2 = 5(\hat{N}+2) + \hat{\Omega}$   
 $\hat{\Omega}^2 = 9 + 16(\hat{N}+2)^2$ 

and  $\hat{N} = \hat{a}^{\dagger} \hat{a}$ 

## References

[1] K. Fujii, K. Higashida, R. Kato, T. Suzuki and Y. Wada, "Explicit Form of Evolution Operator of Three Atoms Tavis-Cummings Model", quant-ph/0404034