Similarity-based Brownian Motion Approach for Remaining Useful Life Prediction

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***Abstract*—This paper proposes a data-driven framework for Remaining Useful Life (RUL) prediction of an operating equipment unit in the case of noisy and limited data. It consists of introducing the Brownian Motion model (BM) in the similarity framework and computing the RUL based on a collection of generated HIs (Health Indicators). For that, the percentile filter is used to pre-process the HIs by generating a collection of profiles from the operating equipment unit's HI and a set of references from a given R2F (Run-to-Failure) indicator. Then, the similarity is computed between each profile of this collection and the references in order to pick the most similar reference to each profile for modeling and RUL prediction. The final RUL is calculated as a weighted aggregation of the obtained RULs of the profile collection. A numerical application using simulated data illustrates the accuracy of this approach.**

***Index Terms*—RUL, HI Modeling, Percentile filter, Similarity**

I. Introduction

Condition-Based Maintenance (CBM) [1] is a maintenance strategy adopted by the manufacturing industries in order to ensure that the equipment units are running efficiently and upholding the highest production quality. CBM is performed based on the current health monitoring of the equipment unit, thus it is important to highlight the use of prognostics to predict the future functioning and the RUL of this equipment in order to schedule maintenance actions and to avoid failure.

Generally, prognostics approaches can be classified into three main categories based on the type of used information. These categories [2] [3] are defined as physical model-based approaches, data-driven approaches and a combination of these two known as fusion-based approaches [4].

The physical model-based approaches [5] use an explicit mathematical model to represent the dynamical system degradation and then to simulate its evolution until failure to estimate the RUL. Data-driven approaches use the monitoring and the historical data to learn the system behaviours and to perform the prognostics. These approaches do not require system models and specific knowledge as the physical model-based ones. They are also faster in implementation and computationally efficient, hence this category of approaches is adopted in this paper.

The prediction of RUL using data-driven approaches can be accomplished based on two sources of information: the HI representing the current degradation of an operating equipment unit, noted in this paper as partial HI, and the R2F (Run-to-Failure) indicators obtained from a population of equipment failed in the same operating conditions. However, the R2F data are generally limited as the equipment is not allowed to be used until failure for finance and safety issues. Besides, these two types of indicators are very noisy in the real world applications due to the measurement process and environments [6]. Hence, to overcome these issues, we propose in this paper a RUL prediction method.

First, the indicators are pre-processed by the 'percentile filter' [6]. This filter generates a set of monotonic profiles from the indicator conserving thus its tendency and isolating the useful information from noises. Consequently, a set of profiles considered as references are extracted from the R2F indicator and, another set of partially degraded profiles representing the current state of the operating equipment unit are obtained from the partial HI. In the literature, some works [7] [8] [9] [10] [11] have been targeted towards similarity-based approaches for RUL prediction when abundant R2F references are available. Generally, these works predict the RUL as a weighted aggregation of local RULs of the references after a similarity computation between these references and the partial HI.

Thus, secondly in our approach for RUL prediction, we adapt this method to the studied case when a collection of partially degraded profiles is available in addition to the references. In the similarity-based approaches, the local RUL of each reference can be predicted without modeling the HI but by mimicking the future evolution of the references until failure as in [8] [12]. Otherwise, some works [7] [13] predict the RUL by modeling the evolution of the partial HI by taking advantage of the references.

In this paper, to predict the RUL, we introduce in the similarity framework, the adaptive Brownian Motion (BM) model [14] which is a statistical method used widely to model degradation processes taking into account the collection of partially degraded profiles and references [15].

The accuracy of this approach is due to several points: (1) important information in the HI is conserved after the pre-processing, (2) comparing to non model similarity-based approaches, the adaptive BM model estimates the RUL of an operating equipment unit using all the past degradation data of the partial HI in addition to its similar reference, (3) considering a set of partially degraded profiles for RUL prediction is more representative compared to the employment of just one as in the existing similarity-based approaches [7] [8] [9] and (4) the modeling of each partially degraded profile is based on the generated reference that is the most similar to it from the available set of references [16] which is beneficial for this approach performance.

