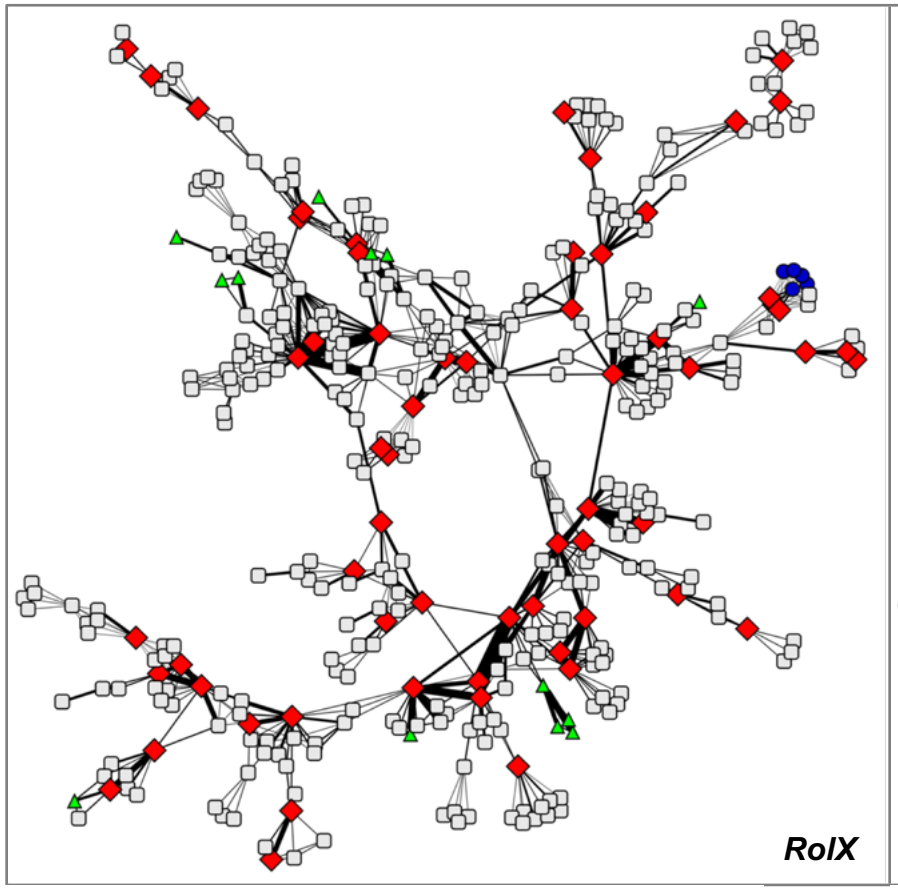
Community Structure in Networks

CS224W: Analysis of Networks
Jure Leskovec, Stanford University
http://cs224w.stanford.edu

Roles and Communities: Example

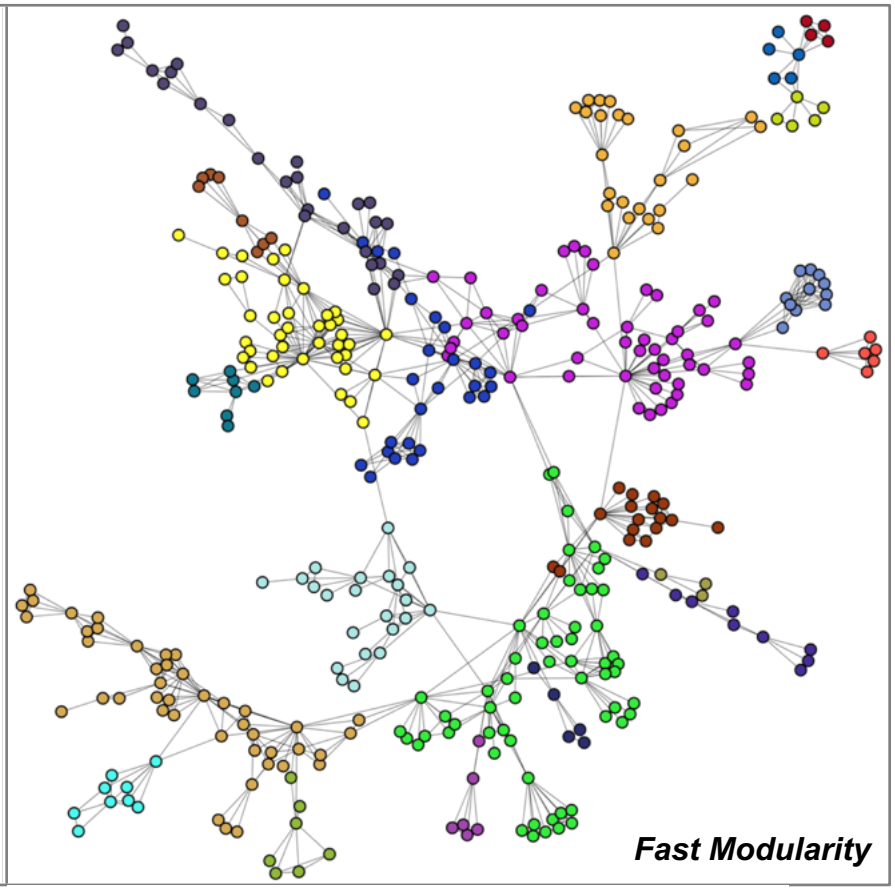
Roles



Henderson, *et al.,* KDD 2012

Nodes with different structural roles (connector node, bridge node, etc.)

Communities



Clauset, *et al.,* Phys. Rev. E 2004

Nodes belonging to the same cluster/community

Plan for Today

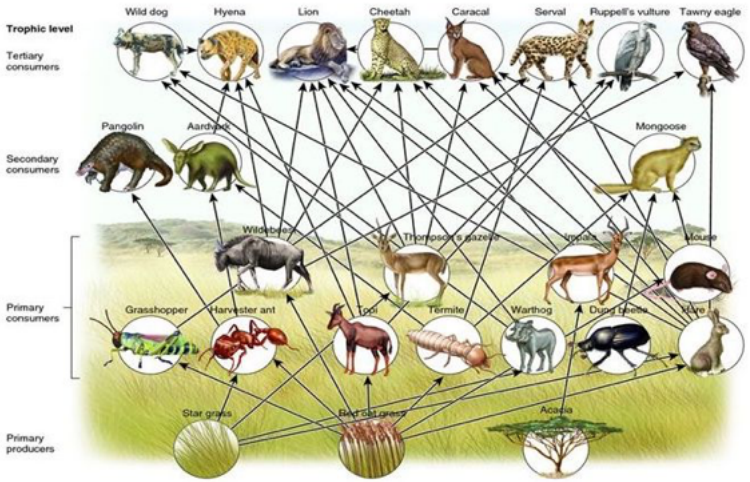
Plan for Today:

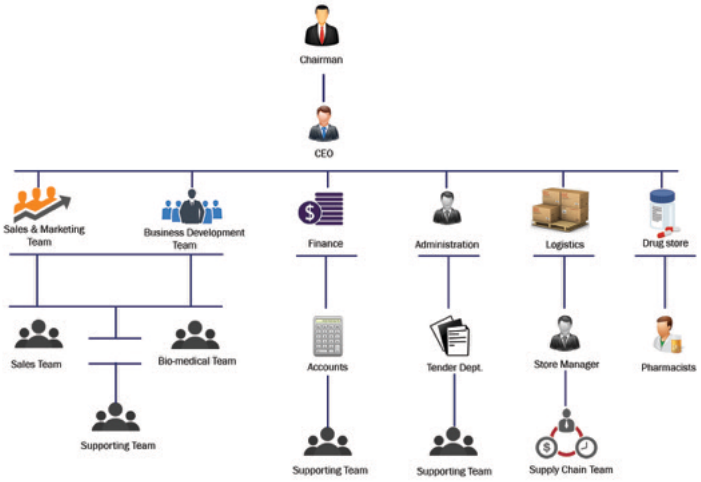
- **Structural role** discovery in networks
- **Community detection** via Modularity optimization

Structural Roles in Networks

What are Roles?

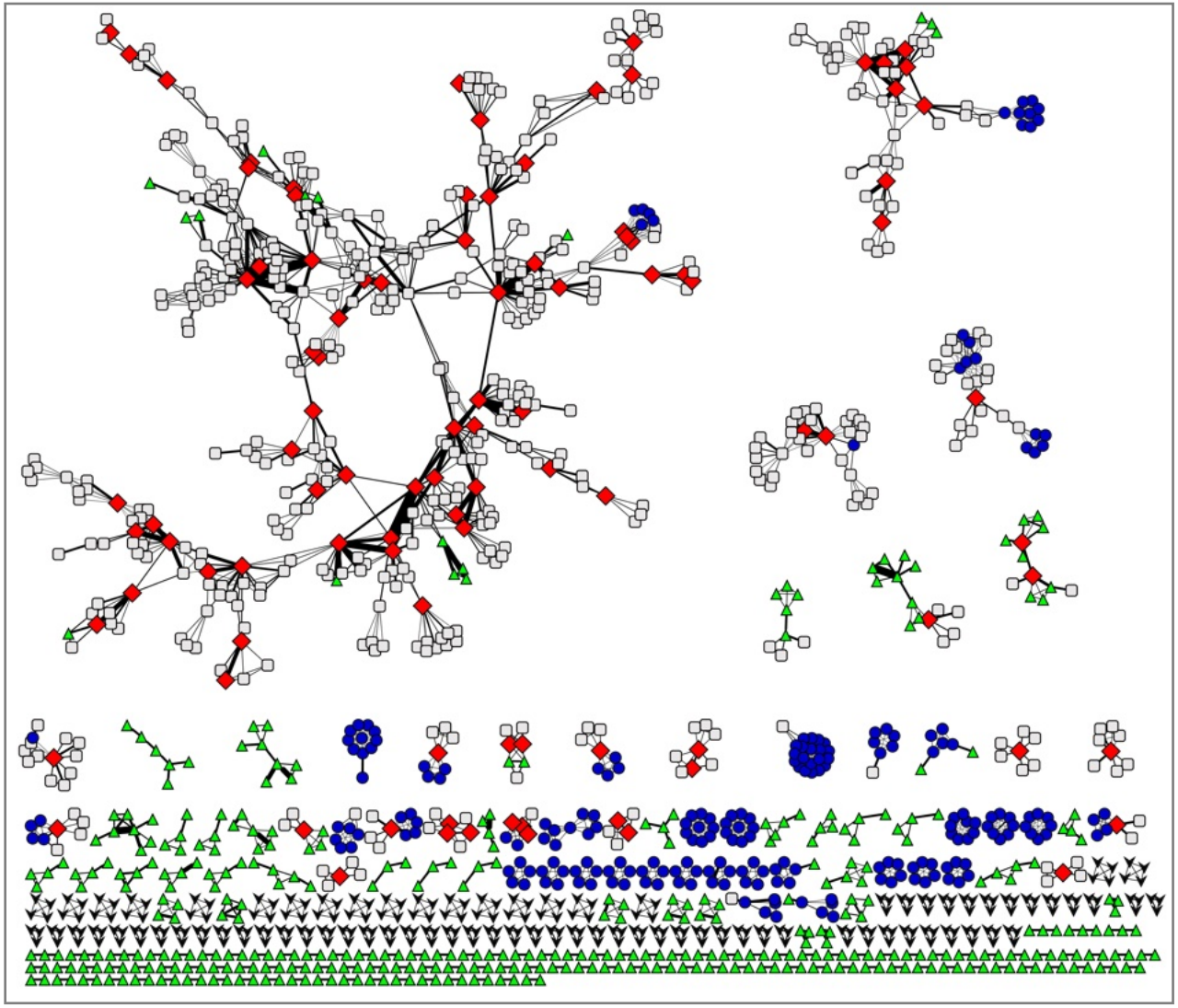
- **Roles are "functions" of nodes in a network:**
  - Roles of **species** in **ecosystems**
  - Roles of **individuals** in **companies**





- **Roles are measured by structural behaviors:**
  - Centers of stars
  - Members of cliques
  - Peripheral nodes, etc.

Example of Roles



- centers of stars
- members of cliques
- peripheral nodes

Network Science
Co-authorship network
[Newman 2006]

Roles versus Groups in Networks

- **Role**: A collection of nodes which have similar positions in a network:
  - Roles are based on the similarity of ties among subsets of nodes
  - Different from **community** (or cohesive subgroup)
    - Group is formed based on adjacency, proximity or reachability
    - This is typically adopted in current data mining

Nodes with the same role need not be in direct, or even indirect interaction with each other

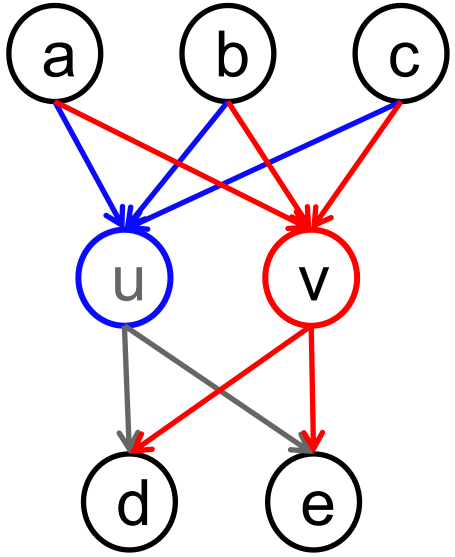
Roles and Communities

- **Roles**:
  - A group of nodes with similar structural properties
- **Communities:**
  - A group of nodes that are well-connected to each other
- Roles and communities **are complementary**

- Consider the social network of a CS Dept:
  - **Roles:** Faculty, Staff, Students
  - **Communities:** AI Lab, Info Lab, Theory Lab

Roles: More Formally

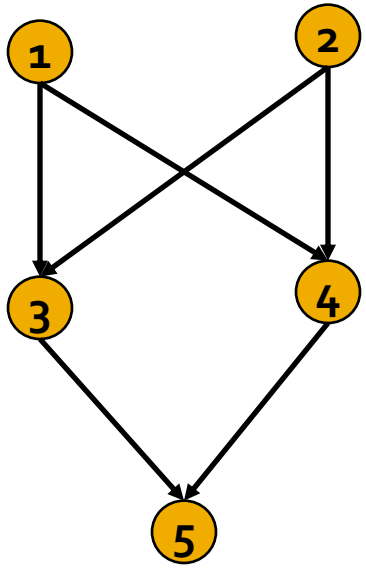
- **Structural equivalence:** Nodes u and v are structurally equivalent if they have the same relationships **to all other nodes** [Lorrain & White 1971]
  - Structurally equivalent nodes are likely to be similar in other ways – *i.e.*, friendships in social networks



Structural Equivalence: Example

- Nodes u and v are **structurally equivalent:**
  - For all the other nodes k, node u has tie to k iff node v has tie to k

- **Example:**



Adjacency matrix

	1	2	3	4	5
1	-	0	1	1	0
2	0	-	1	1	0
3	0	0	-	0	1
4	0	0	0	-	1
5	0	0	0	0	-

- *E.g.,* nodes 3 and 4 are structurally equivalent

Discovering Structural Roles in Networks

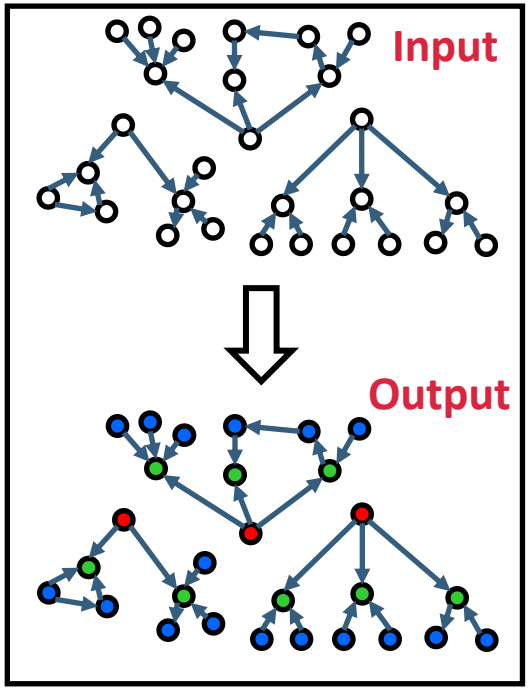
Why Are Roles Important?

