# J.S. Bell's Concept of Local Causality

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John Stewart Bell's famous 1964 theorem is widely regarded as one of the most important developments in the foundations of physics. It has even been described as "the most profound discovery of science." Yet even as we approach the 50th anniversary of Bell's discovery, its meaning and implications remain controversial. Many textbooks and commentators report that Bell's theorem refutes the possibility (suggested especially by Einstein, Podolsky, and Rosen in 1935) of supplementing ordinary quantum theory with additional ("hidden") variables that might restore determinism and/or some notion of an observer-independent reality. On this view, Bell's theorem supports the orthodox Copenhagen interpretation. Bell's own view of his theorem, however, was quite different. He instead took the theorem as establishing an "essential conflict" between the now well-tested empirical predictions of quantum theory and relativistic local causality. The goal of the present paper is, in general, to make Bell's own views more widely known and, in particular, to explain in detail Bell's little-known mathematical formulation of the concept of relativistic local causality on which his theorem rests. We thus collect and organize many of Bell's crucial statements on these topics, which are scattered throughout his writings, into a self-contained, pedagogical discussion including elaborations of the concepts "beable", "completeness", and "causality" which figure in the formulation. We also show how local causality (as formulated by Bell) can be used to derive an empirically testable Bell-type inequality, and how it can be used to recapitulate the EPR argument.

### I. INTRODUCTION

In its most generalized sense, "local causality" is the idea that physical influences propagate continuously through space – i.e., the idea that what Einstein famously called "spooky actions at a distance" are impossible. In addition to originating this catchy slogan, Einstein was also chiefly responsible for the modern, specifically relativistic sense of local causality, according to which causal influences should not only propagate continuously (never hopping across a gap in which no trace is left) but should do so always at the speed of light or slower. The elaboration and precise formulation of this idea will be our central concern here.

It is interesting to first note, though, that the prerelativistic "no action at a distance" sense of local causality has played an important role in the construction and assessment of theories throughout the history of physics.<sup>2,3</sup> For example, some important objections to Isaac Newton's theory of gravitation centered on the theory's alleged positing of such non-local action at a distance. Newton's own view, interestingly, seems to have been that although his theory did claim that (for example) the sun exerted causal influences on the distant planets, this was in principle perfectly consistent with local causality, which he strongly endorsed:

"It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact... That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and

force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it." <sup>4</sup>

Newton's idea<sup>5</sup> was evidently that his gravitational theory simply didn't yet provide a complete description of the underlying (and presumably local) mechanism "by and through which [massive bodies'] action and force may be conveyed from one to another."

Such debates had a philosophical character, though, since at the time nothing was definitely, unambiguously excluded by the requirement of locality. Any apparent action at a distance in a theory could be rendered compatible in principle with local causality by following Newton and simply denying that the theory in question provided a complete description of the relevant phenomena.

This changed in 1905 with Albert Einstein's discovery of Special Relativity (SR), which for the first time purported to identify a certain class of causal influences – namely, those which propagate faster than light – as definitely inconsistent with local causality. As Einstein explained,

"The success of the Faraday-Maxwell interpretation of electromagnetic action at a distance resulted in physicists becoming convinced that there are no such things as instantaneous action at a distance (not involving an intermediary medium) of the type of Newton's law of gravitation. According to the theory of relativity, action at a distance with the velocity of light always takes the place of instantaneous action at a distance or of action at a distance with an infinite velocity of transmission. This is connected with the fact

that the velocity c plays a fundamental role in this theory."

The speed of light c plays a fundamental role vis-à-vis causality in SR because of the relativity of simultaneity. For two events A and B with space-like separation (i.e., such that a signal connecting A and B would have to propagate faster than c), the time ordering is ambiguous: different inertial observers will disagree about whether A precedes B in time, or vice versa. There is thus, according to SR, no objective matter of fact about which event occurs first – and hence no possibility of a causal relation between them, since the relation between a cause and its effect is necessarily time-asymmetric. As J.S. Bell put this point, "To avoid causal chains going backward in time in some frames of reference, we require them to go slower than light in any frame of reference."

After the advent of SR, it didn't take long for the relativistic sense of local causality to be deployed in a criticism of other developing theories, much as the prerelativistic concept had been used against Newton's theory. Indeed, it was Einstein himself - in both the famous, but widely misunderstood, EPR paper<sup>9</sup> and several related but less widely known arguments<sup>10</sup> – who first pointed out that the developing Copenhagen quantum theory violated SR's locality constraint. In particular, according to Einstein, that theory's account of measurement combined with Niels Bohr's completeness doctrine committed the theory to precisely the sort of nonlocal causation that was, at least according to Einstein, prohibited by SR. Einstein thus rejected Bohr's completeness doctrine and supported (something like) what is now (unfortunately<sup>11</sup>) called the local "hidden variables" pro-

Note the interesting parallel to Newtonian gravity here, with the non-locality in some candidate theory being rendered either real or merely apparent, depending on whether or not one interprets the theory as providing a *complete* description of the physical processes in question. Einstein's assessment of Copenhagen quantum theory with respect to local causality is thus logically parallel to Newton's analysis of his own theory of gravitation: the theory, if regarded as complete, violates locality – hence upholding locality requires denying completeness.

This brings us to the main subject of the present paper: the work of J.S. Bell. Bell (unlike many commentators) accepted Einstein's proof of the non-locality of Copenhagen quantum theory. In particular, Bell accepted as valid "the EPR argument from locality to deterministic hidden variables." <sup>14</sup> The setup for this argument involves a pair of specially-prepared particles which are allowed to separate to remote locations. An observation of some property of one particle then permits the observer to learn something about a corresponding property of the distant particle. According to the Copenhagen view, the distant particle fails to possess a definite value for the property in question prior to the observation, and so it is precisely the observation of the nearby particle which – in apparent violation of local causality – triggers the

crystallization of this newly real property for the distant particle.

In Bell's recapitulation of the argument, though, for Einstein, Podolsky, and Rosen (EPR) this

"simply showed that [Bohr, Heisenberg, and Jordan had been hasty in dismissing the reality of the microscopic world. In particular, Jordan had been wrong in supposing that nothing was real or fixed in that world before observation. For after observing only one particle the result of subsequently observing the other (possibly at a very remote place) is immediately predictable. Could it be that the first observation somehow fixes what was unfixed, or makes real what was unreal, not only for the near particle but also for the remote one? For EPR that would be an unthinkable 'spooky action at a distance'. To avoid such action at a distance [one has] to attribute, to the space-time regions in question, real properties in advance of observation, correlated properties, which predetermine the outcomes of these particular observations. Since these real properties, fixed in advance of observation, are not contained in quantum formalism, that formalism for EPR is incomplete. It may be correct, as far as it goes, but the usual quantum formalism cannot be the whole story." 14

Bell thus agreed with Einstein that the local hidden variables program constituted the *only hope* for a locally causal re-formulation of quantum theory.

Bell's historic contribution, however, was a 1964 theorem establishing that no such local hidden variable theory – and hence no local theory of any kind – could generate the correct empirical predictions for a certain class of experiment. <sup>15</sup> According to Bell, we must therefore accept the real existence, in nature, of faster-than-light causation – in apparent conflict with the requirements of SR:

"For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory..." <sup>16</sup>

Indeed, Bell even went so far as to suggest, in response to his theorem and the relevant experimental data, <sup>17,18</sup> the rejection of "fundamental relativity" and the return to a Lorentzian view in which there is a dynamically privileged (though probably empirically undetectable) reference frame:

"It may well be that a relativistic version of [quantum] theory, while Lorentz invariant and local at the observational level, may be necessarily non-local and with a preferred frame (or aether) at the fundamental level."  $^{12}$ 

### And elsewhere:

"...I would say that the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincaré thought that there was an aether – a preferred frame of reference – but that our measuring instruments were distorted by motion in such a way that we could not detect motion through the aether. Now, in that way you can imagine that there is a preferred frame of reference, and in this preferred frame of reference things do go faster than light. .... Behind the apparent Lorentz invariance of the phenomena, there is a deeper level which is not Lorentz invariant... [This] pre-Einstein position of Lorentz and Poincaré, Larmor and Fitzgerald, was perfectly coherent, and is not inconsistent with relativity theory. The idea that there is an aether, and these Fitzgerald contractions and Larmor dilations occur, and that as a result the instruments do not detect motion through the aether – that is a perfectly coherent point of view." 19,20

Here our intention is not to lobby for this radical view, but simply to explain Bell's rationale for contemplating it.