The remainder of this paper is structured as follows: the proposed approach is detailed in section II. Section III presents the approach application on simulated data. Two comparisons that highlight the efficiency of the approach are presented. Finally, conclusion and perspectives are given in section IV.

II. Proposed approach

This section illustrates first an overview of the proposed approach, then the different parts constituting the methodology are explained separately.

A. Outline of the proposed approach

The proposed approach is summarized in Fig.1.

Given the degradation information of an operating equipment unit, the objective is to track its evolution over the lifespan to predict when the failure is going to happen. The availability of a R2F indicator from a failed system which was functioning in the same operating conditions provides information on the future evolution of the equipment unit.

First of all, the percentile filter [6] [17] is applied to each indicator. It generates from the R2F indicator 100 profiles corresponding each to a percentile. They are employed as references in this study for the similarity computation and HI modeling. The profiles extracted from the partial HI are the partially degraded profiles to be extrapolated.

For the RUL prediction, the references that are similar to the partially degraded profiles are more likely to illustrate their evolution over the lifespan [16]. Thus, at each prediction time and for each partially degraded profile of the generated collection, the similarity between this profile and the references is calculated. As there is a high probability that at least one of the references is close (most similar) to that profile, it is taken for modeling. The BM is used next to model the collection profiles and to predict their future evolution. This prediction is based on the past degradation information of the profile and the model parameters of its most similar reference.

Finally, at each prediction time, a set of predicted RUL is obtained from the collection. In order to ensure a more accurate result, the final RUL is computed as a weighted aggregation of this set.

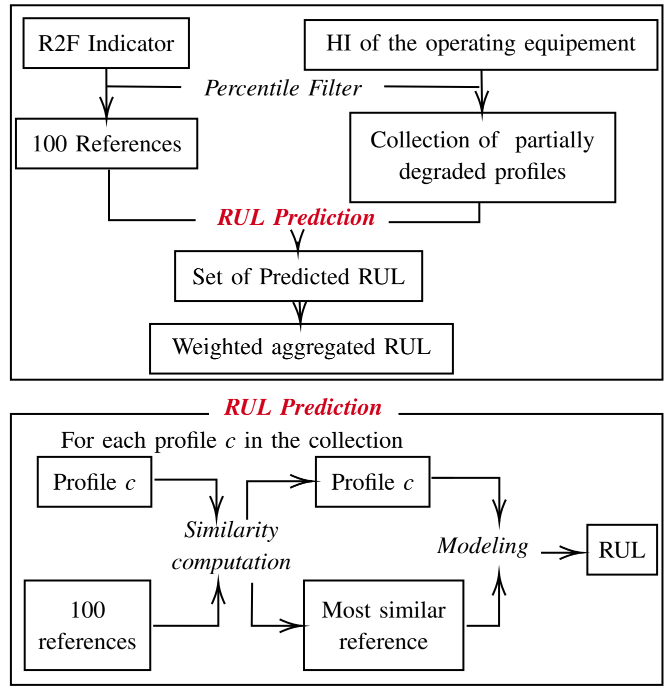


Fig. 1: Schema of the proposed approach

B. Technical Background

1) HI filtering: The percentile filter [6] [17] is used in this paper to process the HI. It is adaptive to indicators consisting of noise, variability and disturbances.

Let $\mathcal{Y}_t$ be the raw HI that measures the degradation over time t, V_N and V_F be respectively the normal and the failure threshold values (Fig. 2). In real applications, these values are set by experts based on their experience and on the well-accepted standards in the domain. This method is summarized in the following steps:

- First, the interval $[V_N, V_F]$ is divided into several sub-intervals with step s (*e.g* $s = 0.01$).

