Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup \overset{A_2}{\cancel{A_1}}) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

$A_1 \cap \cancel{(A_2)}^c \cancel{= \emptyset}$

$P(1 - A_i) = 0$

$\bigcup \; P(1 - A_i) \overset{/}{=} 0$

A $\quad P(\cap_{i=1}^{n} A_i) = 1 - P(\cap_{i=1}^{n} A_i)^c$

$= 1 - P(\cup_{i=1}^{n} A_i^c)$

fix, R 000

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

[redacted]

Date No.

1(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$\le P(A_1) + P(A_2)$ since $P(A_1 \cap A_2) \ge 0$ #

(b) Let $Q(n)$ denote the statement $P(\cup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$

We need to prove $P(1)$ is true and $Q(k) \Rightarrow Q(k+1)$ is true

Basis step $Q(1)$: $P(\cup_{i=1}^{1} A_i) = P(A_1)$, $\sum_{i=1}^{1} P(A_i) = A_i$

It is obvious that $P(\cup_{i=1}^{1} A_i) = \sum_{i=1}^{1} P(A_i)$,

$\therefore$ $Q(1)$ is true.

Inductive step : Assume that $Q(k)$ is true for some $k \in \mathbb{Z}^+$

We need to prove $Q(k) \Rightarrow Q(k+1)$ is true.

$$P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^{k} A_i) \cup P(A_{k+1})$$
$$\le P(\cup_{i=1}^{k} A_i) + P(A_{k+1})$$
$$\le \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$$
$$\le \sum_{i=1}^{k+1} P(A_i)$$
(shown)

$\therefore$ Since $Q(1)$ is true, $Q(k) \Rightarrow Q(k+1)$ is true,

By MI $Q(n)$ is true

(c) $P(\cup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$ | By (b)

≤ 0

(d) $P(\cap_{i=1}^{n} A_i) = 1 - P(\cap_{i=1}^{n} A_i)^c$

$= 1 - P(\cup_{i=1}^{n} A_i^c)$

$\ge 1 - \sum_{i=1}^{n} A_i^c$

$\ge 1 - 0$

≥ 1 $\therefore$ $P(\cap_{i=1}^{n} A_i) = 1$ as $0 \le P \le 1$

Date No.

2 (a) Probability of R in team : $\binom{7}{3} \div \binom{8}{4}$

Probability of E in team: $\binom{8}{3} \div \binom{9}{4}$

~~Probability~~
~~Probability of R/ take/ E/ (:~~

(a) $\frac{1}{2} \times \frac{4}{9} \times \frac{3!}{4!} = \frac{1}{18}$

(b) $\frac{1}{2} \times \frac{4}{9} \times \left(\frac{4!-3!}{4!}\right) = \frac{1}{6}$

(c) $\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$ (both chosen)

$\frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$

$1 - \frac{2}{9} - \frac{5}{18} = \frac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_1^2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

1(a) $0 \le P(A_1 A_2) \le 1$

$0 \le P(A_1 A_2)$

$0 \le P(A_1) + P(A_2) - P(A_1 \cup A_2)$

$P(A_1 \cup A_2) \le P(A_1) + P(A_2)$

(b) when ~~k=2~~, $n=2$,

~~$P(A_1) \le P(A_2)$~~ $P(A_1 \cup A_2) \le \sum_{i=1}^{2} P(A_i)$ (By Part (a))

$= P(A_1) + P(A_2)$

Suppose $n=k$ is true for all $k \in \mathbb{Z}^+$, $P(\cup_{i=1}^{k} A_i) \le \sum_{i=1}^{k} P(A_i)$,

W.T.S that $n=k+1$ is true.

$P(\cup_{i=1}^{k+1} A_i) = P[(\cup_{i=1}^{k} A_i) \cup A_{k+1}]$

$\le P(\cup_{i=1}^{k} A_i) + P(A_{k+1})$ (Part (a))

$\le \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_i)$ (Shown).

(c) Since the value of probability is strictly between 0 and 1 inclusive, hence,

$P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$

$P(\cup_{i=1}^n A_i) \le 0$, ~~this proves that~~ $\sum_{i=1}^n P(A_i) = 0$,

$P(\cup_{i=1}^n A_i) = 0$

(d) $P(\cap_{i=1}^n A_i) = 1 - P[(\cap_{i=1}^n A_i)']$

$= 1 - P(\cup_{i=1}^n A_i')$ — (1)

$P(\cup_{i=1}^n A_i') \le \sum_{i=1}^n P(A_i')$

Since $P(A_i) = 1 - P(A_i')$, for all $1 \le i \le n$,

$P(\cup_{i=1}^n A_i') \le 0$ and probability value is between 0 and 1 inclusive, hence,

$P(\cup_{i=1}^n A_i') = 0$

Hence from (1), $P(\cap_{i=1}^n A_i) = 1$. (Shown).

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

2(a) P(Rebecca and Elise will be paired)

$$= \frac{4}{8} \times \frac{4}{9} \times \frac{1}{4}$$

$$= \frac{1}{18}.$$

(b) P(Rebecca and Elise will be chosen to represent their schools but will not play each other)

$$= \frac{4}{8} \times \frac{4}{9} \times \frac{3}{4}$$

$$= \frac{1}{6}$$

P(Either Rebecca or Elise will be chosen)

$$= \left(\frac{4}{8} \times \frac{5}{9}\right) + \left(\frac{4}{8} \times \frac{4}{9}\right)$$

$$= \frac{1}{2}.$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_1) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

$P(A) + P(B) - P(A \cup B) = P(A \cap B)$

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(AB)$.

$0 \leq P(AB) \leq 1$.

$\therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$ is true for $n=2$

$\Rightarrow P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$ is true for some $n=k$.

~~$P(\cup_{i=1}^{k+1} A_i) =$~~ $\leq$ ~~$\sum_{i=1}^{k+1} P(A_i) = \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$.~~

~~$\leq P(\cup_{i=1}^k A_i) + P(A_{k+1})$.~~

$P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^k A_i \cup A_{k+1})$

$\leq P(\cup_{i=1}^k A_i) + P(A_{k+1})$

$\leq \sum_{i=1}^k P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_i)$

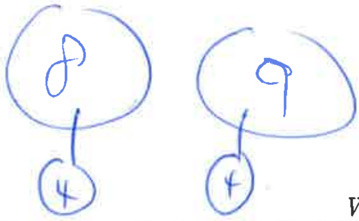
Since true for $n=k+1$, $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$ is true for all k.

(c) $\sum_{i=1}^n P(A_i) = 0$.

$P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i) \leq 0$.

$0 \leq P(\cup_{i=1}^n A_i) \leq 1$

$\Rightarrow P(\cup_{i=1}^n A_i) = 0$



2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) Chances of Rebecca and Elise being paired

$= \frac{4}{8} \times \frac{4}{9} \times \frac{1}{4}$

$= \frac{1}{18}$

(b) Chances will not play each other

$= \frac{4}{8} \times \frac{4}{9} \times \frac{3}{4}$

$= \frac{1}{6}$

(c) Chances either will be chosen.

$= \frac{4}{8} \times \frac{5}{9} + \frac{4}{8} \times \frac{4}{9}$

$= \frac{1}{2}$

NO.: Date:

(d) $\cancel{P(A)=1}$ $P(A_1A_2\ldots A_n)' = P(A_1' \cup A_2' \cup A_3' \ldots A_n')$.

$\cancel{P(B)=1}$

$\cancel{P(A\cap B)=1}$. $= P(\cup_{i=1}^{n} A_i')$.

$1 - P(A_1A_2\ldots A_n)' = P(\cap_{i=1}^{n} A_i')$.

$\cancel{P(A+B}$

$P(A_1A_2\ldots A_n)' \leq 0$.

$P(\cap_{i=1}^{n} A_i) = 1$

NO.: ....

Date: ....

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use $\boxed{\text{induction}}$ to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$.

$\because 0 \leq P(A_1 \cap A_2) \leq 1 \quad \therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

(b) ① when $n=2$ $P(\cup_{i=1}^2 A_i) = P(A_1 \cup A_2) \leq A_1 + A_2 = \sum_{i=1}^n P(A_i)$

② Assume n is true for $n=k$, then for $n=k+1$:

$P(\cup_{i=1}^k A_i) \leq \sum_{i=1}^k P(A_i)$ is true

$P(\cup_{i=1}^{k+1} A_i) = P([\cup_{i=1}^k A_i] \cup A_{k+1}) \leq P(\cup_{i=1}^k A_i) + P(A_{k+1})$

$= \sum_{i=1}^k P(A_i) + P(A_{k+1}) = \sum_{i=1}^{k+1} P(A_i)$ $\therefore P(\cup_{i=1}^{k+1} A_i) \leq \sum_{i=1}^{k+1} P(A_i)$ is also true. #

(c) $P(\cup_{i=1}^n A_i) \leq P(A_1) + P(A_2) + \cdots P(A_n)$ (According to (b))

$= 0$

$\because \begin{cases} P(\cup_{i=1}^n A_i) \leq 0 \\ 0 \leq P(\cup_{i=1}^n A_i) \leq 1 \end{cases} \quad \therefore P(\cup_{i=1}^n A_i) = 0$

(d) ~~① for $n=1$ $P(A_1) = 1$~~

~~② Assume $P(\cap_{i=1}^k A_i) = 1$ is true given $P(A_1) = \cdots P(A_k) = 1$,~~

~~then for $n=k+1$, $P(\cap_{i=1}^{k+1} A_i) = P(\cap_{i=1}^k A_i) + P(A_{k+1}) - 1$~~

$\because P(A_1) = \cdots P(A_n) = 1 \quad \therefore P(A_1^c) = P(A_2^c) = \cdots P(A_n^c) = 0$

$P(\cap_{i=1}^n A_i) = 1 - P((\cap_{i=1}^n A_i)^c) = 1 - P(\cup_{i=1}^n A_i^c)$

From (c) we know $P(\cup_{i=1}^n A_i^c) = 0$ given $P(A_1^c) = \cdots P(A_n^c) = 0$

$\therefore P(\cap_{i=1}^n A_i) = 1 - 0 = 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

R represents Rebecca, E represents Elise

(a) P (R and E both selected and paired) $= \frac{1}{2} \times \frac{4}{9} \times \frac{1 \times 3 \times 2}{4!} = \frac{1}{18}$

(b) P (R and E both chosen but not play each other)

$= \frac{1}{2} \times \frac{4}{9} \times \frac{3 \times 3 \times 2}{4!} = \frac{1}{6}$

(c) P (either Rebecca or Elise be chosen) $= \frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{5}{9} = \frac{1}{2}$

Grp 03

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

$P(A_1 A_2) \geq 0$

$\therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) From (a) we have $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$. (Base case $n=1$,

$P(\cup_{i=1}^{1} A_i) \leq \sum_{i=1}^{1} P(A_i)$

LHS = $P(A_1)$

RHS = $P(A_1)$

$\therefore$ proven.

Assume $P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$ —(1)

$\therefore$ Inductive step :

$P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^{k} A_i \cup A_{k+1})$

$\leq P(\cup_{i=1}^{k} A_i) + P(A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_i)$

$\therefore$ Inequality is true.

d) $P(\bigcap_{i=1}^{n} A_i) = (P(\bigcup_{i=1}^{n} A_i))^c$

$= 1 - P(\bigcup_{i=1}^{n} A_i)$

$= 1$ //

c) $P(A_1) = \ldots = P(A_n) = 0$

$\therefore \sum_{i=1}^{n} P(A_i) = 0$

From (b), $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

Since it is a probability, it is non-negative ≤ 0.