Task	Example Application
Role query	Identify individuals with similar behavior to a known target
Role outliers	Identify individuals with unusual behavior
Role dynamics	Identify unusual changes in behavior
Identity resolution	Identify/de-anonymize, individuals in a new network
Role transfer	Use knowledge of one network to make predictions in another
Network comparison	Compute similarity of networks, determine compatibility for knowledge transfer

Structural Role Discovery Method

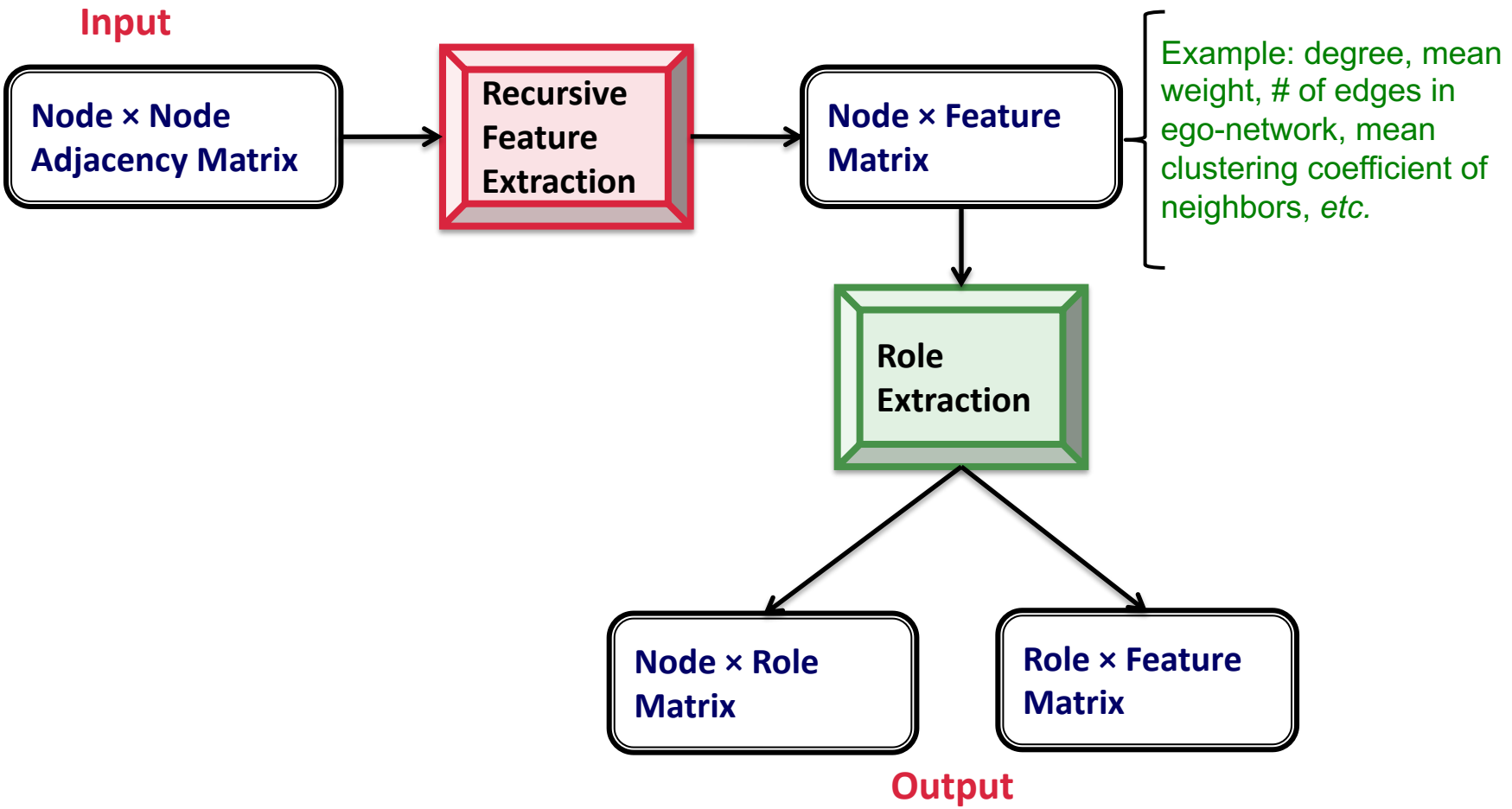
- **RolX:** Automatic discovery of nodes' structural roles in networks
  [Henderson, et al. 2011b]
  - Unsupervised learning approach
  - No prior knowledge required
  - Assigns a mixed-membership of roles to each node
  - Scales linearly in #(edges)

Role Discovery



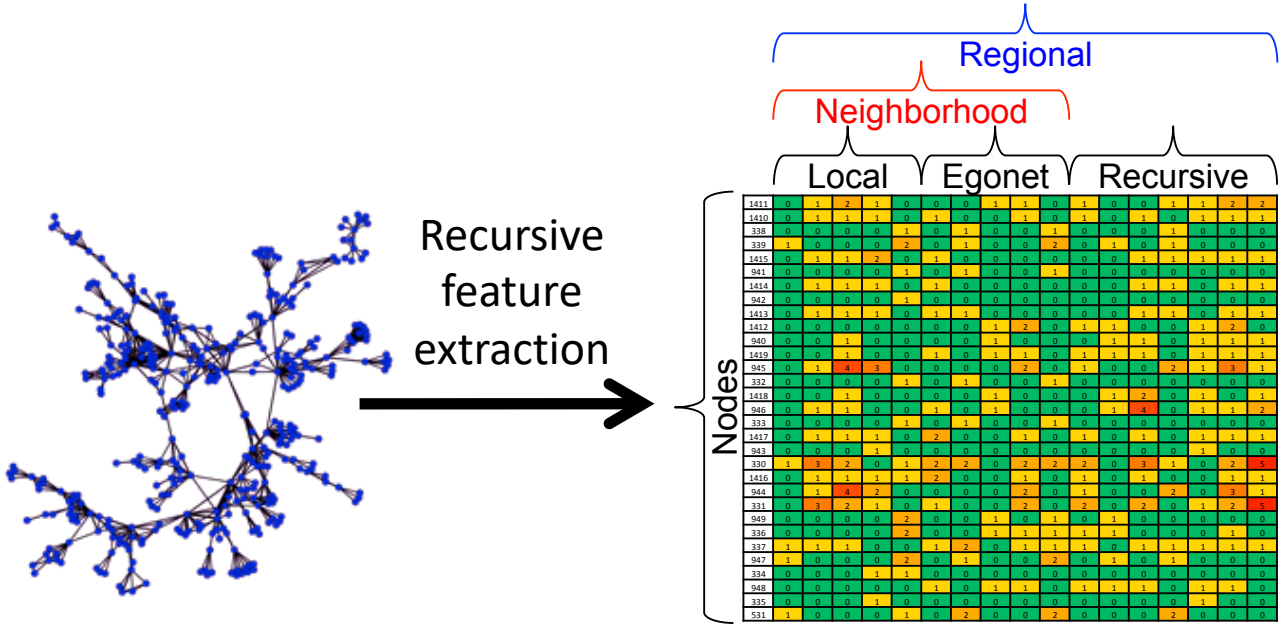
✓Automated discovery
✓Behavioral roles
✓Roles generalize

RolX: Approach Overview



Recursive Feature Extraction

- **Recursive feature extraction** [Henderson, et al. 2011a] turns network connectivity into structural features

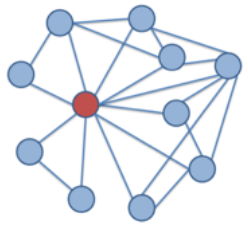


- **Neighborhood features:** What is a node's connectivity pattern?
- **Recursive features:** To what **kinds** of nodes is a node connected?

Recursive Feature Extraction

- **Idea:** Aggregate features of a node and use them to **generate new recursive features**

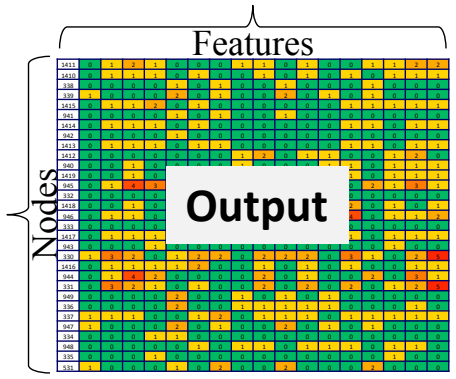
- **Base set of a node's neighborhood features:**
  - **Local features:** All measures of the node degree:
    - If network is directed, include in- and out-degree, total degree
    - If network is weighted, include weighted feature versions
  - **Egonetwork features:** Computed on the node's egonet:
    - **Egonet** includes the node, its neighbors, and any edges in the induced subgraph on these nodes
    - #(within-egonet edges),
      #(edges entering/leaving egonet)



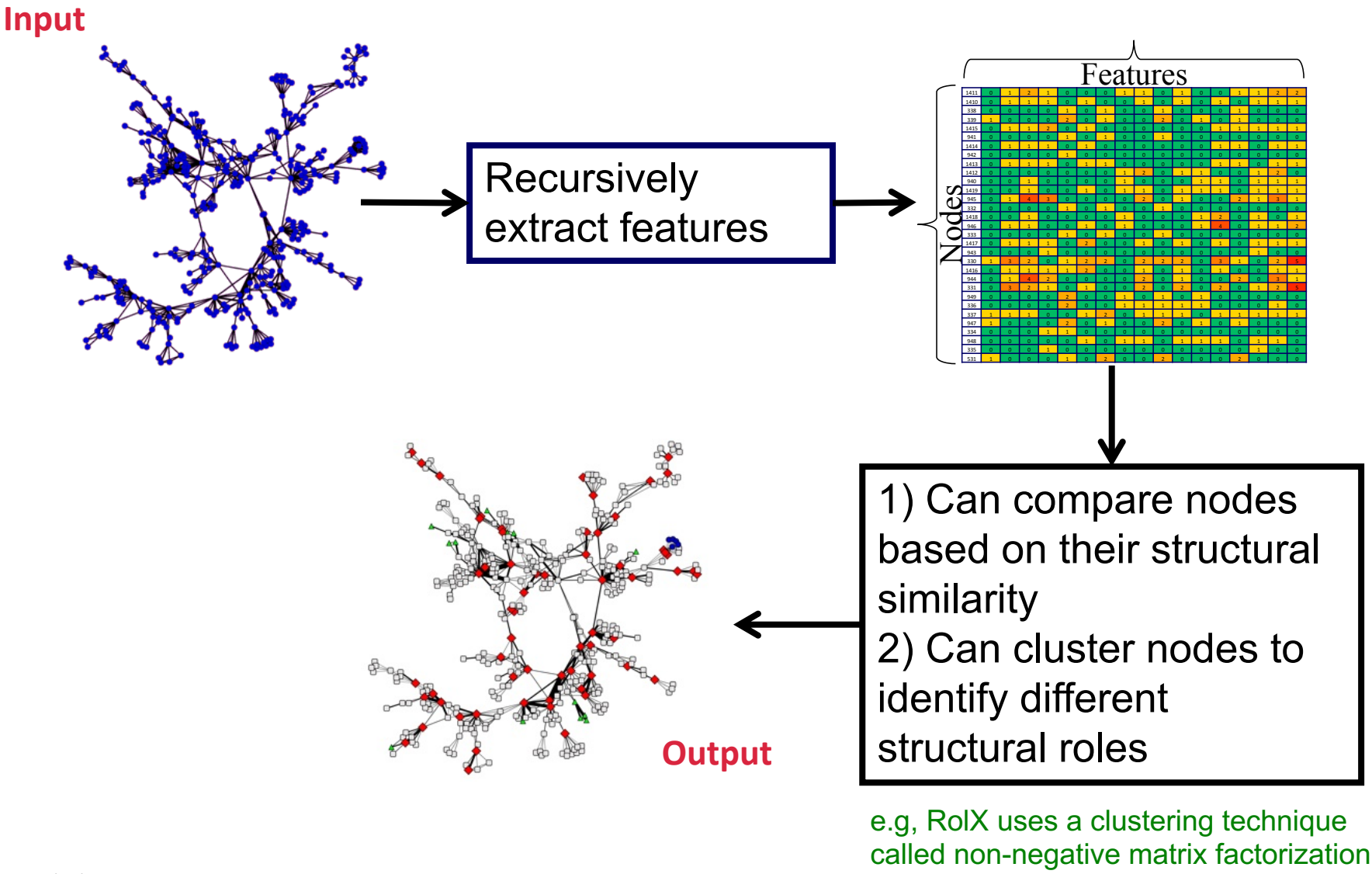
Egonet for **red node**

Recursive Feature Extraction

- Start with the base set of node features
- Use the **set of current node features** to generate **additional features**:
  - Two types of **aggregate functions:** means and sums
    - *E.g.,* mean value of "unweighted degree" feature among all neighbors of a node
    - Compute means and sums over all current features, including other recursive features
  - Repeat
- The number of possible recursive features **grows exponentially** with each recursive iteration:
  - Reduce the number of features using a **pruning technique:**
    - Look for pairs of features that are highly correlated
    - Eliminate one of the features whenever two features are correlated above a user-defined threshold