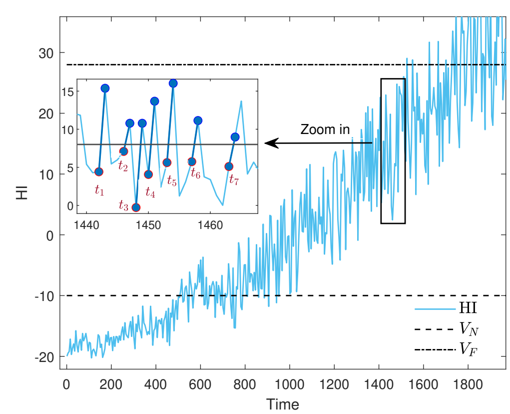


Fig. 2: $\mathcal{Y}_t$ passes through a value v many times

- The splitting values v of $[V_N, V_F]$ are summarized in $\mathcal{V} = \{V_N, V_{N+s}, V_{N+2s}, \ldots, V_F\}$. A set $T_v = \{t_1, t_2, \ldots, t_n\}$ with n the number of its elements is constructed for each value v, by selecting the time t_i verifying the following condition (as in the zoom in Fig. 2):

$$(\mathcal{Y}_{t_i} < v) \ \& \ (\mathcal{Y}_{t_{i+1}} \geq v) \quad (1)$$

This selection is based on the fact that the HI is noisy and it can cross the value v many times.

- In the following, for each $v \in \mathcal{V}$, the goal is to calculate the percentiles $p \in \{1, 2, \ldots, 100\}$ of the set T_v denoted as t_v^p. They are used next to construct the monotonic profiles as (t_v^p, v) relative to each p. These percentiles are computed in two steps:
  1) After placing the values t_i of the set T_v in increasing order, the n corresponding percentiles to these values are computed according to relative position calculation as: $p = 100(j - 0.5)/n$, $j \in \{1, 2, \ldots, n\}$.
  2) The value of the rest $(100 - n)$ percentiles is calculated as follows:

$$t_v^p = \begin{cases} t_1 & \text{for } p < 100(1-0.5)/n \\ t_n & \text{for } p > 100(n-0.5)/n \\ \text{LI} & \text{for } (1-0.5) < \frac{np}{100} < (n-0.5) \end{cases} \quad (2)$$

where LI stands for Linear Interpolation: given $(p_1, t_v^{p_1})$ and $(p_2, t_v^{p_2})$ two pairs of a percentile and its corresponding value respectively, the percentiles p_i between p_1 and p_2 (with $p_1 < p_2$) are computed by LI as follows:

$$t_v^{pi} = t_v^{p_1} + (p_i - p_1)\frac{t_v^{p_2} - t_v^{p_1}}{p_2 - p_1} \quad (3)$$

- 100 profiles are generated (Fig. 3). These profiles Y_t^p are constructed by assimilating to each splitting value v of V, the time t_v^p corresponding to the percentile p of T_v. The t_v^p that breaks the monotony of the profile is omitted.

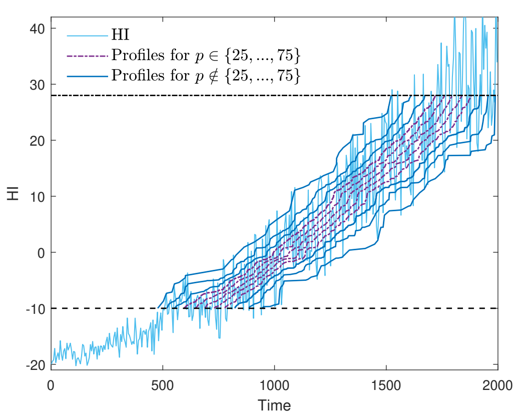


Fig. 3: Raw HI and its extracted profiles by the percentile filter

In this application, the 100 profiles extracted from the R2F indicator are considered in order to provide a large range of possibilities to match the partially degraded profiles. However, for the partial HI, the extracted profiles which their percentiles belong to the set $\mathcal{C} = \{25, \ldots, 75\}$ are only conserved as shown in Fig. 3. This selection is based on the fact that the profiles of the $\mathcal{C}$ cover the central information of the raw HI even if it consists of strong noise as proved in [17]. They also perform its tendency and are not affected by aberrant values as the profiles between percentiles $1 - 25$ and $75 - 100$. Thus, this collection is adopted for RUL prediction.

