Modeling the Universe: Three Essential Topics

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Modeling the Universe: Three Essential Topics

In observing the solar system and universe, three core topics come into play. Represented in their simplest forms, these topics can model the universe in three dimensional space across time. This forms a simple yet comprehensive view of the basic mechanics of the universe. The way that the planets, stars and other celestial bodies interact with each other can be extrapolated from the principles these three topics introduce. Each of these three core topics are introduced chronologically based on the order in which they were scientifically discovered. As such, each topic builds upon and gives context to the previous topics. The first topic introduced comes from Johannes Kepler and his Three Laws of Planetary Motion, which govern the movement of planetary bodies in orbit. The second topic comes from Issac Newton and his laws of gravity and motion. These laws provide a firm theoretical basis for Kepler's Laws because they model the gravitational attractions and movements among objects in space. More importantly, they show how the physical laws observed on earth match those observed in space. The third topic comes from Einstein and his Theories of Special Relativity and General Relativity. While relativity may produce negligible effects on the movements of most planetary bodies operating at non-extreme speeds and masses, it recognizes the limits of Newton's Laws. Einstein's Theories of Relativity further contextualize the concepts of space, time and gravity. Ultimately, these three core topics work together to form a simplified yet powerful view of the mechanics of the universe. Kepler's Laws, Newton's Laws, and Einstein's Theories of Relativity all bring into focus a more complete picture of the universe. Understanding these three core topics fosters a greater understanding and appreciation of the mechanics and behaviors of the observed universe.

Topic 1: Kepler's Laws

Using the Copernican model of a heliocentric (sun-centered) solar system, German astronomer Johannes Kepler derived mostly from experimental data three core laws relating to planetary motion (Current Events, 2011). These laws detail how planetary bodies move while in orbit, something that perplexed astronomers for millennia (Current Events, 2011). Even with the knowledge of a heliocentric solar-system, many found it difficult to reconcile the observed differences in orbital speed and length of planetary bodies. Astronomers did not understand enough about orbit to consistently predict the movement of the planets from a heliocentric perspective. Kepler's Laws allowed scientists to finally predict the motion of planetary bodies accurately and over long periods of time (Davis, 2010). However, it would take until Isaac Newton's Laws of Motion and Gravitation for Kepler's Laws to be explained theoretically (NASA Earth Observatory, 2009). From Kepler's insight into the motion of planetary bodies, scientists finally gained a more accurate and clear picture of the solar system.

Kepler's Law of Ellipses

Kepler's First Law states that planetary orbits are elliptical, with the sun as one of the foci (Kepler, 1995). This law is also commonly referred to as "The Law of Ellipses" (the Physics Classroom 2020). In Kepler's First Law, the foci serve as "centers" of the ellipse, meaning that an orbiting body is always as close to one of the foci as possible. In a circle, both foci rest directly on top of each other. Whereas previously orbits were thought to be purely circular (where both foci would be in the center), Kepler provided further context by showing that orbits actually operate in ellipses, rather than perfect circles.

However, a circular-orbit theory did not match a lot of the experimental data astronomers at the time observed. This inconsistency in data versus theory led many scholars to wildly

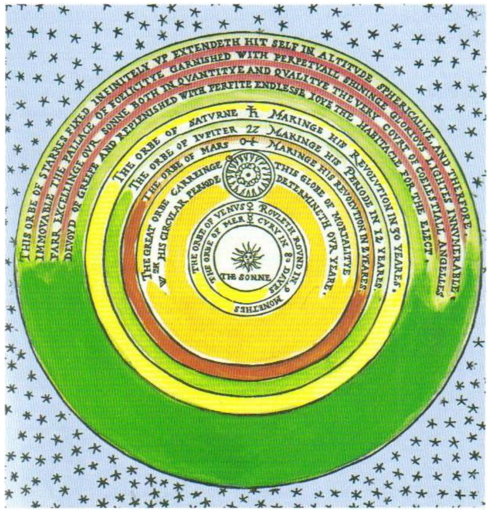

Figure 1: Kepler and his contemporaries called this model the "music of the spheres" because the orbits were similar in proportion to harmonic scales in music (Kepler, 1995).

different conclusions. For instance, some believed that angels flapped their wings at the planets to make them move in elliptical rather than circular orbits (Simanek, 2006).

While Kepler believed in the Copernican model of a sun-centered universe, he could not reconcile the Copernican idea of circular orbits with experimental data (Kepler, 1995). Specifically, Kepler could not reconcile the highly elliptical orbit of Mars with a circular orbit theory (Davis, 2010). He needed a more verifiable answer. This would lead Kepler to develop his first law of planetary motion, showing that planets orbit in ellipses, rather than perfect cirlces.

Kepler's First Law provided a solid mathematical basis for the observed motion of planets. This "Law of Ellipses" best represents itself in polar coordinates by the following equation:

$$r = \frac{p}{1 + e\cos\theta}$$

In this equation, r represents the radius of the planet to the sun, p represents the period (or time it takes for the planet to make a full revolution), and the eccentricity e is defined as a value between 0 and 1.