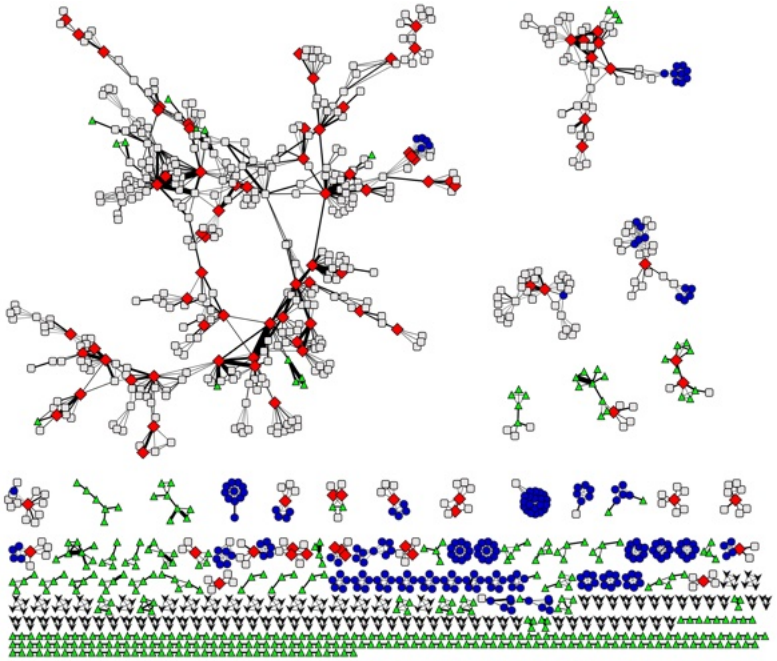
Role Extraction



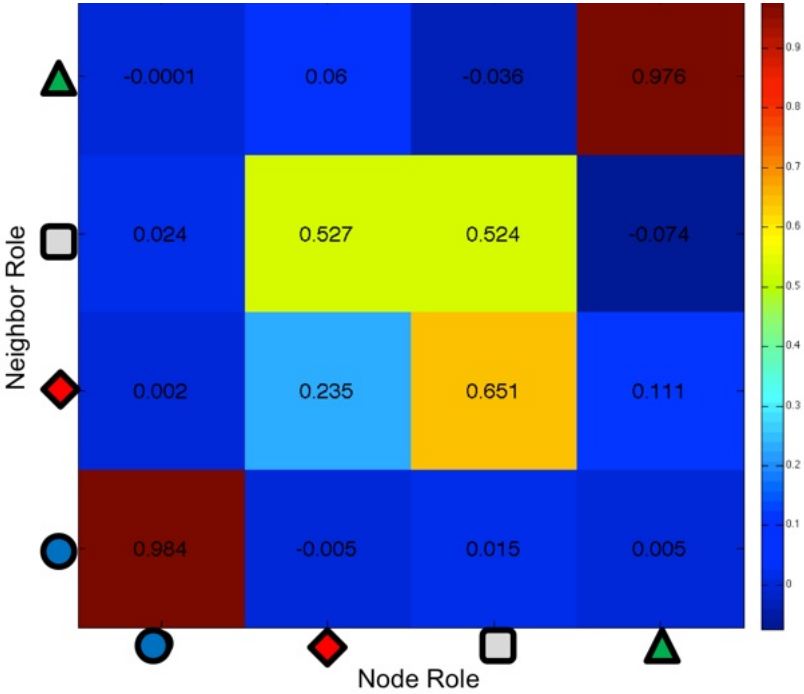
Application: Structural Similarity

- **Task:** Cluster nodes based on their structural similarity
- **Two networks:**
  - Network science co-authorship network:
    - Nodes: Network scientists; Edges: The number of co-authored papers
  - Political books co-purchasing network:
    - Nodes: Political books on Amazon; Edges: Frequent co-purchasing of books by the same buyers
- **Setup:** For each network:
  - Use RolX to assign each node a distribution over the set of discovered, structural roles
  - Determine similarity between nodes by comparing their role distributions

Structural Sim: Co-authorship Net



Role-colored graph: each node is colored by the primary role that RolX finds



Role affinity heat-map

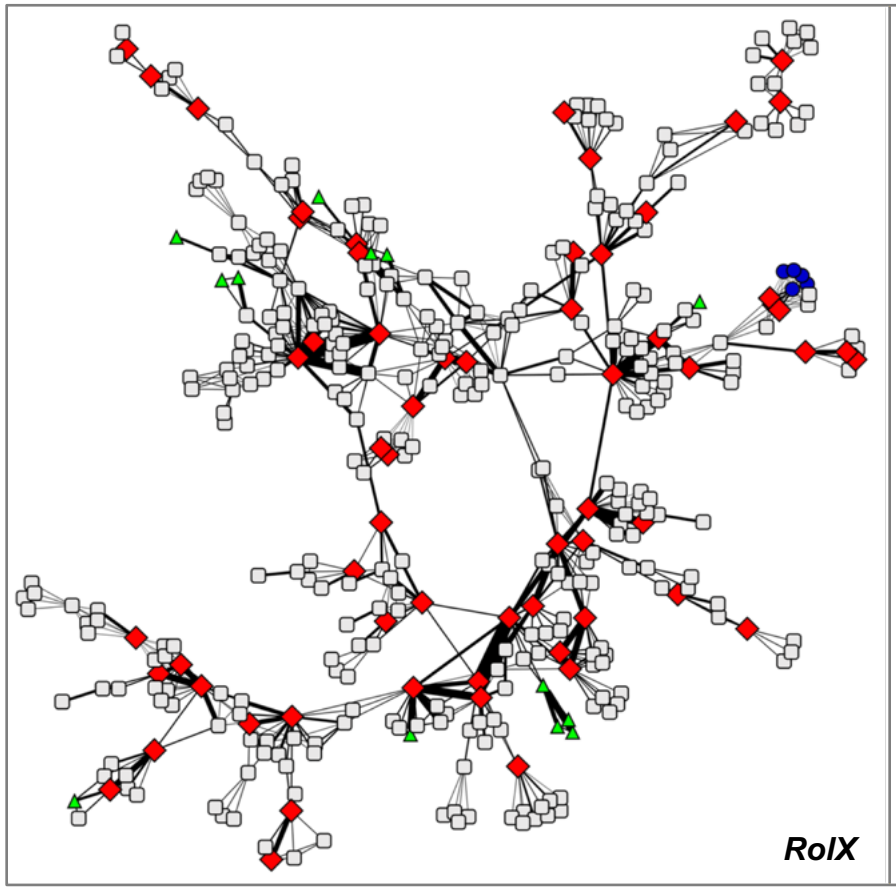
Making sense of roles:

- **Blue circle: Tightly knit**, nodes that participate in tightly-coupled groups
- **Red diamond: Bridge nodes**, that connect groups of nodes
- **Gray rectangle: Main-stream**, most of nodes, neither a clique, nor a chain
- **Green triangle: Pathy**, nodes that belong to elongated clusters

Community Structure in Networks

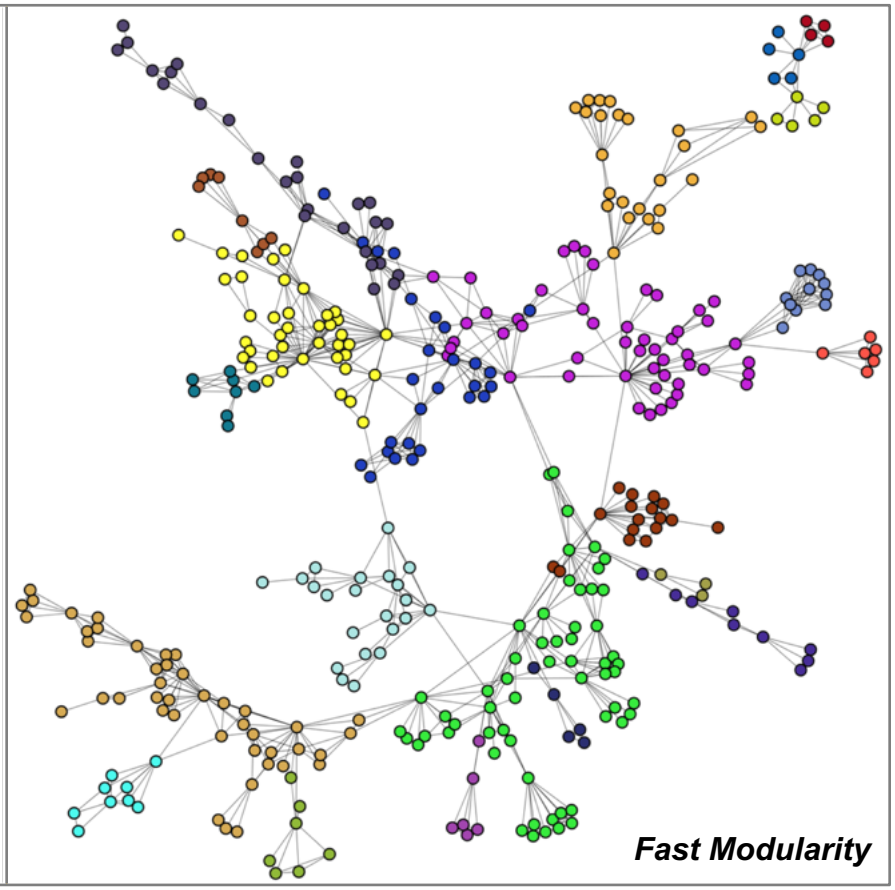
Roles and Communities: Example

Roles



Henderson, *et al.,* KDD 2012

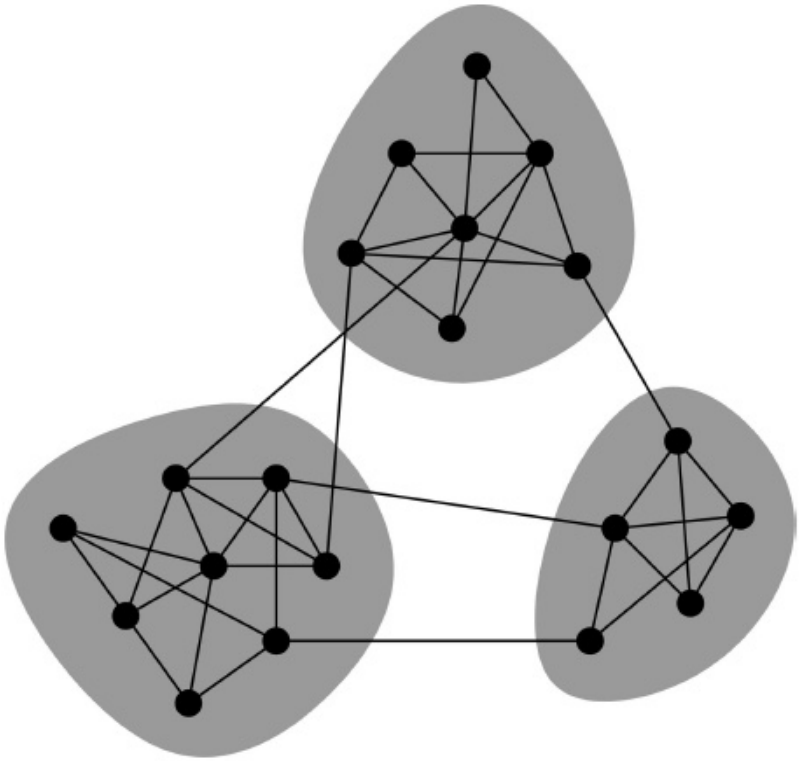
Communities



Clauset, *et al.,* Phys. Rev. E 2004

Networks & Communities

- **We often think of networks "looking" like this:**



- **What led to such a conceptual picture?**

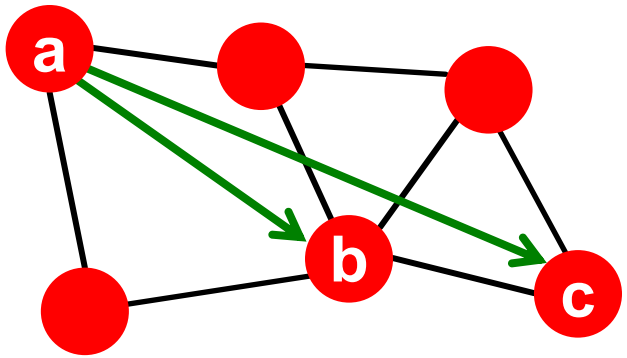
Networks: Flow of Information

- **How does information flow through the network?**
  - **What structurally distinct roles do nodes play?**
  - **What roles do different links ("short" vs. "long") play?**
- **How do people find out about new jobs?**
  - Mark Granovetter, part of his PhD in 1960s
  - People find the information through personal contacts
- **But:** Contacts were often **acquaintances** rather than close friends
  - **This is surprising:** One would expect your friends to help you out more than casual acquaintances
- **Why is it that acquaintances are most helpful?**

[Granovetter '73]

Granovetter's Answer

- **Two perspectives on friendships:**
  - **Structural:** Friendships span different parts of the network
  - **Interpersonal:** Friendship between two people is either **strong** or **weak**
- **Structural role: Triadic Closure**

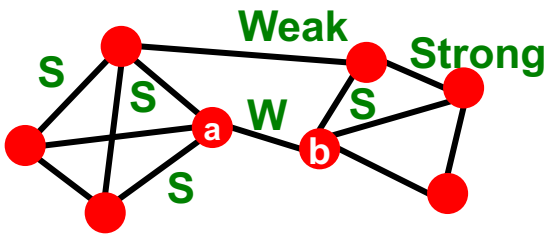


Which edge is more likely, a-b or a-c?

If two people in a network have a friend in common, then there is an increased likelihood they will become friends themselves.