2) Similarity Computation: In this subsection, the key elements of the similarity computation are presented. Let Y_c, $c \in \mathcal{C}$, be an extracted profile from the partial HI and Z_r be a reference profile where $r \in \{1, \ldots, 100\}$. We note $S_{c\leftrightarrow r}(t_i, N_c)$ the similarity function chosen as the Euclidean distance as in [8] [9] that computes the similarity between Y_c and Z_r at the time t_i over the interval $I = N_c\Delta t$, where N_c is the number of monitoring points of Y_c and Δt is the condition monitoring time interval:

$$S_{c\leftrightarrow r}(t_i, N_c) = \sum_{n=0}^{N_c-1} [Y_c(t_i - n\Delta t) - Z_r(t_i - n\Delta t)]^2 \quad (4)$$

Only references that verify $N_c \leq N_r$ are candidate to be selected where N_r is the number of condition monitoring points of Z_r.

At each prediction time, the most similar reference to Y_c is determined. It is modeled by the BM model and is used as a support to Y_c in the prediction. This reference, noted as r_{sim}^c, is chosen in such a way to minimize the similarity function:

$$r_{sim}^c = \arg \min_{1<r<100} S_{c\leftrightarrow r}(t_i, N_c) \quad (5)$$

After determining to each partially degraded profile of $\mathcal{C}$ its similar reference, we compute its weight contribution to RUL. This weight is calculated according to the similarity between the collection's profiles and their most similar references as:

$$w_c(t_i) = \frac{S_{c\leftrightarrow r_{sim}^c}(t_i, N_c)}{\sum_{q=25}^{75} S_{q\leftrightarrow r_{sim}^q}(t_i, N_q)} \quad (6)$$

3) HI modeling: The BM model with an adaptive drift parameter is used to model the extracted profiles noted as Y_t over time t as in [14] and [15]. The strength of this method relies on the introduction of the current and the historical degradation information of the profile into the model to forecast its future trend. This is done by updating the drift parameter of the model whenever a new observation is available. The HI is modeled as follows:

$$y_{t_i} = y_{t_{i-1}} + \mu_{t_{i-1}}(t_i - t_{i-1}) + \sigma\varepsilon_{t_{i-1},t_i} \quad (7)$$

where, y_{t_i} is the degradation information at time t_i, $\mu_{t_{i-1}}$ is the updated drift parameter at t_{i-1} after observing $y_{t_{i-1}}$, σ is a constant and $\sigma\varepsilon_{t_{i-1},t_i}$ is the error term such that $\varepsilon_{t_{i-1},t_i} \sim \mathcal{N}(0, t_i - t_{i-1})$ by the BM. The variance $t_i - t_{i-1}$ makes sense as the random quantity should somehow increases when $t_i - t_{i-1}$ increases.

The Kalman Filter (KF) [18] is a recursive procedure used in this case to estimate and update the drift parameter. It computes the optimal estimator of μ_t at time t, based on the available observations up to y_t and including it. In order to apply KF, the system and the observation equations are respectively given in the following:

$$\mu_{t_i} = \mu_{t_{i-1}} + \nu \quad (8)$$

$$y_{t_i} - y_{t_{i-1}} = \mu_{t_{i-1}}(t_i - t_{i-1}) + \sigma\varepsilon_{t_{i-1},t_i} \tag{9}$$

where, ν is the system error normally distributed such as $\nu \sim \mathcal{N}(0, Q)$.

