Design of Chaotic Pendulum with a Variable Interaction Potential

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1 Introduction

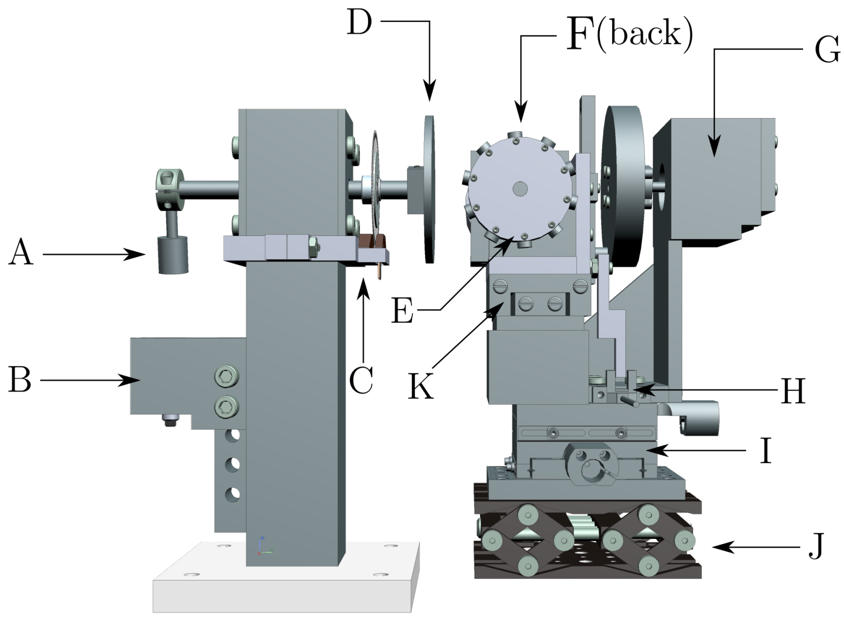


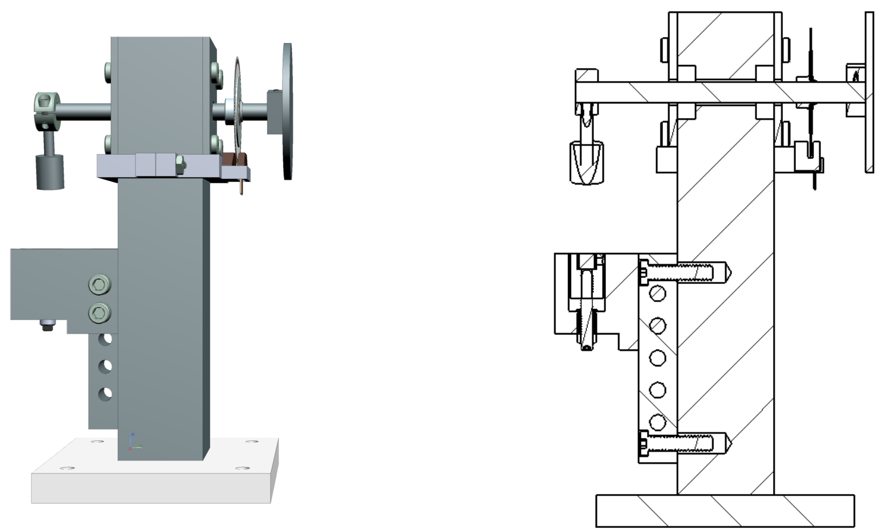
Figure 1: The device we designed for collecting data from a variable interaction potential. This design was inspired by the one described in [1]. Labels A and B represent magnets. C is a rotary encoder for the rotary disk labeled D and E is spinning magnet holder.F is a DC motor and G is step motor. H is the photoelectric sensor. I is the translation table while J is a lab jack. K is a linear slider and G is a stepper motor.

The purpose of this report is to describe the design methodology of a bench top apparatus for studying chaotic motion of a pendulum. Tran et. al used such a device for studying the dynamic behavior of the pendulum especially in the chaotic regime [1]. Fig. 1 depicts the pendulum which is driven by eddy motor and is subject to a varying interaction potential due to the repelling magnets A and B.

In order to replicate and expand upon their work, the machine they used to collect their experimental data must be recreated. However, while the general working principle of the device was explained in Ref. [1], most of the necessary dimensions and part numbers are missing. This makes it impossible to manufacture the apparatus in Ref. [1]. Since none of the authors of Ref. [1] were accessible to provide more information about their design, this report aims to describe a reproducible device for replicating the experimental results in Ref. [1]. This report will display both the computer aided designs of the machined parts as well as an explanation for each parts design. With this information it will be possible to continue the research on the behavior of chaotic magnetic pendulums. The parts in the CAD files are designed in inches, however, all the calculations within this manuscript are in SI units.

