

PLANET FORMATION, ORBITAL EVOLUTION AND PLANET-STAR TIDAL INTERACTION

D.N.C. LIN

UCO/Lick Observatory, University of California, Santa Cruz

J.C.B. PAPALOIZOU

Astronomy Unit, Queen Mary & Westfield College, London

G. BRYDEN

UCO/Lick Observatory, University of California, Santa Cruz

S. IDA

Dept. of Earth and Planetary Sciences, Tokyo Institute of Technology

and

C. TERQUEM

UCO/Lick Observatory, University of California, Santa Cruz

We consider several processes operating during the late stages of planet formation that can affect observed orbital elements. Disk-planet interactions, tidal interactions with the central star, long term orbital instability and the Kozai mechanism are discussed.

I. The formation of protoplanets

Observations. It is generally accepted that the planets in our solar system were formed in a flattened gaseous nebula centered around the Sun. In typical star-forming molecular clouds, dense cores are observed to have specific angular momentum larger than $6 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$ (Goodman et al. 1993) such that their collapse leads to rotationally supported disks analogous to the primordial solar nebula (Terebey et al. 1984). Between 25 to 75% of young stellar objects (YSOs) in the Orion nebula appear to have disks (Prosser et al. 1994; McCaughrean and Stauffer 1994) with typical mass $M_d \sim 10^{-2 \pm 1} M_\odot$, temperature $\sim 10^{2 \pm 1} \text{ K}$ and size $\sim 40 \pm 20 \text{ AU}$ (Beckwith and Sargent 1996; see chapters by McCaughrean et al, Calvet et al. and Wilner et al.). The common existence of protostellar disks around YSOs, with properties

similar to those expected for the solar nebula, suggests that the conditions for planetary formation may be generally satisfied.

Recent observational breakthroughs have led to the discovery of Jupiter mass (M_J) planets around at least a few percent of nearby solar-type stars (see chapter by Cochran et al.). With the present data, we can assert that planetary formation is robust.

In *conventional planetary formation models* (Pollack et al. 1996), the first stage of protoplanetary formation is the rapid build up of solid cores through the coagulation of planetesimals (Safronov 1969; Wetherill and Stewart 1989; Lissauer and Stewart 1993; Aarseth et al. 1993; Kokubu and Ida 1996). When the core mass increases above a critical value (\sim a few M_\oplus), quasi-static evolution is no longer possible and a rapid accretion phase begins (Mizuno 1980; Bodenheimer and Pollack 1986) leading to the formation of gaseous giant planets (Pollack et al. 1996; also see chapter by Wuchterl et al.).

The details of this model are not yet fully worked out, but if we suppose the protoplanet can accrete gas as efficiently as possible, it will first take in gas in the neighborhood of its orbit assumed circular at radius r_p , until it fills its Roche radius $R_R = (q/3)^{1/3} r_p$ where $q = M_p/M_*$ is the protoplanet central star mass ratio, while orbiting in an empty annulus. The mass of the protoplanet is then given by $M_p = 3[4M_d(r)/3M_*]^{3/2} M_*$, where $M_d(r) = \pi \Sigma r^2$ is the characteristic disk mass within radius r , with Σ being the surface density. At 5.2 AU and $\Sigma = 200 \text{ g cm}^{-2}$, this gives a mass $M_p \sim 0.4 M_J$.

Further mass growth now depends on whether the disk has a kinematic viscosity ν capable of producing a mass accretion rate \dot{M}_p onto the protoplanet. If ν is finite, then all of the mass flow through the outer disk should go to the protoplanet. To model the disk viscosity, we adopt the prescription of Shakura and Sunyaev (1973) in which $\nu = \alpha H^2 \Omega$, where α is a dimensionless constant, H is the disk semi-thickness and Ω is the disk angular velocity. The most likely mechanism for providing an effective viscosity in stellar accretion disks is MHD turbulence (Balbus and Hawley 1991), which produces $\alpha \gtrsim 10^{-2}$ in a fully ionized disk. Note however that α may vary throughout the disk and be much smaller in its intermediate parts (see chapter by Stone et al.).

Observationally inferred values of the disk accretion rate $\dot{M}_d \sim 10^{-8 \pm 1} M_\odot \text{ yr}^{-1}$ (Hartmann et al. 1998) are model dependent and highly uncertain. Nonetheless, for such a fiducial \dot{M}_d , a protoplanet may attain a mass $M_p > 10 M_J$ within 10^6 yr . The mass of extra solar planets is $\sim 1 M_J$ (see chapter by Cochran et al.). Unless these planets are preferentially formed in low-mass disks, their growth needs to be terminated/inhibited such that they are unable to accept all the mass that flows through the disk. In this paper we focus on disk/protoplanet tidal interactions as a mechanism for accomplishing this.

A protoplanet exerts tidal perturbations which, for $M_p \sim 1 M_J$, may be adequate to induce the formation of a gap in the disk near its orbit and thereby start to limit accretion flow onto it (Lin and Papaloizou 1993). In general, the gravitational potential due to the protoplanet may be Fourier decomposed in the form:

$$\psi = \sum_l \sum_{m=0}^{\infty} \psi_{l,m} \cos[m(\phi - \omega_{l,m}t)], \quad (1)$$

where ϕ is the azimuthal angle, l and the azimuthal mode number m are integers and $\omega_{l,m} = \omega + (l - m)\kappa_p/m$ is the pattern speed, with ω and κ_p being the angular and epicyclic frequency of the protoplanet, respectively. If the orbit of the planet has a small eccentricity e , then $\psi_{l,m} \propto e^{|l-m|}$ (Goldreich and Tremaine 1980; Shu 1984). For circular orbits, only $l = m$ need be considered.

Both outgoing and ingoing density waves are excited in the disk at the Lindblad resonances located at $r = r_L$ and where, for $l = m$, $\Omega = \omega \pm \kappa/m$ (Goldreich and Tremaine 1978). Here κ is the epicyclic frequency of the gas and Ω is the disk angular velocity. In a Keplerian disk, $\kappa = \Omega$. The ingoing/outgoing waves carry a negative/positive angular momentum flux measured in their direction of propagation as they move away from the protoplanet into the disk interior/exterior. The waves thus carry a positive, outward propagating, conserved angular momentum flux or wave action F_H . In most cases it is reasonable to assume the waves are dissipated at some location in the disk where their angular momentum density is deposited (see Lin and Papaloizou 1993 and references therein). In the limit of a cold two dimensional disk, Goldreich and Tremaine (1978) found for a particular m that:

$$F_H = \left[\frac{m\pi^2 \Sigma r^2}{3\Omega^2(m-1)} \left(\frac{d\psi_{m,m}}{dr} + \frac{2\Omega\psi_{m,m}}{r(\Omega - \omega_{m,m})} \right)^2 \right]_{r=r_L}. \quad (2)$$

The back reaction torque exerted on the disk interior (exterior) to the planet is $-F_H$ (F_H). Thus the inner disk loses angular momentum while the outer disk gains it. Hence the tendency to form a gap. Evaluation of the total torque acting on each side of the disk requires the summation of contributions from all the resonances which for circular orbits amounts to summing over m . As $m \rightarrow \infty$, the location of the resonance approaches the orbit.

However, for a non self-gravitating disk and large m , the waves are sonic in character and thus can only exist further than a distance $\Delta \sim 2H/3$ from the orbit beyond which the relative disk flow is supersonic. This results in a *torque cut off* (Goldreich and Tremaine 1980; Artymowicz 1993a; Korycansky and Pollack 1993; Ward 1997) for $m > r_p/H$. The dynamics in the coorbital region $\Delta \lesssim 2H/3$ is not wave-like and is considered below. Summing over over all resonances,

the total angular momentum flux carried by the waves is essentially the same as that given by Papaloizou and Lin (1984), namely:

$$\dot{H}_T = 0.23q^2\Sigma r_p^4\omega^2(r_p/H)^3, \quad (3)$$

which was obtained by direct calculation of the torques for a model disk in which the protoplanet orbited in a gap of width $\sim H$.

Using equation (3) to estimate tidal torques, and then considering the competition between viscous torques, which tend to fill a gap, and tidal torques, which tend to empty it, Lin and Papaloizou (1979a, 1980, 1986a, 1993) found a viscous condition for gap opening: $q > 40\nu/(\Omega r_p^2)$. They proposed that gap formation would lead to the limitation of accretion but were only able to consider empty gaps. With the introduction of powerful numerical finite difference techniques and computers, recently it has become possible to study gap formation more fully numerically than previously (Artymowicz et al. 1998; Bryden et al. 1998; Kley 1998; and see below). The first results of this work indicate that there is a transitional regime in viscosity for which a gap exists, but some accretion still occurs, but then being essentially switched off for small enough viscosity.