Under the normality assumption, the initial μ_{t_0} has a normal distribution with mean $\hat{\mu}_{t_0}$ and variance P_0. Thus, $\mu_{t>0}$ is a linear combination of two random variables, both with normal distributions, hence it is itself normally distributed and its mean (10) and variance (11) are updated by KF as follows:

$$\begin{aligned} \hat{\mu}_{t_i} &= \hat{\mu}_{t_{i-1}} + P_{i|i-1}(t_i - t_{i-1})F_i^{-1} \\ &\quad (y_{t_i} - y_{t_{i-1}} - \hat{\mu}_{t_{i-1}}(t_i - t_{i-1})) \quad (10) \\ P_i &= P_{i|i-1} - P_{i|i-1}(t_i - t_{i-1})^2 F_i^{-1} P_{i|i-1} \quad (11) \end{aligned}$$

where,

$$\begin{cases} P_{i|i-1} = P_{i-1} + Q \\ F_i = (t_i - t_{i-1})^2 P_{i|i-1} + \sigma^2 (t_i - t_{i-1}) \end{cases} \tag{12}$$

Thus, the future behavior of the profile is modeled after observing y_{t_i} as:

$$y_t = y_{t_i} + \hat{\mu}_{t_i}(t_i - t_{i-1}) + \sigma\varepsilon_{t_{i-1},t_i} \tag{13}$$

where $\hat{\mu}_{t_i}$ is a function of $(y_1, \ldots, y_{t_i})$ for $t_1 < t_2 < \ldots < t_i \leq t$.

4) RUL Prediction: The objective of our work is to predict accurately the time required for the system to fail. It is done by forecasting the path of the degradation indicators Y_c ($c \in \mathcal{C}$) up to the failure threshold. At each prediction time, the partial degradation information is available until t_i and the prognostic is triggered when it exceeds the normal operating threshold.

At each t_i and for each $Y_c, c \in \mathcal{C}$, the RUL computation is resumed by the following steps [15]:

- The most similar reference $Z_{r^c_{sim}}$ to Y_c is picked as in II-B2. Its behavior is taken into account in order to forecast Y_c until failure as detailed just below.
- The corresponding time t_c on which $Z_{r^c_{sim}}$ has almost the same degradation state as Y_c is determined as :

$$Z_{r^c_{sim}}(t_c) \leq Y_c(t_i) \leq Z_{r^c_{sim}}(t_{c+1}) \tag{14}$$

- The drift parameter $\hat{\mu}^c$ of Y_c and $\hat{\mu}^{sim}$ of $Z_{r^c_{sim}}$ are calculated using the KF (equations 10-12).
- The residual is computed as:

$$r_i = \hat{\mu}^c_{t_i} - \hat{\mu}^{sim}_{t_c} \tag{15}$$

- The predicted values of $\hat{\mu}^c$ for $t > t_i$ are:

$$P^c_{t_i} = \{\hat{\mu}^{sim}_{t_c} + |r_i|, \hat{\mu}^{sim}_{t_c+1} + |r_i|, \ldots \hat{\mu}^{sim}_{t_{Max}} + |r_i|\} \tag{16}$$

  where t_{Max} is the time of the last monitoring point of $Z_{r^c_{sim}}$. The absolute value of r_i is taken in order to keep the evolution of the indicator monotonous and subsequently to avoid its divergence from V_F.
- The expected value of Y_c is predicted iteratively over t:
  - ◇ for $0 \leq j < t_{Max} - t_c$

$$\begin{aligned} \mathbb{E}\left[Y_c(t_{i+j+1})\right] &= \mathbb{E}\left[Y_c(t_{i+j}) + \hat{\mu}^{sim}_{t_c+j} + |r_i| + \sigma\epsilon_{0,1}\right] \\ &= \mathbb{E}\left[Y_c(t_{i+j})\right] + \hat{\mu}^{sim}_{t_c+j} + |r_i| \end{aligned} \tag{17}$$

  - ◇ for $j \geq t_{Max} - t_c$

$$\begin{aligned} \mathbb{E}\left[(Y_c(t_{i+j+1})\right] &= \mathbb{E}\left[Y_c(t_{i+j}) + \hat{\mu}^{sim}_{t_{Max}} + |r_i| + \sigma\epsilon_{0,1}\right] \\ &= \mathbb{E}\left[Y_c(t_{i+j})\right] + \hat{\mu}^{sim}_{t_{Max}} + |r_i| \end{aligned} \tag{18}$$