$\therefore P(\bigcup_{i=1}^{n} A_i) = 0$ //.

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a)

$$\text{probability} = \frac{\binom{7}{3}\binom{8}{3} \times 3!}{\binom{8}{4}\binom{9}{4} \times 4!}$$

$$= \frac{1}{18}$$

b)

$$\text{Probability} = \frac{\binom{7}{3}\binom{8}{3}4! - \binom{7}{3}\binom{8}{3}\times 3!}{\binom{8}{4}\binom{9}{4} \times 4!}$$

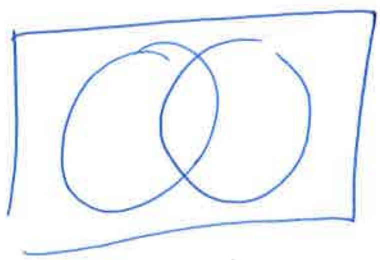
$$= \frac{1}{6}$$

c) $P(\text{R chosen but not E}) = \frac{4}{8} \times \frac{5}{9} = \frac{5}{18}$

$P(\text{E chose but not R}) = \frac{4}{8} \times \frac{4}{9} = \frac{2}{9}$.

Total probability $= \frac{5}{18} + \frac{2}{9}$.

$= \frac{1}{2}$



Grp 03

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A0[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_1) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

we know $P(A_1 \cap A_2) \geq 0$

So,

$P(A_1 \cup A_2) + P(A_1 \cap A_2) = P(A_1) + P(A_2)$

$\therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) We have $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$$P\left(\bigcup_{i=1}^{k} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

$$P\left(\bigcup_{i=1}^{k+1} A_i\right) \leq P\left(\bigcup_{i=1}^{k} A_i\right) + P(A_{k+1})$$

$$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$$

$$= \sum_{i=1}^{k+1} (P(A_i) \quad //$$

c) $P(A_1) = \cdots = P(A_n) = 0$

$\therefore \sum_{i=1}^{n} (A_i) = 0$

$\rightarrow$ a) $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$ 1

$\Rightarrow P\left(\bigcup_{i=1}^{n} A_i\right) = 0$ since it cannot be negative //

d) $P(A_1) = \cdots = P(A_n) = 1$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = 1 - P\left(\left(\bigcap_{i=1}^{n} A_i\right)^c\right)$$

$$= 1 - P\left(\bigcup_{i=1}^{n} A_i^c\right)$$

$$= 1 - 0$$

$$= 1 \quad //$$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) $$= \frac{\binom{8}{3}\binom{7}{3} \times 3!}{\binom{9}{4}\binom{8}{4} \times 4!} = \boxed{\frac{1}{18}}$$

b) $$= \frac{\binom{9}{3}\binom{7}{3}\cdot 4! - \binom{8}{3}\binom{7}{3}\cdot 3!}{\binom{9}{4}\binom{8}{4}\cdot 4!} = \boxed{\frac{1}{6}}$$

c) $$\underset{\text{Rebecca chosen}}{P(A} \cup \underset{\text{Elise chosen}}{B)} = P(A) + P(B) - P(A \cap B)$$

$$\underbrace{P(A \cup B) - P(A \cap B)}_{\text{either but not both}} = P(A) + P(B) - 2 \cdot P(A \cap B)$$

$$= \frac{4}{8} + \frac{4}{9} - 2\cdot\left(\frac{4}{8}\right)\left(\frac{4}{9}\right) = \boxed{\frac{1}{2}}$$

Grp 03

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$. → look at formula in 1(b).
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$. → how to change from ∩ to ∪?

(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

Since $0 \leq P(X) \leq 1$, $P(A_1 \cup A_2) \geq 0$ ~~Since A_1 and A_2 are mutually exclusive, $A_1 \cap A_2 = \phi$, so $P(A_1 A_2) = 0$.~~

$\therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ //

(b) ~~when $n=1$, $P(\bigcup_{i=1}^{1} A_i) \leq \sum_{i=1}^{1} P(A_i)$ $P(A_1) = \sum_{i=1}^{1} P(A_i)$~~

~~when $n=2$, from part (a), $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$~~

~~Assume that $n=k$ is true, when $n=k+1$, $P(\bigcup_{i=1}^{k+1} A_i) = P(A_1) + P(A_2) + \ldots + P(A_k) + P(A_{k+1})$~~

~~$= \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$~~

~~$\geq P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1})$~~

Let the proposition be n.

$P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

Let $n=2$, $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$P(\bigcup_{i=1}^{2} A_i) \leq \sum_{i=1}^{2} P(A_i)$

Assume $n=k$ is true, $P(\bigcup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

When $n=k+1$, $P(\bigcup_{i=1}^{k+1} A_i) \leq \sum_{i=1}^{k+1} P(A_i)$

LHS $= P(\bigcup_{i=1}^{k+1} A_i)$

~~$= P(A_1) + P(A_2) + \ldots + P(A_k) + P(A_{k+1})$~~

~~$= \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$~~

~~$\leq \sum_{i=1}^{k+1} P(A_i)$~~

$= P((\bigcup_{i=1}^{k} A_i) \cup A_{k+1})$

$\leq P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$ 1

$= \sum_{i=1}^{k+1} P(A_i)$ //

(recursion)
$P(A)=1$
$P(B)=1$
$P(AB)=1$
$P(A \cup B) = P(A) + P(B) - P(AB)$
$P(A_1 \cap A_2) = 1$
$P(\underbrace{A_1 \cap A_2}_{B} \cap A_3) = 1$

(c) ~~$P(\bigcup_{i=1}^{n} A_i) = P(A_1) + P(A_2) + \ldots + P(A_n)$~~

~~$\leq \sum_{i=1}^{n} P(A_i)$~~

~~$= 0$.~~

~~$P(A_1) = P(A_2) = 0$~~

~~$P(A_1) P(A_2) = 0$~~

$P(A_1) = 0 = P(A_2)$

$P(A_1) + P(A_2) = P(A_2) = 0$

$P(A_1 \cup A_2) = 0$

Hence, ~~when~~ $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

$= 0$.

(d) ~~when $P(A_1) = 1 = P(A_2)$,~~

demorgan's law

~~$P(A_1) = P(A_2) = P(A_1 A_2)$~~

~~$P(\bigcap_{i=1}^{n} A_i) = P(A_1 A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$~~

~~$P(\bigcap_{i=1}^{n} A_i) = 1$~~

$P(\bigcap_{i=1}^{n} A_i) = P(\bigcup_{i=1}^{n} A_i)^c = 1 - 0 = 1$ //

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) $\cancel{\dfrac{^8C_4 \times {}^9C_4 \times 2 \times {}^6C_2}{^{17}C_8 \times {}^8C_2} = \dfrac{945}{2431}}$

A	B
Rebecca	Elise

$^6C_2$ ways to pair the rest.

~~choose R&E, fix one of them = $^4C_1$~~

~~total outcomes with R & E chosen = $\dfrac{^7C_3 \times {}^8C_3}{^8C_4 \times {}^9C_4}$~~

~~P(R&E paired) = $\frac{2}{9} \div 4 = \frac{1}{18}$ //~~

$$\frac{^7C_3 \times {}^8C_3 \times 3!}{^8C_4 \times {}^9C_4 \times 4!} = \frac{1}{18}$$

(b) $\cancel{^7C_3 \times {}^8C_3}$

Fix rebecca

R	_
_	_
_	_
_	_

$^3C_1$ ways to place Elise.

P(R&E chosen | not playing) = $\dfrac{P(A \cap B)}{P(B)}$ = ~~…~~ = $\dfrac{3}{490}$

~~R&E chosen (total outcomes) = $^7C_3 \times {}^8C_3$~~

~~P(don't play with each other) = 4 × 3 = 12~~

~~R - 4 options _ _ _ _~~

~~E - 3 options _ _ _~~

P(~~chosen but not playing each other~~) = $^7C_3 \times {}^8C_3$

P(not playing each other) = 4 × 3

~~P(A∩B) :~~

$$\frac{(^7C_3 \times {}^8C_3 \times 4!) - (^7C_3 \times {}^8C_3 \times 3!)}{^8C_4 \times {}^9C_4 \times 4!} = \frac{1}{6}$$

(c) Case 1 : R chosen

$\cancel{^7C_3 \times {}^8C_4 = 2450}$ $\quad \frac{4}{8} \times \left(1 - \frac{4}{9}\right) = \frac{5}{18}$

Case 2 : E chosen

$\cancel{^7C_4 \times {}^8C_3 = 1960}$ $\quad \left(1 - \frac{4}{8}\right) \times \frac{4}{9} = \frac{2}{9}$

~~total probability~~

P (E/R chosen but not both) = $\cancel{\dfrac{2450}{^{17}C_8} + \dfrac{1960}{^{17}C_8}}$ = $\dfrac{5}{18} + \dfrac{2}{9} = \dfrac{1}{2}$ //

Grp 03

Tutorial 01 (Week 03)

Name: [redacted]

Matric No.: [redacted]

Original Group: [redacted]

Attended Group: [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.

$\bigcup_{i=1}^{n} A_i \leq \sum_{i=1}^{n} P(A_i)$

(b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.

(c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.

(d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

(a) $P(A_1 \cup A_2) \leq 1$

$P(A_1) \leq 1$, $P(A_2) \leq 1$

$\because$ $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$,

and $\because$ $P(A_1 A_2) \geq 0$,

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (proved)//.

(b) For $n=2$, $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (from part (a)). Hence, base case holds true.

Assuming formula is true for $n=k$, we must now prove formula holds true for $n=k+1$.

ie assume $P\left(\bigcup_{i=1}^{k} A_i\right) \leq \sum_{i=1}^{k} P(A_i)$ true, prove : $P\left(\bigcup_{i=1}^{k+1} A_i\right) \leq \sum_{i=1}^{k+1} P(A_i)$.

$P\left(\bigcup_{i=1}^{k+1} A_i\right) = P\left(\bigcup_{i=1}^{k} A_i \cup A_{k+1}\right) \leq P\left(\bigcup_{i=1}^{k} A_i\right) + P(A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_i)$ (shown)//.

(c) $\because$ $\sum_{i=1}^{n} P(A_i) = P(A_1) + P(A_2) + \ldots + P(A_n)$

$= 0 + 0 + \ldots + 0$

$\sum_{i=1}^{n} P(A_i) = 0$

$\because$ $\bigcup_{i=1}^{n} P(A_i) \leq \sum_{i=1}^{n} P(A_i)$ from part (b),

then $\bigcup_{i=1}^{n} P(A_i) = 0$ (shown)//.

(d) By De Morgan's Law,

$P\left(\bigcup_{i=1}^{n} A_i\right)^c = (A_1 \cup A_2 \cup \ldots \cup A_N)^c$

$= A_1^c A_2^c A_3^c \ldots A_N^c$

$P\left(\bigcup_{i=1}^{n} A_i\right)^c = P\left(\bigcap_{i=1}^{n} A_i\right)$

$= 1 - 0$

$= 1$ // (shown).