In addition, Lin and Papaloizou (1993) pointed out that in a disk with very small viscosity, in order to form a gap it is necessary that $H < R_R = (q/3)^{1/3}r_p$. This thermal condition can be viewed in several ways. It means that the protoplanet's gravity is more important than pressure at a distance R_R from the protoplanet (generating large enough hydrostatic pressure forces would require gradients that would cause a violation of Rayleigh's criterion). From Korycansky and Papaloizou (1996) it is apparent that the thermal condition is required in order that the flow in the neighborhood of the protoplanet be nonlinear enough that shocks be produced which can provide the dissipation which is associated with gap opening (see below). This condition was found for a simple 2D-model disks with a barotropic equation of state.

It may be possible with more complicated physics and the introduction of three dimensional effects (Lin et al. 1990a, 1990b) to alter wave dissipation patterns and thus angular momentum deposition. However, because of the difficulty of clearing material near the protoplanet (it is difficult to see how linear waves could be dissipated closer than several vertical scale heights), it is likely that gap formation will occur much less readily if the thermal condition is not satisfied.

The flow around a partially embedded protoplanet has been simulated in the low- M_p limit in a shearing-sheet approximation (Korycansky and Papaloizou 1996; Miyoshi et al. 1998). These simulations provide useful clues on the flow pattern near the protoplanet. A global illustrative model is shown in Figure 1, in which we set $q = 10^{-4}$, $H/r = 0.07$, and $\alpha = 10^{-3}$. The flow pattern can be divided into

three regions. (1) A protoplanetary disk is formed within R_R of the protoplanet with $H \sim |r - r_p|$. In such a geometrically thick disk, the tidal perturbations of the host star induce a two-arm spiral shock wave with an open pitch angle. Angular momentum is efficiently transferred from the disk to the star's orbit around the planet (equivalent to the planet's orbit around the star) such that gas is accreted onto the core of the protoplanet within few orbital periods $P_o = 2\pi/\omega$. (2) An extended arc is formed in the coorbital region near r_p with gas streaming in horseshoe orbits around the L_4 and L_5 points. Because these are local potential maxima, viscous dissipation leads to the depletion of this region (Lin et al. 1987). (3) In those regions of the disk which have $|r - r_p| > R_R$, the protoplanet's tidal perturbation results in the convergence of stream lines to form pronounced trailing wakes, both inside and outside r_p . These high density ridges have been identified by some (Artymowicz and Lubow 1996) as gas streams along which material flows from the disk to the protoplanet. However, the velocity field viewed in a frame rotating with the companion (Figure 1) clearly indicates that gas in the post-shock region along the ridge line is moving away from the protoplanet.

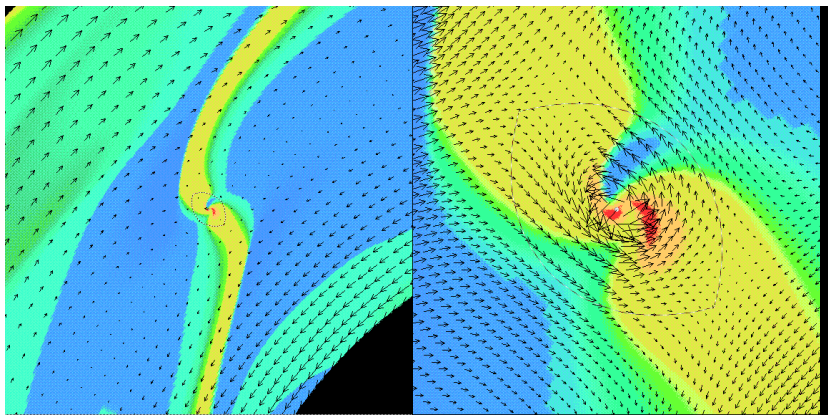


Figure 1. The flow pattern around an embedded protoplanet. On the left is the global trailing wave (the star is located toward the lower right). On the right is a close up view of the circulatory region immediately surrounding the protoplanet. Grey scale indicates surface density and the protoplanet's Roche lobe is shown with a dotted line. Model parameters are $q = 10^{-4}$, $H/r = 0.07$, and $\alpha = 10^{-3}$.

Simulations of gap formation need to consider model evolution over many orbital periods and have to deal with a large density contrast between the gap region and other parts of the disk. Special care is

needed to minimize the tendency of numerical viscosity to produce spurious diffusion into the gap region and thus significantly affect the results. In Figure 2, we illustrate the excitation and propagation of waves and the existence of a clear gap for a planet (with $q = 10^{-3}$) interacting with a disk (with $H/r = 0.04$ and $\alpha = 10^{-3}$).

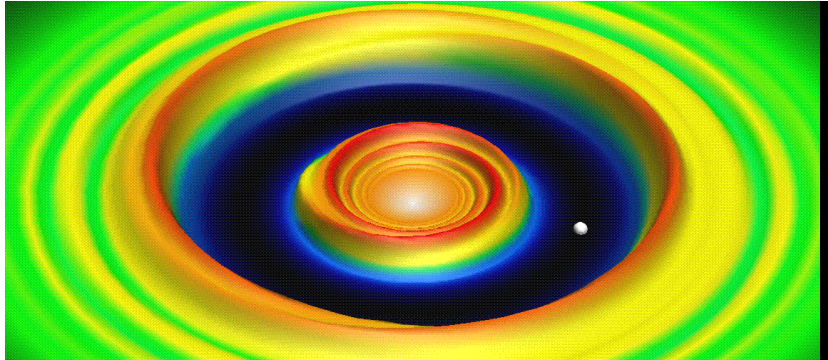


Figure 2. Surface density distribution for a gap opening protoplanet. The surface density near the planet (indicated by a white sphere) has been reduced by four orders of magnitude. Waves are clearly seen propagating both inward and outward away from the protoplanet. Model parameters are $q = 10^{-3}$, $H/r = 0.04$, and $\alpha = 10^{-3}$.

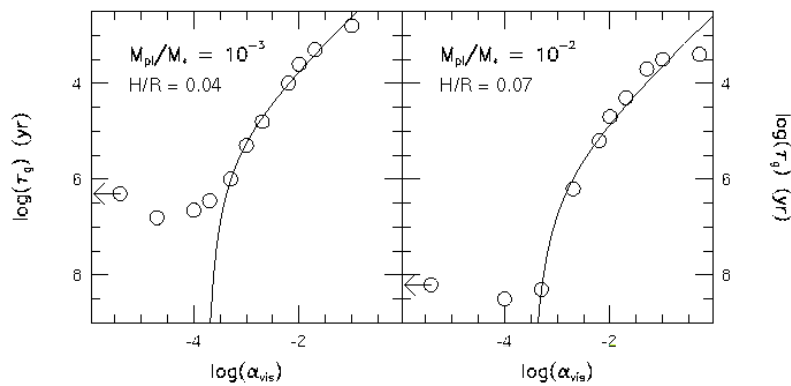


Figure 3. The mass doubling timescale τ_g as a function of viscosity parameter α for one Jupiter mass and $H/r = 0.04$ (left) and 10 Jupiter masses and $H/r = 0.07$ (right). As the gas viscosity is decreased, the accretion rate onto the protoplanet drops off sharply ultimately being determined by numerical effects for $\alpha < 10^{-4}$.

In Figure 3, we plot the growth timescale M_p/\dot{M}_p as a function of α and H/r for protoplanets of mass 1 and 10 M_J . We comment that it is not necessary for accretion to stop entirely for gap formation to affect the final mass of a protoplanet. All one needs is that the mass doubling time should become longer than either the disk lifetime or the time for the interior disk to accrete. In the latter case, the planet would approach the central star before it could increase its mass significantly (Ivanov et al. 1998). From Figure 3 we see that, for $\alpha \sim 10^{-3}$ and $H/r = 0.04$, the mass doubling time for 1 M_J is 10^6 yr. For 10 M_J , the same results apply for $H/r = 0.07$. The results in Figure 3 thus suggest that the tidal truncation process operates during planetary formation and is important in determining the final mass of the planet M_p .