Fig. 1 shows the main components which consist of the magnets, rotary encoder with its rotary disk, DC motor, translation table, lab jack, linear slider, and stepper motor. Magnet B can adjust with major or very specific minor increments. By adjusting magnet B, it is possible to adjust its interaction with magnet A. Magnet A will be spurred into motion by the spinning aluminum disk labeled D. A DC motor will spin the magnets in the holder labeled E across the aluminum disk. A scotch yoke, will transform the rotational motion of a stepper motor, G, to linear motion. This linear motion will push and pull the DC motor, and spinning magnets across the aluminum disk on a linear slider. The magnetic field caused by the spinning magnets moving across the aluminum disk will cause the aluminum disk to rotate. The rotary encoder labeled C will record the speed of the aluminum disk and magnet A. The photogate labeled H will record the linear motion of the linear slider. The translation tables, I, will move the spinning magnets in the XY plane. The lab jack J will adjust the height of the spinning magnets. The majority of the magnets are secured with set screws as shown in Fig. 1, however pressure fit calculations are also provided in this manuscript.

2 Magnetic Pendulum Tower



(a) Left Tower Assembly

(b) Left Tower Assembly Section

Figure 2: Left Tower

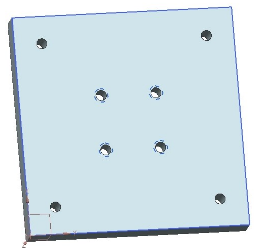
The left tower assembly is displayed in Fig. 2. Unless otherwise stated, the parts used for the structural components of the tower are made out of Delrin plastic.The bearings used in the tower are 8mm inner diameter Bones Swiss bearings. They are press fit into place with a covering plate to keep them in place. The main axle through the tower is a stainless steel 8mm rod. The rotary encoder used is a EM2-2-10000-I and the rotary disk used is a HUBDISK-2-10000-IE. The rotary encoder is attached into a clamp to secure it to the left tower.

2.1 Tower Base

The base of the left tower in Fig. 1 is made separate from the tower itself to facilitate manufacturing. The structure of the left tower can be assembled from one piece of Delrin plastic with its sides milled down leaving a wider base and narrower tower piece. However, a much easier method is to separate the left tower from its base and buying two pieces of Delrin to match their specifications thus reducing the manufacturing time and wasting of material. The base is cut from a corner of a half inch thick sheet of Delrin plastic.

Fig. 3 shows the left tower base which has a square, four inch cross section. There are four quarter inch through holes on each of the four corners of the base. Each hole is one and a half inches from the center and three inches apart. Because of the table, the holes needed to be whole inches apart. This left half an inch from the center of each hole to sides of the base. Half an inch is enough distance to make the hole without secure mounting of the base. These four holes are through holes so the screw can pass through the base and screw into the table to secure it.

In addition to the four holes on the corners of the base to secure it to the table, there are four holes in the center to secure the base to the tower. Each hole is 0.25 inch through hole and is half an inch from the center and one inch from each other. The holes are counterbored as shown in Fig. 3 so that the low-profile


(a) Left Tower Base Top

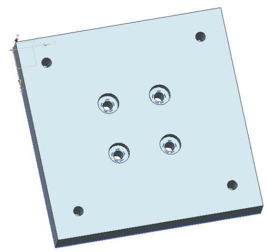

(b) Left Tower Base Bottom

Figure 3: Base for Magnetic Pendulum Tower

socket screw heads are hidden inside the base allowing the base to lay flat on the table. The counterbore is placed under the base so the screw can screw up through the base into the tower securing the base to the tower. The typical recommendation for screw depth is 1.5 $\times$ screw diameter. The base is half an inch thick so there is enough space for the counterbore while leaving enough threading to secure the screw to the base as well as to the tower.

2.2 Pendulum Pedestal

The main component of the left tower is a large rectangular brick made from Delrin plastic. The tower's cross section is square with 1.5 inch sides, and its height is 7.5 inches. Delrin can be purchased in bricks 1.5'x 1.5' and be cut down to six and a half inches. The tower is 1.5" x 1.5" because it has to be able to fit four 1/4" 20 threaded holes in its base so that it can be secured to the base described in Section 2.1 (see Fig. 3). Each hole is one inch apart and half an inch from the center of the tower. The tower is seven and a half inches tall because it is required to have enough room for the slider holding magnet B in Fig. 1 to be raised and lowered depending on the desired interaction with magnet A in the same figure. The tower must also be tall enough for magnet A to swing on its pendulum without conflicting with the base of magnet B.

The left side of the tower is shown in Fig. 5c. The large counter bore hole towards the top is for housing a bearing and shaft. The large hole is 0.9 inches, which is just larger than the outer diameter of an 8mm inner diameter Bones Swiss bearing. The smaller hole is 0.4 inches, which is slightly larger than the 8 mm inner diameter of the bearing. Since the shaft hole of 0.4 inches is larger than the 8mm inner diameter of the bearing, the inner rod will not be in contact with the tower and will not cause additional friction. In addition to the counter bored hole for the bearing and shaft, there are also four holes for 1/4" 20 threaded inserts around the bearing hole. These holes are to secure a bearing cap over the bearing. The bearing cap will keep the bearing from coming out of the tower during movement. The bearing cap, shown in Fig 5a, is a simple rectangle of 1/8" thick 1.5" wide and 2.1" tall block of delrin with four through holes. The holes are quarter inch from the sides of the tower. The top row of holes are 0.6 inches from the top and the second lower row of holes are 1.3 inches below the top row of holes. There is one last set of holes for threaded inserts in the left side of the tower. These two holes are to secure the slider displayed in Fig. 5b.