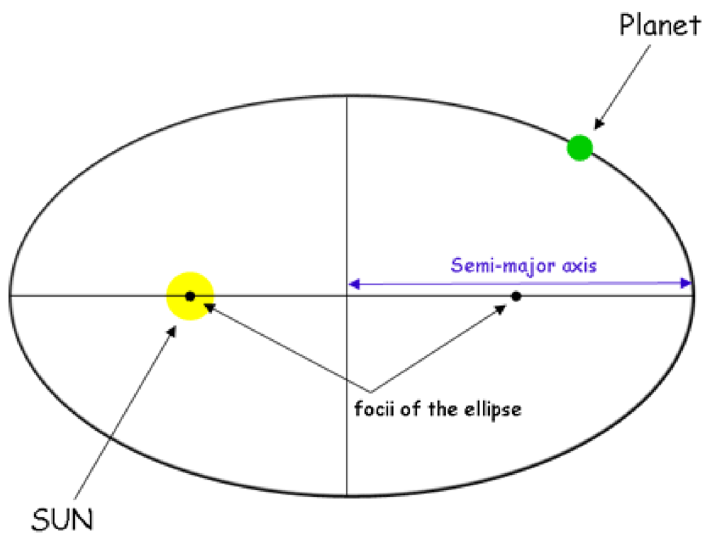

Figure 2: Kepler's First Law states that an object orbits in an ellipse with the sun at one of the foci of the ellipse as represented above

When e stays between 0 and 1, the polar equation produces an ellipse representing planetary orbit. When eccentricity e is zero, the ellipse becomes a perfect circle. When e becomes greater than 1, the ellipse breaks into a hyperbolic shape. This explains why some celestial bodies do not orbit the sun at all, but rather swing past it. The eccentricity e of their orbit would be greater than 1. Kepler's equation shows how the movement of such a body would produce a hyperbola, not an ellipse around a given star.

Kepler's Law of Equal Areas

Kepler's Second Law states that the radius from the sun to a planet sweeps an equal area for an equal time interval (the Physics Classroom, 2020). This law is also referred to as "The Law of Equal Areas" and it best represents itself best by *Figure 3*

From *Figure 3* it is observed that both blue regions have an equal amount of area, but they cover up different sized chunks of the ellipse's circumference. When the planet is further away from the sun, it is able to sweep more area while covering less of the ellipse's circumference. In other words, an orbiting object travels faster when it is closer to the sun,

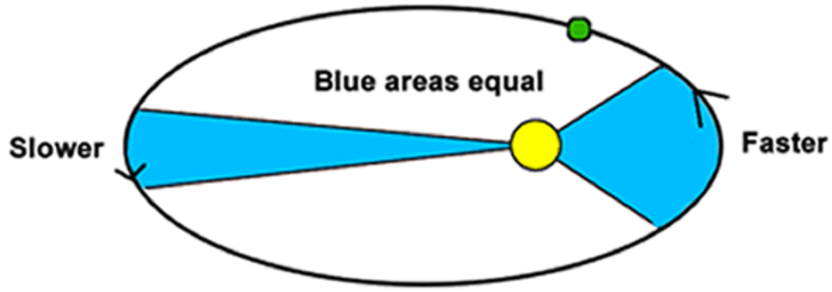

Figure 3: It takes an orbiting body an equal amount of time to cover both blue areas.

and slower when it is further away from it. Whereas Kepler's First Law gives only the shape of an orbit, Kepler's Second Law also gives the orbital speed of a planet, represented by the equation:

$$\frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{rv_\theta}{2}$$

Where the change in area over time equals one-half of the radius multiplied by the velocity of the angle θ. Area refers to the blue region in *Figure 3*, and the velocity of the angle represents the speed at which the planet orbits around one of the foci.

Put simply, Kepler's Second Law (or Law of Equal Area) dictates the differences in speed an object takes based on its position in orbit.

Kepler's Law of Harmonics

Kepler's Third Law, also referred to as "The Law of Harmonies", states that the square of a planet's orbital period and the cube of the length of the semi-major axis of its orbit are proportional to each other (Astronomy, 2015). This law came nearly 10 years after Kepler's first two laws, and it can be represented as follows:

$$P^2 = \frac{4\pi^2 a^3}{k^2(M_{sun} + M_{earth})}$$

Where P represents the period (how far it takes for the planet to complete a revolution), a represents the distance from the sun and k represents a constant referred to as the Gaussian constant (NASA Earth Observatory, 2009). From this equation, it can be determined how far any planetary body is from the sun given the masses of both objects and the orbital period. Whereas the first two laws focus on the movement of one particular planet, Kepler's Third Law focuses on the relationship between multiple planets (Astronomy, 2015). This allowed astronomers to estimate the distances between each of the planets and the sun. Because the proportionality of the period to the length of the semi-major axis is known, more knowledge can be gained into the orbits of other planets in the same solar system as earth. Kepler's Third Law allowed astronomers to estimate how far each of the planets were from the sun and the earth (Astronomy, 2015).