II. Protoplanetary orbital evolution

Resonant interaction between the disk and the planet can lead to changes in the orbital eccentricity e (Lin and Papaloizou 1993; Artymowicz 1993b). The way in which Lindblad torques cause a small eccentricity to grow has been reviewed by Lin and Papaloizou (1993). If the Lindblad torques only are considered, the disk matter is able to tidally pump eccentricity in a similar way to a rapidly rotating star (see below). However, corotation resonances in the disk (at which the pattern speed of a Fourier component of the tidal perturbation corotates with the disk) also need to be considered (see above). When these resonances exist, a torque is applied to the disk at the corotation radius r_C where $\Omega = \omega_{l,m}$ at a rate given by:

$$T_{l,m}^C = -\frac{m\pi^2}{2} \left[\Sigma \left(\frac{d\Omega}{dr} \right)^{-1} \frac{d\varsigma^{-1}}{dr} (\psi_{l,m})^2 \right]_{r=r_C}, \quad (4)$$

where $\varsigma = \kappa^2/(2\Omega\Sigma)$ is the vortensity. The role of corotation resonances depends strongly on the ς -distribution. When Σ vanishes at some disk edge or when there is a gap near a low mass perturber, Goldreich and Tremaine (1980) showed that the action of torques from corotation resonances results in damping e more effectively than effects from Lindblad resonances could excite it. Note that this *does not* apply to the situation where a protoplanet is embedded in a disk with no edges or gap and where tidal interaction is linear.

Assuming $d\ln\varsigma/d\ln r \sim 1$ in the unperturbed disk, Artymowicz (1993b) suggested that the eccentricity of low-mass embedded protoplanets would be damped by tidal interaction. However, in the minimum mass solar nebula model where $\Sigma \propto r^{-3/2}$, vortensity is independent of r and then all corotation torques vanish (Ward 1993). Thus, whether low-mass embedded protoplanets suffer eccentricity damping (Artymowicz 1992) is model dependent. For a planet with a sufficiently

large mass to open a wide gap, corotation resonances would be excluded and e would increase on a timescale $e^2/(de^2/dt) \sim q^{-1}(M_*/M_d)P_o$ (Lin and Papaloizou 1993) which for massive planets can be shorter than the inferred disk evolution timescale.

If the growth of protoplanets is limited by the formation of a gap of width Δ_g , tides may cause e to increase until $e \sim \Delta_g/r_p$ when a potential expansion valid for small e assumed in the above analysis breaks down. In a cold disk, the protoplanet's angular velocity at peri/ap-helion would then be greater/less than $\Omega(r_p \mp \Delta_g)$, resulting in a torque reversal. Such a process could provide a limit to the growth of e . But a protoplanet's increased radial excursion may also cause the gap to widen, in which case e might increase to yet larger values. Analytic treatments and self consistent numerical simulations in the large e limit have not been carried out so far.

The characterization of extrasolar planets' orbits from forthcoming observations will provide some useful constraints on the origin of eccentricity. The coexistence of several planets with similar e but very different M_p (or similar M_p but very different e) is not a natural outcome of the excitation of eccentricity by disk tidal interaction, as this would tend to produce a (e, M_p) relation (Artymowicz 1992).

Planetary orbital migration is induced by the difference between the inner and outer disk torques, ΔT , which react back on the planet, assumed to remain in a circular orbit, such that:

$$\frac{dr_p}{dt} = \frac{2\Delta T}{r_p \Omega M_p}. \quad (5)$$

Based on recent work indicating that ΔT is almost always negative, Ward (1997) suggested that embedded planets migrate inwards on a timescale $\sim 10^5(M_p/M_\oplus)^{-1}$ yr. If this process occurs in protostellar disks, protoplanets would migrate towards the stellar surface at $r = R_*$ once $M_p > 1 M_\oplus$ because their growth timescale (Lissauer and Stewart 1993) would then become longer than this migration timescale. Resolution of this rapid migration dilemma may require the complete and nonlinear analysis of the disk response to the protoplanet in the corotation regions which may be quite complex (see Figures 1 and 2). However, supposing that migration occurs and can be stopped near the star (see below), a scenario leading to the formation of short-period planets has been proposed (see chapter by Ward and Hahn).

After gap formation, there may still be some residual accretion depending on the value of α (see above). Torques are still exerted between disk and protoplanet and the imbalance due to differing properties of the disk on either side of the protoplanet causes orbital migration (Goldreich and Tremaine 1980; Lin and Papaloizou 1986b; Takeuchi et al. 1996). The estimated effects of advection of angular momentum through the accretion flow on this process are usually found to

be small. For small M_p , the protoplanet behaves like a disk particle and so migrates towards the star. For larger masses, the evolution is slower. Nonetheless, the protoplanet is always expected to reach the star before it has time to double its mass (Ivanov et al. 1998).

The above discussion naturally leads to the suggestion that short-period planets were formed at several AU away from their host stars, and subsequently migrated to their present location (Lin et al. 1996). However, *in situ* formation cannot be completely ruled out (Bodenheimer 1997).

A protoplanet's migration may be terminated near R_ if (1) the disk does not extend down to R_* or (2) the host star induces angular momentum transfer to the protoplanet's orbit via tidal effects (see section III).* For the first possibility, interaction between the disk and an intense (> 1 kG) stellar magnetic field has been suggested (Konigl 1991) as the cause for the modest observed rotation period ($P_* \sim 8$ days) of classical T Tauri stars (Bouvier et al. 1993; Choi and Herbst 1996). Such a magnetic field strength may be consistent with recent measurements (Guenther 1997). In this scenario, the stellar field is assumed to induce a cavity out to the magnetospheric radius ($r_m \sim$ a few 10^{11} cm) where P_* is equal to the local Keplerian period of the disk. When a protoplanet migrates interior to the 2:1 resonance of the gas at r_m , the protoplanet is decoupled from the disk and its migration is temporarily stalled. The main uncertainties here are in the observed distribution of P_* (Mathieu et al. 1998).

III. Planet–star tidal interaction

As the planet approaches R_* , the tides raised on the star or planet by its companion become strong enough so that their dissipation leads to orbital evolution. If the distribution of mass of the perturbed object has the same symmetry as the perturbing potential, there is no tidal torque resulting from the interaction. However, in general, dissipative processes (e.g. radiative damping or turbulent viscosity) acting on the tides produce a lag between the response of the star or planet and the perturbing potential, enabling mechanical energy to be lost and angular momentum to be exchanged between the rotation of the perturbed object and the orbital motion. Only if the system is circular, synchronous and coplanar (i.e. the spin of the binary components and that of the orbit are parallel) does the tidal torque vanish (Hut 1980). However, such an equilibrium state is not necessarily stable (Counselman 1973; Hut 1980, 1981).

The response of a star/planet to a tidal perturbation is the sum of two terms, an equilibrium tide and a dynamical tide. The equilibrium tide (Darwin 1879) is the shape that the perturbed object would have if it could adjust instantly to the tidal potential. It is obtained by

balancing the pressure and gravity forces. The dynamical tide contains the oscillatory response of the star/planet. It takes into account the fact that gravity or g modes can be excited in the convectively stable layers of the star/planet and that resonances between the tidal disturbance and the normal modes of the star/planet can occur (Cowling 1941).

In massive close binaries, which have a convective core and a radiative envelope, the dynamical tide cannot be neglected because tidal friction is caused predominantly by the radiative damping of the tidally-excited modes (Zahn 1975, 1977; Savonije and Papaloizou 1983, 1984, 1997; Papaloizou and Savonije 1985, 1997; Goldreich and Nicholson 1989; Savonije et al. 1995; Kumar et al. 1995). In that case, g modes, which are excited mainly near the convective core boundary, propagate out through the envelope to the atmosphere, carrying energy and angular momentum. They are damped close to the surface, where the radiative diffusion time becomes comparable to the forcing period, thus enabling a net tidal torque to be exerted on the star or planet.

In the case of solar-type binaries, which have a radiative core and a convective envelope, it was thought until recently that tidal friction could be well described by the theory of the equilibrium tide, in which only turbulent dissipation of the equilibrium tide in the convective envelope is taken into account (Zahn 1977). However, recent studies have indicated that this is not the case. Even if radiative damping can be ignored, the torque derived using the equilibrium tide only is 4–6 times larger than that taking into account the dynamical tide for binary periods of several days (Terquem et al. 1998a). This is due to the fact that the theory of the equilibrium tide can in principle be applied only when the characteristic timescales of the perturbed object are small compared to the forcing period. In a solar-type star however, the convective timescale in the interior region of the convective envelope is as large as a month. Furthermore, because of the uncertainty over the magnitude of the turbulent viscosity associated with convection, it is not clear that the torque due to turbulent dissipation acting on the full tide in the convective envelope is more important than that due to radiative damping acting on the g modes which propagate inwards in the radiative core (Terquem et al. 1998a; Goodman and Dickson 1998). Here, we focus on the case of a planet orbiting around a solar-type star.