This rationale involves a complex chain of reasoning including at least these four steps: (i) arguing that SR prohibits causal influences between space-like separated events, (ii) constructing a mathematically precise formulation of this prohibition, i.e., of relativistic local causality, (iii) deriving an empirically-testable inequality from this formulation of local causality, and then (iv) establishing that the inequality is inconsistent with actual empirical data. There is an extensive "Bell literature" in which each of these steps is subjected to probing critical analysis. The time-asymmetric character of causal relations – used in the argument for (i) that was sketched above – has for example been challenged by Huw Price<sup>21</sup> and, in a rather different way, by recent work of Roderich Tumulka<sup>22</sup> (which was, incidentally, based on earlier work by Bell<sup>23</sup>). And there remain certain "loopholes" in the experiments demonstrating violations of Bell's inequality, such that one might conceivably doubt (iv).<sup>24</sup>

For the most part, though, physicists do not seriously question (i) and regard (iv) as established with reasonable conclusiveness. The controversies about the meaning and implications of Bell's theorem have thus centered on points (ii) and (iii). Clearly, though, what one says about point (iii) – the question of whether and how a Bell-type inequality is entailed by local causality – will depend strongly on whether and how one has addressed (ii). And sadly, Bell's own views in regard to (ii) have been almost entirely invisible in the Bell literature. (The

review article cited in Ref. 24, for example, doesn't acknowledge Bell's formulation of local causality at all but instead proposes an alternative formulation very different from Bell's.) It is thus not terribly surprising that so many commentators and textbook authors have summarized the upshot of Bell's theorem in ways so different from Bell's own. Typically, for example, one encounters the claim that Bell's inequality follows not from local causality alone, but from the conjunction of local causality with some additional premises; some of the usual suspects here include "hidden variables," "determinism," "realism," "counter-factual definiteness", or an improper insistence on a vaguely-defined "classical" way of thinking. One or more of these (rather than relativistic local causality) is then invariably blamed for the inconsistency with experiment. <sup>29–36</sup>

A full presentation of Bell's alternative to these widespread views would include a systematic treatment of point (iii). We will sketch this derivation in Section VI, but the bulk of our discussion will focus instead on Bell's formulation of local causality, i.e., his views on point (ii). This discussion will be based primarily on (but will also in several ways interpret and extend beyond) Bell's 1990 paper "La nouvelle cuisine" (published in the same year as his untimely death). Clearly explaining Bell's formulation of locality, however, will require also sketching Bell's interesting and refreshingly unorthodox views on a number of related issues in the foundations of quantum theory. The discussion will thus be elaborated and supported with excerpts from Bell's many other papers.

The main audience for the paper is students and physicists with little or no prior knowledge of Bell's theorem beyond what they've read in textbooks. It should be understood, though, that virtually all of the issues raised here are included because some kind of misunderstanding (or simple ignorance) of them has been present and influential in the Bell literature. We will provide occasional citations to works which we think exemplify the various important misunderstandings. But length considerations (and the desire to keep this a self-contained, positive presentation of Bell's views) forbid any extensive polemical discussions. Still, those familiar with the Bell literature will have no trouble appreciating where Bell's own views challenge the conventional wisdom. It is hoped that simply bringing Bell's views more out into the open will stimulate fruitful debates among those with interests in these areas.

The paper is organized as follows. In the following section, we jump quickly from some of Bell's preliminary, qualitative statements to his final, quantitative formulation of relativistic local causality. Then, in Sections III-V, we will highlight and explore various aspects by clarifying some perhaps-unfamiliar or suspicious terms which appear in Bell's formulation and by contrasting them to various other ideas with which they have sometimes been confused. Section VI will show how local causality (as formulated by Bell) can be used to derive an empirically testable Bell-type inequality, and also how it can be used

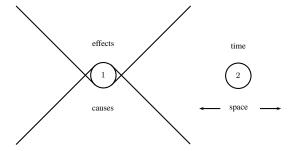


FIG. 1: "Space-time location of causes and effects of events in region 1." (Figure and caption are from Ref. 7.)

to recapitulate the EPR argument. Finally, in Section VII we will summarize the arguments presented and acknowledge some limitations of and open questions about Bell's formulation.

### II. LOCAL CAUSALITY: OVERVIEW

Let us begin with a qualitative formulation of Bell's concept of local causality. In a 1988 interview, in answer to the question "What does locality mean?" Bell responded:<sup>37</sup>

"It's the idea that what you do has consequences only nearby, and that any consequences at a distant place will be weaker and will arrive there only after the time permitted by the velocity of light. Locality is the idea that consequences propagate continuously, that they don't leap over distances." <sup>38</sup>

Bell gave a slightly more careful (but still qualitative) formulation of what he called the "Principle of local causality" in his 1990 paper:

"The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light." <sup>7</sup>

Then, citing a figure which we have reproduced here as Figure 1, Bell continues:

"Thus for events in a space-time region 1 ... we would look for causes in the backward light cone, and for effects in the future light cone. In a region like 2, space-like separated from 1, we would seek neither causes nor effects of events in 1." <sup>7</sup>

This should be relatively uncontroversial. Bell immediately notes, however, that "[t]he above principle of local causality is not yet sufficiently sharp and clean for mathematics."

Here, then, is Bell's sharpened and cleaned formulation of special relativistic local causality. (The reader should

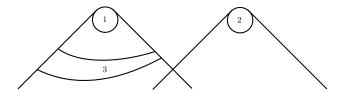


FIG. 2: "Full specification of what happens in 3 makes events in 2 irrelevant for predictions about 1 in a locally causal theory." (Figure and caption are from Ref. 7).

remember that this is, at this point, merely a 'teaser' which those to whom it is not already familiar should only expect to understand after further reading.)

"A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3..."

The space-time regions referred to are illustrated in Figure 2. We may translate Bell's formulation into mathematical form as follows:

$$P(b_1|B_3, b_2) = P(b_1|B_3), \tag{1}$$

where  $b_i$  refers to the value of some particular beable in space-time region i and  $B_i$  refers to a *sufficient* (for example, a complete) specification of *all* beables in the relevant region. The Ps here are the probabilities assigned to event  $b_1$  by the candidate theory in question. Eq. (1) thus asserts mathematically just what Bell states in the caption of his accompanying figure (reproduced here as Figure 2): "full specification of [beables] in 3 makes events in 2 irrelevant for predictions about 1 in a locally causal theory."

Let us then jump right in to a closer examination of the several perhaps-puzzling features of this formulation.

#### III. BEABLES

For those to whom the term is new, the first question about the word "beable" is: how to pronounce it? The word has three syllables. It does not rhyme with "feeble," but with "agreeable." Bell invented the word as a contrast to the "observables" which play a fundamental role in the formulation of orthodox quantum theory, so let us begin there.

### A. Beables vs. Observables

Beables (as contrasted to observables) are those elements of a theory which are supposed to correspond to

something that is *physically real*, independent of any observation. Bell elaborates:

"The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on 'observation'. Indeed observation and observers must be made out of beables." <sup>25</sup>

Or as he explains elsewhere,

"The concept of 'observable' .... is a rather woolly concept. It is not easy to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables ... because they are there."  $^{26}$ 

Bell's reservations here (about the concept "observable" appearing in the fundamental formulation of allegedly fundamental theories) are closely related to the so-called "measurement problem" of orthodox quantum mechanics, which Bell encapsulated by remarking that the orthodox theory is "unprofessionally vague and ambiguous" in so far as its fundamental dynamics is expressed in terms of "words which, however legitimate and necessary in application, have no place in a formulation with any pretension to physical precision" – such words as "system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, observable, information, measurement." <sup>27</sup> As Bell elaborates,

"The concepts 'system', 'apparatus', 'environment', immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split. The notions of 'microscopic' and 'macroscopic' defy precise definition. So also do the notions of 'reversible' and 'irreversible'. Einstein said that it is theory which decides what is 'observable'. I think he was right – 'observable' is a complicated and theory-laden business. Then the notion should not appear in the *formulation* of fundamental theory." <sup>27</sup>

As Bell points out, even Bohr (a convenient personification of skepticism regarding the physical reality of unobservable microscopic phenomena) recognizes certain things (for example, the directly perceivable states of a classical measuring apparatus) as unambiguously real, i.e., as beables:

"The terminology, be-able as against observable, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some

notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.' It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk." <sup>26</sup>

The unprofessional vagueness and ambiguity of orthodox quantum theory, then, is related to the fact that its formulation presupposes these (classical, macroscopic) beables, but fails to provide clear mathematical laws to describe them. As Bell explains,

"The kinematics of the world, in [the] orthodox picture, is given by a wavefunction ... for the quantum part, and classical variables – variables which have values – for the classical part... [with the classical variables being] somehow macroscopic. This is not spelled out very explicitly. The dynamics is not very precisely formulated either. It includes a Schrödinger equation for the quantum part, and some sort of classical mechanics for the classical part, and 'collapse' recipes for their interaction." <sup>27</sup>

There are thus two related problems. First, the posited ontology is rather different on the two sides of (what Bell calls) "the shifty split" <sup>27</sup> – that is, the division between "the quantum part" and "the classical part." But then, as a whole, the posited ontology remains unavoidably vague so long as the split remains shifty – i.e., so long as the dividing line between the macroscopic and microscopic remains undefined. And second, the *interaction* across the split is problematic. Not only is the account of this dynamics (the "collapse" process) inherently bound up in concepts from Bell's list of dubious terms, but the very existence of a special dynamics for the interaction seems to imply inconsistencies with the dynamics already posited for the two realms separately.

As Bell summarizes,

"I think there are professional problems [with quantum mechanics]. That is to say, I'm a professional theoretical physicist and I would like to make a clean theory. And when I look at quantum mechanics I see that it's a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes... So you have a theory which is fundamentally ambiguous..." <sup>19</sup>

The point of all this is to clarify the sort of theory Bell had in mind as satisfying the relevant standards of professionalism in physics. It is often thought, by those who do not understand or do not accept Bell's criticisms of orthodox quantum theory, that the very concept of "beable" (in terms of which his concept of local causality is formulated) commits one already to hidden variables or determinism or some sort of naive realism or some other physically or philosophically dubious principle.

