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16 720
HW 4

1.1 Show that if im coord : $(0,0)$, $F_{33} = 0$

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

$Pt_1 \rightarrow x_1^T = [0,0,1]$
$Pt_2 \rightarrow x_2^T = [0,0,1]$

$$x_2^T F x_1 = 0$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{1\times3} \underbrace{\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{3\times1} = 0$$

$$\begin{bmatrix} F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$\therefore F_{33} = 0$ (shown)

1.2 2^{nd} camera pure-translates along the x-axis.
Show that epipolar lines are parallel to x-axis

Translational $t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$ & Parallel to x-ax, $t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix}$

Rotational (none) $\Rightarrow R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Vector to matrix $\Rightarrow T_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$

$E = T \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \rightarrow$ Ess matrix

$x_1 \rightarrow$ Pts 1
$x_2 \rightarrow$ Pts 2

$x_2^T E x_1 = 0$

Epipolar lines, $l_2 = E x_1$ ①
$x_2^T l_2 = 0$ ②

①, $l_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix}$

②, $\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix} = 0$

$-y_2 t_1 + t_1 y_1 = 0 \rightarrow$ epi line

The epipolar line eqn has only y-value that represents x-axis parallel.

1.3 R_i & t_i. Known: K_1 & K_2 Find: E & F?

~~[illegible]~~ What's R_{rel} & t_{rel}?

$x \rightarrow$ pts

Pinhole eqn, $\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = K[R_i | t_i]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

Find pts, ~~[illegible]~~ to pts, correlation,

$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K[R_1 | t_1]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ & $\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K[R_2 | t_2]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

$= K\left(R_1\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + t_1\right)$ $= K\left(R_2\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + t_2\right)$

$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = R_2^{-1}\left(K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2\right)$

$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K\left(R_1\left(R_2^{-1}\left(K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2\right)\right) + t_1\right)$

$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \underbrace{KR_1R_2^{-1}K^{-1}}_{R_{rel}}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \underbrace{- KR_1R_2^{-1}t_2 + Kt_1}_{t_{rel}}$

$$\boxed{\begin{aligned} R_{rel} &= KR_1R_2^{-1}K^{-1} \\ t_{rel} &= Kt_1 - KR_1R_2^{-1}t_2 \end{aligned}}$$

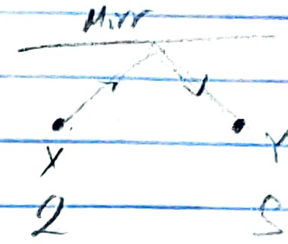
$\boxed{E = t_{rel} \cdot R_{rel}}$

$F = K^{-1}EK = \boxed{K^{-1}(t_{rel} \cdot R_{rel})K}$

1.4 Camera views object in mirror.

Show Eqn to 2 images by skew-symmetric F matrix.

Ass: Object is flat & pts are equal dist



Skew sym: $A^T = -A$

$$\hookrightarrow \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

$E = R[t]_\times$

Mirror, $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ → No rotation $\quad t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

1: Image & 2: Reflection ; $w \to$ pts $= \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$w_2 = K(Rw_1 + t)$

$w_2^T F w_1 = 0$

$F = K^{-1} E K$

$= K^{-1} R \cdot t \, K$

For skew symmetric F matrix, K is ignored

$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [$

Partial cred please...

$$F = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = -F^T \rightarrow \text{skew Symmetric (shown)}$$

3.2 Triangulate

$A_i W_i = 0$

$W_i = [x_i, y_i, z_i]^T$

$$A_i = \begin{bmatrix} pts1_{i1} \cdot C1_{3:} - C1_{1:} \\ pts1_{i2} \cdot C1_{3:} - C1_{2:} \\ pts2_{i1} \cdot C2_{3:} - C2_{1:} \\ pts2_{i2} \cdot C2_{3:} - C2_{2:} \end{bmatrix}$$

$pts\,u_{iv} \implies$ u: Either point 1 or 2
v: x or y coord of points

$C\,W_{y:} \implies$ w: Camera matrix of point 1 or 2
y: which row of camera matrix.