Tides raised on the star by the planet can be analyzed in the limit that the rotational frequency of the star is usually small compared to the orbital frequency. Then, as a result of tidal friction, the star spins-up, the orbit decays (the planet spirals in) and its eccentricity, if any, decreases. To calculate the timescales on which this evolution occurs, we need to quantify the dissipative mechanisms which act on the tides. Recent studies (Claret and Cunha 1997; Goodman and Oh 1997; Terquem et al. 1998a) have found that the turbulent viscosity in the convective envelope that is required to provide the observed circu-

larization rates of main-sequence solar-type binaries (Mathieu 1994) is at least 50 times larger than that simply estimated from mixing-length theory for non-rotating stars. This indicates either (1) that the observations are questionable, (2) that solar-type binaries are not circularized through turbulent viscosity acting on tidal perturbations (see Tassoul 1988 and Kumar and Goodman 1996 for other suggested tidal mechanisms), or (3) that dissipation in the convective envelope of solar-like stars is significantly more efficient than is currently estimated (see Terquem et al. 1998a for a more detailed discussion). Here we assume that circularization of solar-type binaries does occur through the action of turbulent viscosity on the tides, and we then calibrate its magnitude so as to account for the observed timescales. Under these circumstances, when the response of the star is not in resonance with one of its global normal modes, the tides are dissipated more efficiently by turbulent viscosity than by radiative damping. In a resonance, radiative damping dominates and limit the response of the star at its surface. Although the planetary companion may go through a succession of resonances as it spirals in under the action of the tides, for a fixed spectrum of stellar normal modes its migration is controlled essentially by the non-resonant interaction. For a non-rotating star, the orbital decay timescale, spin-up timescale of the star and circularization timescale, in Gyr, are (Terquem et al. 1998a):

$$t_{\text{orb}}^* (\text{Gyr}) = 2.763 \times 10^{-4} \frac{(M_p/M_* + 1)^{5/3}}{M_p/M_*} \left(\frac{P_o}{1 \text{ day}} \right)^{13/3}, \quad (6)$$

$$t_{\text{sp}}^* (\text{Gyr}) = 1.725 \times 10^{-6} \left(\frac{M_p + M_*}{M_p} \right)^2 \left(\frac{P_o}{1 \text{ day}} \right)^3, \quad (7)$$

$$t_{\text{circ}}^* (\text{Gyr}) = 4.605 \times 10^{-5} \frac{(M_p/M_* + 1)^{2/3}}{M_p/M_*} \left(\frac{P_o}{1 \text{ day}} \right)^{13/3}, \quad (8)$$

where P_o is the orbital period. This circularization timescale is valid only if the initial eccentricity is not too large. If only the convective envelope of the star, where tidal dissipation occurs, is spun-up during tidal evolution, then the spin-up timescale has to be multiplied by I_c/I , where I_c and I are the moments of inertia of the convective envelope and the entire star, respectively. For the Sun, $I_c/I \simeq 0.14$.

Tides are also raised on the planet by the star. Contrary to the giant planets of our Solar System, Jupiter-like planets on a close orbit are expected to have an isothermal, and thus radiative (convectively stable) envelope (Guillot et al. 1996; Saumon et al. 1996). For these planets, both turbulent dissipation in the convective core and radiative damping in the envelope act on the tides. So far it is not clear which mechanism is more important.

We express the timescales associated with turbulent dissipation of the tides in terms of the parameter Q , the inverse of which is the effective tidal dissipation function (MacDonald 1964). Damping of the tides in the convective core of the planet leads to the synchronization of the planet rotation with the orbital rotation on the following characteristic timescale, in Gyr (Goldreich and Soter 1966):

$$t_{\text{sp}}^{\text{p},1} (\text{Gyr}) \sim 4.4 \times 10^{-13} Q \left| \frac{1 \text{ day}}{P_p} - \frac{1 \text{ day}}{P_o} \right| \frac{M_p}{M_*} \left(\frac{P_o}{1 \text{ day}} \right)^2 \left(\frac{a}{R_p} \right)^3, \quad (9)$$

where R_p is the planet radius, a is the semi-major axis of the orbit and P_p is the initial value of the planet rotational period (before it undergoes tidal interaction with the star). Since the rotational angular momentum of the planet is in general small compared to its orbital angular momentum, synchronization of the planet occurs before any significant orbital evolution can take place. Once synchronization is achieved, damping of the tides raised in the planet always leads to the decay of the orbital eccentricity on a characteristic timescale which is, in Gyr (Goldreich and Soter 1966):

$$t_{\text{circ}}^{\text{p},1} (\text{Gyr}) \sim 2.8 \times 10^{-14} Q \frac{M_p}{M_*} \frac{P_o}{1 \text{ day}} \left(\frac{a}{R_p} \right)^5. \quad (10)$$

The tides raised on the planet do not lead to the decay of the orbit once the planet is synchronized. We note that Q may depend on the tidal frequency as seen by the planet, and therefore on the rotational frequency of the planet. Since $t_{\text{circ}}^{\text{p},1}$ is calculated assuming synchronization, the value of Q in the above equation may be different from that used for calculating $t_{\text{sp}}^{\text{p},1}$. Orbital evolution of Jupiter's satellites leads to an estimate of $Q \sim 10^5 - 10^6$ for this planet (Goldreich and Soter 1966; Lin and Papaloizou 1979b). However, it is not yet understood where this value of Q comes from, since turbulent viscosity arising from convection would produce $Q \simeq 5 \times 10^{13}$ (Goldreich and Nicholson 1977; see Stevenson 1983 for an alternative). There is of course no reason to assume that the Q -value of Jupiter is similar to that of the extra-solar planets. First, some of these planets orbit very close to their parent star, and therefore are much hotter than Jupiter. Also, Q may depend on the magnitude and the frequency of the tidal oscillation, in which case it would be different for Jupiter if this planet were synchronously rotating on a closer orbit. Therefore we can only speculate when applying the above formulae to extra-solar planets.

Radiative damping of the tides in the envelope, as described above for massive binaries, gives rise to the synchronization and circularization timescales $t_{\text{sp}}^{\text{p},2}$ and $t_{\text{circ}}^{\text{p},2}$, respectively. These timescales have been evaluated (in particular for 51 peg) by Lubow et al. (1997). However,

as they pointed out, the asymptotic analysis they use is valid only if the initial spin rate of the planet is smaller than half that of Jupiter. Besides, this analysis neglects the effect of rotation on the tides, which is important for near-synchronous planets.

In the context of extrasolar planets, we first comment on the magnitude of the perturbed velocity induced by the tides at the stellar surface. Terquem et al. (1998a, 1998b) have found that in the case of 51 Peg, this velocity is too small to be observed. This result is insensitive to the magnitude of the stellar turbulent viscosity and is not affected by the possibility of resonance. It also holds for the other extra-solar planets which have been detected so far.

The discussion of the previous subsection indicates that the rotation of planets on a close orbit is almost certainly synchronous with the orbital rotation. For the star to be synchronized in less than 5 Gyr, i.e. $t_{\text{sp}}^* < 5$ Gyr, a 1 (3) Jupiter mass planet would have to be on an orbit with a period less than 1.4 (3) day(s). We note that in the case of τ Boo, the rotation of the star may then have been synchronized as a result of tidal effects. If only the convective envelope of the star is spun-up, these periods have to be multiplied by about a factor 2. Therefore, the observation of anomalous rapid rotators could give further evidence of the presence of close planets and provide some indication on how the dissipation of tides affect the rotation of the star (Marcy et al. 1997).

The circularization timescale for the orbit is t_{circ} such that $1/t_{\text{circ}} = 1/t_{\text{circ}}^* + 1/t_{\text{circ}}^{\text{p},1} + 1/t_{\text{circ}}^{\text{p},2}$. We note that, according to the expression of the timescales we have given above, $t_{\text{orb}}^* \simeq 6t_{\text{circ}}^*$ and $t_{\text{orb}}^{\text{p},1} > 10t_{\text{circ}}^{\text{p},1}$, so that circularization can be achieved without a significant orbital decay taking place (in contrast to the statement by Rasio et al. 1996). So far it is not clear which of the timescales t_{circ}^* , $t_{\text{circ}}^{\text{p},1}$ and $t_{\text{circ}}^{\text{p},2}$ is the shortest. For 51 Peg, $t_{\text{circ}}^{\text{p},1} < t_{\text{circ}}^*$ requires $Q < 2 \times 10^7$ (see also Rasio et al. 1996 and Marcy et al. 1997 for estimates of $t_{\text{circ}}^{\text{p},1}$). We point out that for a Jupiter mass planet, $t_{\text{circ}}^* < 5$ Gyr for $P_o < 3$ days. Therefore, *all the Jupiter mass planets orbiting with a period less than about 3 days should be on a circular orbit*. This cutoff period is a lower limit, since t_{circ}^* is an upper limit of the circularization timescale.