But this is not correct. The requirement here, ultimately, is only that candidate fundamental theories – at least, those "with any pretension to physical precision" <sup>27</sup> – be formulated *clearly* and *precisely*. And this requires, according to Bell, that the theories provide a *uniform* and *consistent* candidate description of physical reality. In particular, there should be no ambiguity or inconsistency regarding what a given candidate theory is fundamentally *about* (the beables), nor regarding precisely how those posited physically real elements are posited to act and interact (the laws).

#### B. Beables vs. Conventions

So far we have explained the term "beable" by contrasting it to the "observables" of orthodox quantum theory. We must now also contrast the concept of "beables" with those elements of a theory which are, to some degree, conventional:

"The word 'beable' will also be used here to carry another distinction, that familiar already in classical theory between 'physical' and 'non-physical' quantities. In Maxwell's electromagnetic theory, for example, the fields  ${\bf E}$  and  ${\bf H}$  are 'physical' (beables, we will say) but the potentials  ${\bf A}$  and  $\phi$  are 'non-physical'. Because of gauge invariance the same physical situation can be described by very different potentials. It does not matter [i.e., it is not a violation of local causality] that in Coulomb gauge the scalar potential propagates with infinite velocity. It is not really supposed to be there. It is just a mathematical convenience."  $^{26}$ 

Or, as Bell puts the same point in another paper,

"...there are things which do go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King.... And there are things like that in physics. In Maxwell's theory, the electric and magnetic fields in free space satisfy the wave equation

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$
$$\frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

...corresponding to propagation with velocity c. But the scalar potential, if one chooses to work in 'Coulomb gauge', satisfies Laplace's equation

$$-\nabla^2\phi = 0$$

...corresponding to propagation with infinite velocity. Because the potentials are only mathematical conveniences, and arbitrary to a high degree, made definite only by the imposition of one convention or another, this infinitely fast propagation of the Coulomb-gauge scalar potential disturbs no one. Conventions can propagate as fast as may be convenient. But then we must distinguish in our theory between what is convention and what is not."

Thus, in order to cleanly decide whether a given theory is or is not consistent with local causality,

"you must identify in your theory 'local beables'. The beables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real. The concept of 'reality' is now an embarrassing one for many physicists.... But if you are unable to give some special status to things like electric and magnetic fields (in classical electromagnetism), as compared with the vector and scalar potentials, and British sovereignty, then we cannot begin a serious discussion." <sup>7</sup>

This explains why, according to Bell: "It is in terms of local beables that we can hope to formulate some notion of local causality."  $^{26}$ 

#### C. Beables and Candidate Theories

It is important to appreciate that a beable is only a beable relative to some particular candidate theory which posits those elements as physically real (and, presumably, gives precise mathematical laws for their dynamics). For example, the fields **E** and **B** (and not the potentials) are beables according to classical Maxwellian electrodynamics as it is normally understood. But one could imagine some alternative theory which (perhaps motivated by the Aharanov-Bohm effect) posits the Coulomb gauge potentials as beables instead. Note that, although it would be empirically and in some sense mathematically equivalent to the usual theory, the alternative theory would evidently violate local causality (because, say, wiggling a charge would instantaneously affect the physically-real scalar potential at distant locations) while the usual, Maxwellian theory would respect it.

Thinking in terms of such candidate theories helps us separate any questions about what the "real beables" are — what really exists out there in physical reality — into two distinct parts: first, what elements does a given candidate theory posit as beables; and second, which candidate theory do we think is true? The point is, you don't have to be able to answer the second question in order to answer (for a given theory) the first. This should provide some comfort to those who (perhaps influenced by positivist or instrumentalist philosophy) think we can't (and/or shouldn't try to) establish some theoretical picture of external reality as true. Such people may still accept Bell's characterization of when "a theory will be said to be locally causal."

But even those who are not skeptical on principle recognize that, because of the complexity in practice of settling questions about the truth status of scientific theories, some tentativeness is often in order. Bell recognizes this too:

"I use the term 'beable' rather than some more committed term like 'being' or 'beer' to recall the essentially tentative nature of any physical theory. Such a theory is at best a *candidate* for the description of nature. Terms like 'being', 'beer', 'existent', etc., would seem to me lacking in humility. In fact 'beable' is short for 'maybe-able'." <sup>25</sup>

The crucial point is that the "maybe" here pertains to the epistemological status of a given candidate theory. By contrast, the "beable status" <sup>26</sup> of certain elements of a theory relative to that theory should be completely straightforward and uncontroversial. If there is any question about what elements a theory posits as beables, it can only be because the proponents of the theory have not (yet) sufficiently clarified what the theory is about, what the theory is. Whether the theory is true or false is an orthogonal question.

All of that said, Bell does take certain elements largely for granted as beables – that is, as beables that any serious candidate theory would have to recognize as such: "The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments." <sup>26</sup> As noted before, even Bohr acknowledges the real existence (the beable status) of these sorts of things. And, as suggested by Bohr, since our primary cognitive access to the world is through "switches and knobs on experimental equipment" and other such directly perceivable facts - i.e., since such facts must always constitute the primary evidence on which we will have to rest any argument for the truth of a particular candidate physical theory - it is hard to imagine a serious theory which doesn't grant such facts beable status: a theory which didn't would evidently have to regard our perception as systematically delusional, and hence would have to regard any alleged empirical evidence – for anything, including itself – as invalid. In short, such a theory would evidently be selfrefuting.<sup>39</sup>

We stress this point for two related reasons. First,

anyone who is uncomfortable with the apparently "metaphysical" positing of ultimate "elements of reality" (even in a tentative way, through the tentative positing of a candidate physical theory) should be relieved to find that the concept "beable" is merely a placeholder for whatever entities we (tentatively) include in the class which already, by necessity, exists and includes certain basic, directly-perceivable features of the world around us. And second, these particular beables – e.g., the settings of knobs and the positions of pointers – have a particularly central role to play in the derivation (from Bell's concept of local causality) of the empirically testable Bell inequalities. This will be developed in Section VI.

### IV. COMPLETENESS

Having clarified the concept of "beables" which appears in Bell's formulation of local causality, let us now turn to the last phrase in that formulation, italicized here:

"A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3..."

In a word, the key assumption here is "that events in 3 be specified *completely*" (emphasis added).

Let us first see why this requirement is necessary. Consider again Figure 2, and suppose that  $\bar{B}_3$  denotes an *incomplete* specification of beables in region 3. It can then be seen that a violation of

$$P(b_1|\bar{B}_3, b_2) = P(b_1|\bar{B}_3) \tag{2}$$

does not entail the existence of any super-luminal causal influences. For suppose some event "X" in the overlapping backwards light cones of regions 1 and 2 causally influences both  $b_1$  and  $b_2$ . It might then be possible to infer, from  $b_2$ , something about X, from which one could in turn infer something about  $b_1$ . Suppose, though, that the incomplete description of events in region  $3 - \bar{B}_3$  omits precisely the "traces" of this past common cause X. Then  $b_2$  could usefully supplement  $\bar{B}_3$  – i.e., Eq. (2) could be violated – even in the presence of purely local causation.

Thus, as Bell explains, in order for Eq. (1) to function as a valid locality criterion,

"it is important that events in 3 be specified completely. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely." <sup>7</sup>

And here is the same point from an earlier paper:

"Now my intuitive notion of local causality is that events in 2 should not be 'causes' of events in 1, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones [in a local theory]. It is perfectly intelligible then that if  $[B_3]$  in [region 3] does not contain a complete record of events in that [region], it can be usefully supplemented by information from region 2. So in general it is expected that  $[P(b_1|b_2,\bar{B}_3) \neq P(b_1|\bar{B}_3)]$ However, in the particular case that  $[B_3]$  contains already a *complete* specification of beables in [region 3], supplementary information from region 2 could reasonably be expected to be redundant." <sup>26</sup>

It is important to stress that, like the concept of beables itself, the idea of a sufficient (full or complete) specification of beables is relative to a given candidate theory. What Bell's local causality condition requires is that - in order to assess the consistency between a given candidate theory and the relativistic causal structure sketched in Figure 1 – we must include, in  $B_3$ , everything that candidate theory says is present (or relevant) in region 3. It is not, by contrast, necessary that we achieve omniscience regarding what actually exists in some spacetime region.

The appearance of the word "completeness" tends to remind commentators of the EPR argument, and hence apparently also tends to suggest that Bell smuggled into his definition of local causality the unwarranted assumption that orthodox quantum theory is incomplete. (See, for example, Ref. 44.) As mentioned earlier, Bell did accept the validity of the EPR argument. But this means only that, according to Bell, local causality – plus some of QM's empirical predictions – entail the incompleteness of orthodox QM. His view on that point, however, is no part of his formulation of local causality. (See Section VI for further discussion of the relation between local causality and the EPR argument.)

What Bell's formulation does say is only this: whatever your theory posits as physically real (in region 3), make sure you include all of that when calculating the relevant probabilities to test whether your theory respects or violates Eq. (1), i.e., whether your theory is or isn't locally causal in the sense of Figure 1. No assumptions are made about the type of theory to which the locality criterion can be applied. In particular, the incompleteness of orthodox quantum theory (i.e., the existence of "hidden variables") is not assumed. The virtue of Bell's formulation lies precisely in this generality.