2. The chess clubs of two schools consist of, respectively, $\boxed{8}$ and $\boxed{9}$ players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) $$\frac{\binom{7}{3}\times\binom{8}{3}\times 3!}{\binom{8}{4}\times\binom{9}{4}\times 4!} = \frac{1}{18}$$

(b) $$\frac{\left[\binom{7}{3}\times\binom{8}{3}\times 4!\right]-\left[\binom{7}{3}\times\binom{8}{3}\times 3!\right]}{\binom{8}{4}\times\binom{9}{4}\times 4!} = \frac{1}{6}$$

$(1-\frac{4}{9})$

$\left({}^{7}C_{3} \times {}^{8}C_{4}\right)$

(c) Case 1 (Rebecca chosen, Elise not) $= \frac{4}{8} \times \frac{5}{9} = \frac{5}{18}$

Case 2 (Rebecca not chosen, Elise chosen) $= \frac{4}{8} \times \frac{4}{9} = \frac{2}{9}$

$\left({}^{7}C_{4} \times {}^{8}C_{3}\right)$

Probability $= \frac{5}{18} + \frac{2}{9} = \frac{1}{2}$

$(1-\frac{4}{8})$

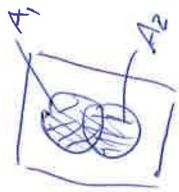
${}^{8}C_{4} \times {}^{9}C_{4}$

Grp 03

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T______



1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

1a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

For all $P(X)$, X any variable, it is always $0 \leq P(X) \leq 1$

b) Let Proposition n be the following:

$$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$$

Prove Proposition for $n=2$,

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$$

For $n=k$, $P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{n} P(A_i)$

For $n=k+1$: $P(\cup_{i=1}^{k+1} A_i) \leq \sum_{i=1}^{k+1} P(A_i)$

$$LHS = P(\cup_{i=1}^{k} A_i \cup A_{k+1}) = P(\cup_{i=1}^{k} A_i) + P(A_{k+1}) - P(\cup_{i=1}^{k} A \cap A_{k+1})$$

$$* \leq P(\cup_{i=1}^{k} A_i) + P(A_{k+1})$$

$$= \sum_{i=1}^{k+1} P(A_i)$$

\#

c) $P(A_1) = \ldots = P(A_n) = 0$

$P(A_1) - P(A_2) = 0$

$P(A_1) - [P(A_1 \cup A_2) - P(A_1) + P(A_1 \cap A_2)] = 0$

Since $P(A_1) = P(A_2) = 0$, $P(A_1 \cap A_2) = 0$

$\cancel{P(A_1)} - P(A_1 \cup A_2) = 0$

$P(A_1 \cup A_2) = 0$

Same for $A_n \cup A_{n-1}$

\#

OR

$P(\cup_{i=1}^{n} A_i) = 0$

$\leq \sum_{i=1}^{n} P(A_i)$

$= P(A_1) + P(A_2) \ldots$

$= 0 + 0 + 0$

Since $P(X) \geq 0$, $P(\cup_{i=1}^{n} A_i) = 0$

d)

$P(A_1) - [P(A_1 \cup A_2) - P(A_1) + P(A_1 \cap A_2)] = 0$

Since $P(A_1) = P(A_2) = 1$, $P(A_1 \cup A_2) = 1$.

$1 - [1 - 1 + P(A_1 \cap A_2)] = 0$

$1 = P(A_1 \cap A_2)$

\#

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

2a) ~~P(~~ Rebecca & Elise $= {}^{4}C_1 \times {}^{4}C_1$ $= 16$

Total probable outcomes $=$ $\dfrac{P(R \wedge E \text{ chosen})}{{}^{8}C_4 \times {}^{9}C_4} = \dfrac{{}^{7}C_3 \times {}^{8}C_3}{{}^{8}C_4 \times {}^{9}C_4} = \dfrac{2}{9}$

$P(R \wedge E) = \cancel{\frac{72}{4410}} \approx \cancel{0.0163}$ ~~or~~ $\cancel{\frac{4}{245}}$ $\quad \frac{2}{9} \div 4 = \frac{1}{18}$

b) $P(\overset{A}{R \wedge E \text{ chosen}} \mid \overset{B}{\text{Not playing}}) = \dfrac{P(A \cap B)}{P(B)}$

$R \wedge E$ chosen $= {}^{7}C_3 \times {}^{8}C_3 = 1960$ cases

They do not play each other $= 4 \times 3 = 12$ cases

$\dfrac{12}{1960}$

c) $P(R \text{ but no } E) = {}^{8}C_4 \times {}^{7}C_4$ $\quad$ Total choices $= {}^{8}C_4 \times {}^{9}C_4 = 8820$

E but no $R = {}^{8}C_3 \times {}^{7}C_4$

~~Total cases = 6860~~

$P(\text{One of them}) = \dfrac{6860}{8820} = \dfrac{{}^{8}C_4 \times {}^{7}C_3 + {}^{8}C_3 \times {}^{7}C_4}{8820}$

$= \dfrac{1}{2}$

1(d) $P(\cap_{i=1}^{n} A_i) = 1 - P((\cap_{i=1}^{n} A_i)^c)$

$= 1 - P(\cup_{i=1}^{n} A_i^c)$ ⟹ De Morgan's Law ⟹ works both ways

$= 1 - 0$ As we get from 1(c) $P(\cup_{i=1}^{n} A_i) = 0$

$= 1$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$.

(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

Since $P(A_1 \cap A_2) \geq 0$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

(b) $P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n)$

$= \cancel{P(A_n \cup A}\ P(A_1 \cup A_2 \cup A_3 \cup \cdots A_{(n-1)}) + P(A_n) - P\{[P A_1 \cup A_2 \cup A_3 \cup \cdots A_{(n-1)}] \cap A_n\}$

~~By Induction~~ since ⟹ ≥ 0

$P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n)$

$\leq P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{(n-1)}) + P(A_n)$

By Induction

$P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n)$

$\leq P(A_1) + P(A_2) + \cdots + (P(A_n))$

$\leq \sum_{i=1}^{n} P(A_i)$ (shown)

(c) Since $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

and $P(A_1) = P(A_2) = \cdots = P(A_n) = 0$

$P(\cup_{i=1}^{n} A_i) \leq 0 + 0 + \cdots + 0$

since $P(\cup_{i=1}^{n} A_i)$ cannot be negative

$P(\cup_{i=1}^{n} A_i) = 0$

(d) $P(\cap_{i=1}^{n} A_i)$

$= \cancel{P[(A_1 \cap A_2 \cap A_3 \cap A_4 \cdots) \cap A_n]}$

$= \cancel{P(A_1 \cap A_2 \cap A_3 \cdots \cap A_{n-1}) + P(A_n) - P(A_1 \cap A_2 \cap \cdots) \cup P(A_n)}$

~~By Induction~~

$\because P(A_1 \cap A_2) = 1 \quad P(A_3) = 1$

~~⟹ P(~~Similarly $P(A_1 \cap A_2 \cap A_3) = 1$

By Induction

$P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) = 1$

$P(\cap_{i=1}^{n} A_i)$

Therefore $P(\cap_{i=1}^{n} A_i) = 1 \Longleftarrow P(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n) = 1$

(d) $\cancel{P(\cap_{i=1}^{n} A_i)}$

$= \cancel{1 - P(\cup_{i=1}^{n} A_i^c)}$

~~P(~~

$P(A) = 1$

$P(B) = 1$

$\because P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$P(A) = 1, P(A) = 1$

$\therefore P(A \cup B) + P(A \cup B)$

$\because$ Neither can be greater than 1

$\therefore P(A \cup B) = P(A \cup B) = 1$

$8 \to 3 \to 56$
$7 \to 3 \to 35$
$8 \to 4 \to 70$
$9 \to 4 \to 126$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

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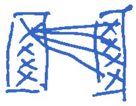


~~(a) $P = (\frac{1}{8})(\frac{1}{9})(\frac{1}{2})(\frac{56}{126})(\frac{1}{4!}) = \frac{1}{108}$~~

~~(b) $P(\text{not play} \mid \text{chosen}) = \dfrac{\frac{107}{108}}{\frac{2}{9}}$~~

(a) $P = \left(\dfrac{{}^7C_3 \times {}^8C_3}{{}^8C_4 \times {}^9C_4}\right)\left(\dfrac{3!}{4!}\right) = \dfrac{1}{18}$

(b) ~~$P =$~~ ~~$\dfrac{\frac{3\times4\times4\times4}{4!}}{(\frac{2}{9})}$~~

$P = \dfrac{{}^7C_3 \times {}^8C_3}{{}^8C_4 \times {}^9C_4}\left(1 - \dfrac{3!}{4!}\right) = \dfrac{1}{6}$

(c) $P =$ ~~$\dfrac{{}^7C_3 \times {}^8C_3}{{}^8C_4 \times {}^9C_4}$~~ ~~$\dfrac{{}^7C_3}{{}^8C_4} + \dfrac{{}^8C_3}{{}^9C_4} - \dfrac{{}^7C_3 \times {}^8C_3}{{}^8C_4 \times {}^9C_4}$~~

~~$= \frac{1}{2} + \frac{4}{9} - \frac{2}{9}$~~

~~$= \frac{13}{18}$~~

① Rebecca only

$P = \dfrac{{}^7C_3}{{}^8C_4} \times \dfrac{{}^8C_4}{{}^9C_4}$

$= (\frac{1}{2})(\frac{5}{9})$

② Elise only

$P = \dfrac{{}^7C_4}{{}^8C_4} \times \dfrac{{}^8C_3}{{}^9C_4}$

$= (\frac{1}{2})(\frac{4}{9})$

$P = (\frac{1}{2})(\frac{5}{9}) + (\frac{1}{2})(\frac{4}{9}) = \frac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

1a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2) \le P(A_1) + P(A_2)$ $(\because P(A_1 A_2) \geq 0)$

b) For $n=2$, $P(\cup_{i=1}^2 A_i) = \cancel{P(A_1 A_2)}$ $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$ $(\because 1a)$
$\cancel{= P(A_1) \times P(A_2 | A_1)}$ $\le \sum_{i=1}^2 P(A_i)$

$n=1$, $P(\cup_{i=1}^1 A_i) = P(A_1) \le \sum_{i=1}^1 P(A_i)$

Assuming the inequality holds for $k \in \mathbb{Z}^+$,

$k+1$: $P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^k A_i \cup A_{k+1})$
$= P(\cup_{i=1}^k A_i) + P(A_{k+1}) - P(\cup_{i=1}^k A_i \cap A_{k+1})$
$\le \sum_{i=1}^k P(A_i) + P(A_{k+1}) - P(\cup_{i=1}^k A_i \cap A_{k+1})$
$\le \sum_{i=1}^k P(A_i) + P(A_{k+1}) = \sum_{i=1}^{k+1} P(A_i)$

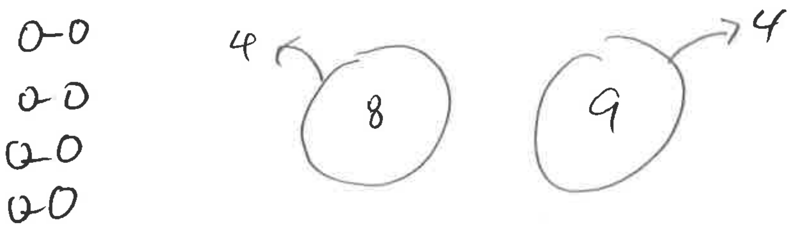
Since the statement is true for $k \in \mathbb{Z}^+$, and $k=1,2$ is true, then the statement is true for $n \in \mathbb{Z}^+$

c) $P(A_1) = \ldots = P(A_n) = 0 \Rightarrow \sum_{i=1}^n P(A_i) = 0$

$0 \le P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i) = 0$,

$\therefore P(\cup_{i=1}^n A_i) = 0$

d) $P(\cap_{i=1}^n A_i) = 1 - P(\cap_{i=1}^n A_i)^c$
$= 1 - P(\cup_{i=1}^n A_i^c)$ $(P(A_1^c) = \ldots = P(A_n^c) = 0)$
$= 1 - 0$
$= 1$



2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school? 2520 8820

2a) P(Rebecca and Elise paired) $= \cancel{1 - P(\text{Rebecca and Elise})^c}$ $= \dfrac{\binom{7}{3}\binom{8}{3}\times 3!}{\binom{8}{4}\binom{9}{4}\times 4!}$

$\cancel{= 1 - \dfrac{\binom{7}{4}\binom{8}{4}\times 4!\times 4!}{\binom{8}{4}\binom{9}{4}\times 4!\times 4!}}$

$\cancel{= \dfrac{17}{48}}$ $= \dfrac{1}{18}$

b) P(R and E) chosen but not play w each other $= \dfrac{\binom{7}{3}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} \times \dfrac{\binom{3}{1}\times 3!}{4!}$

$= \dfrac{1}{6}$

c) Probability $= \dfrac{\binom{7}{3}\binom{8}{4}}{\binom{8}{4}\binom{9}{4}} + \dfrac{\binom{7}{4}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}}$

$= \dfrac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^n A_i\right) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^n A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^n A_i\right) = 1$.