Topic 2: Newton's Laws

Newton's Laws of Motion and Gravitation form the basis for how objects and gravity act at most scales. Previously, scientists such as Galileo had derived equations for calculating the movement of objects (Toshiya, 2013). These equations are known as kinematics, and they introduced the associations between velocity, acceleration, and time. However, pre-Newtonian kinematics left many problems relating to the movement of objects unsolvable. That all changed when in 1686 a student named Issac Newton published his three Laws of Motion and Law of Gravitation in his work titled *Philosophiæ Naturalis Principia Mathematica* (Allain, 2013). Through the use of Calculus, Newton could better map the relationships between acceleration, velocity and time. Newton's Three Laws of Motion also introduced the concept of calculable force, measured in Newtons (Allain, 2013). With this discovery, Newton added much more capabilities to the field of kinematics. With both kinematics and Newton's equations, one can predict the direction, force, acceleration, and velocity of an object over time. From his work in *Philosophiæ Naturalis Principia*

Mathematica, Newton also introduced the concept of gravity (Newton & Cajori, 1974). Newton found that all objects exert a certain amount of gravitational force on each other. Newton modeled this force with a simple equation that would take nearly a century to experimentally verify (Nordtvedt & Faller, 2019). Both Newton's Law of Gravitation and Newton's Laws of Motion form the basis of the field of classical mechanics, which details and describes much of the interactions between objects in the universe.

Newton's Second Law

Arguably the most famous of Newton's Three Laws of Motion is Newton's Second Law. Newton's Second Law states that the sum of the forces on an object is equal to an object's mass times its acceleration, as shown below (Newton & Cajori, 1974).

$$\sum F = ma$$

The equation works with $\sum F$ representing the sum of the forces acting on an object, m representing the mass of the object, and a representing the acceleration (change in speed over time, or $\frac{d(v)}{dt}$). This equation is perhaps one of the most important in all of physics, and it forms a fundamental part of classical (also referred to as Newtonian) mechanics. Using Newton's Second Law, one can solve for many variables relating to an object's motion. When an object's acceleration is zero, the sum of the forces is also zero. This is why Newton's First Law is represented by $\sum F = 0$ for objects moving at a constant acceleration. However, as both mass and acceleration increase, Newton's Second Law becomes less and less experimentally accurate (Bais, 2007). For objects operating at higher speeds and masses, Einstein's Theories of Relativity must be used (Bais, 2007).

Newton's Third Law

Newton's Third Law states that "for every action, there is an equal and opposite reaction" (Allain, 2013). In other words, the magnitude of the force an object A exerts on an object B equals the magnitude of the force an object B exerts on an object A. If two objects collide with each other, they both exercise the same amount of force on each other. In Newton's own words translated into English, "All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction" (Newton & Cajori, 1974). This allows scientists to perform accurate measurements on the force two planetary bodies exert on each other, because both forces are equivalent.

Newton's Law of Universal Gravitation

Another law that is very relevant to planetary motion is Newton's Law of Universal Gravitation (Newton & Cajori, 1974). This law allows one to calculate the gravitational force two objects exert on each other (remember, force acts equivalently on both parties per Newton's Third Law). Newton's Law of gravitation best represents itself below:

$$F_g = \frac{G \cdot m_1 \cdot m_2}{r^2}$$

Where F_g represents the gravitational force, m_1 and m_2 represent the two objects, and r represents the distance between both objects' centers of mass. Notice that G (commonly known as "big G" or "the gravitational constant" or "Newton's constant") is a constant that seems to come out of nowhere (Stern, 2006). When Newton initially derived the formula for universal gravitation, he knew he had a constant missing, but did not know how to find it (Stern, 2006). Without knowing the mass of the earth, Newton could not solve the rest of his equation for G. Modern calculations estimate that this gravitational constant G equals around $6.67 * 10^{-11} Nm/kg^2$, but it took over 100 years after Newton introduced gravity for scientists to find an accurate value for G (Nordtvedt & Faller, 2019).

Calculating G

Before, Newton believed he could potentially calculate G using a pendulum on both a hill and a valley, but he believed the changes between both pendulums due to gravity would end up too tiny to measure (Newton & Cajori, 1974). British experimenter and physicist Henry Cavendish had a better idea. He used a torsional balance to calculate a value for the mass of the earth, which would later be used to solve for G (Levere, 2020). Cavendish set up the experiment similar to *Figure 4*.