Although it is simplest to understand Bell's local causality condition as requiring a *complete* specification

of beables in some spacetime region, there is an important reason why Bell explicitly leaves open the possibility that a specification of "what happens in the backward light cone of 1" might be "sufficiently specified" by something less than a complete specification of the beables there. This has to do with the fact, to be discussed more in Section VI, that in order to derive an empirically-testable Bell-type inequality from the local causality condition, one needs a subsidiary assumption. sometimes called "experimental freedom" or "no conspiracies". This is in essence the assumption that, in the usual EPR-Bell kind of scenario in which a central source emits pairs of specially-prepared particles in opposite directions toward two spatially-separated measuring devices, it is possible for certain settings on the devices (determining which of several possible measurements are made on a given incoming particle) to be made "freely" or "randomly" – that is, independently of the state of the incident particle pair.

In the more recent versions of these experiments, the relevant settings are made using independent (quantum) random number generators. 18 According to orthodox QM, there is therefore nothing in the past light cone of an individual measurement event foretelling which of the possible measurements will be performed. But there of course exist alternative candidate theories (such as the de Broglie - Bohm pilot-wave theory, which is deterministic) according to which those same settings are influenced by events in their pasts. But then, the relevant pasts of the device settings necessarily overlap with the pre-measurement states of the particles being measured. A complete specification of beables in the relevant region containing those pre-measurement states will therefore inevitably include facts relevant to (if not determining) the device settings. And so, in deriving the Bell inequality from local causality, there is a kind of subtle tension between the requirement "that events in 3 be specified completely" and the requirement that device settings can be made independent of the states of the particlesto-be-measured.

To resolve the tension, one need merely allow that the beables in the relevant region can be divided up into disjoint classes: those which are influenced by the preparation procedure at the source (and which thus encode the "state of the particle pair") and those which are to be used in the setting of measurement apparatus parameters. And note that these two classes are likely to be far from jointly exhaustive: in any plausible candidate theory, there will have to exist many additional beables (corresponding, for example, to stray electromagnetic fields, low energy relic neutrinos, etc.) which are in neither of the mentioned classes. One thus expects a considerable "causal distance" between the two classes of beables (at least in a well-designed experiment). This makes the "freedom" or "no conspiracies" assumption – namely, the absense of correlations between the two classes of beables quite reasonable to accept.

This issue will be addressed in some more detail in Section VI. For now, we simply acknowledge its existence as a way of explaining why Bell's formulation of local causality mentions "complete" descriptions of events in region 3 as merely an example of the kind of description which is "sufficient". One might summarize the discussion here as follows: what is required for the validity of the local causality condition is a complete specification of beables in region 3 – but only those beables which are relevant in some appropriate sense to the event  $b_1$  in question in region 1.

### V. CAUSALITY

Recall the transition from Bell's preliminary, qualitative formulation of local causality to the final, "sharp and clean" version. And recall in particular Bell's statement that the preliminary version was *insufficiently* sharp and clean for mathematics. What is it, exactly, that Bell considered inadequate about the qualitative statement? It seems likely that it was the presence there of the terms "cause" and "effect" which are notoriously difficult to define mathematically. Indeed, about his final formulation Bell says: "Note, by the way, that our definition of locally causal theories, although motivated by talk of 'cause' and 'effect', does not in the end explicitly involve these rather vague notions." <sup>7</sup>

How exactly does Bell's "definition of locally causal theories" fail to "explicitly involve" the "rather vague notions" of cause and effect? On its face this sounds paradoxical. But the resolution is simple: what Bell's definition actually avoids is any specific commitment about what physically exists and how it acts. (Indeed, any such commitments would seriously restrict the generality of the locality criterion, and hence undermine the scope of Bell's theorem.) Instead, Bell's definition shifts the burden of providing some definite account of causal processes to theories and itself merely defines a spacetime constraint that must be met if the causal processes posited by a candidate theory are to be deemed locally causal in the sense of SR.

The important mediating role of candidate theories visà-vis causality will be further stressed and clarified in the following subsection. Subsequent subsections further clarify the concept of "causality" in Bell's "local causality" by contrasting it with several other ideas with which it has often been confused or conflated.

### A. Causality and candidate theories

As already discussed, according to Bell it is the job of physical theories to posit certain physically real structures (beables) and laws governing their interactions and evolution. Thus Bell's definition of locally causal theories is not a specification of locality for a particular type of theory, namely, those that are "causal" – with

the implication that there would exist also theories that are "non-causal." A theory, by the very nature of what we mean by that term in this context, is automatically causal. "Causal theory" is a redundancy. And so, as noted earlier, one must understand Bell's "definition of locally causal theories" as a criterion that theories – i.e., candidate descriptions of causal processes in nature – must satisfy in order to be in accord with special relativistic locality. In short, the causality in the "definition of locally causal theories" is simply whatever a given candidate theory says, about whatever it says it about.

As Bell explains, the practical reason for defining local causality in terms of the physical processes posited by some candidate theory (as opposed to the physical processes which actually exist in nature) has to do with our relatively direct access to the one as opposed to the other:

"I would insist here on the distinction between analyzing various physical theories, on the one hand, and philosophising about the unique real world on the other hand. In this matter of causality it is a great inconvenience that the real world is given to us once only. We cannot know what would have happened if something had been different. We cannot repeat an experiment changing just one variable; the hands of the clock will have moved, and the moons of Jupiter. Physical theories are more amenable in this respect. We can calculate the consequences of changing free elements in a theory, be they only initial conditions, and so can explore the causal structure of the theory. I insist that [the theory of local beables, i.e., the local causality concept] is primarily an analysis of certain kinds of physical theory." 28

Bell's view, contra several commentators<sup>45,46</sup>, is thus that no special philosophical account of causation is needed to warrant the conclusion that violation of the locality condition implies genuine non-local causation. For Bell, it is a rather trivial matter to decide, for some (unambiguously formulated) candidate physical theory, what is and is not a genuinely *causal* influence. We can simply "explore the causal structure of" the candidate theory. This of course raises the question of how we might go from recognizing the non-locality of some particular candidate theory, to the claim that nature is non-local. But this is just Bell's theorem: all candidate theories which respect the locality condition are inconsistent with experiment. (See Section VI.) So the "one true theory" (whatever that turns out to be!) - and hence nature itself – must violate relativistic local causality.

### B. Causality vs. determinism

The previous subsection stressed that the "causal" in "locally causal theories" simply refers to the physically real existents and processes (beables and associated laws) posited by some candidate theory, whatever exactly those might be. We in no way restrict the class of theories (whose locality can be assessed by Bell's criterion) by introducing "causality." In particular, the word "causal" in "locally causal theories" is not meant to imply or require that theories be deterministic as opposed to irreducibly stochastic.

"We would like to form some [notion] of *local* causality in theories which are not deterministic, in which the correlations prescribed by the theory, for the beables, are weaker." <sup>26</sup>

Bell thus uses the word "causal" quite deliberately as a wider abstraction which subsumes but does not necessarily entail determinism.

This is manifested most clearly in the fact that Bell's mathematical formulation of "local causality" – Eq. (1) – is stated in terms of probabilities. Indeed, in his 1976 paper "The Theory of Local Beables" <sup>26</sup> Bell discusses "Local Determinism" first, arguing that, in a "local deterministic" theory the actual values of beables in region 1 (of Figure 2) will be determined by a complete specification of beables in region 3 (with additional specification of beables from region 2 being redundant). In our mathematical notation, local determinism means

$$b_1(B_3, b_2) = b_1(B_3) \tag{3}$$

where, as before,  $b_1$  and  $b_2$  are the values of specific beables in regions 1 and 2, while  $B_3$  denotes a sufficient (e.g., complete) specification of beables in region 3.

In a (local) stochastic theory, however, even a complete specification of relevant beables in the past (e.g., those in region 3 of Figure 2) may not determine the realized value of the beable in question (in region 1). Rather, the theory specifies only probabilities for the various possible values that might be realized for that beable. Of course, determinism is not really an alternative to, but is rather merely a special case of, stochasticity:

"Consider for example Maxwell's equations, in the source-free case for simplicity. The fields **E** and **B** in region 1 are completely determined by the fields in region 3, regardless of those in 2. Thus this is a locally causal theory in the present sense. The deterministic case is a limit of the probabilistic case, the probabilities becoming delta functions." <sup>7</sup>

The natural generalization of the above mathematical formulation of "local determinism" is precisely Bell's local causality condition:

$$P(b_1|B_3, b_2) = P(b_1|B_3), (4)$$

i.e.,  $b_2$  is irrelevant – not for determining what happens in region 1 because that, in a stochastic theory, is simply not determined – but rather for determining the probability for possible happenings in region 1. Such probabilities are the "output" of stochastic theories in the same sense that the actual realized values of beables are the "output" of deterministic theories. Thus, Bell's local causality condition for stochastic theories – Eq. (4) – and the analogous condition – Eq. (3) – for deterministic theories, are imposing precisely the same locality requirement on the two kinds of theories: information about region 2 should be irrelevant in regard to what the theory says about region 1, once the beables in region 3 are sufficiently specified.