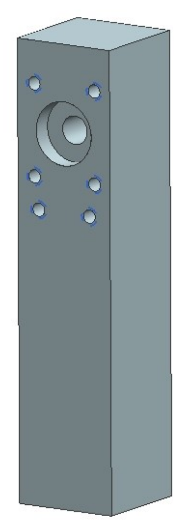
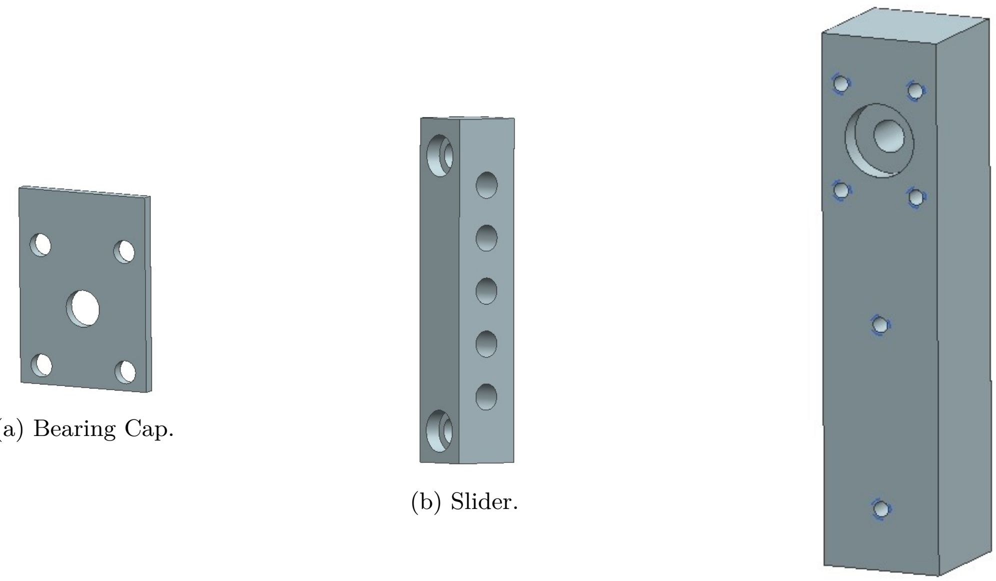


Figure 4: Left Tower Right Side.

The slider is used to adjust the height of magnet B in Fig. 1. The multiple holes on the sides of the slider are for a bolt to pass through and secure the table for magnet B at different heights. The two holes for 1/4” 20 threaded inserts in the middle of the tower are used to secure the slider to the left side of the tower.

The right side of the tower is displayed in Fig. 4. Similar to the left side, it has a counter bore hole for the bearing to fit tightly into the tower as well as a smaller diameter hole that goes through the tower for the axle to fit through the tower. The right side also has the four holes for 1/4” 20 threaded inserts around the bearing for the bearing cap to screw over the bearing securing it into the tower.



(a) Bearing Cap.

(b) Slider.

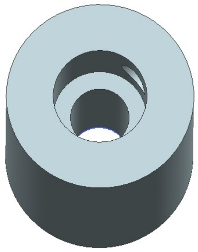
(c) Left Tower Left Side.

Figure 5: Main components of the magnetic pendulum tower

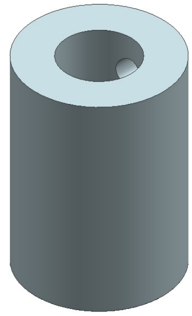
2.3 Magnet B and Magnet Adjustor

Magnet B in Fig. 1 needs to be able to be adjusted in order to control its interaction with magnet A. This magnet will sit in an aluminum holder displayed in Fig. 6a. Magnet B will be secured into place with a set screw. This is the only magnet in the design that is not secured by a pressure fit, because a set screw allows magnet B to be removed more easily than a pressure fit would. This provides more flexibility in terms of reverting the polarity of B or removing it altogether. Fig. 6c shows a finely threaded 3/16"-100 screw that screws up into the holder from below. A 3/16"’-100 threaded bushing is placed into a through hole at the bottom of the holder. The magnet will sit on this finely threaded bushing. The 3/16"’-100 threaded screw will screw up underneath the magnet and a nut for the screw will be tightened onto the bottom of the holder once the magnet and screw are in place. The nut will keep the screw from unscrewing when adjusting the height of magnet B. The final assembly of the magnet, magnet holder, screw, and nut is displayed in Fig. 6c.

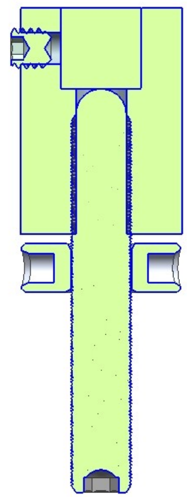
Once the system for holding magnet B in Fig. 6c is assembled the whole system is screwed down into the B magnet attachment where the latter is shown in Fig. 7a. The attachment has two sides with a slot in between. The slot is for the slider in Fig. 5b. Each side of the attachment has two holes which will line up with the holes in the slider. Two bolts will slide through the attachment and slider to secure it into place. The magnet system displayed in Fig. 6c will screw in the hole on the right side of the B magnet attachment. A bushing for the 3/16"’-100 screw will slide through the narrow hole on the right side of the attachment displayed in Fig. 6c. This bushing will allow the whole B magnet system in Fig. 6c to screw up or down by very fine increments. The whole B magnet adjustor is displayed in Fig. 7c.