Granovetter's Explanation

- **Granovetter makes a connection between social and structural role of an edge**
- **First point: Structure**
  - Structurally embedded edges are also socially strong
  - Long-range edges spanning different parts of the network are socially weak
- **Second point: Information**
  - Long-range edges allow you to gather information from different parts of the network and get a job
  - Structurally embedded edges are heavily redundant in terms of information access

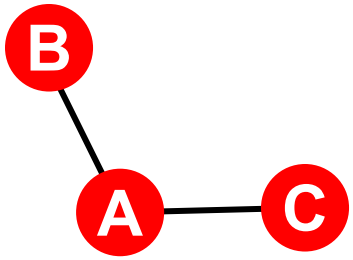


Triadic Closure

- **Triadic closure == High clustering coefficient**

Reasons for triadic closure:

- If B and C have a friend A in common, then:
  - B is **more likely to meet** C
    - (since they both spend time with A)
  - B and C **trust** each other
    - (since they have a friend in common)
  - A has **incentive** to bring B and C together
    - (since it is hard for A to maintain two disjoint relationships)



- **Empirical study by Bearman and Moody:**
  - Teenage girls with low clustering coefficient are more likely to contemplate suicide

Tie strength in real data

- **For many years Granovetter's theory was not tested**
- But, today we have large who-talks-to-whom graphs:
  - Email, Messenger, Cell phones, Facebook
- **Onnela et al. 2007:**
  - Cell-phone network of 20% of country's population
  - **Edge strength:** # phone calls

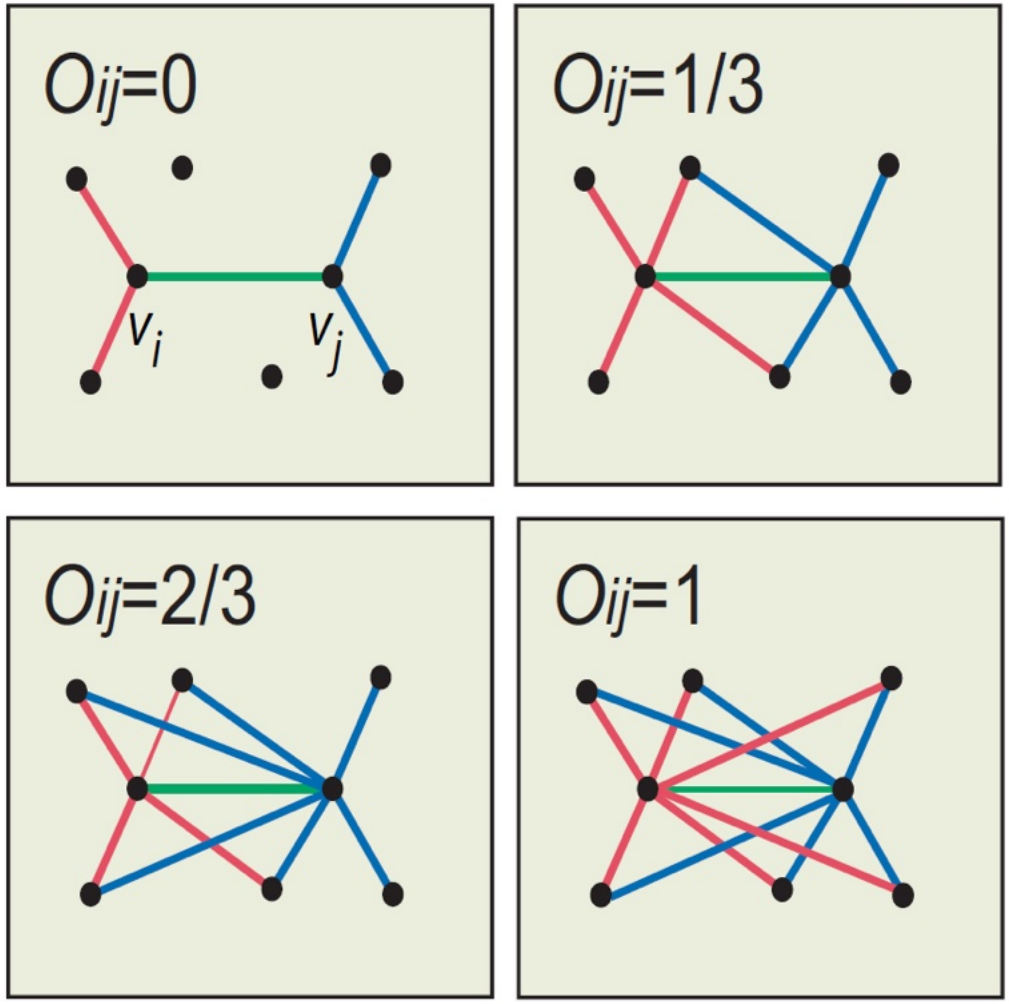
Neighborhood Overlap

- **Edge overlap:**

$$O_{ij} = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$$

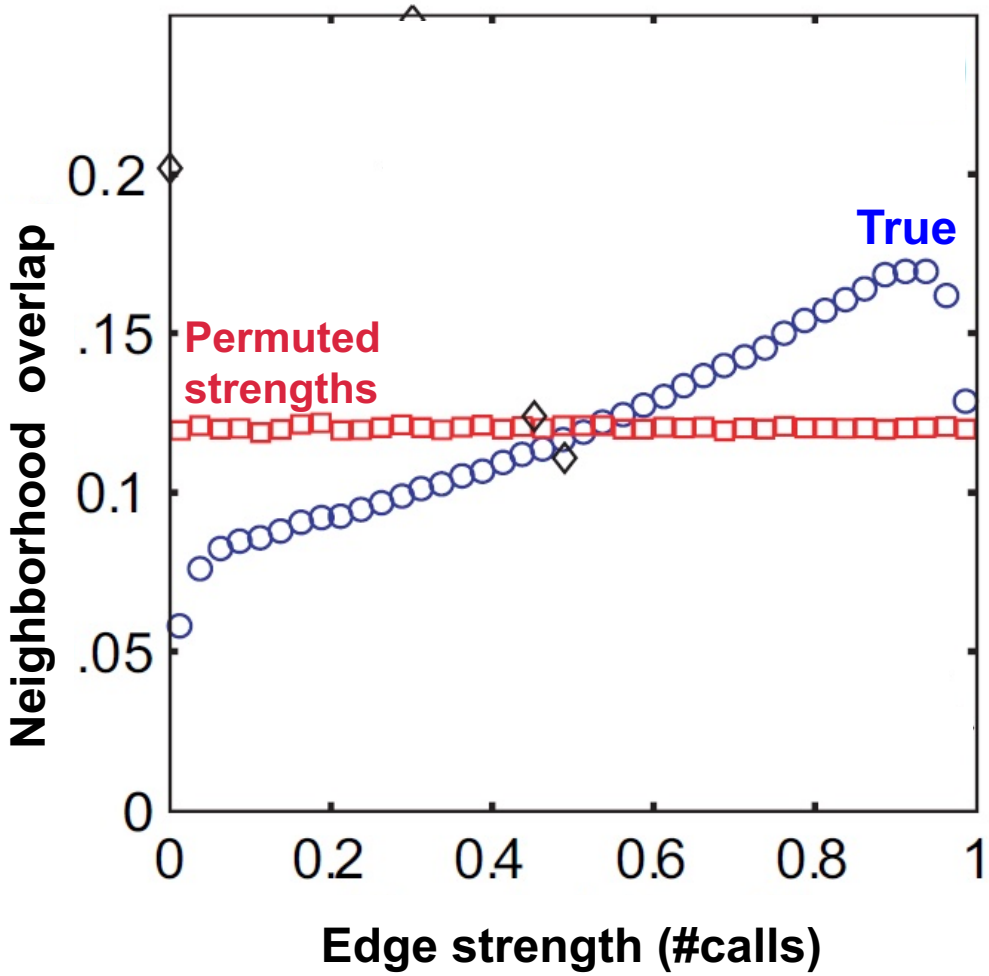
  - $N(i)$ … a set of neighbors of node i

- **Overlap = 0** when an edge is a **local bridge**

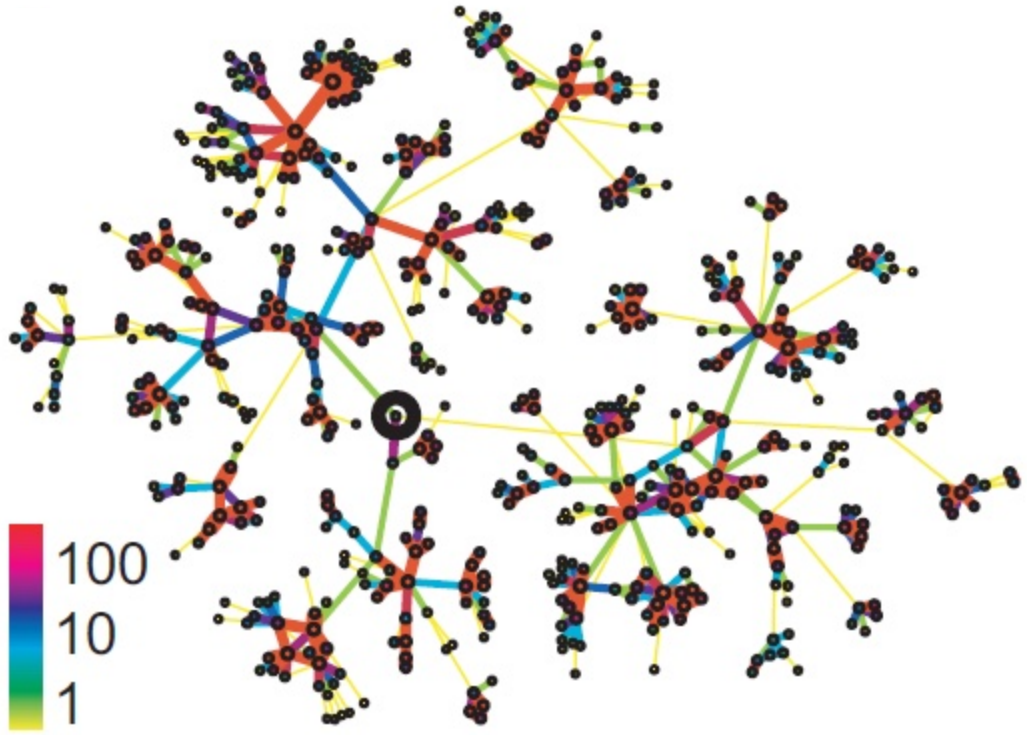


Phones: Edge Overlap vs. Strength

- **Cell-phone network**
- **Observation:**
  - **Highly used links have high overlap!**
- **Legend:**
  - **True:** The data
  - **Permuted strengths:** Keep the network structure but randomly reassign edge strengths