Of course, if one insists that any stochastic theory is ipso facto a stand-in for some (perhaps unknown) underlying deterministic theory (with the probabilities in the stochastic theory thus resulting not from indeterminism in nature, but from our ignorance), Bell's locality concept would cease to work. The requirement of a complete specification of beables in region 3 would then contradict the allowance that such a specification does not necessarily determine the events in region 1. But this is no objection to Bell's concept of local causality. Bell is not asking us to accept that any particular theory (stochastic or otherwise) is true; he's just asking us to accept his definition of what it would mean for a stochastic theory to respect relativity's prohibition on superluminal causation. And this requires us to accept, at least in principle, that there could be such a thing as a genuinely, irreducibly stochastic theory, and that the way "causality" appears in such a theory is that certain beables do, and others do not, influence the probabilities for specific events.

We have been stressing here that "causality" is wider than, and does not necessarily entail, determinism. Bell has deliberately and carefully formulated a local causality criterion that does not tacitly assume determinism, and which is thus stated explicitly in terms of probabilities – the fundamental, dynamical probabilities assigned by stochastic theories to particular happenings in spacetime. Note in particular that the probabilities in Eq. (1) are not subjective (in the sense of denoting the degree of someone's belief in a proposition about  $b_1$ ), they cannot be understood as reflecting partial ignorance about relevant beables in region 3, and they do not (primarily) represent empirical frequencies for the appearance of certain values of  $b_1$ . They are, rather, the fundamental "output" of some candidate (stochastic) physical theory.

#### C. Causality vs. correlation

Everyone knows that correlation doesn't imply causality. Two events (say, the values taken on by beables  $b_1$  and  $b_2$  in Bell's spacetime regions 1 and 2, respectively) may be correlated without there necessarily being any implication that  $b_1$  is the cause of  $b_2$  or vice versa:

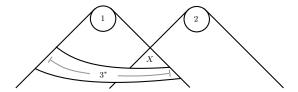


FIG. 3: Similar to Figure 2, except that region 3\* (unlike region 3 of Figure 2) fails to shield off region 1 from the overlapping backward light cones of regions 1 and 2. Thus, (following the language of Figure 2's caption) even full specification of what happens in 3\* does not make events in 2 irrelevant for predictions about 1 in a locally causal theory.

"Of course, mere *correlation* between distant events does not by itself imply action at a distance, but only correlation between the signals reaching the two places." <sup>14</sup>

And Bell describes the issue motivating his 1990 paper "La nouvelle cuisine" as "the problem of formulating ... sharply in contemporary physical theory" "these notions, of cause and effect on the one hand, and of correlation on the other".<sup>7</sup>

It is sometimes reported that Bell's local causality condition is really only a "no correlation" requirement, such that the empirical violation of the resulting inequalities establishes only "non-local correlations" (as opposed to non-local causation). (See, e.g., Ref. 33.) But this is a misconception. Bell uses the term "causality" (e.g., in talking about his "definition of locally causal theories") to highlight that a violation of this condition (by some theory) means that the theory posits non-local causal influences, as opposed to mere "non-local correlations."

It will be clarifying to illustrate this by relaxing a point that Bell has carefully built into his formulation of local causality, and showing that violation of the resulting, weakened condition may still entail *correlations* between space-like separated events, but no longer implies that there are faster-than-light causal influences. Actually, we have done this once already, in the previous section, when we explained why a violation of Eq. (2) would not (unlike a violation of Eq. (1)) entail any violation of the causal structure of Figure 1. We now consider a second modified version of Bell's criterion.

Consider again the spacetime diagram sketched in Figure 2. Bell notes that

"It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2."

Why is this so important? For example, why couldn't we replace region 3 of Figure 2 with a region like that labelled  $3^*$  in Figure 3? This region, just like 3 in Figure 2, closes off the back light cone of 1 and hence – it might seem – would be perfectly sufficient for defining the probabilites associated with  $b_1$  in a locally causal theory.

But a more careful analysis shows that a violation of

$$P(b_1|B_{3^*}, b_2) = P(b_1|B_{3^*}) \tag{5}$$

(the same as Eq. (1) but with region 3 of Figure 2 replaced by region 3\* of Figure 3) does not entail any nonlocal causation. Here, there is a perfectly local causal mechanism by which  $b_1$  and  $b_2$  can be correlated, in a way that isn't "screened off" by conditionalization on  $B_{3*}$ , thus violating Eq. (5) in a situation which involves no violation of relativistic local causation. The mechanism is this: in a stochastic theory, an event may occur at the space-time point labelled "X" in Figure 3 which was not determined by the complete specification of beables  $(B_{3*})$  in region 3\*. But despite not having been determined by beables in its past, that event really comes into existence and may in principle have effects throughout its future light cone - which includes both region 1 and region 2. Event X may, so to speak, broadcast sub-luminal influences which bring about correlations between  $b_1$  and  $b_2$ , such that information about  $b_2$  is not redundant in regard to defining what happens in region 1 (even after conditionalizing on  $B_{3*}$ ). Thus we may have a violation of Eq. (5) – i.e., the candidate theory in question could attribute different values to  $P(b_1|B_{3^*},b_2)$  and  $P(b_1|B_{3^*})$ - despite there being, according to the theory in question, no non-local causation at work. While Eq. (5) may perhaps be described as some kind of "no correlations" condition for regions 1 and 2, it definitely fails as a "no causality" condition.

If we return, however, to the original region 3 (of Figure 2) which does "completely [shield] off from 1 the overlap of the backward light cones of 1 and  $2^{n}$  it becomes clear that no such correlation-without-non-local-causality can occur. Here, if some X-like event (not determined by even a complete specification of beables in region 3) occurs somewhere in the future light cone of region 3, it will necessarily fail to lie in the overlapping past light cones of regions 1 and 2 (which would be necessary for it to in turn locally influence both of those events).

Bell has carefully set things up so that a violation of Eq. (1) entails that there is some non-local causation. It isn't necessarily that something in region 2 is causally influencing something in region 1, or vice versa. It is always possible that there is some other event, neither in region 1 nor region 2, which was not determined by  $B_3$ , and which itself causally influences both  $b_1$  and  $b_2$ . The point is, though, that *this* causal influence would have to be non-local (i.e., would have to violate the special relativistic causal structure sketched in Figure 1).<sup>47</sup>

To summarize the point that a violation of Eq. (1) entails non-local causation (rather than mere correlations between space-like separated events) it is helpful to recall Bell's example of the correlation between the ringing of a kitchen alarm and the readiness of a boiling egg. That the alarm rings just as the egg is finished cooking obviously does not entail or even suggest that the ringing caused the egg to harden. Correlation does not imply causality. As Bell completes the point,

"The ringing of the alarm establishes the readiness of the egg. But if it is already given that the egg was nearly boiled a second before, then the ringing of the alarm makes the readiness no more certain."

Reading  $b_2$  for "the ringing of the alarm,"  $b_1$  for "the readiness of the egg," and  $B_3$  for "the egg was nearly boiled a second before," we have a simple intuitive example of Eq. (1): although  $b_1$  and  $b_2$  may be *correlated* such that information about  $b_2$  can tell us something about  $b_1$ , that information will be redundant (in a locally causal theory) once  $B_3$  is specified.

## D. Causality vs. signaling

One final idea that is often confused with local causality is local (i.e., exclusively slower-than-light) signaling. <sup>40</sup> Signaling is, of course, a certain human activity in which one person transmits information, across some distance, to another person. Such transmission clearly requires a causal connection between the sending event and the receiving event, but it requires more as well: namely, the ability of the people involved to send and receive the information. That is, signaling requires some measure of control (over appropriate beables) on the part of the sender, and some measure of access (to appropriate beables) on the part of the recipient.

The requirement that theories prohibit the possibility of faster-than-light signaling – which incidentally is all that is imposed in relativistic quantum field theory by the requirement that field operators at spacelike separation commute<sup>26</sup> – is thus a much weaker condition than the prohibition of faster-than-light causal influences. Theories can exhibit violations of relativistic local causality and yet (because certain beables are inadequately controllable by and/or inadequately accessible to humans) preclude faster-than-light signals. Orthodox quantum mechanics (including ordinary relativistic quantum field theory) is one example of such a theory. Another example is the pilot-wave theory of de Broglie and Bohm, in which

"...the consequences of events at one place propagate to other places faster than light. This happens in a way that we cannot use for signaling. Nevertheless it is a gross violation of relativistic causality." <sup>16</sup>

One of the most prevalent mistakes made by commentators on Bell's theorem is to conflate local causality with local signaling. <sup>41</sup> Often this takes the form of a kind of double-standard in which alternatives to ordinary QM are dismissed as "non-local" (and therefore unacceptable) on the grounds that they include (either manifestly, as in the case of the pilot-wave theory, or just in principle, as established by Bell's theorem) "gross violations of relativistic causality" – but then ordinary QM itself is

argued by comparison to be perfectly "local" (where now only "local signaling" is meant or proved). Such reasoning, though, is obviously equivocal once one appreciates that "local causality" and "local signaling" simply mean different things.

Clearly differentiating these two notions does raise the question of which, after all, SR should be understood to prohibit. But the idea that the relativistic causal structure, sketched in Figure 1, should somehow apply exclusively to this narrowly human activity, seems highly dubious:

"...the 'no signaling...' notion rests on concepts which are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question:

Who do we think we are?