(a) Top View of B Magnet Holder



(b) Side View of B Magnet Holder



(c) B Magnet System Section View

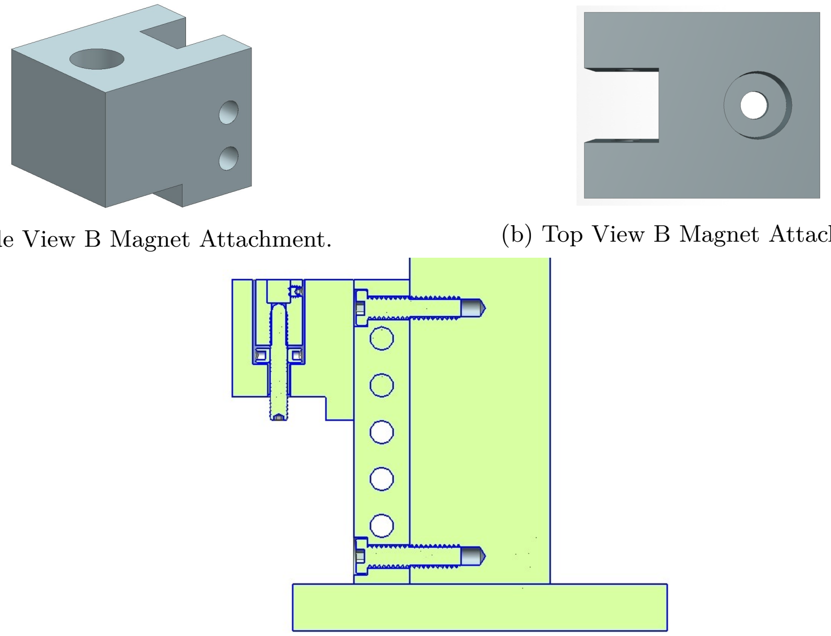
Figure 6: Magnet B Holder.

Once Magnet B is attached, it will be adjusted both in major half inch increments and minor hundredths of an inch increments. The major increments will be when the attachment is slid up and down the slider. The slider will give an adjustment of half an inch. The fine adjustment screw will give an adjustment of hundredths of inches.

2.4 Magnet A and Attachment to Axle

As displayed in Fig. 1, magnet A acts as a pendulum attached to the main axle. Magnet A hangs down and interacts with magnet B. When the aluminum disk, on the right end of the figure, starts to spin, the axle will also spin causing magnet A to spin in a circle around the main axle. Magnet A will be pressure fit into place in a holder. As determined in Section 4, the force from the pressure fit is more than enough to hold the magnet in place even at maximum speeds.

The holder for magnet A is displayed in Fig. 8a. This magnet is pressure fit into the hole at the top of the holder. On the bottom of the holder there is a 10-32 threaded hole as shown in Fig. 8b. A 1 inch long 10-32 threaded rod will be screwed into the bottom of the holder. The top of this rod will be screwed into an 8mm diameter clamp on shaft collar. This collar is composed of two half circles which clamp around the main axle. There are two screws on either side of the collar. When these screws are tightened, the collar tightens around the main axle. A 10-32 threaded hole will be drilled into the collar so that the 10-32 rod can be screwed into it. Once completed, magnet A will be pressure fit into a holder which is screwed into a collar and the latter is then clamped onto the main axle. The magnet A assembly is displayed in Fig. 8c.



(a) Side View B Magnet Attachment.

(b) Top View B Magnet Attachment.

(c) B Magnet Adjustor

Figure 7: Magnet B Attachments.

When the main axle starts to spin the collar will spin causing magnet A to also spin.

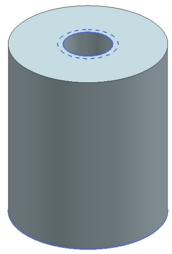
2.5 Aluminum Disk Holder

An aluminum disk is attached to the far right side of the main axle see point D in Fig. 1. When the spinning magnets from the right tower pass over the surface of the aluminum disk, the magnetic field will cause the aluminum disk to start spinning. When the aluminum disk starts to spin, it will also spin magnet A attached on the left side of the tower.

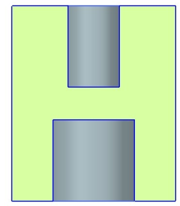
The aluminum disk is 1/8" thick with an outer diameter of 2.5". The aluminum disk is attached to to a universal mount by an adhesive. Screws are not used because the magnetic field of the screws could interact with the magnetic fields of the spinning magnets passing over the aluminum disk causing unwanted effects. The adhesive's shear strength must be adequate for securing the aluminum disk to its holder.