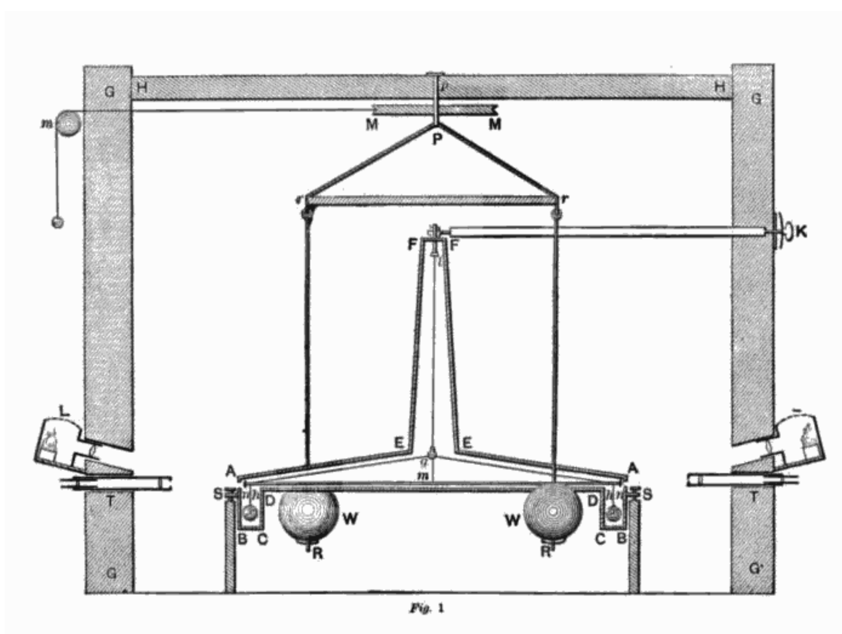

Figure 4: Model of Cavendish's experiment. Modern experiments follow this same basic model to calculate G to greater accuracy.

The Cavendish's Machine calculated the gravitational effect the little lead balls had on the larger lead balls using a very delicate balance (Gregersen, 2019). By measuring the angle the rod moved to, Cavendish determined the gravitational force between the two balls. Because of the sensitivity of Cavendish's instrument, he could obtain accurate measurements for the mass of the earth, which could be plugged into Newton's equation and solved for G (Gregersen, 2019). However, this meant that the simple bustle of traffic or unnoticeable seismic shifts would affect the results of the experiment drastically (Walker, 1997). This contributed much annoyance to Cavendish, eventually forcing him to move his experiment to a basement where such disturbances became less frequent (Walker, 1997). Luckily, this extra precaution helped Cavendish find a fairly accurate value for the mass of the earth, and therefore G.

Today, scientists use modern versions of Cavendish's experiment to calculate G more accurately (Gregersen, 2019). However, in part because of the difficulty in calculating G to a higher degree, some scientists believe that G may in fact not be a constant at all, but rather something that slightly changes over time (Mann, 2016).

With this G constant, one can calculate the gravitational force between any two bodies. This constant is relevant to planetary motion, where objects behave according to the amount of gravitational force they exert on each other. The addition of this G constant also proves Newton's Law of Gravity with experimental evidence, because the observed gravitational force on earth matches the predicted gravitational force from Newton's Law of Universal Gravitation.

Topic 3: Einstein and Relativity

The third and final set of principles relevant to the motions of celestial bodies in the universe are Einstein's Theories of both Special and General Relativity. Relativity, while crucial to the understanding of the universe, harbors negligible effects on the observations of most astronomical occurrences. However, it ties together and addresses observed flaws in Newton's view of physics (Susskind & Friedman, 2019).

Unfortunately, classical mechanics cannot describe all objects in motion. Newton himself observed certain flaws in his theory, such as a lack of an explanation for how gravity actually works (Lamb, 2010). Many reconciled this disparity by describing gravity as traveling through an immaterial "ether" (Nordtvedt & Faller, 2019). However, it would take until Albert Einstein's Theories of both Special and General Relativity for these issues to be fully reconciled. Einstein's Theories of Relativity detail the relative nature by which time operates at objects operating at extreme speeds and masses. Put simply, time depends on the observer (Howell, 2017). On most scales, Newton's equations can be used to accurately calculate velocity with respect to time. However, as velocity and mass approach extreme values, Newton's laws start to fall apart (Susskind & Friedman, 2019). Relativity addresses these problems happening at extrema.

Special Relativity

Einstein's first published theory on relativity, named the Theory of Special Relativity, mainly deals with objects moving at speeds close to the speed of light. Previously, scientists such as Galileo postulated that variables such as speed are relative (Toshiya, 2013). An object's speed depends on where it is measured. For instance, a person moving towards the front of the train may feel that they are moving only a few meters per second. However, an observer from outside of the train would say that the person moves at a rate around the velocity of the train. Conversely, an observer from outer-space would also notice that the earth carrying the train is in motion and rotates relative to the sun. Put simply, in order to assess the speed at which the passenger travels, one must pick an "inertial reference frame", because the passenger's speed changes depending on what it is being compared to. Notice that even though the speeds may be different, from each inertial reference frame, the same rules of physics apply and are used to calculate the speed. In both calculating the speed from the train, outside the train, and in outer-space, the same equations from Newton's classical mechanics are used. While this may seem intuitive to things like speed, inertial reference frames are also important in relativity.

Experiments have verified that the speed of light remains constant no matter the inertial reference frame (Redd, 2018). This means that calculating the speed of light from a train, on the earth, and in space would all yield the exact same value. This conflicts with the idea that speed depends on the observer. As stated above, the laws of physics should not change based on the reference frame. Also, the speed of light should not change based on the inertial reference frame. Up until Einstein's Theory of Special Relativity, scientists could not reconcile these two conflicting observations.

Einstein solved this dilemma of inertial reference frames at extreme speeds by showing that time itself is relative. If time differs based on the inertial reference frame it is measured from, then objects moving at extreme velocities do not surpass the speed of light and do not violate any laws of physics. Put simply, there is no "universal time", it all depends on where time is measured. Special Relativity reconciles this point. Light travels not through some ether, but through space and time itself (Susskind & Friedman, 2019). This allows the speed of light and mechanics of the universe to remain constant regardless of the reference frame. While the idea that an object moving at a higher velocity experiences time more quickly than an object moving at a slower velocity may not make intuitive sense, many experiments have verified this fact (Greene, 2020).