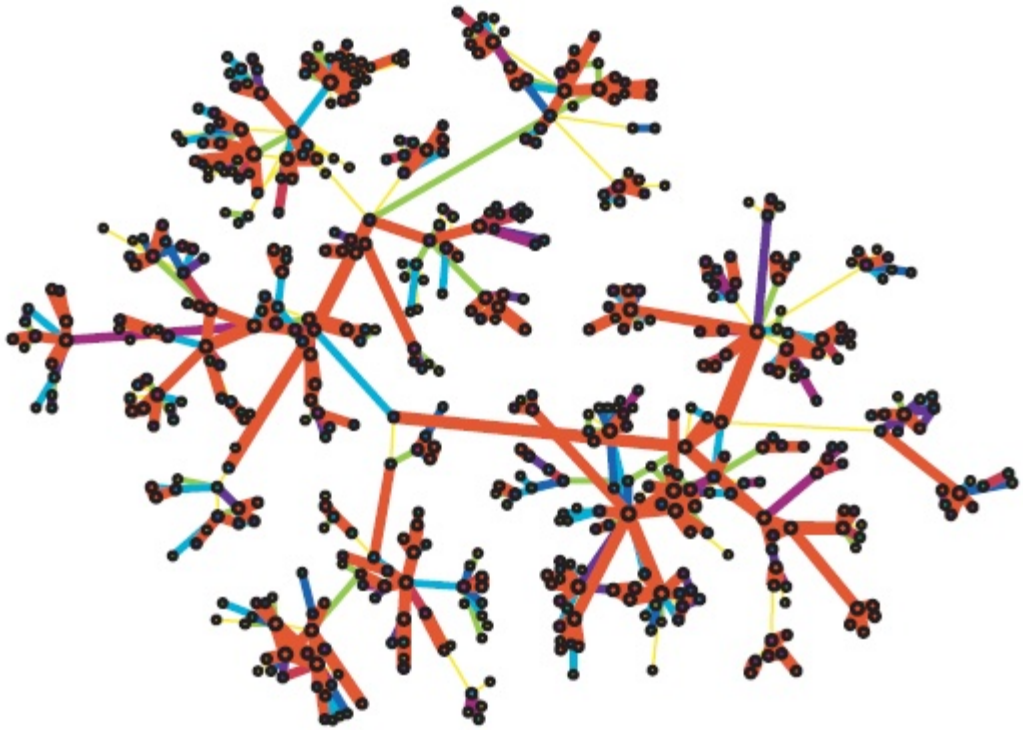


Real Network, Real Tie Strengths



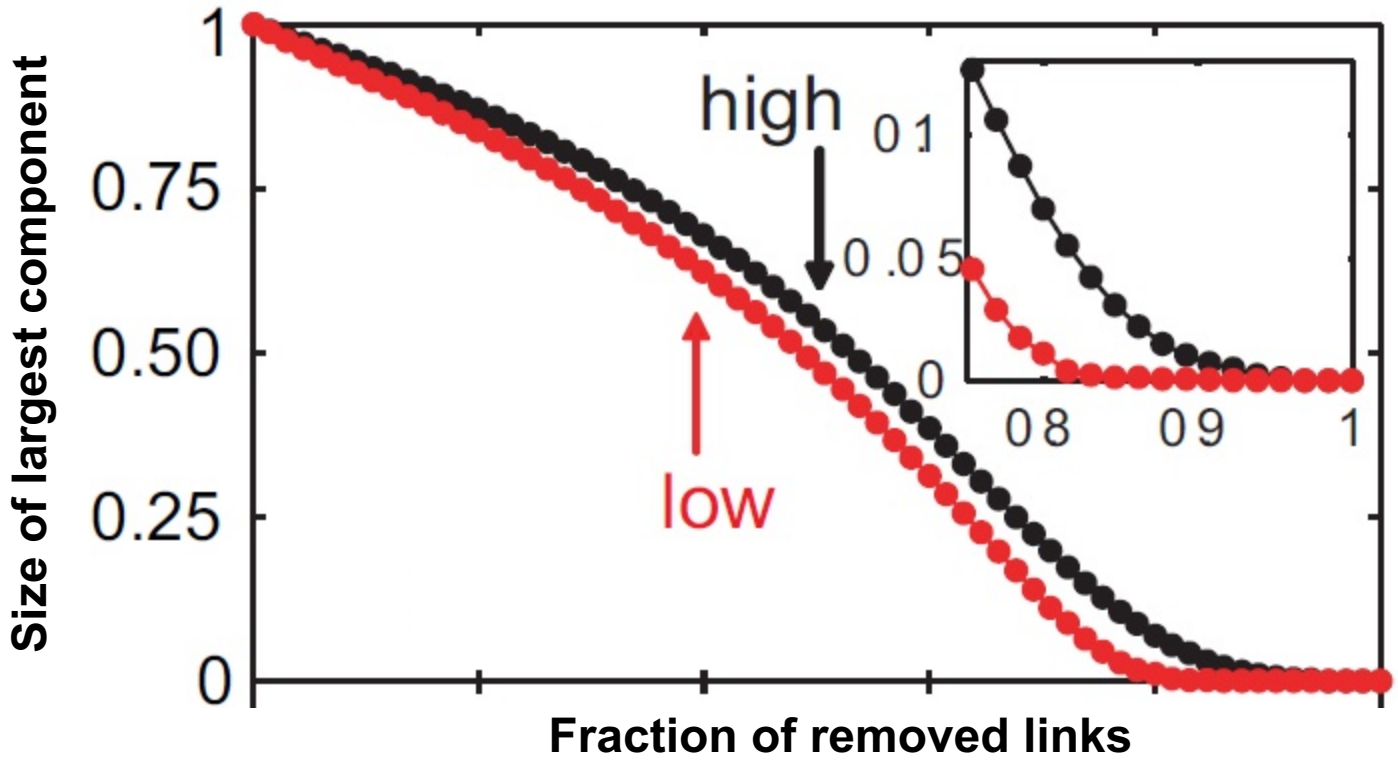
- **Real edge strengths in mobile call graph**
  - Strong ties are more embedded (have higher overlap)

Real Net, Permuted Tie Strengths



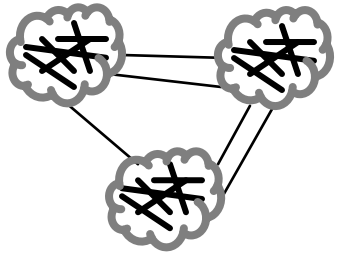
- **Same network, same set of edge strengths**
  but now **strengths are randomly shuffled**

Link Removal by Strength



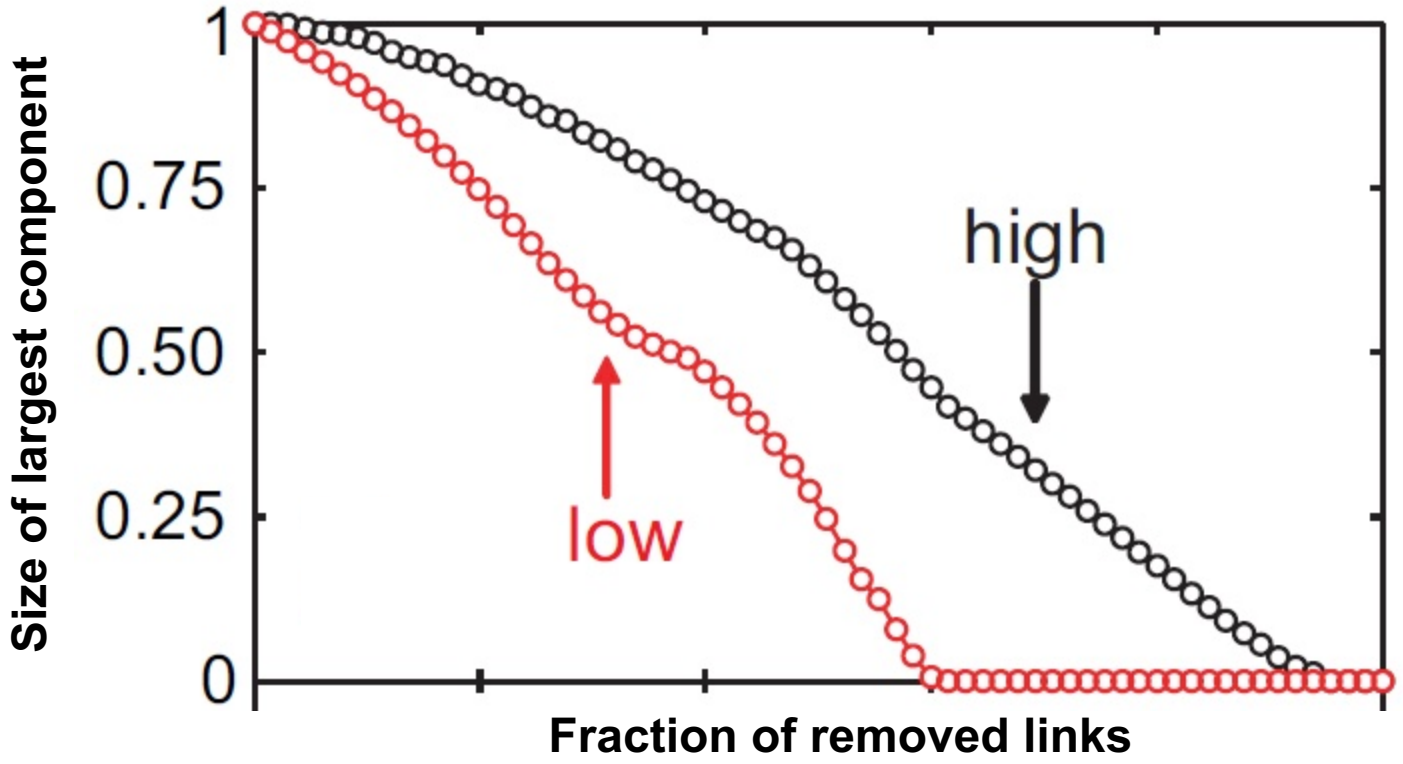
Low disconnects the network sooner

- Removing links by **strength (#calls)**
  - Low to high
  - High to low



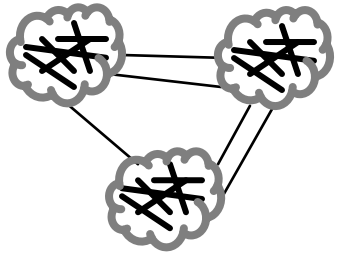
Conceptual picture of network structure

Link Removal by Overlap



Low disconnects the network sooner

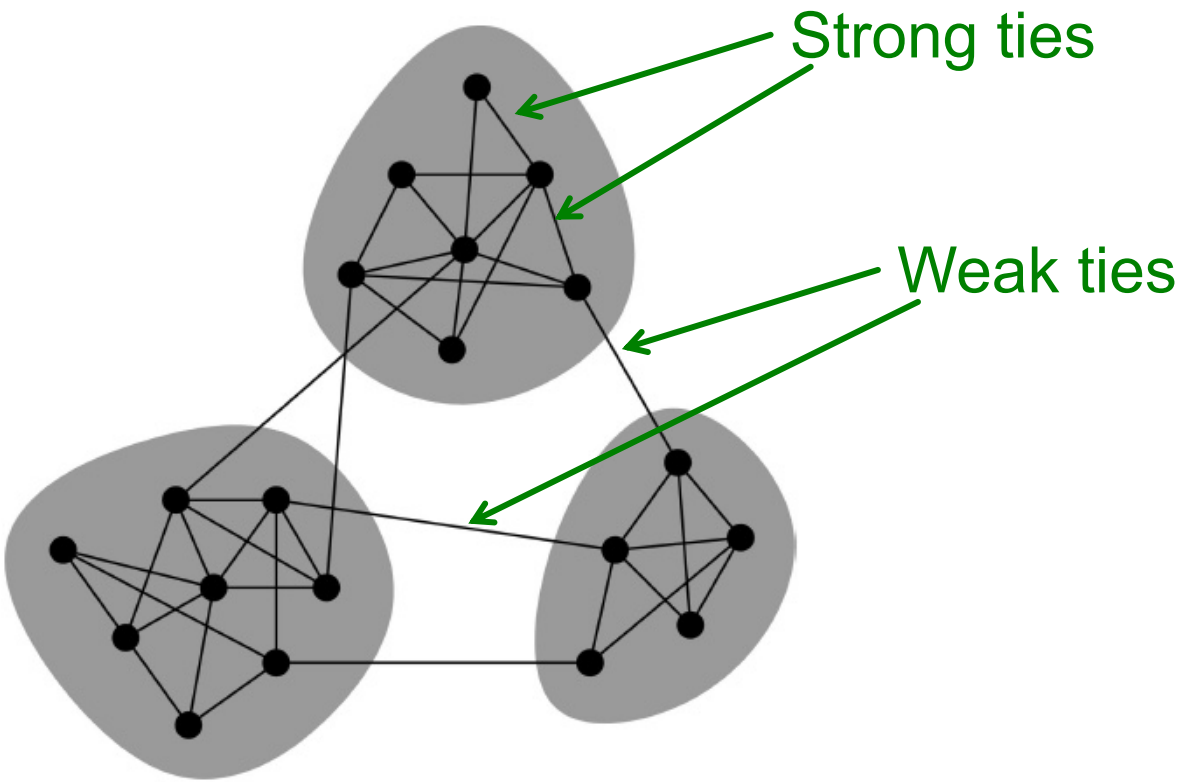
- Removing links based on **overlap**
  - Low to high
  - High to low



Conceptual picture of network structure

Conceptual Picture of Networks

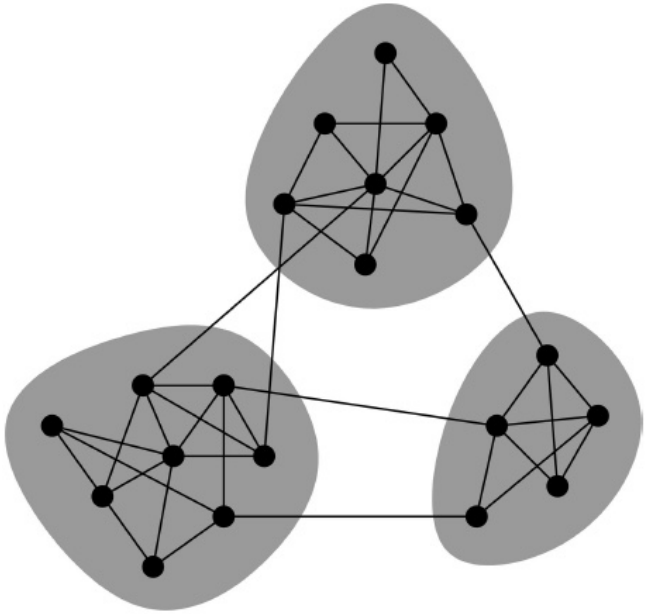
- **Granovetter's theory leads to the following conceptual picture of networks**



Network Communities

Network Communities

- Granovetter's theory suggest that networks are composed of **tightly connected sets of nodes**

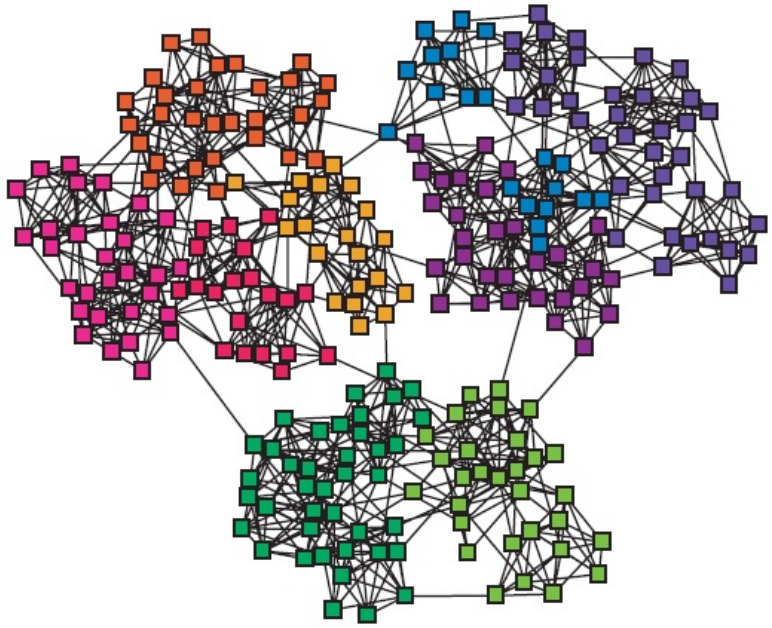


- **Network communities:**
  - Sets of nodes with **lots** of **internal** connections and **few external** ones (to the rest of the network).