3 Actuator Tower Assembly

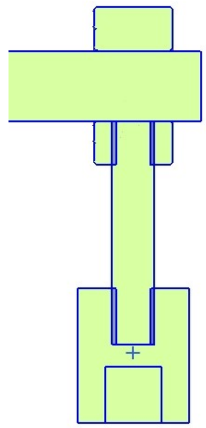
The right tower in Fig. 1 is the actuator tower. This tower is called the actuator tower because as the spinning magnets pass over the aluminum disk in the pendulum tower, the magnetic field from the spinning magnets cause the aluminum disk to start to spin and in turn spins magnet A. The actuator tower consists of two motors, a DC motor and a stepper motor. The DC motor spins the magnets in the holder labeled E in Fig. 1. The stepper motor is labeled G in the figure and it uses a scotch yoke mechanism, labeled F, to push and pull the DC motor and its spinning magnets along the linear slider, labeled K. As the spinning magnets are pumped back and forth on the linear slider, they pass across the aluminum disk labeled D. The height of the spinning magnets can be adjusted by the lab jack labeled J, which raises and lowers the entire actuator tower. The spinning magnets can also be adjusted in the X and Y directions using the translation



(a) A Magnet Holder.



(b) A Magnet Holder Section View.



(c) A Magnet Assembly

Figure 8: A Magnet Attachment.

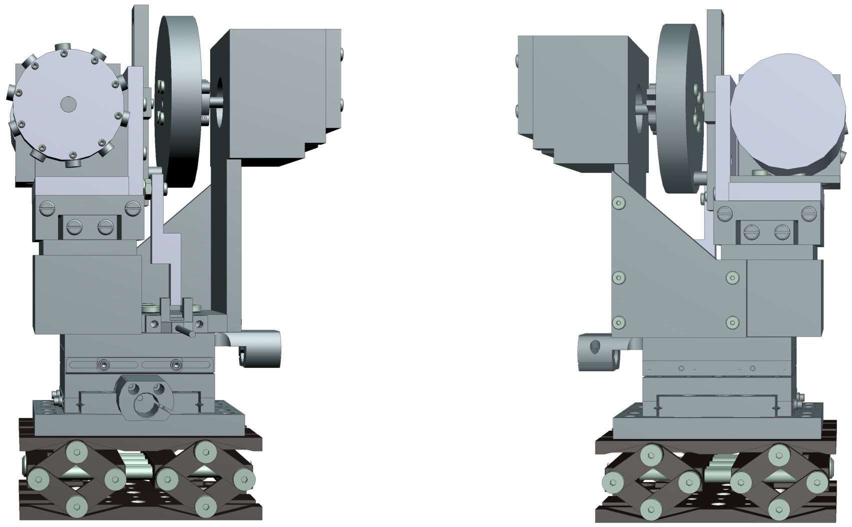
stage labeled I. This stage allows precise movement of the spinning magnets in the XY plane. There is also a photoelectric sensor labeled H, which is triggered each time the DC motor passes the center-line of the actuator tower. This is used to both (a) check the frequency at which the spinning magnets pass over the aluminum disk, and (b) to enable once_per_period sampling of the pendulum's angle.

3.1 Actuator Tower Base

The right tower base in Fig. 10 screws into an XY translation stage and a lab jack. The right tower base is the foundation for the right tower. The raised portion of the base is for attaching the linear slider. The two holes for 6-32 threaded inserts in the raised portion of the base is for the linear slider. The chunk out of the left corner is for a photoelectric sensor. The two holes for M3 screws on the left side of the base are to secure the photoelectric sensor in position. The three 0.25 inch holes on the right lower side of the base are for securing the base to the XY translation stages. Four 1/4"-20 screws will screw through the 0.25 inch holes from the base and into the transitional stage.

3.2 DC Motor Mounting Parts

The DC motor sits on a L bracket shown in Fig. 11a. The DC motor and its bracket sits on a linear slider which is screwed onto the right tower base in Fig. 10. The floor of the L bracket in Fig. 11c has four counterbored holes for 6-32 screws. Because of the counterbored holes, the screws will be hidden in the L bracket floor. The floor of the L bracket also has two 6-32 screw holes on the left side. These holes are for the right side wall shown in Fig. 11b, to screw into place. The right side wall for DC motor has three sets of 6-32 screw holes. The screw holes on the bottom left of the right side screws into the floor of the L bracket.



(a) Actuator Tower Assembly.

(b) Actuator Tower Assembly Back.

Figure 9: Actuator Tower Assembly.

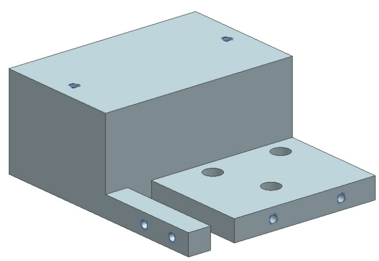
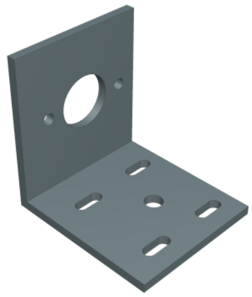


Figure 10: Actuator Tower Base

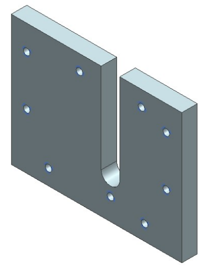
The set of holes on the back left are optional. In addition to the three sets of screws, there is a slot running through the right side wall in Fig. 11b. This slot is for the pin in the scotch yoke wheel. As the wheel spins, the pin will slide up and down in the slot pushing the DC motor back and forth on the linear slider.