Einstein's Theory of Special Relativity also showed that at extreme velocities approaching the speed of light, mass begins to change along with time (Howell, 2017). Specifically, mass quickly approaches infinity. Einstein represented this relationship between mass and the speed of light with his arguably most famous equation:

$$E = mc^2$$

In Einstein's equation, E represents the energy of an object, m represents the mass of the object and c represents the speed of light. This equation also allows scientists to calculate the amount of energy stored in an object. Because of the size of c^2, Einstein's equation shows that all objects contain an immense amount of energy. This energy is harnessed through nuclear power plants and weapons (Howell, 2017). Einstein explored the relation between mass and time deeper in his subsequent Theory of General Relativity (Greene, 2020).

General Relativity

Einstein further expounded on his Theory of Special Relativity by introducing the idea that time also interacts differently on objects operating at extreme masses. Einstein's Theory of General Relativity details the fabric of the universe. It introduces the relationship between space and time, commonly referred to as the space-time continuum (Greene, 2020). This is the medium by which gravity travels. Whereas previously many scientists failed to capture the medium through which gravity travels (often resorting labelling it as an "ether"), Einstein showed that gravity travels through space and time itself (Greene, 2020). Newton himself knew that his idea of gravity needed a medium to travel through (Greene, 2020). It took until Einstein's theory that gravity travels through time and space itself for Newton's point to be fully reconciled (Kaku, 2020). If Special Relativity represents how mechanics work at extreme velocities, then General Relativity represents how gravity works at extreme masses.

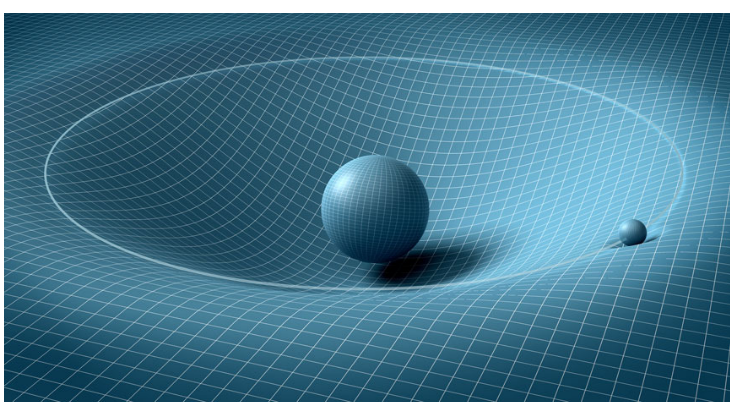

Figure 5: This "bend" would happen in three dimensions instead of two.

The core essence of General Relativity best represents itself in *Figure 5*. Larger massed objects “bend” space-time, making the smaller massed object attracted to it. This explains why objects experience gravitational attraction in the first place. However, this bend that gravity makes also affects time as well as space (Greene, 2020). Thus, the term "space-time continuum" is used to communicate these features brought up by General Relativity.

Theoretically, enough mass over a small enough area can produce a “hole” in this continuum so deep that not even something moving at the speed of light can escape it. This is what produces black holes (Kaku, 2020). The escape velocity of a black hole surpasses the cosmological speed limit of the speed of light.

Conclusion

Through the combination of Kepler’s Laws, Newton’s Laws, and Einstein’s Theories of Special and General Relativity, a heuristic understanding of the universe and its core mechanics is obtained. From Newton’s Laws of Motion and Gravitation, the concepts of gravity, force and motion are established. Kepler’s Laws complement Newton’s laws by accurately describing the motions of planetary bodies in orbit, such as in the solar system. Adding further context to the work of Kepler and Newton, Einstein’s Theories of Relativity marry the concepts of mass, space, time and gravity together, completing a comprehensive picture of the universe and its mechanics. By gaining the ability to interact with the mathematics and principles behind the inner-workings of the universe, one can achieve a greater understanding and admiration for the cosmos. With this understanding comes a greater appreciation for the physics of reality and the ability for science to describe the natural world. All can benefit from a sturdy footing in the observed principles that control the natural world. By recognizing the simple yet fundamental laws that govern the universe, one can cherish life at a more in-depth level.

References

Allain, R. (2013, October 3). *A Closer Look at Newton's Third Law*. Retrieved December 3, 2020, from https://www.wired.com/2013/10/a-closer-look-at-newtons-third-law/.

This source goes over the many ways in which Newton's Law can be represented, including the way Newton and modern physicists describe it. Wired posts a variety of articles relating to science and technology. Author Rhett Allain is an associate professor of physics at Southeastern Louisiana University. His article concisely points out the flaws of an "action vs reaction" way of explaining Newton's Third Law.

Bais, S. (2007). *Very special relativity: An illustrated guide*. Harvard University Press.

Sander Bais is an author and professor of theoretical physics at the University of Amsterdam. His book focuses on bringing the concepts of special relativity to a digestible level to his audience. It goes over the basics of special relativity, with relevant examples and pictures that can be converted into playable simulations. Overall, the book does an excellent job making relativity easy to understand and interpret.