Communities, clusters, groups, modules

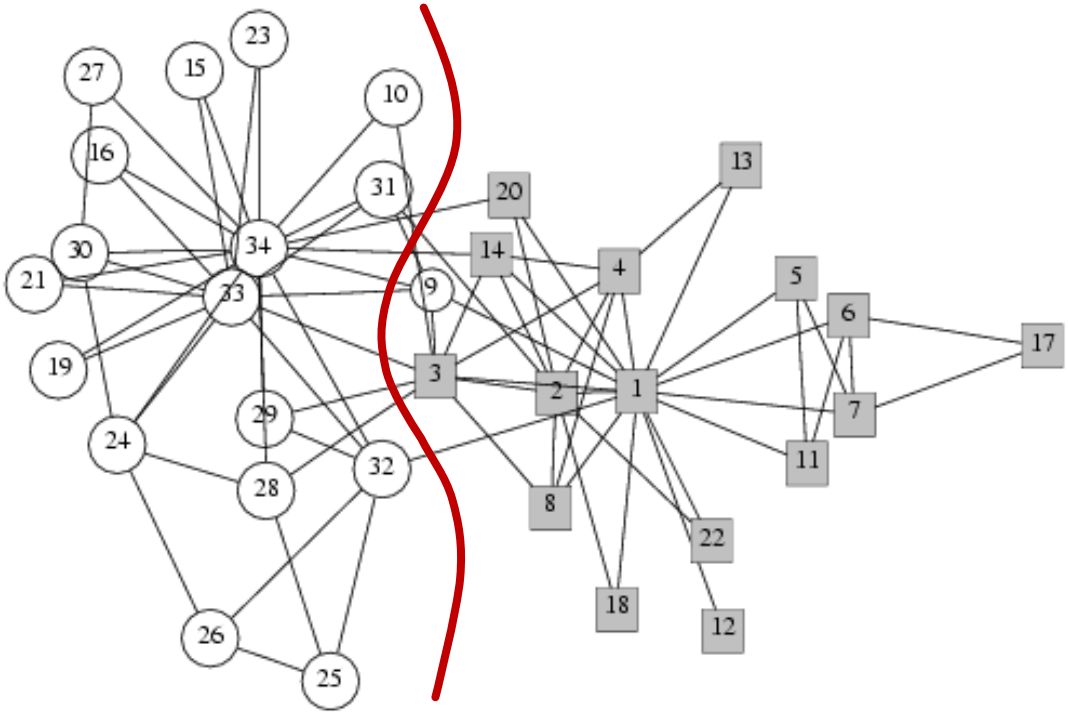
Finding Network Communities

- **How to automatically find such densely connected groups of nodes?**
- Ideally such automatically detected clusters would then correspond to real groups
- **For example:**



Communities, clusters, groups, modules

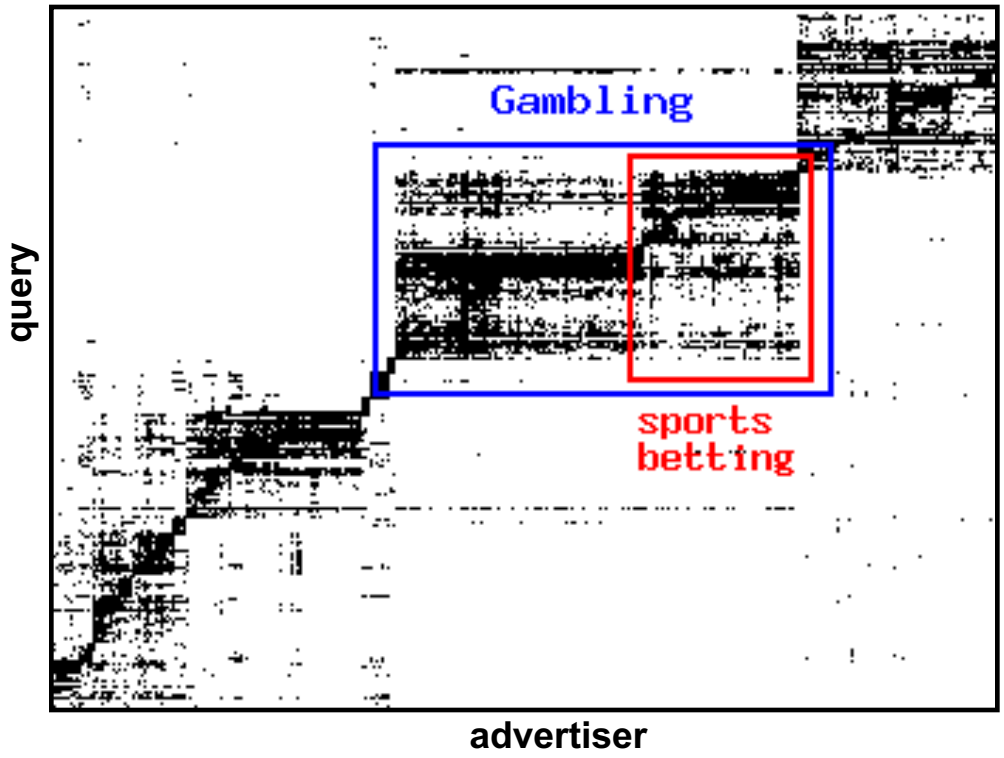
Social Network Data



- **Zachary's Karate club network:**
  - Observe social ties and rivalries in a university karate club
  - During his observation, conflicts led the group to split
  - Split could be explained by a minimum cut in the network

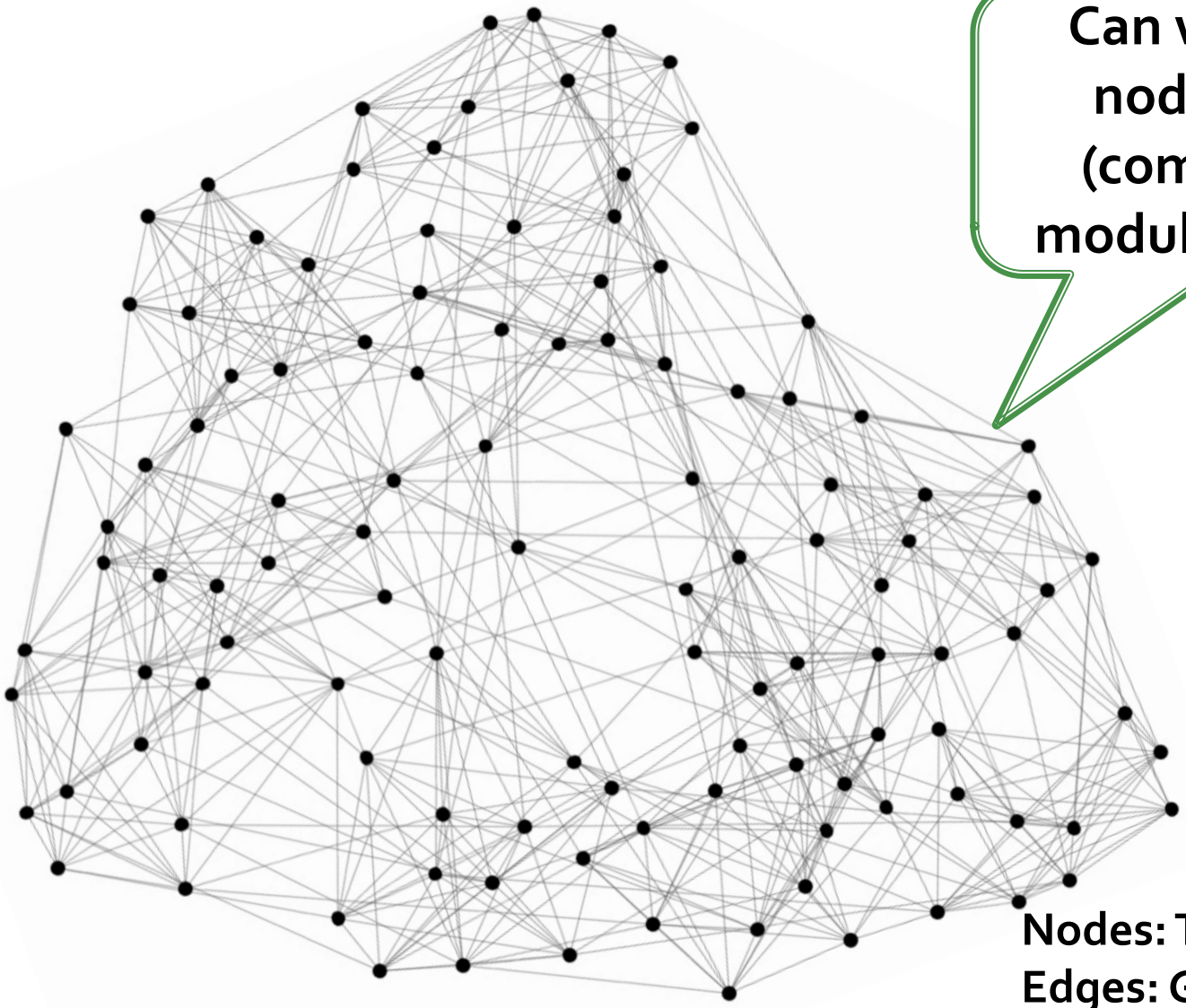
Micro-Markets in Sponsored Search

Find micro-markets by partitioning the "query-to-advertiser" graph in web search:



Nodes: advertisers and queries/keywords; Edges: Advertiser advertising on a keyword.

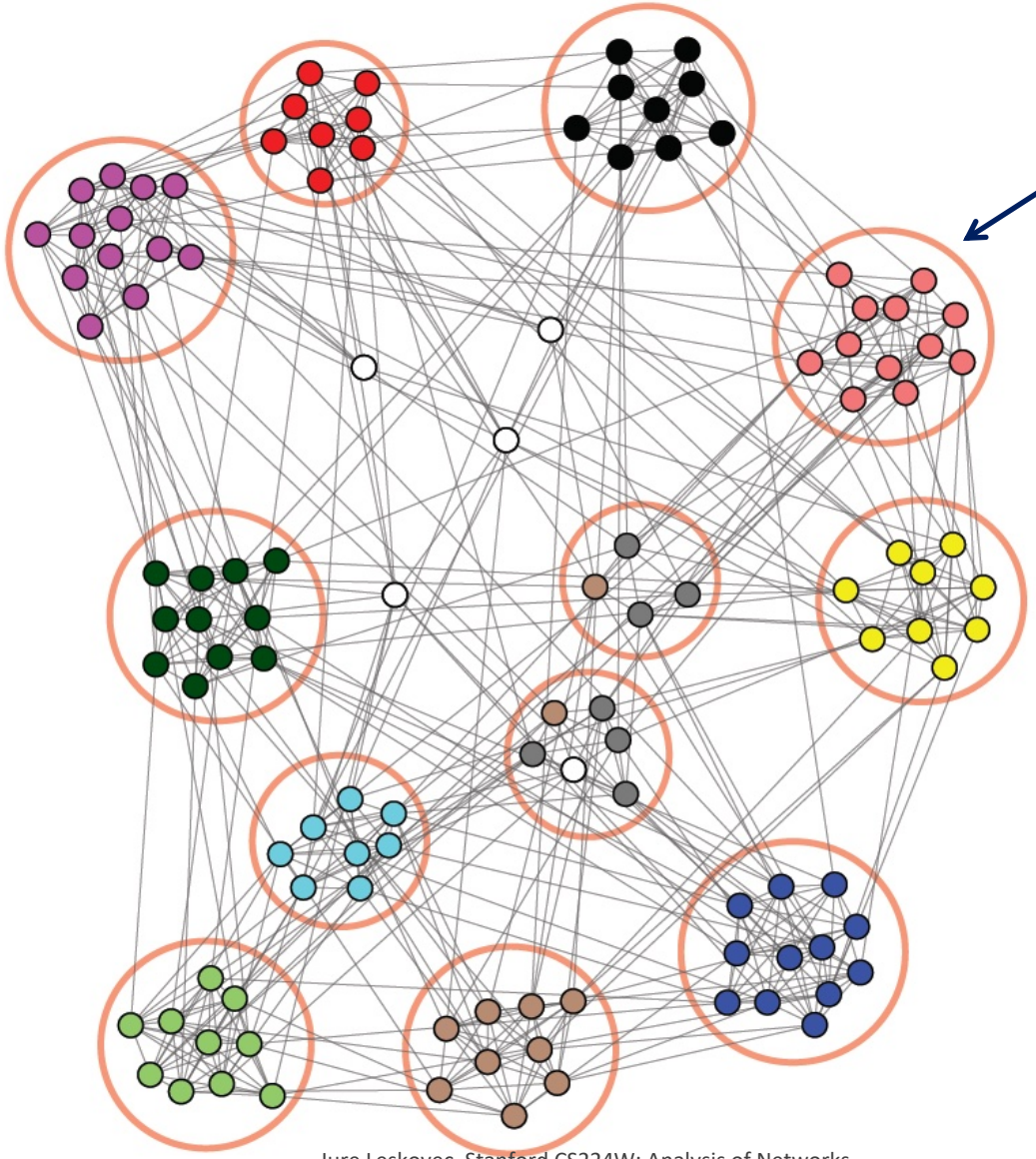
NCAA Football Network



Can we identify node groups? (communities, modules, clusters)

Nodes: Teams
Edges: Games played

NCAA Football Network

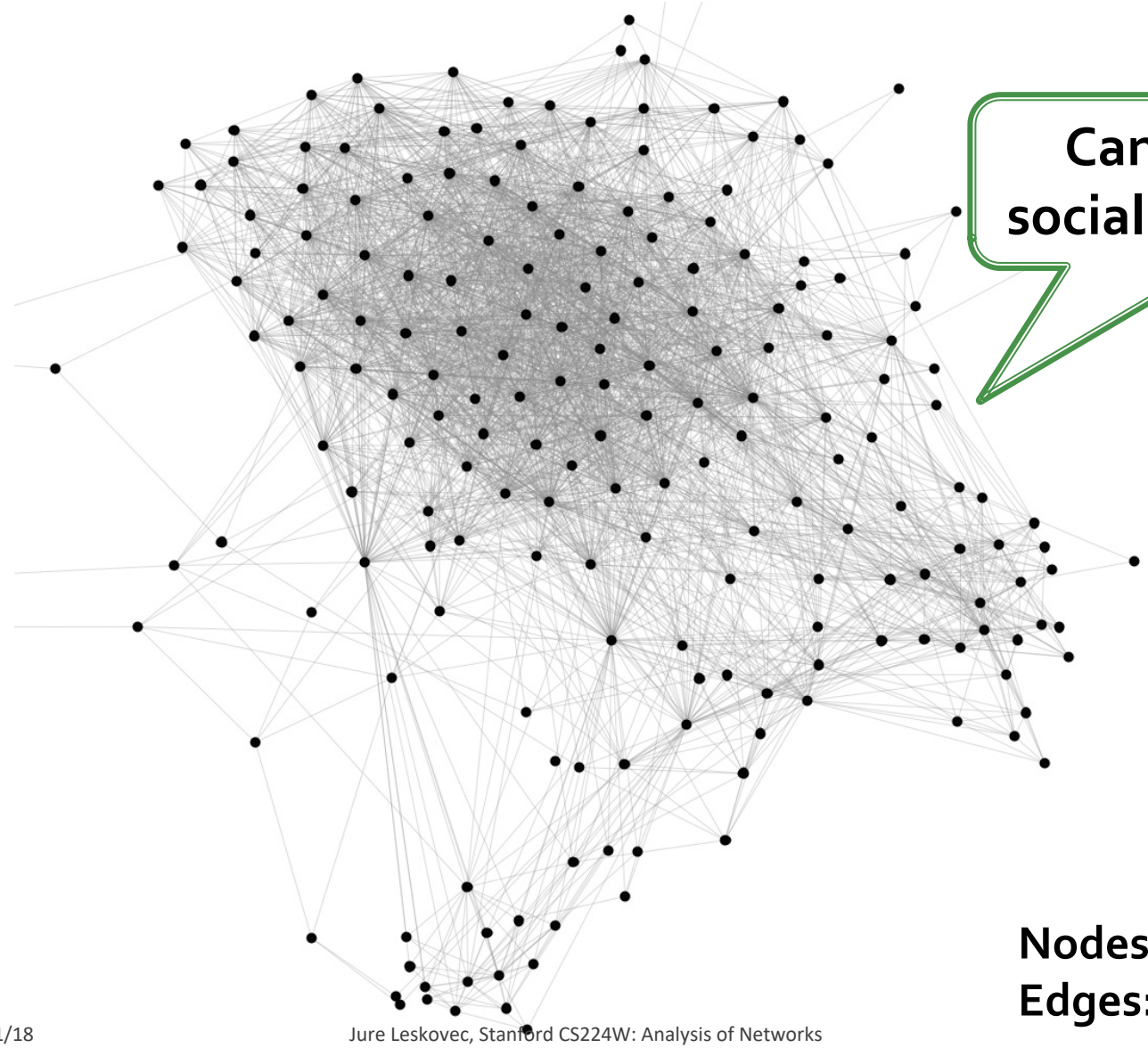


NCAA conferences

- Mid American
- Big East
- Atlantic Coast
- SEC
- Conference USA
- Big 12
- Western Athletic
- Pacific 10
- Mountain West
- Big 10
- Sun Belt
- Independents