We who can make 'measurements', we who can manipulate 'external fields', we who can 'signal' at all, even if not faster than light? Do we include chemists, or only physicsts, plants, or only animals, pocket calculators, or only mainframe computers?"<sup>7</sup>

That is, the idea that SR is compatible with non-local causal influences (but only prohibits non-local signaling) seems afflicted by the same problem (reviewed in Section III) that necessarily afflicts theories whose formulations involve words like "observable", "microscopic", "environment", etc. In particular, the notion of "signaling" seems somehow too superficial, too anthropocentric, to adequately capture the causal structure of Figure 1.

### VI. IMPLICATIONS OF LOCAL CAUSALITY

Having reviewed Bell's careful *formulation* of relativistic local causality, let us now more briefly indicate some of its important *applications*.

### A. Factorization

In the typical EPR-Bell setup, we have separated observers (traditionally Alice and Bob) making spin-component measurements (using, say, Stern-Gerlach devices oriented spatially along the  $\hat{a}$  and  $\hat{b}$  directions, respectively) on each of a pair of spin-entangled particles. The outcomes of their individual measurements (manifested in the final location of the particle, or the position of some pointer, or some fact about some other beable) may be denoted by A and B respectively.

The beables pertaining to a given run of the experiment may then be cataloged as in Figure 4. Roughly, we may think of  $\hat{a}$  and  $\hat{b}$  (which live in regions 1 and 2, respectively) as referring to the spatial orientations of the

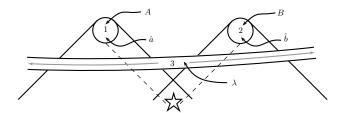


FIG. 4: Space-time diagram illustrating the various beables of relevance for the EPR-Bell setup. (Cf. Bell's diagram in Ref. 7.) Separated observers Alice (in region 1) and Bob (in region 2) make spin-component measurements (using apparatus settings  $\hat{a}$  and  $\hat{b}$  respectively) on a pair of spin- or polarization-entangled particles (represented by the dashed lines). The measurements have outcomes A and B respectively. The state of the particle pair in region 3 is denoted  $\lambda$ . Note that what we are here calling region 3 extends across the past light cones of both regions 1 and 2. It thus not only "completely shields off from 1 the overlap of the backward light cones of 1 and 2"7, but also vice versa. Bell's local causality condition therefore requires both that  $\hat{b}$  and B are irrelevant for predictions about the outcome A, and that  $\hat{a}$ and A are irrelevant for predictions about the outcome B, once  $\lambda$  is specified.

two pieces of measuring apparatus (this being the basis for the notation), and  $\lambda$  (in region 3) as referring to the state of the particle pair emitted by the source. (Note that the phrase "state of the particle pair" should not be taken too seriously; no actual assumption is made about the existence, for example, of literal particles.)

Unlike region 3 of Figure 2, region 3 of Figure 4 extends across the past light cone not only of region 1, but of region 2 as well. It particular, this extended region 3 closes off the past light cones of both regions 1 and 2 and shields both regions from their overlapping past light cones. A complete specification of beables in this region 3 will therefore, according to Bell's concept of local causality, "make events in 2 irrelevant for predictions about 1"7 and will also make events in 1 irrelevant for predictions about 2:

$$P(A|\hat{a}, \hat{b}, B, \lambda) = P(A|\hat{a}, \lambda) \tag{6}$$

and

$$P(B|\hat{a}, \hat{b}, \lambda) = P(B|\hat{b}, \lambda). \tag{7}$$

From these and the identity

$$P(A, B|\hat{a}, \hat{b}, \lambda) = P(A|\hat{a}, \hat{b}, B, \lambda) \cdot P(B|\hat{a}, \hat{b}, \lambda) \tag{8}$$

the so-called "factorization" of the joint probability for outcomes A and B immediately follows:

$$P(A, B|\hat{a}, \hat{b}, \lambda) = P(A|\hat{a}, \lambda) \cdot P(B|\hat{b}, \lambda). \tag{9}$$

This factorization condition is widely recognized in the Bell literature to be sufficient for the derivation of empirically-testable Bell-type inequalities. As Bell notes, however.

"Very often such factorizability is taken as the starting point of the analysis. Here we have preferred to see it not as the *formulation* of 'local causality', but as a consequence thereof." <sup>7</sup>

## B. The EPR Argument

In their famous 1935 paper, Einstein, Podolsky and Rosen argued that a local explanation for the perfect correlations predicted by quantum theory (in a certain kind of situation of which the above EPR-Bell setup is an example) required the existence of locally pre-determined values for the measurement outcomes. Since, as mentioned in the introduction, ordinary QM contains no such elements of reality, EPR concluded that ordinary QM (and in particular the wave function) did not provide a complete description of physical reality. They suggested that an alternative, locally causal theory which did provide a complete description of physical reality might be found.

Taking for granted that the relevant empirical predictions of quantum theory are correct, one can summarize the logic of EPR's argument this way:

$$locality \rightarrow incompleteness$$
 (10)

where "incompleteness" means specifically the incompleteness of the orthodox quantum mechanical description of the particles in question (in terms of their quantum state alone). This is of course logically equivalent to the statement that

completeness 
$$\rightarrow$$
 non-locality (11)

which explains why the EPR argument is sometimes characterized as an argument for the incompleteness of orthodox QM, and sometimes instead as pointing out the non-locality of that candidate theory.

In their 1935 paper, EPR appealed to an intuitive notion of local causality which was not precisely formulated. It is therefore of interest that their argument can be recapitulated and made rigorous by using Bell's formulation of local causality. It is clarifying to begin with the EPR argument in the form of Statement 11. The proof then consists in simply using the formulated notion of local causality in its directly-intended way – namely, to assess whether a particular candidate theory is or is not local.

Take again the situation indicated in Figure 4. Here it is important to appreciate that because of the structure of region 3 – and note that it could be extended into a space-like hypersurface crossing through the region 3 depicted there, and still satisfy the requirements discussed earlier – the relevant complete specification of beables does not presuppose that the "state of the particle pair" itself must "factorize" into two distinct and independent states for the two particles. The state can instead be characterized in a way that is essentially inseparable, as

in ordinary QM, and the argument still goes through: "It is notable that in this argument nothing is said about the locality, or even localizability, of the variable  $\lambda$ . These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time. It is assumed only that the outputs A and B, and the particular inputs a and b, are well localized." Let us suppose in particular that the "preparation procedure" at the particle source (the star in Figure 4) gives rise to a pair in (what ordinary QM describes as) the spin singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2) \tag{12}$$

where  $|\uparrow\rangle_1$  means that particle 1 is spin-up along (say) the z-direction, etc.

Suppose also that  $\hat{a} = \hat{b} = \hat{z}$ , i.e., both Alice and Bob (freely) choose to measure the spins of the incoming particles along the z-direction. Then QM predicts (letting A = +1 denote the result that Alice finds her particle to be spin up, etc.) that either A = +1 and B = -1 (with probability 50%) or that A = -1 and B = +1 (with probability 50%).

Thus, noting that for orthodox QM  $\lambda$  in Figure 4 is simply the quantum mechanical wave function, we have for example that

$$P(A = +1|\hat{a}, \lambda) = \frac{1}{2},$$
 (13)

but also that

$$P(A = +1|\hat{a}, \hat{b}, \lambda, B = -1) = 1, \tag{14}$$

in violation of Eq. (1). Orthodox QM is not a locally causal theory:

"The theory requires a perfect correlation of [results] on the two sides. So specification of the result on one side permits a 100% confident prediction of the previously totally uncertain result on the other side. Now in ordinary quantum mechanics there just is nothing but the wavefunction for calculating probabilities. There is then no question of making the result on one side redundant on the other by more fully specifying events in some space-time region 3. We have a violation of local causality."

As pointed out, Statements 10 and 11 are logically equivalent, so it is clear that a locally causal explanation for the perfect correlations predicted by QM will require a theory with more (or perhaps just different) beables than just the quantum mechanical wave function. But it is possible to show directly from Bell's concept of local causality that one must in particular posit beables which pre-determine the outcomes of both measurements.

To begin with, let us drop the assumption (which applied to ordinary QM) that it is possible to fully control

the state  $\lambda$  produced by the preparation procedure at the source. Instead, we allow that  $\lambda$  may perhaps take several distinct values from one run of the experiment to another. Let us also continue to assume that Alice and Bob both freely choose to make measurements along the  $\hat{z}$  direction.

The argument is then simple: we have already shown that local causality entails the factorization of the joint probability for outcomes A, B, once  $\lambda$  is specified. Considering for example the case A=+1, B=+1, whose joint probability vanishes, we therefore have that, for each specific value of  $\lambda$  that might (with nonzero probability) be produced by the preparation procedure, one of  $P(A=+1|\hat{a},\lambda)$  and  $P(B=+1|\hat{b},\lambda)$  must vanish. But since there are only two possible outcomes for each measurement, each of these possibilities entails that the opposite outcome is pre-determined. For example,

$$P(A = +1|\hat{a}, \lambda) = 0 \rightarrow P(A = -1|\hat{a}, \lambda) = 1$$
 (15)

which means that those particular values of  $\lambda$  to which this applies must "contain" or "encode" the outcome A=-1 which will then be revealed with certainty if a measurement along  $\hat{a}$  is performed. It is easy to see that the possible values of  $\lambda$  must therefore fall into two mutually exclusive and jointly exhaustive categories – those which encode the pre-determined outcomes A=+1 and B=-1, and those which encode the pre-determined outcomes A=-1 and B=+1.