Boys, C. V. (1894). *Proceedings* (Vol. 14). Royal Institution of Great Britain.

This source is an excerpt from the proceedings of the Royal Institution of Great Britain. In it, C. Vernon Boys argues for the acceptance of G, and why it is so difficult to calculate. The Royal Institution of Great Britain has produced many famous scientists, as well as essential research for understanding the natural world.

Cavendish, H. (1798, June 21). From https://upload.wikimedia.org/wikipedia/commons/d/dd/Cavendish_Experiment.png.

Drawing of torsion balance apparatus used by Henry Cavendish in the 'Cavendish Experiment' to measure the gravitational constant in 1798. This is a vertical section through the apparatus, including the building that housed it. Copy of Figure 1 from his 1798 paper 'Experiments to determine the Density of the Earth' published in Philosophical Transactions of the Royal Society of London, (part II) 88 p.469-526 (21 June 1798). Alterations: removed frame and caption, compensated for shear distortion caused by scanning book, converted to PNG.

Davis, E. (2010, June). *Deriving Kepler's Laws of Planetary Motion*. Union University Department of Mathematics. Retrieved November 6, 2020, from https://www.uu.edu/dept/math/SeniorPapers/09-10/DavisEmily.pdf.

Emily Davis is a math major at Union University. Her project focused on presenting proofs for Kepler's 3 Laws of Planetary Motion. It markets to those who want to know the history and derivations for Kepler's laws. The presentation goes step by step through Kepler's laws and their applications. The presentation combines many aspects of Kepler and his work, making the presentation more interesting and helpful to the lay viewer.

Digges, T. (1595). *Thomas Digges Map* [Illustration; jpg]. https://upload.wikimedia.org/wikipedia/commons/thumb/3/3e/ThomasDiggesmap.JPG/573px-ThomasDiggesmap.JPG.

Forveille, T. (n.d.). *Kepler's first law* [Illustration; gif]. https://www.aanda.org/images/stories/PressRelease/PRaa200411/glo_kepler1.gif.

Fowler, M. (2002, February). *Deriving Kepler's Laws from the Inverse-Square Law*. Galileo UVa. Retrieved November 6, 2020, from http://galileo.phys.virginia.edu/classes/152.mf1i.spring02/KeplersLaws.htm.

Michael Fowler is a professor of physics at the University of Virginia. His article comes in companion with one of the classes he teaches at UVa, where students have to rederive Kepler's laws. Any student of math, astronomy or physics looking for information about Kepler's laws finds relevant information in his article. The article goes over step-by-step how Kepler's 3 laws can be derived using the inverse square law. The laws and derivations are concise and to the point mathematically.

Greene, B. (Director). (2020). *Your Daily Equation #26: Einstein's General Theory of Relativity: The Essential Idea* [Film]. World Science Festival.

This video goes over in detail Einstein's Theory of General Relativity. It shows how it works in 2 and 3 dimensions, as well as the motivation behind it's derivation. Brian Greene is a physicist and science popularizer. His books have appeared on the New York Times' Bestseller List. The World Science Fair collaborated with him to produce a YouTube video detailing the innerworkings of General Relativity. Overall, this has to be one of the best source on the internet for understanding the basics of General Relativity.

Gregersen, E. (2019, January 17). Cavendish experiment. *Encyclopaedia Britannica Online*. Retrieved November 6, 2020, from https://www.britannica.com/science/Cavendish-experiment.

Author Erik Gregersen is an editor for Encyclopedia Britannica. He has written numerous articles for Britannica and worked at McMaster University and University of Chicago Press. The article details Henry Cavendish's experiment to derive G. The entry goes over Cavendish's experiment and the significance of "big G". It provides concise and relevant information to one of the universe's most important constants.

Hawking, S. (1998). *A brief history of time: Updated and expanded tenth anniversary edition*. Bantam Books.

Stephen Hawking was a famous physicist and mathematician best known for his work with black holes and the big bang, as well as his popularization of science. His book primarily focuses on an audience that wants to understand what is called "big history" and other major astronomical events. His book goes over the cosmological history of the universe, all explained in simple terms. The book provides great insight and introduction to many of the mechanics of the universe.

Howell, E. (2017, March 30). *Einstein's Theory of Special Relativity*. Space.com. Retrieved November 6, 2020, from https://www.space.com/36273-theory-special-relativity.html.

Elizabeth Howell is a regular contributor to Space.com. She is also a Ph.D candidate at UNC-Chapel Hill. Her article focuses on those interested in relativity and those who want to know the motivations behind Einstein's theory. The article goes over the history of relativity and Einstein's famous $E = mc^2$ equation. The article provides in-depth insight to relativity and its applications.

Johannes Kepler. (2011, February 28). *Current Events, a Weekly Reader Publication*, 110(18), 5.

The magazine Current Events publishes weekly articles from nameless authors pertaining to a motley of subjects. The magazine markets to high schoolers interested in a variety of topics. This excerpt goes over a biography of Kepler's life and his influence. It is clear, concise, and helpful to understanding the man behind the math.

Kaku, M. (2020, April 14). Albert Einstein. *Encyclopaedia Britannica Online*. Retrieved November 6, 2020, from https://www.britannica.com/biography/Albert-Einstein.