- The predicted local RUL of Y_c at t_i, noted as RUL^c_{pred}, is equal to $\hat{j}\Delta t$ unit of time, such that:

$$\hat{j} = \inf\{j : E(Y_c(t_{i+j})) > V_F\} \tag{19}$$

Hence, the RUL at time t_i is calculated as a weighted aggregation of RULs of the collection's profiles:

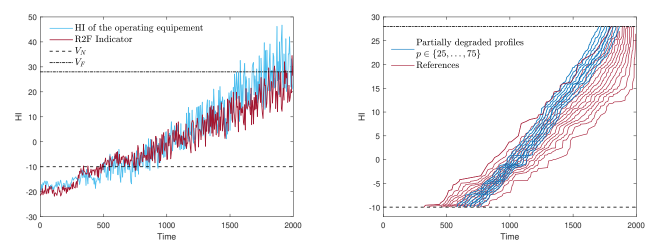
$$RUL_{pred}(t_i) = \sum_{c \in \mathcal{C}} w_c RUL^c_{pred}(t_i) \tag{20}$$

III. Application

The proposed approach is evaluated by numerical experiments on data simulated on *Matlab*. The data corresponds to an equipment unit of the semiconductor manufacturing process. It consists of measurements collected from 6 sensors and 351 observations for a recipe that was performed on 2000 wafers. To demonstrate the accuracy of our method, a comparison with existing similarity-based ones [8] [12] is performed. Besides, a study is also established to justify the choice of using a collection of the partially degraded profiles and its impact on RUL prediction. The results are presented graphically and evaluated by the metrics dedicated to prognostic.

A. Experimental setup

The raw HI of the operating equipment unit and the R2F indicator are presented in Fig. 4a. The R2F indicator is extracted from the simulated data set as in [17], and the partial HI is generated by taking into account the dynamics of the R2F with the aim of being in the same operating conditions. The V_N and the V_F are set to 10 and 28 respectively.



(a) Raw HIs (b) Extracted profiles after filtering

Fig. 4: HIs before and after filtering

In order to remove the noise, these two HI are filtered. The extracted profiles between the percentiles 25 and 75 from the partial HI in addition to the 100 profiles representing the R2F indicators are illustrated in Fig. 4b.

In this application, Y_{50} is chosen as the representative of the collection profiles as it is the median of the extracted profiles. The prognostic is performed between the time when this profile exceeds the phase of normal functioning ($T_{init} = 700$) and the time when it reaches the failure threshold ($T^{50}_{end} = 1807$).

To evaluate the approach, the real RUL is calculated and compared to the predictions. It is the time required by the collection profiles to reach the failure threshold as:

$$RUL_{real}(t_i) = \frac{1}{|\mathcal{C}|}\sum_{c\in\mathcal{C}} RUL^{c}_{real}(t_i) \qquad (21)$$

such that, $RUL^c_{real}(t_i) = T^c_{end} - t_i$, $\mathcal{C} = \{25,\ldots,75\}$ and $|\mathcal{C}| = \{\#c, c \in \mathcal{C}\}$.

B. Results and discussion

The evaluation metrics and the obtained experimental results are presented in this subsection.

1) Metrics: Among the proposed metrics for RUL evaluation [19] [20], we have used two specific metrics to evaluate the performance of our method as they evaluate the RUL prediction from differents ways: the Root Mean Square Error (RMSE) and the $(\alpha - \lambda)$ accuracy.