Planets found on eccentric orbits at smaller periods are indicative that the stellar rotation frequency (assumed zero up to now) is large compared to the orbital frequency. Indeed, in that case tidal friction in the star pumps up the orbital eccentricity, opposing the effect of tidal friction in the planet. However, the above calculations indicate that the planet may have to be on a very close orbit for the tidal friction in the star to be able to increase significantly the orbital eccentricity. Whether the planet can get to such a close orbit is questionable if the disk in which the planet formed did not extend down to R_* .

The timescale on which the orbit decays is t_{orb}^* . The condition

$t_{\text{orb}}^* < 5$ Gyr requires $P_o < 2$ days for a Jupiter mass planet. If a planet has not been able to get to such a close orbit because, e.g., the inner parts of the disk were truncated, orbital decay may not occur as a result of tidal friction, and it may not plunge onto their parent star.

However, *several planets may attain short periods*. After the first-born protoplanet emerges and migrates to a few R_* , because the formation timescale increases with r , additional, subsequently formed protoplanets may migrate inwards pushing protoplanetary cores ahead of their inward path until they become trapped at Lindblad resonances. Such resonant trapping is found for the Galilean satellites (e.g. Goldreich and Peale 1966; Lin and Papaloizou 1979b) and it occurs if the migration rate is sufficiently slow. It raises the intriguing possibility that planets temporarily parked close to the star because of absence of disk material could be forced to plunge into the host star.

On the other hand, if the star is a rapid rotator (with period shorter than that of the orbit) and has a weak magnetic field, tidal effects transfer angular momentum from the star to the orbit with an increasing efficiency as the orbit is pushed out until a tidal barrier is produced such that the migration time and inverse orbital decay time balance. Tentatively using estimates derived from equation (7), an inward migration rate of 10^6 yr could only be balanced by tidal effects acting on a $1 M_J$ planet if the period was ~ 0.3 days, which increases to ~ 0.7 days for a $3 M_J$ planet and a migration time of 10^7 yr. If planets can survive at such short periods (more likely for massive planets and slow migration rates) and if eccentricity growth is suppressed by tides within the planet, tidal effects might temporarily cause the halting of migration by transferring angular momentum from the star to the inner orbit and from there to any residual disk material via Lindblad torques and so on to outer planets (cf. the situation for Saturn's rings). Such processes have not yet been worked out in detail. However, a solar-type star is likely to have a small amount of angular momentum in comparison to the disk/planet system in total, and once the star slows down sufficiently, the inward migration must continue.

As a planet eventually becomes engulfed by the host star, it is disrupted by tidal breakup, heating, and ram pressure stripping (Sandquist et al. 1998). Supporting evidence for such phenomena may be found in the super-solar metal abundance of 51 Peg (G2V), 55 ρ Cnc (G8V), and τ Boo (F7V) (Butler et al. 1997; Gonzalez 1997, 1998). The convective envelope for each of these stars contains a few $10^{-2} M_\odot$. The mixing of 10–40 M_\oplus of planetary heavy element material within the stellar envelope (Zharkov and Gudkova 1991) would lead to a significant metal enrichment there. Since the depth of the convection zone of main sequence stars decreases significantly with increasing stellar mass, the routine engulfing of planets by their hosts might lead to a tendency for hotter planetary hosts to show a general overall metallicity

enhancement with respect to cooler ones.

IV. Long term stability of planetary systems

In a relatively massive disk, several giant planets may be formed with $M_p \sim 1\text{--}3 M_J$ and $a > 1$ AU. Their long term orbital stability determines the dynamical evolution of the system. After the depletion of the disk gas, mutual gravitational perturbation between the planets may gradually increase their eccentricities until their orbits cross each other on a timescale τ_x . Extrapolation of existing numerical results (e.g. Franklin et al. 1990; Chambers et al. 1996) give $\tau_x \sim 10^{12\text{--}18}$ yr for the solar system. But τ_x would reduce to $\sim 10^{8\text{--}12}$ or $10^{5\text{--}8}$ yr if all the gas giants had $M_p = 1$ or $2 M_J$, respectively.

Since systems of massive planets formed with similar values of a and small e eventually suffer orbit crossing (Lin and Ida 1997), massive eccentric planets may have acquired their orbital properties as a consequence of orbit crossing (Rasio and Ford 1996; Weidenschilling and Marzari 1996; Lin and Ida 1997). Once orbit crossing occurs, close encounters eventually take place. When these involve equal mass planets, they produce a velocity perturbation V with magnitude limited by the surface escape velocity $V_{\text{esc}} \simeq 60 (M_p/M_J)^{1/3} (\rho_p/1 \text{ g cm}^{-3})^{1/6} \text{ km s}^{-1}$, where ρ_p is the planet's internal density (Safronov 1969). This results in $e \lesssim V_{\text{esc}}/(a\Omega) \sim 2 (M_p/M_J)^{1/3} (a/1 \text{ AU})^{1/2}$. Thus observed eccentricities can be accounted for.

Supposing that giant planets form at ~ 1 AU, massive planets ($M_p \gtrsim 5 M_J$) with moderately high $e \sim 0.3$ and moderately small $a \sim 0.5$ AU can be accounted for by orbit crossing and merging. Eccentric planets with $a > 1$ AU and $M_p \sim M_J$ can be produced by orbit crossing followed by ejection. Merger, resulting in $e \lesssim 0.5$, is favored at small r whereas ejection occurs preferentially at large r because the cross-section for direct collisions is independent of r while that for close encounters is $\propto V^{-4} \propto (r\Omega)^{-4} \propto r^2$ (Lin and Ida 1997). Ida and Lin (1998) find that the orbital properties of a merged body are consistent with those of the planets in eccentric orbits around 70 Vir and HD 114762.

Numerical simulations of many planet systems indicate that although some may be ejected, a residual population of eccentric planets ($e \gtrsim 0.5$) may remain bound to the central star at large distances ($a > 100$ AU). Close encounters also excite the relative inclination up to $\sim e/2$ radians. Detection of additional companions around stars with an eccentric planet would provide tests of these scenarios.

Rasio and Ford (1996) suggested that the short period planets were dynamically scattered into a region close to the central star. Although the short-period planets may be circularized (see above), numerical sim-

ulations indicate that the scattering scenario would lead to a population of planets with high eccentricity at large distances (10–100 AU) from their central stars. A comprehensive search for these planets would provide a useful test of this scenario as against that of disk–planet interaction which produces orbits with smaller e .

V. Effects of secular perturbations due to a distant companion

Some extrasolar planets are found in binary systems. For example, a planet is found around 16 Cyg B which orbits at a distance of 1.7 AU from the central star which is known to have a binary companion 16 Cyg A. It has been suggested that the high e ($=0.67$) of 16 Cyg B is excited by the gyroscopic perturbation due to 16 Cyg A (Holman et al. 1997). A similar effect may be caused by the gravitational perturbations from other planetary bodies. Here we discuss the effect of secular perturbations on a long timescale such that a time average may be performed. The orbiting bodies may then be considered as having their mass distributed continuously around their orbits as in the classical theories of Laplace and Lagrange (see Hagihara 1972 and references therein). Semi-major axes do not change under secular perturbation. However, changes to the eccentricity and inclination may be produced.

If orbits are coplanar, in general only modest changes to eccentricities are produced if the perturbing bodies are widely separated. An exception to this might occur if secular resonances sweep through the system because of a changing gravitational potential due to disk dispersal or changing stellar oblateness (eg. Ward 1981). However, if orbits are allowed to have high inclination, large eccentricity changes can occur to the orbit of an inner planet perturbed by a distant body or binary companion.

To consider this *gyroscopic perturbation effect*, let us consider the simplest case of the motion of an inner planet with mass M_p perturbed by an outer companion with mass M' assumed to be in a circular orbit with semi-major axis a' . The outer companion is assumed to contain sufficient angular momentum that its orbit remains fixed defining a reference plane to which the inner planet orbit has inclination i . We note ω_a the longitude of the apsidal line measured in the orbital plane from the line of nodes.