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

$\because P(A_1 A_2) \geq 0$

$\therefore P(A_1) + P(A_2) - P(A_1 \cup A_2) = P(A_1 A_2) \geq 0$

$P(A_1) + P(A_2) \geq P(A_1 \cup A_2)$

b) for $n=1$

$P(\cup_{i=1}^{1} A_i) = P(A_1)$

$\sum_{i=1}^{n} P(A_i) = P(A_1)$.

Thus $P(A_1) = P(A_1)$

which is true for $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$.

$n=2$ is proved in part a).

Assume $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$ is true for $n=k$.

$P(\cup_{i=1}^{k} A_i) \le \sum_{i=1}^{k} P(A_i)$.

Then for $n=k+1$

$P(\cup_{i=1}^{k} A_i) \cup P(A_{k+1}) \le P(\sum_{i=1}^{k} P(A_i) + P(A_{k+1}))$

$\Rightarrow P(\cup_{i=1}^{k+1} A_i) \le P(\sum_{i=1}^{k+1} P(A_i))$

$(\because P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^{k} A_i) \cup P(A_{k+1})$.

$P(\sum_{i=1}^{k+1} P(A_i)) + P(A_{k+1}) = \sum_{i=1}^{k+1} P(A_i)$

c). $\sum_{i=1}^{n} P(A_i) = P(A_1) + \cdots P(A_n)$
$= 0 + \cdots + 0 = 0$.

As $P(\cup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$

Thus $P(\cup_{i=1}^n A_i) \le 0$.

As $P(\cup_{i=1}^n A_i) \geq 0$

Thus $P(\cup_{i=1}^n A_i) = 0$.

d) $P(\cap_{i=1}^n A_i) = 1 - P[(\cap_{i=1}^n A_i)^c]$

$= 1 - P(\cup_{i=1}^n A_i^c)$

$= 1 - 0$ $(\because P(A_1^c) = P(A_2^c) = \cdots = P(A_n^c) = 0$, $P(\cup_{i=1}^n A_i^c) = 0)$

$= 1$

8 → 4 (7 1)
9 → 4 (8 1)

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) P (Rebecca and Elise paired) $= \dfrac{\binom{7}{3}\times\binom{8}{3}\times 3!}{\binom{8}{4}\times\binom{9}{4}\times 4!} = \dfrac{35}{70}\times\dfrac{56}{126}\times\dfrac{1}{4} = \dfrac{1}{18}$.

P (Rebecca and Elise will be chosen to represent but not play each other)

b) $= \dfrac{\binom{7}{3}\times\binom{8}{3}}{\binom{8}{4}\times\binom{9}{4}}\times\dfrac{\binom{3}{1}\times 3!}{4!} = \dfrac{35}{70}\times\dfrac{56}{126}\times\dfrac{18}{24} = \dfrac{1}{6}$

c). P (either Rebecca or Elise but not both chosen to represent their school).

= P (Rebecca choosen but not Elise) + P (Elise choosen but not Rebecca).

$= \dfrac{\binom{7}{3}\times\binom{8}{4}}{\binom{8}{4}\times\binom{9}{4}} + \dfrac{\binom{7}{4}\times\binom{8}{3}}{\binom{8}{4}\binom{9}{4}}$

$= \dfrac{35\times 70}{126} + \dfrac{35\times 56}{70\times 126}$

$= \dfrac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_1) \leq P(A_1) + P(A_2)$. $\longrightarrow P(A_1)+P(A_2)-P(A_1 \cup A_2) \geq 0$
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

1.a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$P(A_1) + P(A_2) - P(A_1 \cup A_2) = P(A_1 \cap A_2)$

$\because P(A_1 \cap A_2) \geq 0$

$\therefore P(A_1) + P(A_2) - P(A_1 \cup A_2) \geq 0$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) Let G denote the statement that $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

when $n=1$, $LHS = P(\bigcup_{i=1}^{1} A_i) = P(A_1)$

$RHS = \sum_{i=1}^{1} P(A_i) = P(A_1)$

$LHS = RHS$

$\therefore$ when $n=1$, G is true

Assume that G is true for $n=k$,

then $P(\bigcup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

$\therefore P(\bigcup_{i=1}^{k+1} A_i) = P((\bigcup_{i=1}^{k} A_i) \cup A_{k+1})) \leq P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_i)$

$\therefore G$ is true for $n=k+1$ $\therefore P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$ $\square$

c) $\sum_{i=1}^{n} P(A_i) = P(A_1) + P(A_2) + \ldots + P(A_n) = 0$

$\because$ from b), $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) = 0$

$\therefore P(\bigcup_{i=1}^{n} A_i) = 0$ (since $P(\bigcup_{i=1}^{n} A_i) \geq 0$)

d) $P(\bigcap_{i=1}^{n} A_i) = 1 - P((\bigcap_{i=1}^{n} A_i)^c)$

$= 1 - P(\bigcup_{i=1}^{n} A_i^c)$

$\because P(A_1) = P(A_2) = \ldots = P(A_n) = 1$

$\therefore P(A_1^c) = P(A_2^c) = \ldots = P(A_n^c) = 0$

$\therefore$ by c), $P(\bigcup_{i=1}^{n} A_i^c) = 0$

$\therefore P(\bigcap_{i=1}^{n} A_i) = 1 - 0 = 1$ $\square$

$P(\bigcup_{i=1}^{n} A_i^c) = 0$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) P(Rebecca and Elise will be paired)

$$= \frac{\binom{7}{3}}{\binom{8}{4}} \times \frac{\binom{8}{3}}{\binom{9}{4}} \times \frac{3!}{4!}$$

$$= \frac{35}{70} \times \frac{56}{126} \times \frac{1}{4} = \frac{1}{18}$$

b) P(Rebecca and Elise will not be paired)

$$= \frac{\binom{7}{3}}{\binom{8}{4}} \times \frac{\binom{8}{3}}{\binom{9}{4}} \times \left(\frac{4!-3!}{4!}\right)$$

$$= \frac{35}{70} \times \frac{56}{126} \times \frac{18}{24} = \frac{1}{6}$$

c) P(either Rebecca or Elise but not both be chosen)

= P(Rebecca is chosen) + P(Elise is chosen) − 2P(both are chosen)

$$= \frac{\binom{7}{3}}{\binom{8}{4}} + \frac{\binom{8}{4}}{\binom{9}{4}} - \frac{\binom{7}{3}\times\binom{8}{4}}{\binom{8}{4}\times\binom{9}{4}} \times 2$$

$$= \frac{35}{70} + \frac{56}{126} - \frac{35}{70} \times \frac{56}{126} \times 2$$

$$= \frac{1}{2} + \frac{4}{9} - \frac{1}{2} \times \frac{4}{9} \times 2$$

$$= \frac{1}{2}$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup \cancel{A_1}\,A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$.

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2)$

$P(A_1 \cup A_2) + P(A_1A_2) = P(A_1) + P(A_2)$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) Assume formula is true for $n = k$ $P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

$P(\cup_{i=1}^{k} A_i) + P(A_{k+1}) \leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k) + P(A_{k+1}) \leq P(A_1) + P(A_2) + \ldots + P(A_{k+1})$

$P(\cup_{i=1}^{k} A_i) - (A_{k+1}) \cap (A_1 \cup A_2 \cup \ldots \cup A_k)$

$\leq P(A_1) + P(A_2) + \ldots + P(A_{k+1})$

$P(\cup_{i=1}^{k+1} A_i) \leq \sum_{i=1}^{k+1} P(A_i)$

c) From part b)

$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.

given $P(A_1) = \ldots = P(A_n) = 0$,

$\Rightarrow \sum_{i=1}^{n} P(A_i) = 0$.

$P(\cup_{i=1}^{n} A_i) \leq 0$.

since probability cannot be a negative number,

$P(\cup_{i=1}^{n} A_i) = 0$.

d) $P(\cap_{i=1}^{n} A_i) = 1$

$1 - P(\cap_{i=1}^{n} A_i) = 0$

$P(\cup_{i=1}^{n} A_i^c) = 0$

given $P(A_1) = \ldots = P(A_n) = 1$

$P(A_1)^c = \ldots = P(A_n)^c = 0$ $\Rightarrow P(A_1) = \ldots = P(A_n)$

$= P(\cup_{i=1}^{n} A_i) = 1$

$\Rightarrow P(A_1)^c = \ldots P(A_n)^c$

$= P(\cup_{i=1}^{n} A_i)^c = 0$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) Rebecca and Elise are paired $= \dfrac{\binom{7}{3}}{\binom{8}{4}} \times \dfrac{\binom{8}{3}}{\binom{9}{4}} \times \dfrac{3!}{4!} = \dfrac{1}{18}$

b) $P\left(\text{Both chosen, do not play together}\right) = \dfrac{\binom{7}{3}\binom{8}{3}\cdot(4!-3!)}{\binom{8}{4}\cdot\binom{9}{4}\cdot 4!} = \dfrac{1}{6}$

b) A= not play against each other; B= chosen to represent their schools

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\left(1-\frac{1}{18}\right)\times\frac{2}{9}}{\frac{2}{9}} = \dfrac{17}{18}$

$P(B) = \dfrac{\binom{7}{3}\times\binom{8}{3}}{\binom{8}{4}\times\binom{9}{4}} = \dfrac{2}{9}$

$\dfrac{\binom{7}{3}}{\binom{8}{4}} \times \dfrac{\binom{8}{3}}{\binom{9}{4}} \times \dfrac{4!-3!}{4!} = \dfrac{1}{6}$

c) case 1: choose Elise, not rebeca

$1 \times \binom{7}{3} \times \binom{8}{4} = 2450$

8

9

case 2: choose Rebecca, not Elise

$1 \times \binom{8}{3} \times \binom{7}{4} = 1960$

Probability $= \dfrac{2450+1960}{\binom{8}{4}\times\binom{9}{4}} = \dfrac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted]   Matric No.: A[redacted]

Original Group: T[redacted]   Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_{2}) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$.

$P(A_1^c) = 0$ $P($

1. a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2)$ this does not mean $P(A_i) = P(S)$. Does not at all!

For $P(A_1A_2)$ is a probability, $0 \leq P(A_1A_2) \leq 1 \Rightarrow -1 \leq P(A_1A_2) \leq 0$.