Nodes: Teams
Edges: Games played

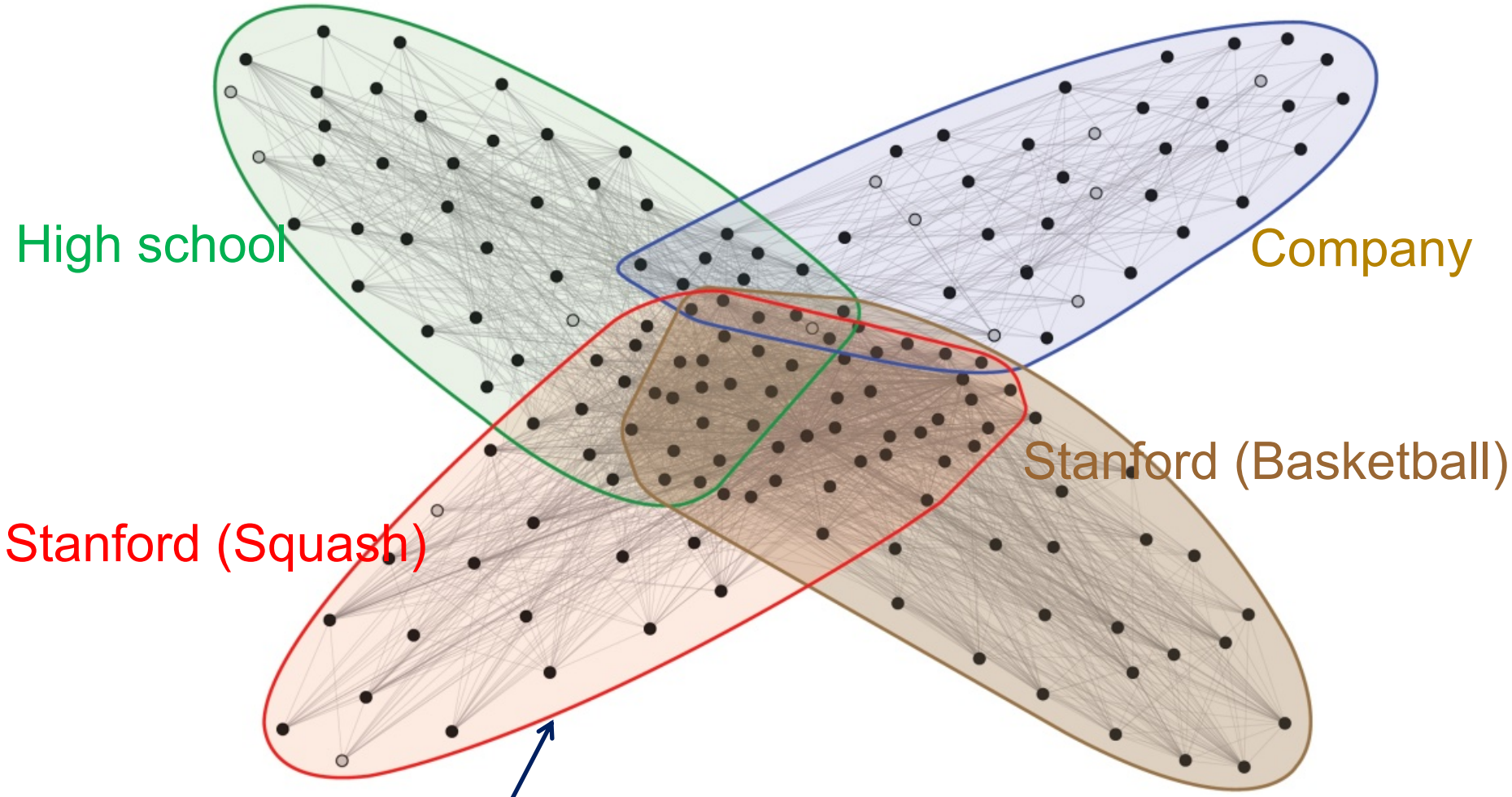
Facebook Ego-network



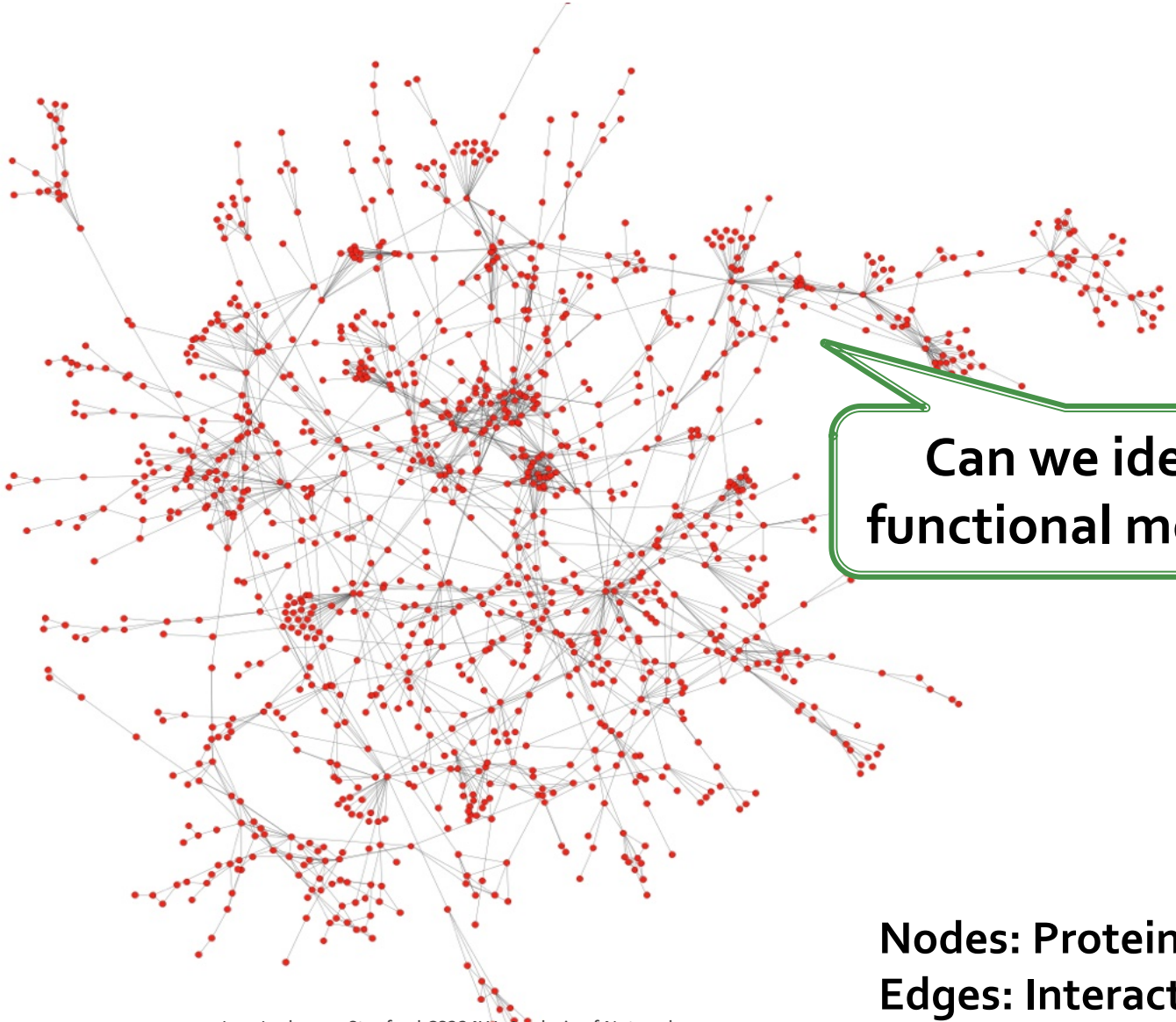
Can we identify social communities?

Nodes: Users
Edges: Friendships

Facebook Ego-network



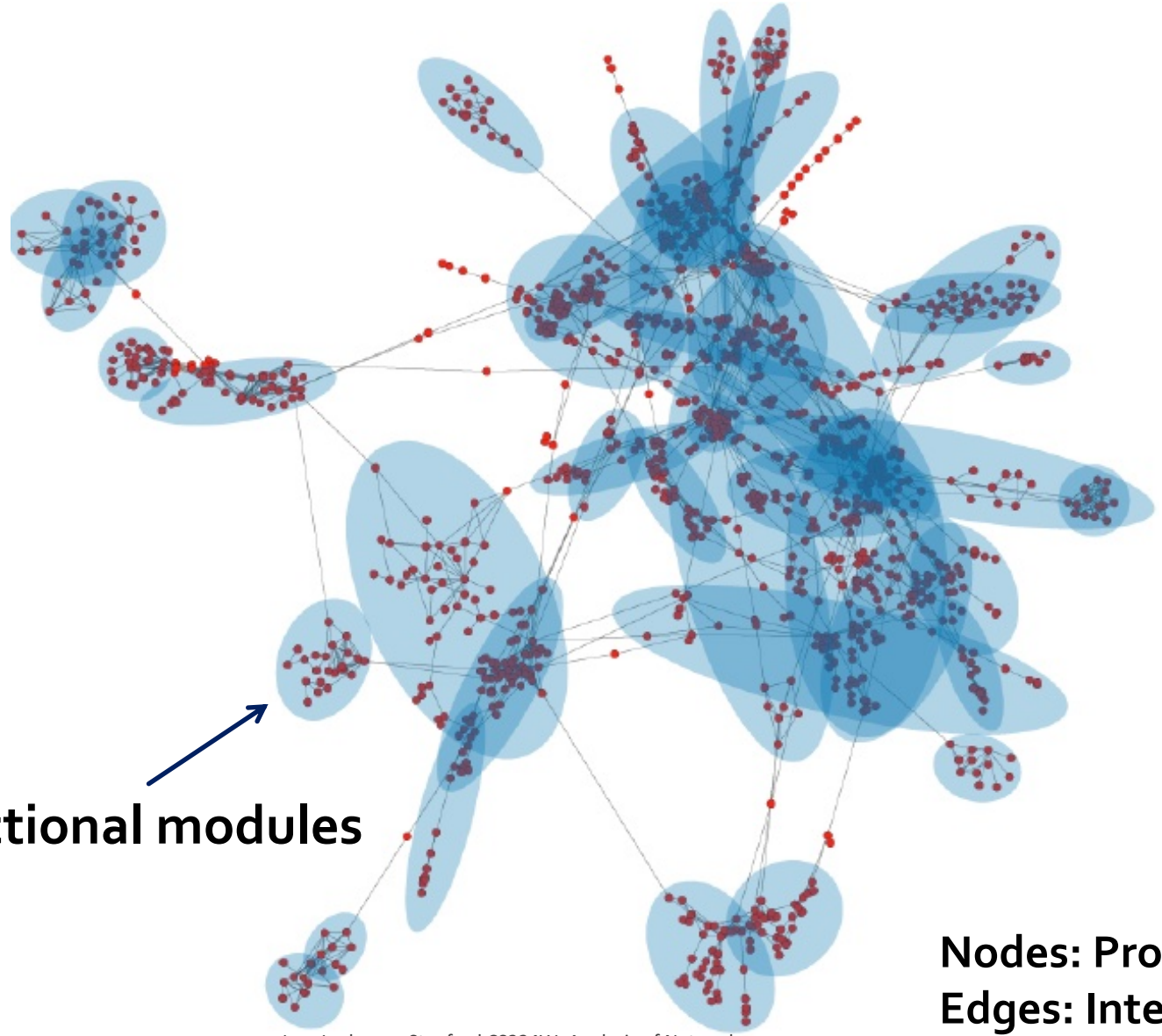
Protein-Protein Interactions



Can we identify functional modules?

Nodes: Proteins
Edges: Interactions

Protein-Protein Interactions

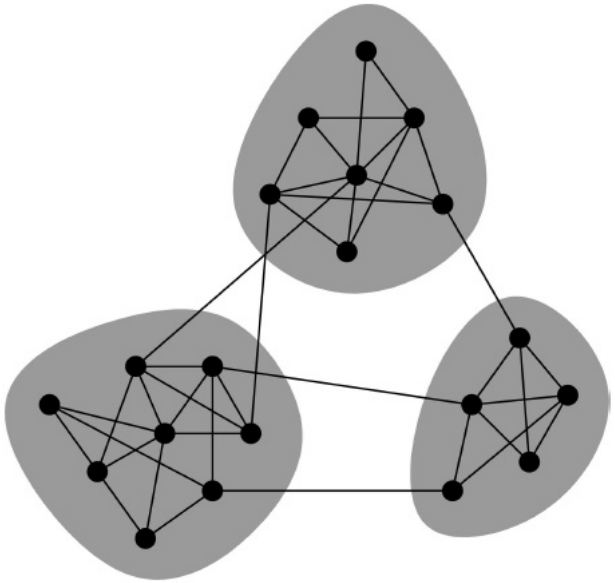


Functional modules

Nodes: Proteins
Edges: Interactions

Network Communities

- **Communities:** sets of **tightly connected nodes**
- Define: **Modularity Q**
  - A measure of how well a network is partitioned into communities
  - Given a partitioning of the network into groups $s \in S$:

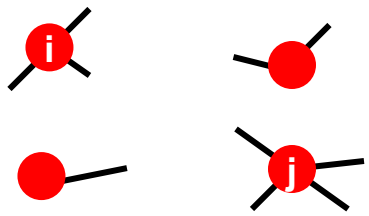


$$Q \propto \sum_{s \in S} [\ (\#\ \text{edges within group } s) - (\text{expected } \#\ \text{edges within group } s)\ ]$$

Need a null model!