If the distance of M_p from M_* is r , its speed is v , the angle between its position vector and that of M' is Θ , and only the dominant quadrupole term in the interaction potential multipole expansion is retained, its motion is governed by the Hamiltonian (Kozai 1962; Hagihara 1972; Innanen et. al. 1997):

$$H = \frac{1}{2}M_*v^2 - \frac{GM_*M_p}{r} - \frac{GM'M_p r^2}{2a'^3} (3 \cos^2 \Theta - 1). \quad (11)$$

After performing a time average, appropriate for secular perturbations:

$$H = -\frac{GM_*M_p}{2a} - \frac{GM'M_p a^2}{16a'^3} \left[(3 \cos^2 i - 1) (2 + 3e^2) + 15e^2 \sin^2 i \cos 2\omega_a \right]. \quad (12)$$

The fact that both H and the component of angular momentum parallel to the outer orbital axis, $L \cos i = \sqrt{GM_* a (1 - e^2)} \cos i$, are constant enables elimination of i and ω_a , so that a complete solution can be found from the equation for the rate of change of e , which can be derived from the canonical equation $dL/dt = -\partial H / \partial \omega_a$, in the form:

$$\frac{de}{dt} = \frac{15nM'a^3 e \sin^2 i \sin 2\omega_a}{8M_* a'^3}, \quad (13)$$

with $2\pi/n$ being the orbital period of the planet M_p .

The above equation indicates that e can oscillate between extremes occurring when $\sin 2\omega_a = 0$. There has been particular interest in finding conditions under which e can start from very small values and then increase unstably to values $\lesssim 1$. To examine conditions for this to occur, we use $H = \text{const}$ and $L \cos i = \text{const}$ to find ω_a and i in terms of e . Assuming initial values $i = i_0$ and $e = e_0$, when $\omega_a = \omega_0$, for infinitesimally small but non zero values of e_0 that can be neglected, we find:

$$\cos 2\omega_a = \frac{1 - 5 \cos^2 i_0 - e^2}{5(1 - e^2 - \cos^2 i_0)}. \quad (14)$$

From the above, we see that when $e \rightarrow 0$:

$$\cos 2\omega_a = \frac{(1 - 5 \cos^2 i_0)}{5(1 - \cos^2 i_0)}. \quad (15)$$

Clearly, for solutions of the type we seek, we require $\cos 2\omega_a > -1$, which requires that the initial inclination exceed a critical value (see Innanen et. al. 1997) such that:

$$\cos^2 i_0 < 3/5. \quad (16)$$

If this inequality is satisfied (if it is not then e_0 cannot be neglected), equation (13) indicates that e grows from a very small value up until the value obtained by setting $\omega_a = \pi/2$, namely:

$$e^2 = 1 - 5 \cos^2 i_0 / 3, \quad (17)$$

attained when $\cos^2 i = 3/5$. A range of eccentricities may be generated in this way. However, values close to unity may be attained for initial

inclinations close to 90^0 . There are then solutions which have large amplitude oscillations in eccentricity (Kozai effect) and adding back in the non secular terms can lead to chaotic behavior. The large eccentricity changes occur independently of the size of the perturbation. However, the characteristic timescale for the eccentricity changes to occur is given from equation (13) as $\sim (M_* a'^3)/(M' a^3)$ orbital periods of the inner planet, which is long for small perturbations.

But note that the effect requires the motion of the apsidal angle ω_a be governed only by the perturbation considered. Other effects may disrupt this, such as general relativistic corrections (e.g. Holman et al. 1997) and the oblateness of the central star. Both could be considered in the discussion below, but here we limit ourselves to considering the effects of relativistic apsidal precession which may lead to constraints on the orbital elements of planets with short periods.

To incorporate *relativistic apsidal precession* to lowest order, we modify the Newtonian potential such that keplerian elliptic orbits are induced to precess at the correct rate. We modify the potential due to the central mass, such that:

$$\frac{-GM_*}{r} \rightarrow \frac{-GM_*}{r} \left(1 + \frac{3GM_*}{rc^2} \right), \quad (18)$$

with c being the speed of light. The additional term can be added into the Hamiltonian H and averaged so that equation (12) becomes:

$$H \rightarrow H - \frac{3(GM_*)^2 M_p}{2a^2 c^2 \sqrt{1-e^2}}. \quad (19)$$

The same procedure as described above can be used to find ω_a and i in terms of e and hence conditions that must be satisfied in order that large values of e might be attained. The condition analogous to (16) for the Kozai effect to work is:

$$\cos^2 i_0 < (3 - 2f)/5. \quad (20)$$

Here the parameter:

$$f = \left(\frac{GM_*}{ac^2} \right) \left(\frac{M_* a'^3}{M' a^3} \right) \quad (21)$$

measures the importance of relativistic precession relative to the perturbation due to the outer companion. From this, if $f > 3/2$ the effect is completely suppressed. When the Kozai effect occurs, the maximum possible eccentricity that can be generated is obtained by setting $\omega_a = \pi/2$ and $i_0 = 90^0$. This gives:

$$e^2 = 1/2 + \sqrt{4f/3 + 1/4} - 4f/3, \quad (22)$$

which indicates that, for $f = 0.1$, e as high as 0.99 can be generated for $i_0 = 90^\circ$. The parameter f may be conveniently expressed as:

$$f = 5.0519 \times 10^{-7} \frac{1}{q'} \left(\frac{P'_o}{P_o} \right)^2 \left(\frac{1 \text{ day}}{P_o} \right)^{2/3}. \quad (23)$$

Here, P_o and P'_o denote the orbital periods of M_p and M' respectively and $q' = M'/M_*$.

Planets with periods less than around 3–4 days are likely to have their orbits circularized as a result of tidal effects (see above). From the requirement $f > 3/2$, we see that for inner planets with somewhat larger periods, but still on the order of days, binary companions with mass ratio of order unity need orbital periods shorter than about 40 yr in order to pump significant eccentricity. If a massive planet with $q' \sim 10^{-2}$ is considered, it must orbit with a period ten times shorter ~ 4 yr. Such an object should be readily detectable.

We comment that for a given distant binary companion with inclination close to 90° to the planet orbit, and no other perturbers, there is a planet with a period long enough such that the Kozai effect works such that it could produce eccentricities close enough to unity that the planet has arbitrarily close approaches to the central star. Then tidal effects may act towards circularization so decreasing the distance to apoapse until a circular orbit is produced. The final period would have to be a few days. Alternatively, should the closest approach to the star be beyond the largest radius at which orbits can be circularized, the orbit would retain a Kozai cycle with high maximum eccentricity.

VI. Summary

Here we summarize the various processes discussed in the paper, their influence on the evolution of planetary systems, and their observational implications.

a) *Planet-disk tidal interaction* excites potentially observable spiral density waves and can create clean gaps in the disk, limits mass growth and drives orbital migration. Growth limitation by gap formation together with inward migration may naturally account for the upper limit in the observed mass ratio distribution of extra solar planets provided they were formed in disks with $H \lesssim 0.1r_p$ and $\alpha \lesssim 10^{-3}$. Since the critical mass for gap formation is an order of magnitude larger than for dynamical gas accretion, a bimodal mass distribution with a depression between these masses may be anticipated. After gap formation, massive protoplanets are expected to migrate with the viscous evolution of the disk until they encounter a stellar magnetospheric cavity or tidal barrier. The existence of short period planets in CTTS would imply that the local disk viscous evolution timescale is shorter than the

typical age ($\sim 10^6$ yr.) Orbital migration may lead to stellar consumption of protoplanets with surviving planets locked into commensurable orbits. Contamination of their host stars (during the latter's main sequence evolution) by plunging planets may have led to a relatively high metallicity. Planet-disk interaction is likely to excite only small $e \lesssim 0.3$ and lead to a relation between M_p and e which could be observed.

Stellar tides raised on short-period planets should rapidly synchronize their spin. Short period planets with $M_p \sim M_J$ and $P \lesssim 3-4$ days (e.g. τ Boo) should have their orbits circularized by tides raised on a slowly rotating star within a few Gyr. In the case of massive planets, the stellar rotation may also be synchronized with the orbit. But, only very short period planets ($P \lesssim 2$ days) are expected to undergo significant orbital decay.

c) *Long term gravitational interaction between planets with initially small e* leads to orbit crossing on a time scale which is sensitively determined by their masses and initial separations. Subsequent close encounters between comparable masses can lead to high orbital eccentricity and planets scattered into extended orbits. This mechanism may provide a supply of planets with a range of short periods, some of which may undergo tidal circularization and/or plunge into the star, together with outlying accomplices. The latter could be imaged by the next generation of IR interferometers.

d) *Secular perturbations* by distant binary, or massive outlying planetary, companions in relatively highly inclined orbits can also excite high e through the Kozai mechanism. Potentially this process might produce a supply of short period planets ($P \lesssim 2-3$ days) that have undergone tidal circularization. But because of the very high relative inclinations required, it is likely to work only in a small number of cases.