- The RMSE evaluates the overall predictions. It is computed as:

$$RMSE = \sqrt{\frac{\sum_{t_i=T_{init}}^{T^{50}_{end}} (\mathrm{RUL}_{real}(t_i) - \mathrm{RUL}_{pred}(t_i))^2}{T^{50}_{end} - T_{init}}} \qquad (22)$$

- $(\alpha-\lambda)$ accuracy evaluates the prognostic at specific moments. It determines whether a prediction result falls or not within the accuracy zone at a specific time t_λ, where $\lambda \in [0,1]$. This zone is defined with respect to a percentage α of accuracy related to the real RUL (α is fixed at 10% for precision). The time t_λ is computed between the beginning of the prediction and the real failure time as:

$$t_\lambda = T_{init} + \lambda(T^{50}_{end} - T_{init}) \qquad (23)$$

2) Comparison with existing similarity-based approaches: Generally in the existing similarity-based approaches [8] [12], the RUL at each prediction time t_i is equal to the number of monitoring points of their similar reference from this time until failure. Thus, the RUL is obtained without modeling the HI and without predicting its future values. In the following, the way of computing the RUL is evaluated in these approaches and compared to our method (as in section II-B3).

For that, the same framework of the proposed approach is considered with regard to the HI filtering, the similarity computation and the RUL prediction as a weighted aggregation of local RULs of the collection profiles. The difference occurs in the methods of computing the local RULs.

The predictions are illustrated in Fig. 5. It is shown that the difference between the real and the predicted RUL is smaller in our approach except for the first period between $[700, 750]$. This remains valid on the last moments close to failure where maintenance actions should be taken as illustrated in the zoom.

It can be concluded that the proposed approach outperforms the existing similarity-based ones. This is explained first by the fact that modeling and predicting the future evolution of the HI as in our approach is more accurate than mimicking the evolution of the similar reference to the partial HI until failure without adapting it to the current situation as in the existing approaches. Second, the BM model used in the approach employs two sources of information: the past degradation information of the partial HI and those of its similar reference. Hence, it provides additional information for the benefit of the prediction contrary to the existing similarity-based approaches where the partial HI is only used in the similarity computation.

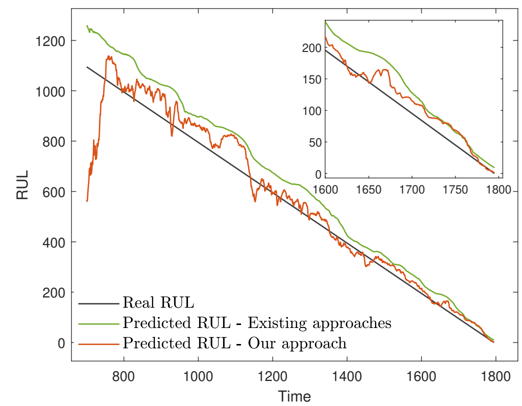


Fig. 5: Predicted RUL (our approach VS existing approaches) and a zoom on the last 200 times

Then, the different approaches are evaluated by prognostic metrics. The RMSE is computed first. The results show that the proposed approach significantly outperforms the existing similarity-based approaches with an improvement of 42.6% (106.9 vs 61.4) in the RUL prediction on the overall interval and 58.8% (63.4 vs 26.1) on the last 200 predictions.

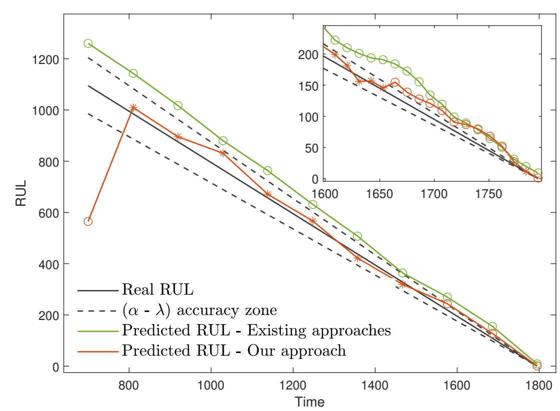


Fig. 6: The result of the metric $(\alpha - \lambda)$ accuracy

For the $(\alpha - \lambda)$ accuracy metric, it can be noticed, that the predicted RUL corresponding to t_λ where $\lambda \in [0, 0.1, 0.2, \ldots, 1]$ falls more in the accuracy zone in the proposed approach (Fig. 6). This remains valid for λ calculated in a step of 1/100 between 0 and 1: from a total of 101 t_λ, 74 predictions in our approach fall in the accuracy zone compared to only 8 in the existing similarity-based approaches. The result of this calculation on the last 200 prediction times is presented in the zoom of Fig. 6.