3.3 Spinning Magnet Holder for DC Motor

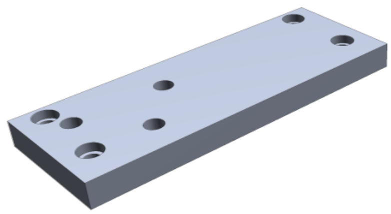
The DC motor spins six magnets outside the DC motor housing. These spinning magnets pass over the aluminum disk from the left tower. The DC motor shaft will be converted from a 5mm shaft into a 0.25"' shaft by a coupler. The 0.25"' shaft will be supported by a bearing in the front of the DC housing providing additional support for the spinning magnets in Fig. 12. The holder will be press fit onto the 0.25"' shaft. The holder for the magnets is a cylinder with ten holes evenly spaced on the outside of the cylinder. Each hole is 0.25 inches for the magnets to be press fit into place. As calculated in section 4 on page 15, the pressure fit is expected to provide enough force to hold the magnets in place at maximum speed. However, one of the magnets came out during one experiment. Therefore, set screw holes are added on both side of the disk. 4-40, 1/8" long set screws are used to secure the magnets inside the holes.



(a) L bracket.



(b) Right side wall for DC Motor.



(c) Floor for L bracket.

Figure 11: DC Motor Housing

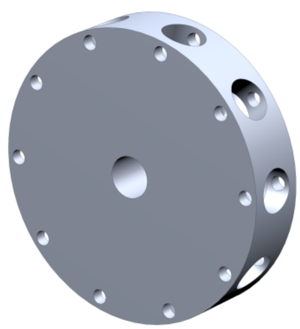
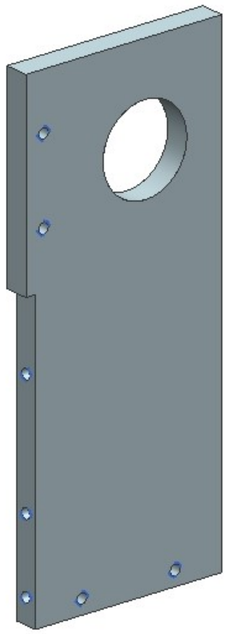


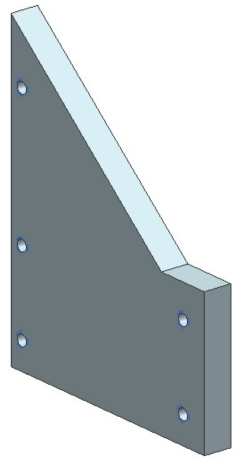
Figure 12: Magnet Holder

3.4 Stepper Motor Support and Stand

The stepper motor is represented by G in Fig. 1. The stepper motor rests against the stepper motor stand in Fig. 13a. The stand is screwed through the holes at the bottom of the stand and into the base from Fig. 10. The screws go through the two holes at the bottom front of the stand and into the two holes on the right side of the tower base. These screws will secure the stepper motor mount to the base of the actuator tower. The stepper motor support in Fig. 13b screws into both the stepper motor stand and the base to give better support to the stepper motor. The three holes on the left side of the support screw into the three holes on the left side of the motor stand in the cut out. The two holes on the right of the support will screw into the base. The support was initially a rectangular shape with only two screw holes on both sides of the support. However, the support was changed to a more triangular shape so that it would reach farther up the stand. By securing the stand farther up, it reduces any bending of the stand and keeps the stepper motor housing from shaking. The large hole in the front top of the stand is for the stepper motor rotating shaft to reach through. The two smaller holes on the left of the large hole is to screw the wall of the stepper motor housing to the stand. All of the screw holes are for 6-32 threaded inserts.



(a) Stepper Motor Stand



(b) Stepper Motor Support

Figure 13: Stepper Motor Stand and Support

3.5 Stepper Motor Housing

The stepper motor is secured into place in a stepper housing. The stepper housing screws into the stepper stand to secure the motor in place. Both sides of the stepper motor housing are 3/8" wide made of Delrin plastic. The screw holes in all the housing parts and the motor stand are for 6-32 screws. Fig. 14b is the right side of the stepper motor housing. The side of the motor housing has one screw hole on the top for the top of the stepper housing to screw into place. It also has two screw holes on the side to screw into the motor stand from Fig. 13a. These two screws from the side of the housing will screw into the right side of the motor stand next to the large hole. The right side of the stepper motor also has two screw holes on the bottom to secure the bottom of the stepper motor housing into place. These two screw holes are closer to the stepper stand than the back of the housing because the bottom of the housing only partially covers the bottom of the motor. The motor has wires attached to the bottom of the motor which need to be connected to a power supply. By having the bottom of the housing only partially cover the motor, it allows room for the wires to be pulled out of the housing.