This work is supported by NSF and NASA through grants AST-9315578 and NAGW-4967.

REFERENCES

- Aarseth, S. J., Lin, D. N. C., and Palmer, P. L. 1993. Evolution of Planetesimals. II. Numerical Simulations. *Astrophys. J.* 403:351–372.
- Artymowicz, P. 1992. Dynamics of binary and planetary-system interaction with disks - Eccentricity changes. *Publ. Astron. Soc. Pacific* 104:769–774.
- Artymowicz, P. 1993a. On the Wave Excitation and a Generalized Torque Formula for Lindblad Resonances Excited by External Potential. *Astrophys. J.* 419:155–165.
- Artymowicz, P. 1993b. Disk-Satellite Interaction via Density Waves and the Eccentricity Evolution of Bodies Embedded in Disks. *Astrophys. J.* 419:166–180.
- Artymowicz, P., and Lubow, S. 1996. Mass Flow through Gaps in Circumbinary Disks. *Astrophys. J.* 467:L77–L80.
- Artymowicz, P., Lubow, S., and Kley, W. 1998. In *Planetary Systems: The Long View*, ed. L. Celnikier (Editions Frontières), *in press*
- Bryden, G., Chen, X., Lin, D. N. C., Nelson, R., and Papaloizou, J. C. B. 1998. Tidally induced gap formation in protostellar disks: gap clearing and suppression of protoplanetary growth. *Astrophys. J. Submitted*.
- Balbus, S. A., and Hawley, J. F. 1991. A powerful local shear instability in weakly magnetized disks. I — Linear analysis. *Astrophys. J.* 376:214–233.
- Beckwith, S. V. W., and Sargent, A. I. 1996. Circumstellar disks and the search for neighbouring planetary systems. *Nature* 383:139–144.
- Bodenheimer, P. and Pollack, J. B. 1986. Calculations of the accretion and evolution of giant planets: The effects of solid cores. *Icarus* 67:391–408.
- Bouvier, J., Cabrit, S., Fernandez, M., Martin, E. L., Matthews, J. M. 1993. COYOTES-I: the photometric variability and rotational evolution of T-Tauri stars. *Astron. Astrophys.* 272:176–206.
- Butler, R. P., Marcy, G. W., Williams, E., Hauser, H., and Shirts, P. 1997. Three New "51 Pegasi-Type" Planets. *Astrophys. J.* 474:L115–L118.
- Chambers, J. E., Wetherill, G. W., and Boss, A. P. 1996. The Stability of Multi-Planet Systems. *Icarus* 119:261–268.
- Choi, P. I., and Herbst, W. 1996. Rotation periods of stars in the Orion

- nebula cluster: the bimodal distribution. *Astron. J.* 111:283–298.
- Claret, A., and Cunha, N. C. S. 1997. Circularization and synchronization times in main-sequence of detached eclipsing binaries — II. Using the formalisms by Zahn. *Astron. Astrophys.* 318:187–197.
- Counselman, C. C. III 1973. Outcomes of tidal evolution. *Astrophys. J.* 180:307–316.
- Cowling, T. G. 1941. The non-radial oscillations of polytropic stars. *Mon. Not. Roy. Astron. Soc.* 101:367–375.
- Darwin, G. H. 1879. *Phil. Trans. Roy. Soc. London* 170:1.
- Franklin, F., Lecar, M., and Quinn, T. 1990. On the stability of orbits in the solar system - A comparison of a mapping with a numerical integration. *Icarus* 88:97–103.
- Goldreich, P., and Nicholson, P. D. 1977. Turbulent viscosity and Jupiter’s tidal Q. *Icarus* 30:301–304.
- Goldreich, P., and Nicholson, P. D. 1989. Tidal friction in early-type stars. *Astrophys. J.* 342:1079–1084.
- Goldreich, P., and Peale, S. 1966. Spin-orbit coupling in the solar system. *Astron. J.* 71:425–438.
- Goldreich, P., and Soter, S. 1966. Q in the solar system. *Icarus* 5:375–389.
- Goldreich, P., and Tremaine, S. 1978. The velocity dispersion in Saturn’s rings. *Icarus* 34:227–239.
- Goldreich, P., and Tremaine, S. 1980. Disk-satellite interactions. *Astrophys. J.* 241:425–441.
- Gonzalez, G. 1997. The stellar metallicity-giant planet connection. *Mon. Not. Roy. Astron. Soc.* 285:403–412.
- Gonzalez, G. 1998. Spectroscopic analyses of the parent stars of extra-solar planetary system candidates. *Astron. Astrophys.* 334:221–238.
- Goodman, A. A., Benson, P. J., Fuller, G. A., and Myers, P. C. 1993. Dense cores in dark clouds. VIII — Velocity gradients. *Astrophys. J.* 406:528–547.
- Goodman, J., and Dickson, E. S. 1998. Dynamical tide in solar-type binaries. *Astrophys. J. Submitted*.
- Goodman, J., and Oh, S. P. 1997. Fast tides in slow stars: the efficiency of eddy viscosity. *Astrophys. J.* 486:403–412.
- Guenther, E. W. 1997. Magnetic fields of T Tauri stars. In *Herbig-Haro Flows and the Birth of Stars*, eds. B. Reipurth and C. Bertout (Kluwer Academic Publishers), pp. 465–474.
- Guillot, T., Burrows, A., Hubbard, W. B., Lunine, J. I., and Saumon, D. 1996. Giant planets at small orbital distances. *Astrophys. J.* 459:L35–L38.
- Hagihara, Y. 1972. *Celestial Mechanics, Vol. II, Part 1: Perturbation theory* (Cambridge: MIT Press), p. 447.
- Hartmann, L., Calvet, N., Gullbring, E., and D’Alessio, P. 1998. Ac-

- cretion and the Evolution of T Tauri Disks. *Astrophys. J.* 495:385–400.
- Holman, M., Touma, J., and Tremaine, S. 1997. Chaotic variations in the eccentricity of the planet orbiting 16 Cyg B. *Nature* 386:254.
- Hut, P. 1980. Stability of tidal equilibrium. *Astron. Astrophys.* 92:167–170.
- Hut, P. 1981. Tidal evolution in close binary systems. *Astron. Astrophys.* 99:126–140.
- Ida, S., and Lin, D. N. C. 1998. *Submitted*.
- Innanen, K. A., Zheng, J. Q., Mikkola, S., and Valtonen, M. J. 1997. The Kozai mechanism and the stability of planetary orbits in binary star systems. *Astron. J.* 113:1915–1919.
- Ivanov, P. B., Papaloizou, J. C. B., and Polnarev, A. G. 1998. The evolution of supermassive binary caused by an accretion disc. *Mon. Not. Roy. Astron. Soc. Submitted*
- Kley, W. 1998. Mass flow and accretion through gaps in accretion disks. *Mon. Not. Roy. Astron. Soc. Submitted*.
- Kokubu, E., and Ida, S. 1996. On Runaway Growth of Planetesimals. *Icarus* 123:180–191.
- Konigl, A. 1991. Disk accretion onto magnetic T Tauri stars. *Astrophys. J.* 370:L39–L43.
- Korycansky, D. G., and Papaloizou, J. C. B., 1996. A Method for Calculations of Nonlinear Shear Flow: Application to Formation of Giant Planets in the Solar Nebula. *Astrophys. J. Supp.* 105:181–190.
- Korycansky, D. G., and Pollack, J. B. 1993. Numerical calculations of the linear response of a gaseous disk to a protoplanet. *Icarus* 102:150–165.
- Kozai, Y. 1962. Secular perturbations of asteroids with high inclination and eccentricity. *Astron. J.* 67:591–598.
- Kumar, P., Ao, C. O., and Quataert, E. J. 1995. Tidal Excitation of Modes in Binary Systems with Applications to Binary Pulsars. *Astrophys. J.* 449:294–309.
- Kumar, P., and Goodman, J. 1996. Nonlinear damping of oscillations in tidal-capture binaries. *Astrophys. J.* 466:946–956.
- Lin, D. N. C., Bodenheimer, P., and Richardson, D. C. 1996. Orbital migration of the planetary companion of 51 Pegasi to its present location. *Nature* 380:606–607.
- Lin, D. N. C., and Ida, S. 1997. On the Origin of Massive Eccentric Planets. *Astrophys. J.* 477:781–791.
- Lin, D. N. C., and Papaloizou, J. C. B. 1979a. Tidal torques on accretion discs in binary systems with extreme mass ratios. *Mon. Not. Roy. Astron. Soc.* 186:799–496.
- Lin, D. N. C., and Papaloizou, J. C. B. 1979b. On the structure of circumbinary accretion disks and the tidal evolution of commensu-