3) Impact of using a collection of partially degraded profiles: As the HI affects the prediction result, we study the impact of using a collection of partially degraded profiles on the RUL. Hence, the RUL is computed using two methods:

- Using one profile illustrating the current health of the equipment unit as adopted in the existing similarity-based approaches [7] [8] [9]. In this application, we choose the 50^{th} partially degraded profile as it is the median of the collection. This method is noted as 'Profile 50'.
- Using a collection of profiles extracted from the partial HI between the percentiles 25 and 75. This method is noted in the following as 'Profile 25-75' (our approach).

An overview of the two predicted RUL in Fig. 7 shows a smaller variation around the ground truth RUL in the presented approach compared to the 'Profile 50'.

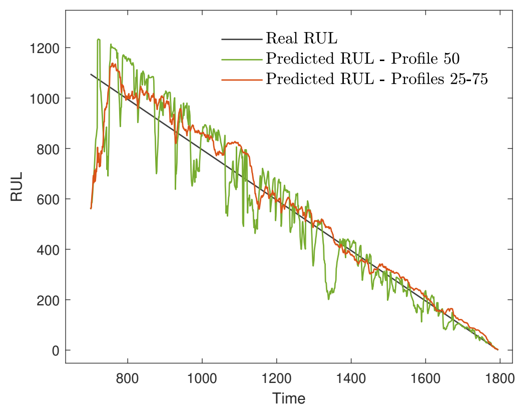


Fig. 7: Predicted RUL (one profile VS a collection of profiles)

For the numerical evaluation, our approach has the smallest RMSE on the overall prediction interval: 61.4 vs 102.1. For the $(\alpha - \lambda)$ accuracy (Fig. 8), the predicted RUL stays more in the accuracy zone in our approach than the 'Profile 50' method for $\lambda \in [0, 0.1, 0.2, \dots, 1]$.

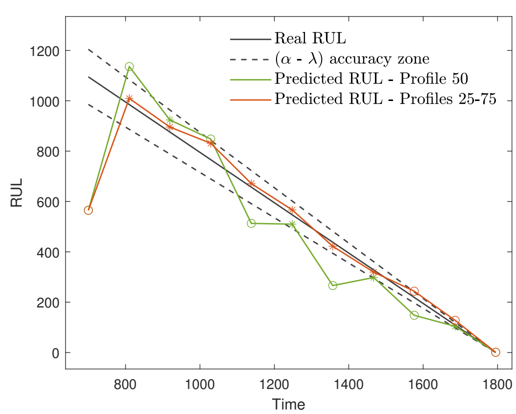


Fig. 8: The result of the metric $(\alpha - \lambda)$ accuracy

A first conclusion concerns the RUL prediction: a more accurate and robust estimation can be obtained by aggregating the RULs of a set of profiles as in the presented approach, than from a single one. This is explained by the fact that the method is more representative and avoids to have a sensitive predicted RUL based on a limited sample. The second conclusion concerns the filtering: using a collection of profiles illustrates better the behavior of the noisy partial HI and conserves much more the useful information than from a single one.

IV. Conclusion

This paper proposes a similarity-based BM method for RUL prediction when the partial HI of the operating equipment unit and the available R2F indicator are noisy. The RUL prediction is based on a collection of profiles extracted from the HIs after a pre-processing step using the percentile filter. Experiments performed on simulated data have proved its accuracy over existing the similarity based methods. In the near future, this work will be tested on real data and on wider applications when particularly the partial HI and the R2F indicators are not exactly collected in the same operating conditions.

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