The left side of the stepper housing in Fig. 14a is very similar to the right side except it has two screw holes on both the front and back instead of front and side. This allows the left side to screw directly into the front of the stepper stand instead of screwing into its side. The reason for having one side screw into the front of the stand and one side to screw into the side of the stand is because it allows for ease of manufacturing. If both sides of the stepper motor housing were attached to the front of the stand like the left side, the stepper stand would have to be 2.475 inches wide. On the other hand, if both sides were screwed onto the side like the right side, the width of the stand would be 1.725 inches. Additionally, with one side attached to the front of the stand and one side attached to the side of the stand, the stepper stand has a width of 2.1 inches. For ease of manufacturing one side is attached to the front and one side is attached the side of the stepper stand.

The back of the stepper motor housing from Fig. 14e has two sets of screw holes on the front. These screws will attach to the right and left side of the housing securing the back into place. The top of the housing's back will have a single screw hole to secure the top of the housing. There are no holes in the bottom of the

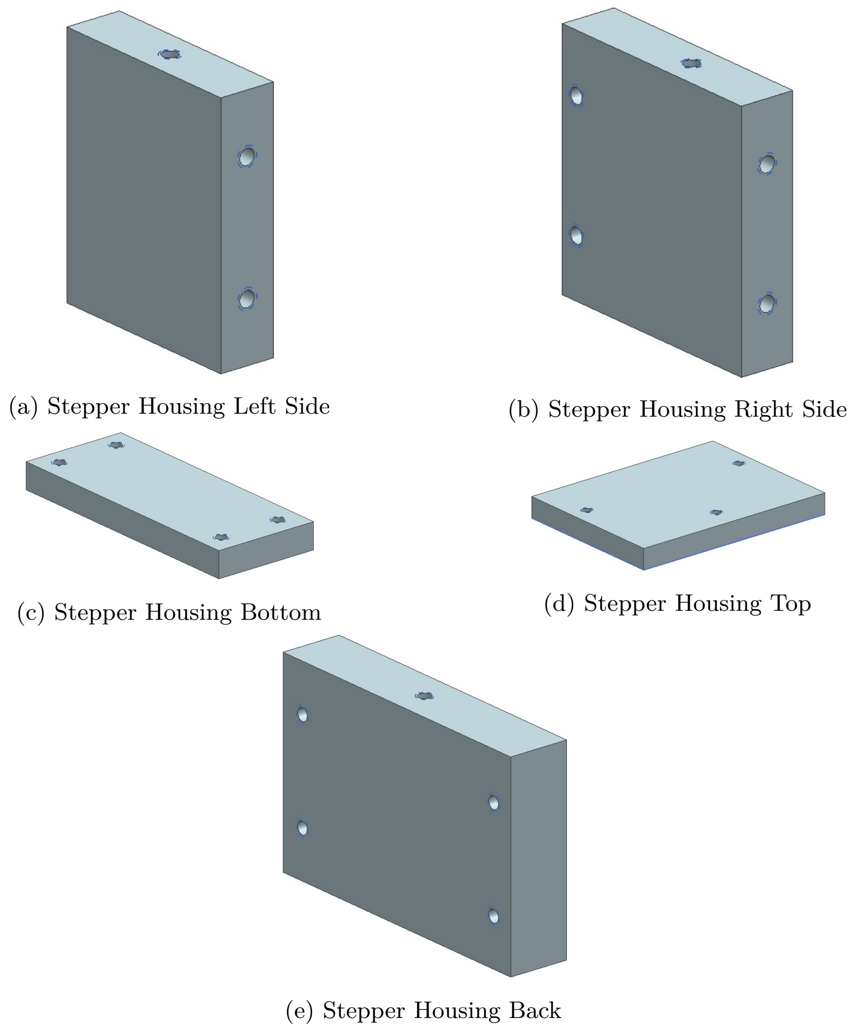


Figure 14: components of the stepper housing

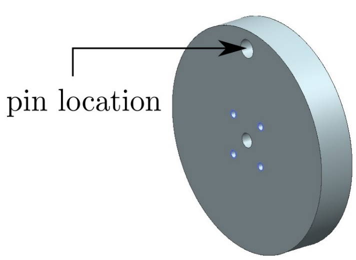
back piece because the bottom of the housing will not reach to the back of the housing because of the gap intentionally introduced for the motor's wires.

The stepper housing bottom in Fig. 14c has two holes on either side of its top. These two pairs of holes screw into the bottom of the right and left sides of the housing. The top of the housing from Fig. 14d has three screw holes through its top. These holes screw into both sides as well as the housing's back. The housing is used to enclose the stepper motor and secure it into place on the stepper motor stand.

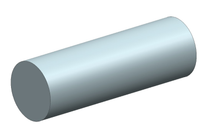
3.6 Scotch Yoke

The scotch yoke apparatus converts the rotational displacement of the stepper motor into linear displacement of the DC motor housing from Section 3.2 on the linear slider from Section 3.7. An aluminum disk from Fig. 15a is attached to the stepper motor by a clamping mount. A clamping mount is used instead of the universal mount used to attach the magnet holder from Section 3.3 because the clamping mount provides more torque than the universal mount. The clamping mount slides around the stepper motor shaft and tightens around the shaft. The universal mount just uses a set screw to secure the mount the shaft. The aluminum disk needs more torque because it has to be able to pull and push the DC housing on the linear slider. The aluminum disk is 0.5 inches thick with another 0.25 inch hole toward the edge of the disk. This 0.25 inch hole is 0.25 inches deep and is used to hold the pin in Fig. 15b.