$\therefore P(A_1) + P(A_2) - P(A_1A_2) \leq P(A_1) + P(A_2)$. $\therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) For $n=1$, $P(\bigcup_{i=1}^{n} A_i) = P(\bigcup_{i=1}^{1} A_i) = P(A_1) = \sum_{i=1}^{1} P(A_i)$

Assume for $k \in \mathbb{N}$, $P(\bigcup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

consider $P(\bigcup_{i=1}^{k+1} A_i) = P(\bigcup_{i=1}^{k} A_i + A_{k+1}) \leq P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1}) \leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

$= \sum_{i=1}^{k+1} P(A_{k+1})$ [shown]

inductive step is shown $\Rightarrow$ Boole's inequality is true $\forall n \in \mathbb{N}$

c) $P(\bigcup_{i=1}^{n} A_i)$ is a probability $\Rightarrow 0 \leq P(\bigcup_{i=1}^{n} A_i)$.

$\therefore P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) = 0$

$\Rightarrow 0 \leq P(\bigcup_{i=1}^{n} A_i) \leq 0 \Rightarrow P(\bigcup_{i=1}^{n} A_i) = 0$. [deduced]

d) $P(\bigcap_{i=1}^{n} A_i) = 1 - P(\bigcup_{i=1}^{n} A_i^c)$

$P(A_i^c) = 0 \quad \forall i \in \{1, 2, \ldots, n\}$.

$\therefore$ From c) $P(\bigcup_{i=1}^{n} A_i^c) = 0$

$\therefore P(\bigcap_{i=1}^{n} A_i) = 1$ // [deduced]

Alternatively, $\because P(A) = P(B) = 1$,

$P(A \cup B) = P(A) + P(B) - P(AB)$
$= 1 + 1$

For $P(AB) \leq 1$, $-1 \leq A - P(AB)$

& $P(A \cup B) \leq 1$ so the only possibility is

$P(A \cup B) = 1$ & $P(AB) = 1$

then we can do it recursively, $P(A_1A_2 \cdots A_n)$
$= P(A_1(A_2 \cdots A_n))$
$= P(A_1A_2(A_3 \cdots A_n))$

4! orderings

${}^{8}C_4$ ${}^{9}C_4$

17 → choose total of 8

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) $$\frac{{}^{7}C_3 \times {}^{8}C_3 \times 3!}{{}^{8}C_4 \times {}^{9}C_4 \times 4!} = \frac{1}{18}$$

b) $$\frac{{}^{7}C_3 \times {}^{8}C_3}{{}^{8}C_4 \times {}^{9}C_4}\left(1 - \frac{3!}{4!}\right) = \frac{1}{6}$$

c) ${}^{8}C_4 \times {}^{9}C_4$ = total no of ways to choose 4 from each team.

case 1: choose Reb but not Elise:

$$1 \times {}^{7}C_3 \times {}^{8}C_4 = 2450$$

case 2: choose Elise not Reb:

$$1 \times {}^{8}C_3 \times {}^{7}C_4 = 1960.$$

~~tota~~ ∴ $$\frac{1960 + 2450}{{}^{8}C_4 \times {}^{9}C_4} = \frac{1}{2}. \quad //$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_1) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

$\because P(A_1 A_2) \geq 0 \quad \therefore P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

(b) From (a), $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ is true for $n=1$.

By induction: $P(\cup_{i=1}^{n} A_i) \leq P(\cup_{i=1}^{n-1} A_i) + P(A_n)$

$\leq P(\cup_{i=1}^{n-2} A_i) + P(A_{n-1}) + P(A_n)$

......

$\leq \sum_{i=1}^{n} P(A_i)$

(c) $P(A_1) = \cdots = P(A_n) = 0$, then $\sum_{i=1}^{n} P(A_i) = 0$.

From (b) $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) = 0$.

$\because P(\cup_{i=1}^{n} A_i) \geq 0 \quad \therefore P(\cup_{i=1}^{n} A_i) = 0$

× not necessarily true

(d) $P(A_1) = \cdots = P(A_n) = 1$, then $A_1 = \cdots = A_n =$ sample space S.

$S = \{1, 2\}$

$P(\{1\}) = 1$

$P(\{2\}) = 0$

$P(\{1\}) \neq S$

$\therefore P(\cap_{i=1}^{n} A_i) = P(S) = 1$.

(d) $P(\cap_{i=1}^{n} A_i)^c = P(\cup_{i=1}^{n} A_i^c)$

$P(A_1) = \cdots = P(A_n) = 1$, then $P(A_1^c) = \cdots = P(A_n^c) = 0$

from (c) $P(\cup_{i=1}^{n} A_i^c) = 0$

then $P(\cap_{i=1}^{n} A_i)^c = 0$, then $P(\cap_{i=1}^{n} A_i) = 1 - 0 = 1$.

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) $P(R \,\&\, E \text{ paired}) = \frac{4}{8} \times \frac{4}{9} \times \frac{1}{4} = \frac{1}{18}$ [redacted]

(b) $P(R \,\&\, E \text{ chosen but not paired}) = \frac{4}{8} \times \frac{4}{9} \times \frac{3}{4} = \frac{1}{6}$

(c) $P(\text{either } R \text{ or } E \text{ chosen but not both}) = \frac{4}{8} \times \frac{5}{9} + \frac{4}{9} \times \frac{4}{8} = \frac{1}{2}.$

(a) $P(R \,\&\, E \text{ paired}) = \dfrac{\binom{7}{3}\binom{8}{3}\,3!}{\binom{8}{4}\binom{9}{4}\,4!} = \frac{1}{18}$ → # when R&E are alr chosen & paired

(b) $P = \dfrac{\binom{7}{3}\binom{8}{3}\,4! - \binom{7}{3}\binom{8}{3}\,3!}{\binom{8}{4}\binom{9}{4}\,4!} = \frac{1}{6}$ → # they randomly paired − # they play together

(c) $P = \dfrac{\binom{7}{3}\binom{8}{4} + \binom{7}{4}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} = \frac{1}{2}$ → # R chosen E not chosen + # E chosen R not chosen

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

a) $P(A_1 \cup A_2) = P[(A_1 \cup A_1) \cup A_2]$
$\leq P(A_1 \cup A_1) + P(A_2)$
$\leq P(A_1) + P(A_2)$ (proven).

b) Proven from (a), $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$P(\cup_{i=1}^{n} A_i) = P([A_1 \cup \ldots \cup A_{n-1}] \cup A_n)$
$\leq P(A_1 \cup \ldots \cup A_{n-1}) + P(A_n)$
$\leq P(A_1) + \ldots + P(A_{n-1}) + P(A_n)$
$\leq \sum_{i=1}^{n} P(A_i)$ (proven).

c) From (b), $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

If $P(A_1) = \ldots = P(A_n) = 0$, then $\sum_{i=1}^{n} P(A_i)$
$= P(A_1) + P(A_2) + \ldots + P(A_n)$
$= 0 + 0 + \ldots + 0 = 0$

~~Since~~ By Axioms of Probability, probability of event occurring must be between 0 and 1. $(0 \leq P(A) \leq 1)$

$\therefore$ Since $\sum_{i=1}^{n} P(A_i) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$ (Deduced).

d) $P(A_1) = 1$
$P(A_2) = 1$
~~If P(A₁A~~

$P(\cap_{i=1}^{n} A_i) = P(A_1 \cap A_2 \cap \ldots \cap A_n)$

$P(A_1 \cap A_2) = 1$

$P(A_1 \cup A_2) = \underbrace{P(A_1)}_{1} + \underbrace{P(A_2)}_{1} - P(A_1 \cap A_2)$

$\therefore P(A_1 \cup A_2) = 1$ and $P(A_1 \cap A_2) = 1$

$P(A_3) = 1$
$\therefore P(A_1 \cap A_2 \cap A_3) = 1$
$\therefore P(A_1 \cap \ldots \cap A_n) = 1$ (proven)

$\frac{3}{-}\ \frac{2}{-}\ \frac{1}{-}$ ^{8} ^{9}

$\underset{R}{{}^{1}C_1}\ {}^{7}C_3$ $\underset{\text{Elise}}{{}^{1}C_1}\ {}^{8}C_3$ R $\underset{E}{4\ 3\ 2}$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

8   9

(a) Rebecca and Elise will be paired? → ${}^{7}C_3 \times {}^{8}C_3 \times 3\times2\times1$ ${}^{8}C_4\,{}^{9}C_4$

(b) Rebecca and Elise will be chosen to represent their schools but will not play each other? ↗ ${}^{7}C_3 \times {}^{8}C_3 \times 4\times3\times2 \times 1$

(c) either Rebecca or Elise (but not both) will be chosen to represent their school? → ① Rebecca represent (from sch A) and Elisa (sch B) (no represent)

${}^{1}C_1 \cdot {}^{7}C_3 \cdot {}^{8}C_4 + {}^{7}C_4 \cdot {}^{8}C_3$

a) P(Rebecca and Elise paired)

$$= \frac{\binom{7}{3}\cdot\binom{8}{3}\cdot 3\cdot 2\cdot 1}{\binom{8}{4}\binom{9}{4}4!} = \frac{1}{18}$$

b) P(R and E ~~play~~ represent but not against each other)

(complements)

$$= \frac{\binom{7}{3}\cdot\binom{8}{3}\cdot 4\cdot3\cdot2\cdot1}{\binom{8}{4}\cdot\binom{9}{4}\cdot 4!} - \frac{\binom{7}{3}\cdot\binom{8}{3}\cdot 3\cdot2\cdot1}{\binom{8}{4}\binom{9}{4}\cdot 4!} = \frac{1}{6}$$

c) P(Rebecca or Elsa represent but not both)

$$= \frac{\binom{7}{3}\binom{8}{4}}{\binom{8}{4}\cdot\binom{9}{4}} + \frac{\binom{7}{4}\binom{8}{3}}{\binom{8}{4}\cdot\binom{9}{4}} = \frac{1}{2}$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^n A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^n A_i\right) = 1$.

(a) By the Inclusion Exclusion Principle, $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$.
Since $P(A_1 A_2) \geq 0$ (Axiom 1), $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (Proven) #

(b) From (a), it is shown that when $n=2$, $P\left(\bigcup_{i=1}^{2} A_i\right) \leq \sum_{i=1}^{2} P(A_i)$.

Suppose $P\left(\bigcup_{i=1}^{k} A_i\right) \leq \sum_{i=1}^{k} P(A_i)$ is true for some $k \in \mathbb{Z}^+$,

consider
$$P\left(\bigcup_{i=1}^{k+1} A_i\right) = P\left(\bigcup_{i=1}^{k} A_i \cup A_{k+1}\right)$$
$$\leq P\left(\bigcup_{i=1}^{k} A_i\right) + P(A_{k+1})$$
$$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$$
$$= \sum_{i=1}^{k+1} P(A_i)$$

$\therefore$ k is true $\Rightarrow$ $k+1$ is true. By mathematical induction, $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$ (Proven) #.

(c) $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$ (from (b))

$\Rightarrow P\left(\bigcup_{i=1}^{n} A_i\right) \leq 0$

$\Rightarrow P\left(\bigcup_{i=1}^{n} A_i\right) = 0$ ($\because$ $P\left(\bigcup_{i=1}^{n} A_i\right) \geq 0$ and $P\left(\bigcup_{i=1}^{n} A_i\right) \leq 0$) (Deduced) #.

(d) $P\left(\bigcap_{i=1}^{n} A_i\right) = 1 - P\left(\bigcup_{i=1}^{n} A_i^c\right)$

$= 1 - 0$ ($\because$ $P(A_i^c) = 0$ since $P(A_i) = 1$)

$= 1$ (Deduced).

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) $P(\text{Rebecca and Elise will be paired}) = \dfrac{\binom{7}{3}\binom{8}{3}3!}{\binom{8}{4}\binom{9}{4}4!}$

$= \dfrac{1}{18}$ #.

