Intro: Asymptotic Notation

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Data Structures and Algorithms Algorithmic Toolbox

Learning Objectives

- Understand the basic idea behind asymptotic runtimes.
- Describe some of the advantages to using asymptotic runtimes.

Last Time

Computing Runtimes Hard

- Depends on fine details of program.
- Depends on details of computer.

Idea

All of these issues can multiply runtimes by (large) constant.

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All of these issues can multiply runtimes by (large) constant. So measure runtime in a way that ignores constant multiples.

Problem

Unfortunately, 1 second, 1 hour, 1 year only differ by constant multiples.

Solution

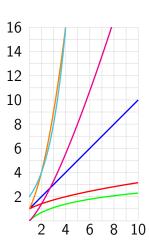
Consider asymptotic runtimes. How does runtime scale with input size.

Approximate Runtimes

| | n | <i>n</i> log <i>n</i> | n^2 | 2 ⁿ |
|--------------|-----------------|-----------------------|-------------------|------------------------|
| n = 20 | 1 sec | 1 sec | 1 sec | 1 sec |
| n = 50 | 1 sec | 1 sec | 1 sec | 13 day |
| $n = 10^2$ | 1 sec | 1 sec | 1 sec | $4 \cdot 10^{13}$ year |
| $n = 10^6$ | 1 sec | 1 sec | 17 min | |
| $n = 10^9$ | 1 sec | 30 sec | 30 year | |
| max <i>n</i> | 10 ⁹ | 10 ^{7.5} | 10 ^{4.5} | 30 |

$\log n \prec \sqrt{n} \prec n \prec n \log n \prec n^2 \prec 2^n$

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