Author Michio Kaku is a scientist famous for popularizing and making science accessible. He holds degrees in physics from both Harvard and Berkeley and has published several bestsellers on the future and physics. This article primarily markets towards those who want to gain a deeper understanding of Albert Einstein and his work. It clearly details his life and works in laymen's terms.

Kepler, J. (1995). *Epitome of Copernican astronomy: & Harmonies of the world*. Promethus Books.

Johannes Kepler was a famous German astronomer, mathematician and thinker. His book, republished and translated in 1994, goes over much of his reasonings and motivations behind Kepler's laws. This book appealed to other astronomers and mathematicians at the time, as well as those looking to understand the works of Copernicus. The book goes over much of Copernicus's astronomy and Kepler's personal opinions. While the language and some of the ideas are antiquated, the book provides a great introduction to Copernican astronomy.

Kepler, J. (2019). *The harmonies of the world* (C. G. Wallis, Comp.). CreateSpace Independent Publishing Platform.

Johannes Kepler was a famous German astronomer, mathematician and thinker. His book, republished and translated in 2019, goes over the various harmonic motions observed in astronomical events. This book appealed to other astronomers and mathematicians at the time. The book goes over the full derivations and Kepler's personal opinions on the reason for unity in astronomical events. While the language and some of the ideas are antiquated, the book provides great insight to the motivations behind Kepler's Laws.

Kepler's third law. (2015, July). *Astronomy*, 43(7), 17.

This publication comes from an Astronomy Journal in the Gale database. For several decades, the journal Astronomy has published concise and well researched articles on a variety of astronomical and cosmological subjects. This excerpt details Kepler's 3rd Law of planetary motion, summarizing the idea in full. The source gives information on applications of Kepler's 3rd Law as well. While brief, this source gives powerful insight and perspective into Kepler's 3rd Law.

Kepler's Three Laws. (n.d.). The Physics Classroom. Retrieved November 18, 2020, from https://www.physicsclassroom.com/class/circles/Lesson-4/Kepler-s-Three-Laws.

This source, produced by the Physics Classroom, explains Kepler's 3 laws and a little bit about their derivations. The Physics Classroom produces physics tutorials and lessons for teachers and students. The source also provide practice problems relating to Kepler's laws. Overall, the source combines both Newton and Kepler's ideas into a concise format that makes it easy for students to understand.

Kurtus, R. (2015, January 25). *Kepler's Second Law* [Illustration; gif]. https://www.school-for-champions.com/astronomy/images/keplers_laws_areas_swept.gif.

Lahoti, S. (2018, April 25). *Build a Virtual Reality Solar System in Unity for Google Cardboard*. Packt. Retrieved October 24, 2020, from https://hub.packtpub.com/virtual-reality-solar-system-unity-google-cardboard/.

Sugandha Lahoti is a content editor at PactHub. She also writes on game development, web development and data science. This article primarily markets to those looking to use unity to make applications for google cardboard. The article goes over step by step how to make a 3D solar system for google cardboard using unity 3D. It is clear, concise and contains lots of code examples.

Lamb, R. (2010). *What is relativity?* How Stuff Works. Retrieved November 21, 2020, from https://science.howstuffworks.com/science-vs-myth/everyday-myths/what-is-relativity1.htm.

The article goes over the key concepts of relativity. Specifically, it displays the relative nature of time, and motivations for Einstein's Theory of Special Relativity. HowStuffWorks.com publishes several high level scientific articles for those looking to learn more about a variety of topics. Overall, the source is concise, and contains many references to other works on the topic of Relativity.

Levere, T. H. (2020, October 6). Henry Cavendish. *Encyclopaedia Britannica Online*. Retrieved November 21, 2020, from https://www.britannica.com/biography/Henry-Cavendish.

The article goes over the life and legacy of Henry Cavendish. Specifically, the type of person Cavendish was, and the experiments that he performed, such as his famous Cavendish Experiment, which calculated the mass of the earth. Author Trevor Levere is a historian focused on the history of science at hte University of Toronto. Encyclopedia Britannica is one of the largest English encyclopedias in the world, with hundreds of peer reviewers and research teams.

Mann, A. (2016). News Feature: The curious case of the gravitational constant. *National Academy of Sciences of The United States of America*, 113(36). https://doi.org/10.1073/pnas.1612597113.

The article goes over G from its derivation to current techniques for measuring it. The PNAS is a national organization dedicated to publishing scientific research, as well as publicizing scientific concepts. It is clear, concise and contains a lot of information referenced several times in the research paper.

Newton, I., & Cajori, F. (1974). *Sir Isaac Newton's mathematical principles of natural philosophy and his system of the world*. University of California press.

Republished and translated version of Isaac Newton's "Principa Mathematica". The book contains much of Newton's work concerning physics and astronomy, including his Three Laws of Motion and Law of Gravitation. This book contains much of the derivation, explanations and motivations behind classical mechanics. Despite the antiquated language, much of it is very easy to understand.

Nordtvedt, K., & Faller, J. E. (2019, June 20). Gravity. *Encyclopaedia Britannica Online*. Retrieved November 6, 2020, from https://www.britannica.com/science/gravity-physics.