- nable satellites. *Mon. Not. Roy. Astron. Soc.* 188:191–201.
- Lin, D. N. C., and Papaloizou, J. C. B. 1980. On the structure and evolution of the primordial solar nebula. *Mon. Not. Roy. Astron. Soc.* 191:37–48.
- Lin, D. N. C., and Papaloizou, J. C. B. 1986a. On the tidal interaction between protoplanets and the primordial solar nebula. II - Self-consistent nonlinear interaction. *Astrophys. J.* 307:395–409.
- Lin, D. N. C., and Papaloizou, J. C. B. 1986b. On the tidal interaction between protoplanets and the primordial solar nebula. III - Orbital migration of protoplanets. *Astrophys. J.* 309:846–857.
- Lin, D. N. C., and Papaloizou, J. C. B. 1993. On the tidal interaction between protostellar disks and companions. In *Protostars and Planets III*, eds. E. H. Levy and J. I. Lunine (Tucson: University of Arizona Press), pp. 749–835.
- Lin, D. N. C., Papaloizou, J. C. B., and Ruden, S. P. 1987. On the confinement of planetary arcs. *Mon. Not. Roy. Astron. Soc.* 227:75–95.
- Lin, D. N. C., Papaloizou, J. C. B., and Savonije, G. J. 1990a. Wave propagation in gaseous accretion disks. *Astrophys. J.* 364:326–334.
- Lin, D. N. C., Papaloizou, J. C. B., and Savonije, G. J. 1990b. Propagation of tidal disturbance in gaseous accretion disks. *Astrophys. J.* 365:748–756.
- Lissauer, J. J., and Stewart, G. R. 1993. Growth of planets from planetesimals. In *Protostars and Planets III*, eds. E. H. Levy and J. I. Lunine (Tucson: University of Arizona Press), pp. 1061–1088.
- Lubow, S. H., Tout, C. A., and Livio, M. 1997. Resonant tides in close orbiting planets. *Astrophys. J.* 484:866–870.
- MacDonald, G. J. F. 1964. Tidal friction. *Rev. Geophys.* 2:467–541.
- Marcy, G. W., Butler, R. P., Williams, E., Bildsten, L., Graham, J. R., Ghez, A. M., and Jernigan, J. G. 1997. The planet around 51 Pegasi. *Astrophys. J.* 481:926–935.
- Mathieu, R. D. 1994. Pre-main-sequence binary stars. *Ann. Rev. Astron. Astrophys.* 32:465–530.
- Mathieu et al. 1998. *Submitted*.
- McCaughrean, M. J., and Stauffer, J. R. 1994. High resolution near-infrared imaging of the trapezium: A stellar census. *Astron. J.* 108:1382–1397.
- Miyoshi, K., et al. 1998. Numerical simulation of tidal interaction between a protoplanet and the solar nebula. *Submitted*.
- Mizuno, H. 1980. Formation of the giant planets. *Prog. Theor. Phys.* 64:544–557.
- Papaloizou, J. C. B., and Lin, D. N. C. 1984. On the tidal interaction between protoplanets and the primordial solar nebula. I - Linear calculation of the role of angular momentum exchange. *Astrophys. J.* 285:818–834.

- Papaloizou, J. C. B., and Savonije, G. J. 1985. On the tidal evolution of massive X-ray binaries — The tidal evolution time-scales for very long orbital periods. *Mon. Not. Roy. Astron. Soc.* 213:85–96.
- Papaloizou, J. C. B., and Savonije, G. J. 1997. Non-adiabatic tidal forcing of a massive, uniformly rotating star — III. Asymptotic treatment for low frequencies in the inertial regime. *Mon. Not. Roy. Astron. Soc.* 291:651–657.
- Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., and Greenzweig, Y. 1996. Formation of the Giant Planets by Concurrent Accretion of Solids and Gas. *Icarus* 124:62–85.
- Prosser, C. F., Stauffer, J. R., Hartmann, L., Soderblom, D. R., Jones, B. F., Werner, M. W., and McCaughrean, M. J. 1994. HST photometry of the trapezium cluster. *Astrophys. J.* 421:517–541.
- Rasio, F. A., and Ford, E. B. 1996. Dynamical instabilities and the formation of extrasolar planetary systems. *Science* 274:954–956.
- Rasio, F. A., Tout, C. A., Lubow, S. H., and Livio, M. 1996. Tidal decay of close planetary orbits. *Astrophys. J.* 470:1187–1191.
- Safronov, V. S. 1969. In *Evolutsiia doplanetnogo oblaka*. (Moscow)
- Sandquist, E., Taam, R. E., Lin, D. N. C., and Burkert, A. 1998. Planet Stripping and Stellar Metallicity Enhancements. *Astrophys. J.* in press.
- Saumon, D., Hubbard, W. B., Burrows, A., Guillot, T., Lunine, J. I., and Chabrier, G. 1996. A Theory of Extrasolar Giant Planets. *Astrophys. J.* 460:993–1018.
- Savonije, G. J., and Papaloizou, J. C. B. 1983. On the tidal spin up and orbital circularization rate for the massive X-ray binary systems. *Mon. Not. Roy. Astron. Soc.* 203:581–593.
- Savonije, G. J., and Papaloizou, J. C. B. 1984. On the tidal evolution of massive X-ray binaries — The spin-up and circularization rates for systems with evolved stars and the effects of resonances. *Mon. Not. Roy. Astron. Soc.* 207:685–704.
- Savonije, G. J., and Papaloizou, J. C. B. 1997. Non-adiabatic tidal forcing of a massive, uniformly rotating star — II. The low-frequency, inertial regime. *Mon. Not. Roy. Astron. Soc.* 291:633–650.
- Savonije, G. J., Papaloizou, J. C. B., and Alberts, F. 1995. Nonadiabatic tidal forcing of a massive uniformly rotating star. *Mon. Not. Roy. Astron. Soc.* 277:471–496.
- Shakura, N. I., and Sunyaev, R. A. 1973. Black holes in binary systems: Observational appearance. *Astron. Astrophys.* 24:337–355.
- Shu, F. H. 1984. Waves in planetary rings. In *Planetary Rings*, eds. R. Greenberg and A. Brahic (Tucson: University of Arizona Press), pp. 513–561.
- Stevenson, D. J. 1983. Anomalous bulk viscosity of two-phase fluids and implications for planetary interiors. *J. Geophys. Res.* 88:2445–2455.

- Takeuchi, T., Miyama, S. M., and Lin, D. N. C. 1996. Gap Formation in Protoplanetary Disks. *Astrophys. J.* 460:832–847.
- Tassoul, J.-L. 1988. On orbital circularization in detached close binaries. *Astrophys. J.* 324:L71–L73.
- Terebey, S., Shu, F. H., and Cassen, P. 1984. The collapse of the cores of slowly rotating isothermal clouds. *Astrophys. J.* 286:529–551.
- Terquem, C., Papaloizou, J. C. B., Nelson, R. P., and Lin, D. N. C. 1998a. On the Tidal Interaction of a Solar-Type Star with an Orbiting Companion: Excitation of g-Mode Oscillation and Orbital Evolution. *Astrophys. J.* 502:788–801.
- Terquem, C., Papaloizou, J. C. B., Nelson, R. P., and Lin, D. N. C. 1998b. Oscillations in solar-type stars tidally induced by orbiting planets. In *Planetary Systems: The Long View*, ed. L. Celnikier (Editions Frontières), *in press*
- Ward, W. R. 1981. Solar nebula dispersal and the stability of the planetary system. I— Scanning secular resonance theory. *Icarus* 47:234–264.
- Ward, W. R. 1993. Disk-planet interactions: Torques from the co-orbital zone. *Anal. NY Acad. Sci.* 675:314–323.
- Ward, W. R. 1997. Protoplanet Migration by Nebula Tides. *Icarus* 126:261–281.
- Weidenschilling, S. J., and Marzari, F. 1996. Gravitational scattering as a possible origin for giant planets at small stellar distances. *Nature* 384:619–621.
- Wetherill, G. W., and Stewart, G. R. 1989. Accumulation of a swarm of small planetesimals. *Icarus* 77:330–357.
- Zahn, J. P. 1975. The dynamical tide in close binaries. *Astron. Astrophys.* 41:329–344.
- Zahn, J. P. 1977. Tidal friction in close binary stars. *Astron. Astrophys.* 57:383–394.
- Zharkov, V. N., and Gudkova, T. V. 1991. Models of giant planets with a variable ratio of ice to rock. *Ann. Geophys.* 9:357–366.