Indeed, since the measurement axis is assumed to be "free" the same argument will establish that  $\lambda$  must encode pre-determined outcomes for all possible measurement directions. One thus sees how theories of deterministic hidden variables (or what N. David Mermin has dubbed "instruction sets"  $^{48}$ ) are in fact required by local causality.

# C. CHSH Inequality

It is well-known that a Bell-type inequality follows from the assumption of local deterministic hidden variables or "instruction sets". That theories of this type are actually required by locality (as explained in the previous sub-section) should therefore already clarify the seriousness with which Bell took the idea of a fundamental conflict between SR and the predictions of QM. This conflict can, however, be brought out in an even more streamlined way, by deriving a Bell-type inequality directly from the factorization of the joint probability as in Eq. (9) – and hence from Bell's local causality (without any additional discussion of determinism or pre-determined values).

Assume that the measurement scenario indicated in Figure 4 is repeated many times, with each setting being selected "freely" or "randomly" on each run, from two possibilities:  $\hat{a} \in \{\hat{a}_1, \hat{a}_2\}, \ \hat{b} \in \{\hat{b}_1, \hat{b}_2\}$ . The procedure which prepares or creates the "particles to be measured" is held fixed for all runs of the experiment. As before, this

will not necessarily imply that  $\lambda$  is constant for all runs, since the relevant beables may be less than fully controllable; we will assume, though, that the distribution of different values of  $\lambda$  across the runs can be characterized by a probability distribution  $\rho(\lambda)$ .

Next we define the correlation of outcomes A and B as the expected value of their product:

$$C(\hat{a}, \hat{b}) = \int \sum_{A,B} A B P(A|\hat{a}, \lambda) P(B|\hat{b}, \lambda) \rho(\lambda) d\lambda$$
$$= \int \bar{A}(\hat{a}, \lambda) \bar{B}(\hat{a}, \lambda) \rho(\lambda) d\lambda$$
(16)

where

$$\bar{A}(\hat{a},\lambda) = P(A = +1|\hat{a},\lambda) - P(A = -1|\hat{a},\lambda) \tag{17}$$

satisfies  $|\bar{A}| \leq 1$ , and similarly for  $\bar{B}$ .

Now we consider several combinations of correlations involving different pairs of settings. To begin with,

$$C(\hat{a}_1, \hat{b}_1) \pm C(\hat{a}_1, \hat{b}_2)$$

$$= \int \bar{A}(\hat{a}_1, \lambda) \left( \bar{B}(\hat{b}_1, \lambda) \pm \bar{B}(\hat{b}_2, \lambda) \right) \rho(\lambda) d\lambda$$
(18)

so that

$$\left| C(\hat{a}_1, \hat{b}_1) \pm C(\hat{a}_1, \hat{b}_2) \right| 
\leq \int \left| \bar{B}(\hat{b}_1, \lambda) \pm \bar{B}(\hat{b}_2, \lambda) \right| \rho(\lambda) d\lambda. \tag{19}$$

Similarly, we have that

$$\begin{vmatrix}
C(\hat{a}_2, \hat{b}_1) \mp C(\hat{a}_2, \hat{b}_2) \\
\leq \int \left| \bar{B}(\hat{b}_1, \lambda) \mp \bar{B}(\hat{b}_2, \lambda) \right| \rho(\lambda) d\lambda.$$
(20)

Adding Equations (19) and (20), and noting that  $|x \pm y| + |x \mp y|$  is one of 2x, -2x, 2y, or -2y, we have that

$$\left| C(\hat{a}_1, \hat{b}_1) \pm C(\hat{a}_1, \hat{b}_2) \right| + \left| C(\hat{a}_2, \hat{b}_1) \mp C(\hat{a}_2, \hat{b}_2) \right|$$

$$< 2$$
(21)

which is the so-called Clauser-Horne-Shimony-Holt (CHSH) inequality. <sup>44</sup> This is in essence the relation tested in the experiments, e.g., Refs. 17 and 18. Quantum theory predicts that (for appropriate preparations of the two-particle state and for appropriate choices of  $\hat{a}_1$ ,  $\hat{a}_2$ ,  $\hat{b}_1$ , and  $\hat{b}_2$ ) the left hand side should be  $2\sqrt{2}$  – more than 40% larger than the constraint implied by local causality. And the experimental results are in excellent agreement with the quantum predictions.

Since the inequality is derived from the local causality condition, what follows from the experimental results is that *any* theory which makes empirically correct predictions will have to violate the local causality condition. As Bell writes, "The obvious definition of 'local causality' does not work in quantum mechanics, and this cannot be attributed to the 'incompleteness' of that theory." <sup>7</sup>

### D. The "Free Variables" Assumption

Let us finally return to the assumption that the settings  $\hat{a}$  and  $\hat{b}$  are "free" or "random". Mathematically speaking, this was the assumption that the probability distribution  $\rho(\lambda)$  for the distribution of possible "states of the particle pair" created by the source is independent of the apparatus settings  $\hat{a}$  and  $\hat{b}$ . For example, in deriving Eq. (18) one assumes that the same probability distribution  $\rho(\lambda)$  characterizes runs in which  $\hat{a}_1$  and  $\hat{b}_1$  are measured, as characterizes runs in which  $\hat{a}_1$  and  $\hat{b}_2$  are measured. As Bell writes in support of this assumption,

"we may imagine the experiment done on such a scale, with the two sides of the experiment separated by a distance of order light minutes, that we can imagine these settings being freely chosen at the last second by two different experimental physicists.... If these last second choices are truly free ..., they are not influenced by the variables  $\lambda$ . Then the resultant values for  $[\hat{a}]$  and  $[\hat{b}]$  do not give any information about  $\lambda$ . So the probability distribution over  $\lambda$  does not depend on  $[\hat{a}]$  or  $[\hat{b}]$ ..."

The real (as opposed to imagined) experiments, however, do not involve "settings being freely chosen at the last second by two different experimental physicists" but instead involve physical random (or pseudo-random) number generators. As mentioned earlier, though, this means that – at least in principle, for some possible candidate theories – a complete description of beables in region 3 of Figure 4 will include not only a complete description of the "state of the particle pair" but also a complete description of whatever physical degrees of freedom are determining or influencing the eventual settings  $\hat{a}$  and  $\hat{b}$  – making it not only possible but rather likely that the candidate theory in question should exhibit (contrary to the assumption that was made) correlations between what we have called  $\lambda$  and those settings.

As suggested earlier, though, one can here appeal to the expectation that serious candidate theories will posit an enormously large number of physical degrees of freedom in a spacetime region like 3, only some tiny fraction of which are actually needed to completely specify the "state of the particle pair" - i.e., the beables which are physically influenced by the preparation procedure at the source. There are then innumerable other beables in region 3 which might be used to determine/influence the apparatus settings. The expectation – more precisely, in the context of the derivation of the locality inequality, the assumption – is that this is done in some way such that there are no correlations, no conspiratorial preestablished harmonies, between the beables chosen to determine the apparatus settings and those which encode the "state of the particle pair".

As Bell acknowledges, therefore, one logical possibility (in the face of the empirical violations of the CHSH inequality) is that

"it is not permissible to regard the experimental settings  $[\hat{a}]$  and  $[\hat{b}]$  in the analyzers as independent of the supplementary variables  $\lambda$ , in that  $[\hat{a}]$  and  $[\hat{b}]$  could be changed without changing the probability distribution  $\rho(\lambda)$ . Now even if we have arranged that  $[\hat{a}]$  and  $[\hat{b}]$  are generated by apparently random radioactive devices, housed in separated boxes and thickly shielded, or by Swiss national lottery machines, or by elaborate computer programmes, or by apparently free willed experimental physicists, or by some combination of all of these, we cannot be *sure* that  $[\hat{a}]$  and  $[\hat{b}]$ are not significantly influenced by the same factors  $\lambda$  that influence A and B. But this way of arranging quantum mechanical correlations would be even more mind boggling than one in which causal chains go faster than light. Apparently separate parts of the world would be deeply and conspiratorially entangled, and our apparent free will would be entangled with them." 14

And here is another relevant statement from Bell:

"An essential element in the reasoning here is that  $[\hat{a}]$  and  $[\hat{b}]$  are free variables. One can envisage then theories in which there just are no free variables for the polarizer angles to be coupled to. In such 'superdeterministic' theories the apparent free will of experimenters, and any other apparent randomness, would be illusory. Perhaps such a theory could be both locally causal and in agreement with quantum mechanical predictions. However I do not expect to see a serious theory of this kind. I would expect a serious theory to permit 'deterministic chaos' or 'pseudorandomness', for complicated subsystems (e.g. computers) which would provide variables sufficiently free for the purpose at hand. But I do not have a theorem about that."7

It is sometimes erroneously thought that the "freedom" or "no conspiracies" assumption being discussed here does – or should – follow from local causality. For example, in their illuminating "Exchange [with Bell] on Local Beables" A. Shimony, M.A. Horne, and J.F. Clauser criticized Bell's derivation for using (in our notation) the assumption  $\rho(\lambda|\hat{a},\hat{b}) = \rho(\lambda)$  which, they correctly pointed out, does not follow from local causality. Bell subsequently clarified that it was a separate assumption, not supposed to follow from local causality. As Shimony, Horne, and Clauser noted, however, the additional assumption seems eminently reasonable:

"we feel that it is wrong on methodological grounds to worry seriously about [the possibility of the kind of conspiracy that would render the assumption inapplicable] if no specific causal linkage [between the beables  $\lambda$ and those which determine the apparatus settings] is proposed. In any scientific experiment in which two or more variables are supposed to be randomly selected, one can always conjecture that some factor in the overlap of the backward light cones has controlled the presumably random choices. But, we maintain, skepticism of this sort will essentially dismiss all results of scientific experimentation. Unless we proceed under the assumption that hidden conspiracies of this sort do not occur, we have abandoned in advance the whole enterprise of discovering the laws of nature by experimentation." <sup>43</sup>

Imagine, for example, an experimental drug trial in which patients are randomly selected to receive either the drug or a placebo. It is always logically possible that the supposedly random selections (made, say, by flipping a coin) are in fact correlated with some pre-existing facts about the health of patients. And such a correlation could skew the results of the trial – say, resulting in a statistically significant improvement in the health of the patients given the genuine drug even though in fact the drug is impotent or worse. The suggestion is that, in general, unless there is some plausible causal mechanism that might conceivably produce the correlations in question (for example, instead of flipping coins, the drug/placebo assignments are made on the basis of patients' blood pressures) it is reasonable to assume that the conspiratorial correlations are absent.