Null Model: Configuration Model

- **Given real G on n nodes and m edges, construct rewired network G'**
  - Same degree distribution but random connections
  - Consider G' as a multigraph
  - **The expected number of edges between nodes i and j** of degrees k_i and k_j equals to: $k_i \cdot \frac{k_j}{2m} = \frac{k_i k_j}{2m}$
    - The expected number of edges in (multigraph) **G'**:
      - $= \frac{1}{2}\sum_{i \in N}\sum_{j \in N}\frac{k_i k_j}{2m} = \frac{1}{2}\cdot\frac{1}{2m}\sum_{i \in N} k_i\left(\sum_{j \in N} k_j\right) =$
      - $= \frac{1}{4m} 2m \cdot 2m = m$



Note:
$\sum_{u \in N} k_u = 2m$

Modularity

- **Modularity of partitioning S of graph G:**
  - **Q** $\propto \sum_{s \in S}$ **[(# edges within group** s**) –**
    (expected # edges within group s**)]**
  - $Q(G,S) = \frac{1}{2m} \sum_{s \in S} \sum_{i \in s} \sum_{j \in s} \left(A_{ij} - \frac{k_i k_j}{2m} \right)$

    Normalizing const.: -1<Q<1

    A_{ij} = 1 if i→j, 0 else
- **Modularity values take range [–1,1]**
  - It is positive if the number of edges within groups exceeds the expected number
  - **Q** greater than **0.3-0.7** means **significant community structure**

RECAP: Modularity

$$Q(G,S) = \frac{1}{2m}\sum_{s\in S}\sum_{i\in s}\sum_{j\in s}\left(A_{ij} - \frac{k_i k_j}{2m}\right)$$

Equivalently modularity can be written as:

$$Q = \frac{1}{2m}\sum_{ij}\left[A_{ij} - \frac{k_i k_j}{2m}\right]\delta(c_i, c_j)$$

- A_{ij} represents the edge weight between nodes i and j;
- k_i and k_j are the sum of the weights of the edges attached to nodes i and j, respectively;
- $2m$ is the sum of all of the edge weights in the graph;
- c_i and c_j are the communities of the nodes; and
- δ is an indicator function

Idea: We can identify communities by maximizing modularity

Louvain Modularity

Louvain Algorithm

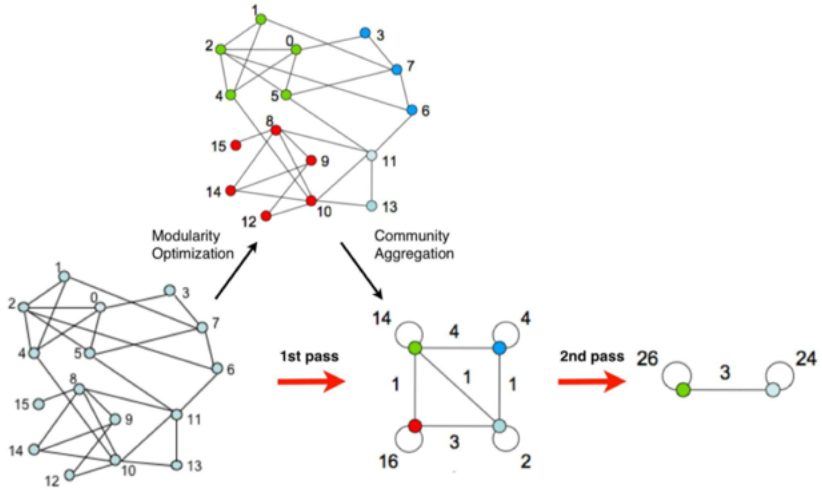
- **Greedy algorithm** for community detection
  - O(n log n) run time
- Supports weighted graphs
- Provides hierarchical partitions
- Widely utilized to **study large networks** because:
  - Fast
  - Rapid convergence properties
  - High modularity output (i.e., "better communities")

"Fast unfolding of communities in large networks" Blondel et al. (2008)

Louvain Algorithm: At High Level

- Louvain algorithm **greedily maximizes** modularity
- **Each pass is made of 2 phases:**
  - **Phase 1:** Modularity is optimized by allowing only local changes of communities
  - **Phase 2:** The identified communities are aggregated in order to build a new network of communities
  - **Goto Phase 1**

The passes are repeated **iteratively** until no increase of modularity is possible!



Louvain: 1st phase (partitioning)

- Put each node in a graph into a distinct community (one node per community)
- For each node *i*, the algorithm performs two calculations:
  - Compute the modularity gain (ΔQ) when putting node i into the community of some neighbor j
  - Move i to a community of node j that yields the largest gain ΔQ
- The loop runs until no movement yields a gain

Louvain: Modularity Gain

What is ΔQ if we move node i to community C?

$$\Delta Q(i \to C) = \left[\frac{\Sigma_{in}+k_{i,in}}{2m} - \left(\frac{\Sigma_{tot}+k_i}{2m}\right)^2\right] - \left[\frac{\Sigma_{in}}{2m} - \left(\frac{\Sigma_{tot}}{2m}\right)^2 - \left(\frac{k_i}{2m}\right)^2\right]$$

- where:
  - Σ_{in}… sum of link weights between nodes in C
  - Σ_{tot}… sum of all link weights of nodes in C
  - $k_{i,in}$… sum of link weights between node i and C
  - k_i… sum of all link weights (i.e., degree) of node i

Σ_{in}:

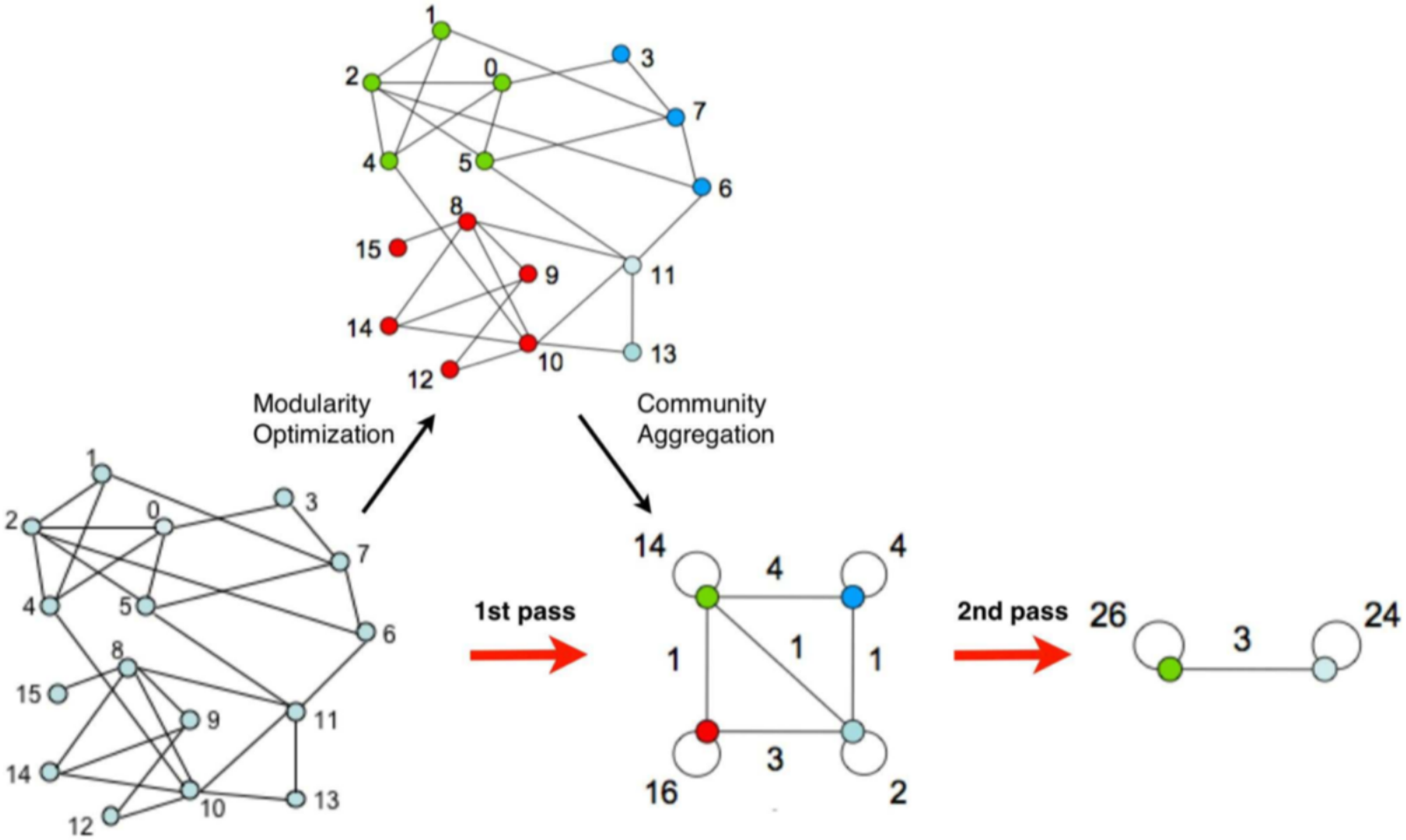
Σ_{tot}:

- Also need to derive $\Delta Q(D \to i)$ of taking node i out of community D.
- And then: $\Delta Q = \Delta Q(i \to C) + \Delta Q(D \to i)$

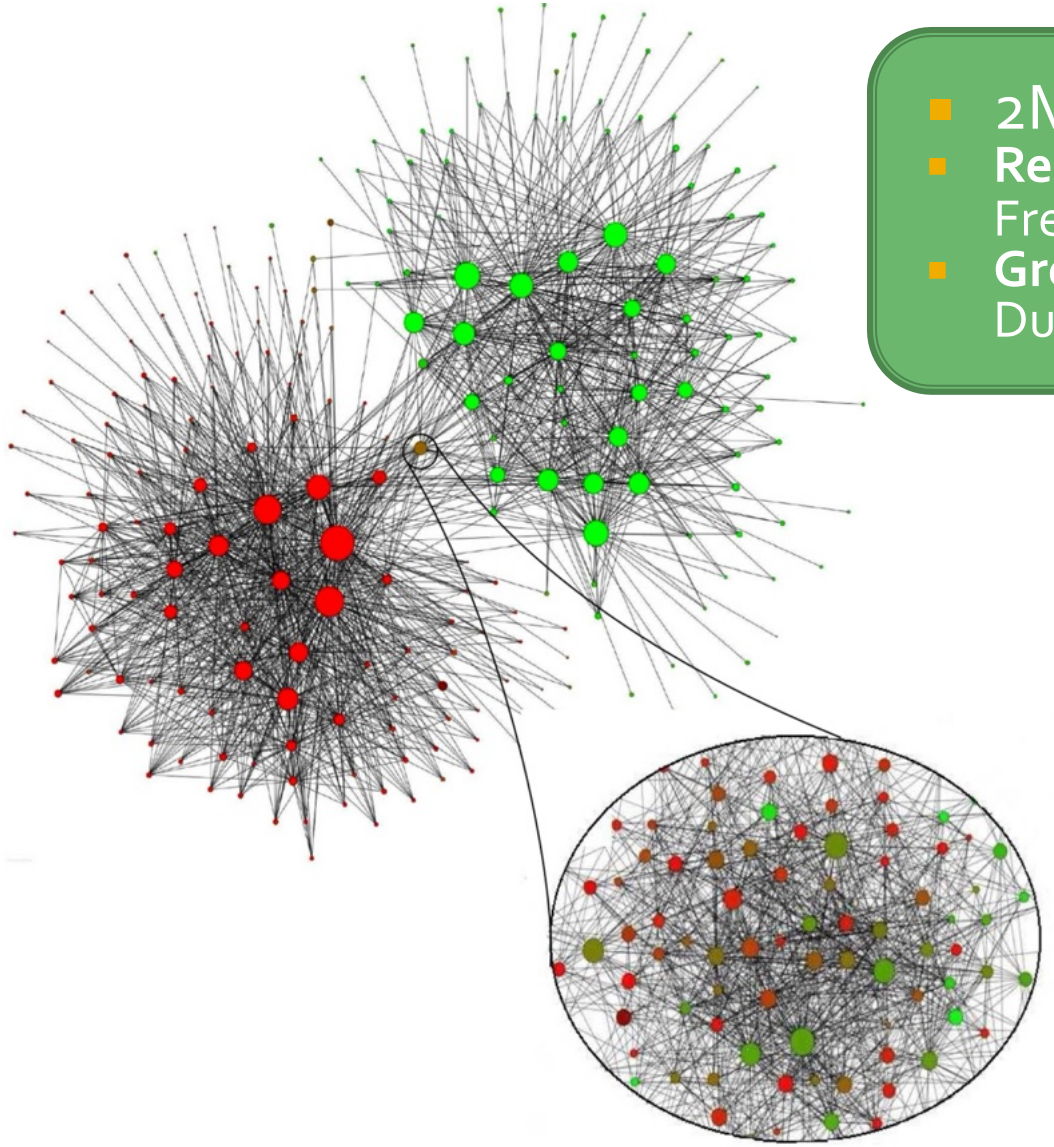
Louvain: 2nd phase (restructuring)

- The partitions obtained in the first phase are contracted into **super-nodes**, and the network is created accordingly
  - Super-nodes are connected if there is at least one edge between nodes of the corresponding partitions
  - The weight of the edge between the two super-nodes is the sum of the weights from all edges between their corresponding partitions
- **The loop runs until the community configuration does not change anymore**

Louvain Algorithm



Belgian Mobile phone network



- 2M nodes
- **Red nodes**: French speakers
- **Green nodes**: Dutch speakers