Kenneth Nordtvet and James Faller are both writes for Encyclopedia Britannica. Faller is a physicist who was heavily involved in the Apollo program. Nordtvet is a famous academic in the physics community, having taught at Harvard University and Stanford University before his retirement. Their article goes over all aspects of gravity, from Newton's law of gravitation to modern approaches for measuring gravity. The article provides a complete and rigorous picture of gravity and the math behind it.

Redd, N. T. (2018, March 7). *How Fast Does Light Travel? | The Speed of Light*. Space.com. Retrieved December 3, 2020, from https://www.space.com/15830-light-speed.html.

Nola Taylor Redd is a writer for Space.com, a website that publishes articles on various topics in astronomy and astrophysics. She holds a degree in both English and Astrophysics from Agnes Scott College. Redd goes over many concepts relating to the speed of light, including how it is measured and how it relates to Special and General Relativity. Redd concisely blends the two topics

in a way that allows the reader to understand the importance and relevance to the speed of light, and its relation to Relativity.

The Science: Orbital Mechanics. (2009, July 7). NASA Earth Observatory. Retrieved November 21, 2020, from https://earthobservatory.nasa.gov/features/OrbitsHistory/page2.php.

This source from NASA gives a high level introduction to orbital mechanics. Specifically, it goes over Newton's Laws, Relativity and Kepler. Because of the conciseness of the article, NASA Earth Observatory can give a brief basis for the three essential principles that govern much of the universe's mechanics.

Simanek, D. E. (2006, February). *Why Not Angels?* Lockhaven.edu. Retrieved November 21, 2020, from https://lockhaven.edu/~dsimanek/philosop/angels.htm.

Author Donald Simanek is a professor at Lock Haven University. His work focuses on the history of science. His article details how religion and science have changed over time and how religion was often used to make up for flaws in scientific theory. The source takes on a somewhat humourous tone, showing how gaps in scientific theory have led scientists to pursue nonsensical conclusions.

Sinicki, A. (2020, June 22). *How to make a game in Unity: it starts with a simple 3D maze game*. Android Authority. Retrieved October 10, 2020, from https://www.androidauthority.com/how-to-make-a-game-in-unity-1130929/.

Author Adam Sinicki has written for Android Authority for several years. He focuses mainly on developer related topics, as well as health and fitness. This article fits well with Android Authority's demographic of people who may be interested in developing a game for android. The article targets those new to unity and game development in general. The article takes readers step-by-step

through the creation of a simple android game in unity. Overall, the article was very concise and helpful to those hoping to learn game development.

Stern, D. P. (2006, March 24). (20) *Newton's theory of "Universal Gravitation"*. NASA. Retrieved November 6, 2020, from https://pwg.gsfc.nasa.gov/stargaze/Sgravity.htm.

David Stern is a researcher emeritus for NASA's Goddard Institute. He studied at both the Hebrew University of Jerusalem and the Israel Institute of Technology. The article primarily targets individuals looking to broaden their understanding of Newton's theory of Universal Gravitation. The article goes over the history and derivation of Newton's theory of Universal Gravitation. This theory is then applied to the moon and earth, showing how gravity is different on each body. The article gives prime examples of the principles it teaches and is very digestible to readers.

Susskind, L., & Friedman, A. (2019). *Special Relativity and Classical Field Theory: The Theoretical Minimum* (2019 ed.). Basic Books.

The book is a transcription of lectures given by Art Friedman and Leonard Susskind. Leonard Susskind is a physics professor at Stanford University and Art Friedman is a longtime student of physics. The book markets primarily to those who want to understand Special Relativity and Classical Field Theory in simple terms. The book goes over the theoretical minimum, or what a student needs to start applying relativity and field theory to real-world problems. The book reads much like a conversation between two friends as student and teacher, and elegantly presents much of the mathematics behind Einstein and Maxwell's Theories.

Thomsen, P. (2019, April 18). *General Relativity* [Illustration; jpg]. https://patriotpower.ogsd.net/wp-content/uploads/2019/04/general-relativity.jpg.

Toshiya, N. (2013, August 29). *How original is Galileo's work on kinematics?* Retrieved December 3, 2020, from https://www.nagaitoshiya.com/en/2013/mean-speed-theorem/.

Nagai Toyoshia is the author of a blog focused on numerous physics topics. In this essay, he displays many different sources from Galileo's time relating to the concepts of acceleration and velocity. This shows that Galileo and other scientists during the Italian Renaissance had a basic idea of kinematics. The source does little other than present basic hard evidence for the capabilities of kinematics during Galileo's time.

Walker, J. (1997, July 8). *Bending Spacetime in the Basement*. Retrieved December 3, 2020, from https://www.fourmilab.ch/gravitation/foobar/.

This article gives an in-depth introduction to how to replicate Cavendish's experiment at home. It highlights the sensitive nature of the experimental setup Cavendish faced, and how to replicate it. Specifically, it addresses the need of a basement for measuring the mass of the earth. John Walker is a blogger from Switzerland who writes on a variety of STEM topics. This source takes on a fun tone, while also explaining how to make a practical set-up of Cavendish's experiment.