(b). $P(\text{R and E chosen but not play each other}) = \dfrac{\binom{8}{3}\binom{7}{3}4! - \binom{7}{3}\binom{8}{3}3!}{\binom{8}{4}\binom{9}{4}4!}$

$= \dfrac{1}{6}$ #

(c) $P(\text{either R or E but not both chosen}) = \dfrac{\binom{7}{3}\binom{8}{4}+\binom{8}{3}\binom{7}{4}}{\binom{8}{4}\binom{9}{4}}$

$= \dfrac{1}{2}$ #.

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^n A_i\right) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^n A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^n A_i\right) = 1$.

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
$\le P(A_1) + P(A_2)$ since $P(A_1 \cap A_2) \ge 0$

b) $P(\bigcup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$

Let $n=2$,
$P(A_1 \cup A_2) \le P(A_1) + P(A_2)$ as proven in (a).

Assuming the formula is true for $n=k$,
ie. $P(\bigcup_{i=1}^{k} A_i) \le \sum_{i=1}^{k} P(A_i)$

Assume it is also true for $n=k+1$, ie.

$P(\bigcup_{i=1}^{k+1} A_i) \le \sum_{i=1}^{k+1} P(A_i)$

LHS $\rightarrow$ $P(\bigcup_{i=1}^{k+1} A_i) = P(\bigcup_{i=1}^{k} A_i) \cup P(A_{k+1}) = P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1}) - P(\bigcup_{i=1}^{k} A_i \cap A_{k+1})$
$\le P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1})$ (as shown in (a))

RHS $\rightarrow$ $\sum_{i=1}^{k+1} P(A_i) = \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$

By induction, LHS $\le$ RHS, hence $P(\cup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$ is true.

1c) From (b), $P(\bigcup_{i=1}^{n} A_i) \le \sum_{i=1}^{n} P(A_i)$

Since $\sum_{i=1}^{n} P(A_i) = P(A_1) + \ldots + P(A_n) = 0 + \ldots + 0 = 0$, $P(\bigcup_{i=1}^{n} A_i) \le 0 \Rightarrow P(\bigcup_{i=1}^{n} A_i) = 0$
since it cannot be a negative number.
(By the 1st axiom)

1d) $(A \cup B)^c = A^c \cap B^c$

Since $P(A_1^c) = \ldots = P(A_n^c) = 1 - 1 = 0$,

$P(\bigcap_{i=1}^{n} A_i^c) = P(\bigcup_{i=1}^{n} A_i)^c = 0$ from part (c).

$\therefore P(\bigcap_{i=1}^{n} A_i) = 1 - P(\bigcap_{i=1}^{n} A_i^c) = 1 - 0$
$= 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) $\frac{4}{8} \times \frac{4}{9} \times \frac{1}{4} = \frac{1}{18}$

b) $\frac{4}{8} \times \frac{4}{9} \times \frac{3}{4} = \frac{1}{6}$

c) ~~Case 1: Reb chosen, Elise not.~~

$\cancel{\frac{4}{8} \times \frac{5}{9} = \frac{5}{18}}$

Case 2: Reb not chosen, Elise chosen.

$\frac{4}{8} \times \frac{4}{9} = \frac{2}{9}$

P(either Reb or Elise (but not both)) $= \frac{5}{18} + \frac{2}{9}$

$= \frac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$.

(a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

Since $P(A_1 \cap A_2) \geq 0 \Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$

(b) Base case: when $n=1$ $P(A_1) = P(A_1)$ Obviously holds

General case: Suppose that k is true, that is

$P(\bigcup_{i=1}^{k} A_k) \leq \sum_{i=1}^{k} P(A_k)$

For $k+1$,

$P(\bigcup_{i=1}^{k+1} A_{k+1}) = P(\bigcup_{i=1}^{k} A_k \cup A_{k+1}) \leq P(\bigcup_{i=1}^{k} A_k) + P(A_{k+1})$ by (a)

$\sum_{i=1}^{k+1} P(A_{k+1}) = \sum_{i=1}^{k} P(A_k) + P(A_{k+1})$, thus

$P(\bigcup_{i=1}^{k} A_k) + P(A_{k+1}) \leq \sum_{i=1}^{k} P(A_k) + P(A_{k+1})$ holds

By Induction, $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$ ☐

(c) $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) = P(A_1) + \cdots + P(A_n) = 0$, Since $P(\bigcup_{i=1}^{n} A_i) \geq 0$, $P(\bigcup_{i=1}^{n} A_i) = 0$

(d) By De Morgan's Law $(A \cap B)^c = A^c \cup B^c$

Thus $(\bigcap_{i=1}^{n} A_i)^c = (\bigcup_{i=1}^{n} A_i)^c$, since $P(A_1^c) = P(A_2^c) = \cdots P(A_n^c) = 0$,

$P((\bigcap_{i=1}^{n} A_i)^c) = 0$, Hence $P(\bigcap_{i=1}^{n} A_i) = 1 - P((\bigcap_{i=1}^{n} A_i)^c) = 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) $\frac{4}{8} \cdot \frac{4}{9} \cdot \frac{1}{4} = \frac{1}{18}$

(b) $\frac{4}{8} \cdot \frac{4}{9} \cdot \left(1 - \frac{1}{4}\right) = \frac{3}{18} = \frac{1}{6}$

(c) Let E_1 denote that Rebecca will be chosen and E_2 denote that Elise will be chosen

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \frac{4}{8} + \frac{4}{9} - \frac{4}{8} \times \frac{4}{9}$$
$$= \frac{13}{18}$$

Then either R, or E is chosen

$$P = P(E_1 \cup E_2) - P(E_1 \cap E_2) = \frac{13}{18} - \frac{4}{8} \times \frac{4}{9} = \frac{1}{2}$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^{n} A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^{n} A_i) = 1$.

1a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

$P(A_1 \cup A_2) + P(A_1 A_2) = P(A_1) + P(A_2)$

As $0 \leq P(A_1 A_2) \leq 1$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ ▫

1b) $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

For $n=2$,

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (from (a))

Assume it is true for $n=k$,

$P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

For $n=k+1$,

$P(\cup_{i=1}^{k+1} A_i) = P(A_1 \cup A_2 \cup \ldots \cup A_k \cup A_{k+1})$

$= P(\cup_{i=1}^{k} A_i \cup A_{k+1})$

$= P(\cup_{i=1}^{k} A_i) + P(A_{k+1}) - P(\cup_{i=1}^{k} A_i A_{k+1})$

$P(\cup_{i=1}^{k} A_i \cup A_{k+1}) \neq P(\cup_{i=1}^{k} A_i) + P(A_{k+1}) - P(\cup_{i=1}^{k} A_i A_{k+1})$

$\leq P(\cup_{i=1}^{k} A_i) + P(A_{k+1})$ (from (a))

$P(\cup_{i=1}^{k+1} A_i) \leq \sum_{i=1}^{k+1} P(A_i)$ ▫

1c) $P(A_1) = \ldots = P(A_n) = 0$

$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

$\leq 0 + 0 + \ldots + 0$

$P(\cup_{i=1}^{n} A_i) \leq 0$

As $0 \leq P(\cup_{i=1}^{n} A_i) \leq 1$,

$P(\cup_{i=1}^{n} A_i) = 0$

1d) De Morgan's Law

~~$(A \cup B)^c = A^c B^c$~~

$P((\cup_{i=1}^{n} A_i)^c) = P((\cap_{i=1}^{n} A_i)^c)$

$= 0$ (from (c))

$P(\cap_{i=1}^{n} A_i) = 1 - P((\cap_{i=1}^{n} A_i)^c)$

$= 1 - 0 = 1$ ▫

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

a) $\frac{4}{8} \times \frac{4}{9} \times \frac{1}{4} = \frac{1}{18}$

b) $\frac{4}{8} \times \frac{4}{9} \times \frac{3}{4} = \frac{1}{6}$

c) $P(R \text{ chosen not } E) = \frac{4}{8} \times \frac{5}{9} = \frac{5}{18}$

$P(E \text{ chosen not } R) = \frac{4}{8} \times \frac{4}{9} = \frac{2}{9}$

$P(\text{either not both}) = \frac{5}{18} + \frac{2}{9}$

$= \frac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

1. (a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2)$

$P(A_1 \cup A_2) + P(A_1A_2) = P(A_1) + P(A_2)$

$\therefore$ Since $P(A_1A_2)$ is non-negative, $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

(b) Let $H(n)$ be the statement that $P(\bigcup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$.

$H(1)$: $LHS = P(\bigcup_{i=1}^1 A_i) = P(A_1)$

$RHS = \sum_{i=1}^1 P(A_i) = P(A_1) \geq LHS$

Assume that the statement is true for $n=k$, $P(\bigcup_{i=1}^k A_k) \leq \sum_{i=1}^k P(A_i)$

When $n = k+1$,

$$LHS = P(\bigcup_{i=1}^{k+1} A_i) = P(\bigcup_{i=1}^{k} A_i \cup A_{k+1}) \leq P(\bigcup_{i=1}^{k} A_i) + P(A_{k+1})$$
$$\leq \sum_{i=1}^k P(A_i) + P(A_{k+1})$$
$$= \sum_{i=1}^{k+1} P(A_i) = RHS$$

$\therefore$ By Mathematical Induction, the statement is true for all $n \in \mathbb{N}$.

(c) $P(A_1) + P(A_2) + \ldots + P(A_n) = 0$

$\Rightarrow \sum_{i=1}^n P(A_i) = 0$

Since $P(\bigcup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$, $P(\bigcup_{i=1}^n A_i) \leq 0$

$\therefore$ But, also, since $P(\bigcup_{i=1}^n A_i) \geq 0$, by Squeeze Theorem, $P(\bigcup_{i=1}^n A_i) = 0$.

(d) Since $P(A_1A_2 \ldots A_n) \geq P(A_1) + P(A_2) + \ldots + P(A_n) - (n-1)$, (proven in worked example 5(b) for week 01b)

$P(\bigcap_{i=1}^n A_i) \geq n - (n-1) = n - n + 1 = 1$

$\therefore$ And since $0 \leq P(\bigcap_{i=1}^n A_i) \leq 1$, so by Squeeze Theorem, $P(\bigcap_{i=1}^n A_i) = 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

~~(a) P(Rebecca and Elise will be paired)~~ $= \dfrac{(1 \times {}^7C_3 \times 1) \times (1 \times {}^8C_3 \times 1)}{({}^8C_4 \times {}^4C_1) \times ({}^9C_4 \times {}^4C_1)}$

~~(b) P(Rebecca and Elise chosen, but not play each other)~~

$= \dfrac{(1 \times {}^7C_3 \times {}^3C_1) \times (1 \times {}^8C_3 \times {}^3C_1)}{({}^8C_4 \times {}^4C_1) \times ({}^9C_4 \times {}^4C_1)}$

~~(c) (i) P(Rebecca~~

(a) P(Rebecca and Elise will be paired)

$$= \frac{(1 \times {}^7C_3) \times (1 \times {}^8C_3) \times 3!}{({}^8C_4) \times ({}^9C_4) \times 4!} = \frac{1}{18}$$

(b) P(Rebecca and Elise chosen, but not play each other)

$$= \frac{(1 \times {}^7C_3) \times (1 \times {}^8C_3) \times 3 \times 3 \times 2}{({}^8C_4) \times ({}^9C_4) \times 4!} = \frac{1}{6}$$

(c) $P(\text{Rebecca is chosen}) = \dfrac{(1 \times {}^7C_3) \times ({}^8C_4) \times 4!}{({}^8C_4) \times ({}^9C_4) \times 4!} = \dfrac{5}{18}$