That is, the additional assumption (beyond local causality) which is needed to derive the CHSH inequality "is no stronger than one needs for experimental reasoning generically".<sup>43</sup> The "no conspiracies" assumption thus falls into the same category as some other things (for example, the validity of logic, and certain mathematical operations) which, while used in the derivation, are not on the table as seriously challengeable. This explains why we sometimes do not even bother to mention this assumption — as, for example, when writing that the CHSH inequality follows from Bell's concept of local causality alone.

## VII. SUMMARY AND OPEN QUESTIONS

We have reviewed J.S. Bell's formulation of relativistic local causality, including a careful survey of its conceptual background and a sketch of its most important implications. We have stressed in particular that Bell's formulation does not presuppose determinism or the existence of hidden variables (or any of the other sorts of

things that are sometimes blamed for the empirical violation of Bell-type inequalities) but instead seems plausibly to just capture the intuitive idea, widely taken as an implication of special relativity, indicated in Figure 1 – namely, the idea that causal influences cannot propagate faster than light. And as we have seen, taking now the "no conspiracies" assumption for granted, the empirically-violated CHSH inequality can be cleanly derived from Bell's concept of local causality without the need for further assumptions involving determinism, hidden variables, "realism" or "classicality" (whatever exactly those ideas mean), etc.

This hopefully clarifies why Bell disagreed with the widespread opinion that his theorem – and the associated experiments – somehow vindicate ordinary quantum theory as against hidden variable alternative theories or somehow vindicate Bohr's philosophy as against Einstein's. Instead, the reader can now appreciate why, for Bell, "the real problem with quantum theory" is the "apparently essential conflict between any sharp formulation and relativity [, i.e., the] apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory..." <sup>16</sup>

It should be noted, however, that the presentation here has been of J.S. Bell's formulation of relativistic local causality. Although we have argued strongly for its reasonableness, this particular formulation should not necessarily be regarded as definitive.

Here we briefly indicate several points on which its applicability to various sorts of "exotic" theories could be questioned and where a more general or distinct formulation of local causality might be sought.

For example, one might worry that a theory with a non-Markovian character (that is, a theory in which causal influences can jump discontinuously from one time to a later time) could violate Bell's local causality condition despite positing no strictly faster-than-light influences. The idea would be that, in such a theory, influences could "hop over" region 3 of Figure 2, leading to correlations in regions 1 and 2 but leaving no trace in 3. This could seemingly be addressed by requiring that Bell's region 3 extend farther back in time, i.e., cover a region of spacetime so "thick" that hopping non-Markovian influences cannot make it across. In the limit of arbitrarily large violations of Markovianity, this would evidently require region 3 to encompass the entire past light cone of the region 3 pictured in Figure 2. But this maneuvre raises further issues: the more of spacetime that gets included in region 3, the more one might start to doubt the reasonableness of the "no conspiracies" assumption, and the more one might worry that the condition would fail to detect certain kinds of non-localities (and so would function merely as a necessary condition for, rather than a formulation of, locality).

Similar problems arise when one contemplates the possibility of theories which posit not only local beables (i.e., those "being associated with definite positions in space" <sup>25</sup>) but also *non-local beables*. The de Broglie -

Bohm pilot-wave theory is probably the clearest example here: its posited ontology includes both particles (which follow definite trajectories in 3-space and are pre-eminent examples of local beables) and a guiding wave (which is simply the usual quantum mechanical wave function but interpreted unambiguously as a beable). But, for an N-particle system (the universe, say) the wave function is a function on the 3N-dimensional configuration space. It is thus, for this theory, a non-local beable.<sup>49</sup>

As mentioned previously, Bell's region 3 can be extended into a space-like hypersurface without spoiling any of the arguments that have been given in this paper. We may then generously include, where Bell's formulation instructs us to use a complete specification of the *local* beables in region 3, also values for any nonlocal beables which, like wave functions, can be associated with hypersurfaces. And it is important that, even when including information about non-local beables in this way, such theories still violate the condition, i.e., are diagnosable as non-local. Still, as formulated, Bell's notion of local causality seems to presuppose that we are dealing with theories positing exclusively local beables.<sup>50</sup> It can be stretched to accommodate certain extant theories which posit also non-local beables, but how to do this with complete generality, and what other issues (like those encountered above in the case of non-Markovian theories) may arise in the attempt, remains unclear. On the other hand, it is also unclear how seriously one can or should take theories with non-local beables in the first place. In particular: should such theories even be considered candidates for "locally causal" status? And perhaps more importantly, could a theory positing non-local beables possibly count as genuinely consistent with special relativity?

Such questions will certainly not be answered here. We raise them simply to give the reader some sense of the concerns that one might have about Bell's formulation of local causality. Their admittedly exotic character should however help explain why Bell felt driven to contemplate "unspeakable" deviations from conventional wisdom. In particular, one can now appreciate how simple everything would become if we simply dropped the insistence on reconciling the Bell experiments with "fundamental relativity" and instead returned to the (empirically equivalent) pre-Einstein view according to which there exists a (hidden) preferred frame of reference. As explained by Bell in the quotes given in the Introduction, such a view can easily accomodate faster-than-light causal influences in a way that Einsteinian relativity, seemingly, can't.

Again, though, our goal here is not to lobby for this view, but merely to explain Bell's rationale for taking it seriously, as a possibility warranting careful attention – not just by philosophers and commentators, but by physicists interested in addressing the puzzles of yesterday, today, and tomorrow.

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- <sup>11</sup> The terminology of "hidden variables" is unfortunate because, at least in the one clear extant example of a "hidden variables theory," the "hidden variables" are precisely the variables which are not, in fact, hidden. About this theory (the de Broglie - Bohm "pilot wave" theory, a.k.a. "Bohmian Mechanics," which adds to the standard quantum mechanical wave function definite particle positions obeying a deterministic evolution law) Bell remarked: "it would be appropriate to refer to the xs as 'exposed variables' and to  $\psi$  as a 'hidden variable'. It is ironic that the traditional terminology is the reverse of this." <sup>12</sup> And similarly: "Although [in Bohmian Mechanics]  $\Psi$  is a real field it does not show up immediately in the result of a single 'measurement,' but only in the statistics of many such results. It is the de Broglie - Bohm variable X that shows up immediately each time. That X rather than  $\Psi$  is historically called a 'hidden' variable is a piece of historical silliness." 13 It is also relevant that the wave function  $\psi$  is "hidden" - in the sense of being not accessible via experiment even in orthodox quantum theory (which is suppsed to be the primary example of a non-hidden-variable theory). For some illuminating further discussion, see: Roderich Tumulka, "Understanding Bohmian Mechanics: A Dialogue"

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- For Bell, there was no important distinction between the terms "locality" and "local causality". For example, Bell first uses the phrase "local causality" in print in his 1976 paper "The theory of local beables". The paper's section 2 (entitled "Local causality") presents an early version of his formulation of this idea. But then later in the same paper he refers to the inequality (which he has shown how to derive from "local causality") as "the locality inequality" and even remarks that the detailed discussion of "local causality" in section 2 had "been an attempt to be rather explicit and general about the notion of locality, along lines only hinted at in previous publications."<sup>26</sup>. This usage is consistent with his later publications: see, for example, his 1987 preface to the first edition of his collected papers (see pp. xi-xii of Ref. 8) and his 1990 paper "La nouvelle cuisine."7
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- <sup>48</sup> N. D. Mermin, "Bringing home the atomic world: Quantum mysteries for anybody" Am. J. Phys., 49, 940-3 (1981)
- About the de Broglie Bohm theory, Bell writes: "No one can understand this theory until he is willing to think of ψ as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in 3N-space." <sup>12</sup> Some proponents of the de Broglie Bohm pilot wave theory, however, prefer to interpret the wave function in that theory not as a beable at all, but rather as a law. See, for example, S. Goldstein and N. Zanghi, "Reality and the Role of the Wavefunction in Quantum Theory," arXiv:1101.4575
- <sup>50</sup> T. Norsen, "The theory of (exclusively) local beables", Foundations of Physics 40 1858 (2010)