$P(\text{Elise is chosen}) = \dfrac{({}^7C_4) \times (1 \times {}^8C_3) \times 4!}{({}^8C_4) \times ({}^9C_4) \times 4!} = \dfrac{2}{9}$

$\therefore$ P(either (but not both) Rebecca or Elise will be chosen)

$$= \frac{5}{18} + \frac{2}{9} = \frac{1}{2}$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A [redacted]

Original Group: T [redacted] Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

(a) $\cancel{P(A_1 \cup A_2) = P(A_1)}$

$\cancel{P(A_2) \geq 0}$

$\cancel{P(A_1) + P(A_2) \geq P(A_1)}$

$\cancel{P(A_1) + P(A_2) \geq P(A_1 \cup A_2)}$ //

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$P(A_1 \cap A_2) \geq 0$

$\therefore \cancel{P(A_1 \cup A_2) - P(A_1) - P(A_2) \geq 0}$

$P(A_1) + P(A_2) - P(A_1 \cup A_2) \geq 0$

$P(A_1) + P(A_2) \geq P(A_1 \cup A_2)$ //

(b) $\cancel{P(A_1) + P(A_2)}$ $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$\cancel{P(A_2 \cup A_3) \leq P(A_2) + P(A_3)}$

$\cancel{P(A_3 \cup A_4) \leq P(A_3) + P(A_4)}$

$\vdots$

$\cancel{P(A_{n-1} \cup A_n) \leq P(A_{n-1}) + P(A_n)}$

$\cancel{P([A_1 \cup A_2] \cup A_3) \leq [P(A_1) + P(A_2)] + P(A_3)}$

$\cancel{P([A_1 \cup A_2 \cup A_3] \cup A_4) \leq [P(A_1) + P(A_2) + P(A_3)] + P(A_4)}$

$\vdots$

$\cancel{P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)}$

$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

when $n = 1$,

$P(A_1) \leq P(A_1)$

which is true

Assuming that $\cancel{P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)}$

P_k is true, ie. $P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$

For P_{k+1},

LHS: $P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^{k} A_i) \cup P(\cup_{i=k}^{k+1} A_i)$

$= P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k) \cup P(A_{k+1})$

$\leq [P(A_1) + P(A_2) + P(A_3) + \cdots + P(A_k)] + P(A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + {}^{1} P(A_{k+1})$

$\leq \sum_{i=1}^{k+1} P(A_i)$ (induction statement) : RHS

//

(c) $P(A_1) = P(A_2) = \ldots = P(A_n) = 0$

$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) = 0$

& $P(\cup_{i=1}^{n} A_i) \geq 0$

$\therefore P(\cup_{i=1}^{n} A_i) = 0$ //

(d) $\cancel{P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)}$

$\cancel{P(\cap_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - P(\cup_{i=1}^{n} A_i)}$

Suppose $P(A_i) = 1$

$P(A_i^c) = 0$

As $P(A_i^c) = 0$,

$P(\cup_{i=1}^{n} A_i^c) = 0$

$\cancel{P(\cup_{i=1}^{n} A_i^c) = 1 - P(\cap_{i=1}^{n} A_i)}$

$P(\cup_{i=1}^{n} A_i^c)$

$= P[(\cap_{i=1}^{n} A_i)^c]$

$= 0$

$P(\cap_{i=1}^{n} A_i) = 1 - 0$

$= 1$ //

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) probability $= \dfrac{1 \times {}^7C_3 \times 1 \times {}^8C_3 \times 3!}{{}^8C_4 \times {}^9C_4 \times 4!} = \dfrac{11760}{211680} = \dfrac{1}{18}$

(b) $\dfrac{1 \times {}^7C_3 \times 1 \times {}^8C_3 \times 4! - 1 \times {}^7C_3 \times 1 \times {}^8C_3 \times 3!}{{}^8C_4 \times {}^9C_4 \times 4!} = \dfrac{35280}{211680} = \dfrac{1}{6}$

(c) $\dfrac{{}^8C_4 \times {}^9C_4 \times 4! - 1 \times {}^7C_3 \times 1 \times {}^8C_3 \times 4! - {}^7C_4 \times {}^8C_4 \times 4!}{{}^8C_4 \times {}^9C_4 \times 4!} = \dfrac{105840}{211680} = \dfrac{1}{2}$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^n A_i\right) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^n A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^n A_i\right) = 1$.

1(a). $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2)$

$0 \le P(A_1A_2) \le 1$

$\therefore\ P(A_1 \cup A_2) \le P(A_1) + P(A_2)$

(b). $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$

When $n=1$,

LHS $= P(A_i)$

$=$ RHS

Assuming P_k is true ie "$P(\cup_{i=1}^k A_i) \le \sum_{i=1}^k P(A_i)$",

For P_{k+1},

LHS $= P(\cup_{i=1}^{k+1} A_i)$

$= P((A_1 \cup A_2 \ldots \cup A_k) \cup A_{k+1})$

$\le P(A_1 \cup A_2 \ldots \cup A_k) + P(A_{k+1})$ (part (a))

$\le \sum_{i=1}^k P(A_i) + P(A_{k+1})$ (induction statement)

$\le \sum_{i=1}^{k+1} P(A_i)$

$P_k \Rightarrow P_{k+1}$ is true.

By MI, $P(\cup_{i=1}^k A_i) \le \sum_{i=1}^k P_1(A_i)$ is true.

1(c) $P(A_1) = \ldots P(A_n) = 0$

$\therefore \sum_{i=1}^n P(A_i) = 0$

From (b), since $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$

$\therefore P(\cup_{i=1}^n A_i) = 0$

1(d) ~~$P((\cup_{i=1}^n A_i)^c) \le P(\cap_{i=1}^n A_i^c)$~~

Suppose $P(A_i) = 1$

Then $P(A_i^c) = 0$

Then in (c) $P(\cup_{i=1}^n A_i^c) = 0$

$\cup_{i=1}^n A_i^c = (\cap_{i=1}^n A_i)^c$

$P((\cap_{i=1}^n A_i)^c) = 0$

$\therefore P(\cap_{i=1}^n A_i) = 1 - 0$

$= 1$

8 9

4 chosen

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

(a) P(Rebecca and Elise paired) = $\cancel{\frac{{}^8C_4\,{}^9C_4}{{}^8C_4\,{}^9C_4 \times 4!}}$ $\cancel{\frac{4}{8}\times\frac{4}{9}\times\frac{1}{4}=\frac{1}{18}}$

$$\frac{1\times{}^7C_3\times1\times{}^8C_3\times3!}{{}^8C_4\times{}^9C_4\times4!} = \frac{11760}{211680} = \frac{1}{18}$$

~~(b) P(Rebecca and Elise chosen but will not play each other)~~

(b) P(Rebecca and Elise will be chosen but will not play each other)

$$= \frac{1\times{}^7C_3\times1\times{}^8C_3\times{}^3C_1\times3!}{{}^8C_4\times{}^9C_4\times4!} = \frac{35280}{211680} = \frac{1}{6}$$

(c) P(Rebecca chosen ∪ Elise chosen) − ~~P~~(Rebecca chosen ∩ Elise chosen)

= $\cancel{1\times{}^7C_3\times1\times{}^8C_3}$ ~~P(A)+~~
$\cancel{1\times{}^7C_3\times}$ ~~P(Rebecca)~~

P(Rebecca chosen) + P(Elise chosen) − 2(Rebecca chosen ∩ Elise chosen)

$$= \frac{1\times{}^7C_3\times{}^9C_4\times4!}{{}^8C_4\times{}^9C_4\times4!} + \frac{{}^8C_4\times{}^8C_3\times1\times4!}{{}^8C_4\times{}^9C_4\times4!} - 2\left(\frac{1\times{}^7C_3\times1\times{}^8C_3\times4!}{{}^8C_4\times{}^9C_4\times4!}\right)$$

$$= \frac{\cancel{11760}\ 94080+105840-2\times47040}{211680} = \frac{1}{2}$$

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: [redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

1a)

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

Since $P(A_1 \cap A_2) \geq 0$,

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

c) $\sum_{i=1}^{n} P(A_i) = 0$, $P(\cup_{i=1}^{n} A_i) \leq 0$ $\therefore P(\cup_{i=1}^{n} A_i) = 0$

d) $\sum_{i=0}^{n} P(A_i^c) = 0$, $P((\cap_{i=1}^{n} A_i)^c) = P(\cup_{i=1}^{n} (A_i^c)) \leq \sum_{i=1}^{n} P(A_i) = 0$

$\therefore P((\cap_{i=1}^{n} A_i)^c) = 0 \to P(\cap_{i=1}^{n} A_i) = 1$

b) let Q_n denote

$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$, $n \in \mathbb{Z}^+ \cup \{0\}$

For Q_1,

$P(\cup_{i=1}^{1} A_i) = P(A_i) = \sum_{i=1}^{1} P(A_i)$

$\therefore Q_1$ is true

Assume Q_k is true, $k \in \mathbb{Z}^+ \cup \{0\}$

case $k+1$,

$LHS = P(\cup_{i=1}^{k+1} A_i) = P((\cup_{i=1}^{k} A_i) \cup A_{k+1})$

$= P(\cup_{i=1}^{k} A_i) + P(A_{k+1}) - P(\cup_{i=1}^{k} A_i \cap A_{k+1})$

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1}) - P(\cup_{i=1}^{k} A_i \cap A_{k+1})$

$\leq \sum_{i=1}^{k+1} P(A_i)$

$\therefore P_k \Rightarrow P_{k+1}$

By MI, $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

~~2a) $P_{RPE} = \frac{4}{8} \times \frac{4}{9} \times \frac{1}{4} = \frac{1}{18}$~~

~~b) $p = \frac{4}{8} \times \frac{4}{9} - \frac{4}{8} \times \frac{4}{9} \times \frac{1}{4} = \frac{1}{6}$~~

2a) Total outcome $= (8C_4)(9C_4)(4!) = 211680$

R pair E $= (7C_3)(8C_3)(3!) = 11760$

$P(R \text{ pair } E) = \frac{11760}{211680} = \frac{1}{18}$

2b)

Outcome $= (7C_3)(8C_3)(4! - 3!) = 35280$

$P(R \; E) = \frac{35280}{211680} = \frac{1}{6}$

2c) Rebecca Outcome $= 8C_4 \; 8C_3 \; 4! = 94080$

Elise Outcome $= 7C_3 \; 9C_4 \; 4! = 105840$

Both outcome $= 7C_3 \; 8C_3 \; 4! = 47040$

$$P(E \text{ or } R) = \frac{94080 + 105840 - 2 \times 47040}{211680} = \frac{1}{2}$$

Tutorial 01 (Week 03)

Name [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

1.(a) By axiom, $P(A_1A_2) \in [0,1]$

$\therefore P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2) \le P(A_1) + P(A_2)$ #

(b) Let $P(k) := P(\cup_{i=1}^k A_i) \le \sum_{i=1}^k P(A_i)$

Base case $(k=1)$ trivial $(P(A_1) \le P(A_1))$

Induction step: Assume $P(k)$ holds for some $k \ge 2$

Show $P(k+1)$ holds

by $P(k)$, $P(\cup_{i=1}^k A_i) \le \sum_{i=1}^k P(A_i)$

$P((\cup_{i=1}^k A_i) \cup A_{k+1}) \le P(\cup_{i=1}^k A_i) + P(A_{k+1})$ $(\because (a))$

$\le \sum_{i=1}^k P(A_i) + P(A_{k+1})$ $(\because P(k))$

$P(\cup_{i=1}^{k+1} A_i) = \sum_{i=1}^{k+1} P(A_i)$

$\Rightarrow P(\cup_{i=1}^{k+1} A_i) \le \sum_{i=1}^{k+1} P(A_i)$ (QED)

(c) Suppose $P(A_i) = 0$ $i = 0, 1, \ldots, n$ — ①

Then by (b) $P(\cup_{i=1}^n A_i) \le \sum_{i=1}^n P(A_i) = 0$ (by ①)

$\therefore P(\cup_{i=1}^n A_i) \le 0$

by axiom, $0 \le P(\cup_{i=1}^n A_i)$ $\Big\} \Rightarrow 0 \le P(\cup_{i=1}^n A_i) \le 0$

$\Rightarrow P(\cup_{i=1}^n A_i) = 0$ #

(d) Suppose $P(A_i) = 1$ $i = 0, \ldots, n$

Then $P(A_i^c) = 0$ $i = 0 \ldots n$

Then by (c) $P(\cup_{i=1}^n A_i^c) = 0$ $\Big\}$ $P((\cap_{i=1}^n A_i)^c) = 0$

$(\cup_{i=1}^n A_i^c) = (\cap_{i=1}^n A_i)^c$ $\therefore P(\cap_{i=1}^n A_i) = 1 - 0 = 1$ #

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

2.(a) $|S| = ({}^8C_4)({}^9C_4)\,4! = 211680$

\# outcomes in which R & E are paired

$= ({}^7C_3)({}^8C_3)\,3! = 11760$

$\Rightarrow$ Ans $= \frac{11760}{211680} = \frac{1}{18}$ #

(b) Let C be the event 'R & E are chosen'

P be the event 'R & E play each other'

then $P \subset C \Rightarrow P(C) = P(P) + P(CP^c)$

$P(CP^c) = P(C) - P(P)$

$P(C) = \frac{({}^7C_3)({}^8C_3)4!}{({}^8C_4)({}^9C_4)\,4!} = \frac{2}{9}$

$P(P) = (a) = \frac{1}{18}$

Ans $= \frac{2}{9} - \frac{1}{18} = \frac{1}{6}$

(c) Let R be the event 'R is chosen'

Let E be the event 'E is chosen'

$P(RE^c) = \frac{({}^7C_3)({}^8C_4)\,4!}{|S|} = \frac{5}{18}$

$P(R^cE) = \frac{({}^7C_4)({}^8C_3)\,4!}{|S|} = \frac{2}{9}$

Ans $= P(RE^c) + P(R^cE) = \frac{1}{2}$ #

Tutorial 01 (Week 03)

Name: [redacted] Matric No.: A[redacted]

Original Group: T[redacted] Attended Group: T[redacted]

1. (a) Prove that $P(A_1 \cup A_1) \le P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P\left(\cup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P\left(\cup_{i=1}^{n} A_i\right) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P\left(\cap_{i=1}^{n} A_i\right) = 1$.

a) 1. $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

2. $P(A_1 \cap A_2) \geq 0$

3. $P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$

4. Thus $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.

b) Base case: $n = 1$. Then $P(\cup_{i=1}^{1} A_i) \leq \sum_{i=1}^{1} P(A_i)$. Hence the base case is true.

Assume that for some $k \in \mathbb{Z}^+$, $P(\cup_{i=1}^{k} A_i) \leq \sum_{i=1}^{k} P(A_i)$.

Then consider that $P(\cup_{i=1}^{k+1} A_i) \leq P(\cup_{i=1}^{k}) + P(A_{k+1})$ (from (a))

$\leq \sum_{i=1}^{k} P(A_i) + P(A_{k+1})$ (from inductive hypothesis)

$= \sum_{i=1}^{k+1} P(A_i)$ QED.

(c) From part (a),

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) \leq P(A_1) + \ldots + P(A_n).$$

From question,

$$P(A_1) + \ldots + P(A_n) = 0.$$

Hence

$P(A_1 \cup \ldots \cup A_n) \leq 0$. Since $P(A_1 \cup \ldots \cup A_n)$ cannot be less than 0 (axiom), $P(A_1 \cup \ldots \cup A_n)$ can only be 0.

(d) Proof by contradiction.

Suppose between any two A_i, i.e A_i and A_j, $P(A_i \cap A_j) < 1$.

Let $P(A_i \cap A_j) = x$. Then $P(A_i \setminus \{A_j\}) = P(A_j \setminus \{A_i\}) = 1 - x$.

Then $P(A_i \cup A_j) = 1 - x + 1 - x + x = 2 - x > 1$.

Contradiction.

Hence between any 2 events A_i and A_j, their intersection must be 1.

Repeated accumulated intersections will cause $P(\cap_{i=1}^{n} A_i) = 1$, i.e.

$P(A_1 \cap \ldots A_{A-1} \cap A_n) = P(A_1 \cap \ldots \cap A_{n-2}) \cap \underline{P(A_{n-1} \cap A_n)} = 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

(a) Rebecca and Elise will be paired?

(b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

(c) either Rebecca or Elise (but not both) will be chosen to represent their school?

$\frac{8!}{4!4!} \times \frac{9!}{4!5!}$

a) $\cancel{\binom{8}{1}\binom{9}{1} = 72}$ ~~total ways.~~

~~P(Rebecca plays Elise) =~~ $\cancel{\frac{1}{72}}$.

~~b) By complement,~~ $\cancel{\frac{71}{72}}$.

$\binom{8}{4}\binom{9}{4} = 70 \times 126$

$= 8820$ (total no. of ways)

~~Assume Rebecca chosen, Elise chosen,~~

$\cancel{\binom{7}{3}\binom{8}{3}}$ $8820 \times 4! = 211680$ (total)

$\binom{7}{3}\binom{8}{3} \times 3! = 35 \times 56 \times 6$

$= 11760$

(a) $= \frac{1176}{21168} = 0.05555\ldots$

$= \frac{1}{18}$

b) ~~Given that~~

$\frac{\binom{7}{3}\binom{8}{3} \times (4! - 3!)}{211680} = \frac{35 \times 56 \times 18}{211680} = 0.166666$ (b)

$= \frac{1}{6}$

c) Rebecca chosen. Elise not.

$\binom{7}{3}\binom{8}{4} = 35 \times 70 = 2450$

Elise chosen. Rebecca not.

$\binom{8}{3}\binom{7}{4} = 56 \times \cancel{70}\ 35 = \cancel{3920}\ 1960$

mutually exclusive

Total ways $= \binom{8}{4}\binom{9}{4} = 70 \times 126$

$= 8820$

Total number of ways to only choose one $= \frac{2450 + \cancel{3920}\ 1960}{8820} = \frac{4410}{8820}$

$= \cancel{\frac{637}{882}} = \cancel{0.722222}$

$= 0.5$

ST2131/MA2216 Wednesday Semester II, 2018/2019

Tutorial 01 (Week 03)

Name: [redacted]

Matric No.: A [redacted]

Original Group: T [redacted]

Attended Group: T [redacted]

1. (a) Prove that $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$.
   (b) Starting with (a), use induction to prove Boole's Inequality: $P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$.
   (c) Deduce that if $P(A_1) = \ldots = P(A_n) = 0$, then $P(\cup_{i=1}^n A_i) = 0$.
   (d) Deduce that if $P(A_1) = \ldots = P(A_n) = 1$, then $P(\cap_{i=1}^n A_i) = 1$.

~~1a) $P(A_1 \cup A_1) = P(A_1)$~~

~~Since $P(A_2) \geq 0$, the $P(A_1 \cup A_1) = P(A_1) \leq P(A_1) + P(A_2)$~~

1a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$, since $P(A_1 \cap A_2) \geq 0$

b) For $n=1$, $P(\cup_{i=1}^1 A_i) = P(A_1) \leq \sum_{i=1}^1 P(A_i) = P(A_1)$

$n=2$ has been proven in 1(a).

Assuming that Boole's inequality holds for some k, $k \in \mathbb{Z}^+$,

$k+1$ case: $P(\cup_{i=1}^{k+1} A_i) = P(\cup_{i=1}^k A_i \cup A_{k+1})$

$= P(\cup_{i=1}^k A_i) + P(A_{k+1}) - P((\cup_{i=1}^k A_i) \cap A_{k+1})$

$\leq \sum_{i=1}^k P(A_i) + P(A_{k+1}) - P((\cup_{i=1}^k A_i) \cap A_{k+1})$

$\leq \sum_{i=1}^k P(A_i) + P(A_{k+1}) = \sum_{i=1}^{k+1} P(A_i)$

Since ~~k holds~~ statement is true for any arbitrary $k \in \mathbb{Z}^+$, and is true for ~~k=2~~, $k=2$ and $k=1$, then the statement is true for all $n \in \mathbb{Z}^+$.

c) $P(A_1) = \ldots = P(A_n) = 0 \Rightarrow \sum_{i=1}^n P(A_i) = 0$

Since $0 \leq P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i) = 0$, then $P(\cup_{i=1}^n A_i)$ must $=0$.

d) $(\cap_{i=1}^n A_i) = (\cup_{i=1}^n A_i^c)^c$

- since $P(A_1) = 1$, $P(A_1^c) = 0$. same for $i=1,\ldots,n$; since $P(A_i) + P(A_i^c) = 1$

By part (c), $P(\cup_{i=1}^n A_i^c) = 0$

Then, $P((\cup_{i=1}^n A_i^c)^c) = P(\cap_{i=1}^n A_i) = 1$, since $P(\cup_{i=1}^n A_i^c) + P((\cup_{i=1}^n A_i^c)^c) = 1$

2. The chess clubs of two schools consist of, respectively, 8 and 9 players. Four members from each club are randomly chosen to participate in a contest between the two schools. The chosen players from one team are then randomly paired with those from the other team, and each pairing plays a game of chess. Suppose that Rebecca and her sister Elise are on the chess clubs at different schools. What is the probability that

   (a) Rebecca and Elise will be paired?

   (b) Rebecca and Elise will be chosen to represent their schools but will not play each other?

   (c) either Rebecca or Elise (but not both) will be chosen to represent their school?

2a) $P(R \text{ is chosen} \cap E \text{ is chosen for their respective teams}) = \dfrac{\binom{1}{1}\binom{7}{3}}{\binom{8}{4}} \times \dfrac{\binom{1}{1}\binom{8}{3}}{\binom{9}{4}} = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$

$P(R \text{ pairs with } E \mid R \text{ and } E \text{ are in their school teams}) = \frac{3!}{4!} = \frac{1}{4}$ $= \dfrac{P(R \text{ pairs with } E \cap \text{in school teams})}{P(R \text{ and } E \text{ are in school teams})}$

$= \dfrac{P(R \text{ pairs with } E)}{P(R \text{ and } E \text{ are in school teams})}$

$P(R \text{ pairs with } E) = \frac{1}{4} \times \frac{2}{9} = \frac{1}{18}$

b) $P(R \text{ does not pair with } E \cap \text{chosen to represent schools}) = \frac{2}{9} - \frac{1}{18}$

$= \frac{1}{6}$

c) $P(\text{both chosen to represent their school}) = \dfrac{\binom{1}{1}\binom{7}{3}}{\binom{8}{4}} \times \dfrac{\binom{1}{1}\binom{8}{3}}{\binom{9}{4}} = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$

$P(\text{both not chosen to represent their school}) = \left(1 - \dfrac{\binom{1}{1}\binom{7}{3}}{\binom{8}{4}}\right) \times \left(1 - \dfrac{\binom{1}{1}\binom{8}{3}}{\binom{9}{4}}\right)$

$= \frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$

$P(\text{either } R \text{ or } E\text{, not both, is chosen to represent their school}) = 1 - \frac{2}{9} - \frac{5}{18}$

$= \frac{1}{2}$