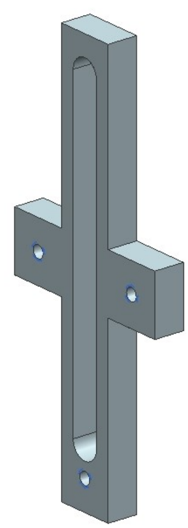
The slider in Fig. 15c will be screwed onto the outside of the DC motor housing in Fig. 11b. The slots from the scotch yoke slider and the side of the DC motor housing will line up but the slot in the housing will be slightly larger than that in the scotch yoke slider. This is so that the scotch yoke pin in the aluminum disk does not rub against the DC housing, only the scotch yoke slider. The slot from the scotch yoke slider is also slightly longer than it needs to be. This is so that the pin does not hit the end of the slot with each rotation. Once the slider is secured to the housing, the scotch yoke pin from the aluminum disk will be put in the slider. When the aluminum disk rotates because of the stepper motor, the pin will slide up and down in the slider pulling and pushing the DC motor housing back and forth on the linear slider. Both the scotch yoke slider and the pin are made of oil impregnated bronze to reduce contact friction and minimize wear.



(a) Aluminum Disk.



(b) Scotch Yoke Pin.



(c) Scotch Yoke Slider.

Figure 15: Components of the scotch yoke mechanism.

3.7 Linear Slide, Lab Jack, XY translation stage, and DC Motor

The linear slide used to attach the L bracket (see Fig. 11a) to the base of the actuator tower has a linear displacement of 1.5 inches in each direction. It is mounted on a spacer shown in Fig. 16. This spacer is placed between the linear slider and the the base of the actuator tower (see Fig. 9).

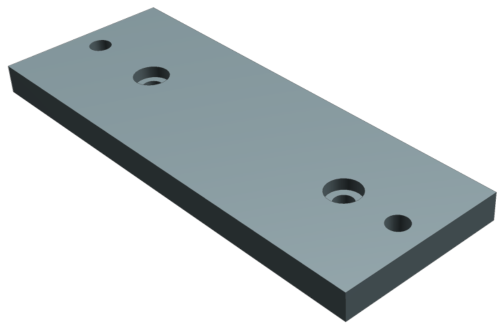


Figure 16: Spacer for slider.

The scotch yoke pin is 1.25 inches from the center of the aluminum disk. This is because the rotating magnets in Section 3.3 need to pass over the entire 2.5 inch diameter of the aluminum disk on the magnetic pendulum tower causing it to spin. Since the total displacement of the spinning magnets is 2.5 inches, their movement is 1.25 inches from the center of the aluminum disk. If the spinning magnets start at the center of the aluminum disk from the magnetic pendulum tower, the 1.5 inch movement in each direction from the linear slider is more than enough. The slider is also 0.75 inches high with four 6-32 screw holes through the top of the slider. These four holes screw through the L bracket floor and into the slider to secure the DC motor to the slider. The DC motor's maximum rotational speed is 12000 RPM and it is powered by a 12 volt DC source.

The lab jack is used to raise and lower the entire actuator tower. When fully extended, the lab jack has a maximum height of 1.88 inches but when fully retracted, its height is 0.84 inches. The surface of the lab jack is 4.00 inches long and 3.00 inches wide, and it has 4-40 screw holes which are used to secure the XY translation stage to the lab jack.

The XY translation stage is 44 mm (1.73 inches) tall. The stage has a base to secure it but the base only has slots for two 1/4"-20 screws. Because the lab jack uses 4-40 screws, the slots for the 1/4"-20 screws will have to be filled and then 4-40 screw holes will be drilled through the filling. Two 4-40 holes will be drilled into the filling to replace the two 1/4"-20 holes. These fillers will be 3D printed.

3.8 Stepper Motor

In order to determine a suitable stepper motor for this machine, the minimum torque on the motor has to be calculated. The motor has to be able to push and pull the DC housing and the DC motor on the linear slide. Therefore, it is necessary to check that using the stepper motor will be able to handle the movement of the DC motor housing.

The required torque can be calculated from

$$T = F \times L \times sin(\theta) \tag{1}$$

where F is the linear force acting on L which is the length of the lever arm, and θ is the angle between the linear force and the lever arm. The maximum force will be when the force is perpendicular to the lever arm causing θ to be 90 degrees. We are interested in using the maximum torque

$$T_{max} = F \times L \tag{2}$$

to obtain the minimum required torque for the stepper motor.

The force can be calculated by

$$F = m\,a \tag{3}$$

where m is the total mass of the DC motor assembly, and a is its acceleration.

Because the stepper motor will be moving the DC housing and its contents in sinusoidal motion via the scotch yoke mechanism according to

$$x = A\,sin(\omega t) \tag{4a}$$

$$v = A\omega\,cos(\omega t) \tag{4b}$$

$$a = -A\omega^2\,sin(\omega t), \tag{4c}$$

the maximum acceleration of the DC housing is given by

$$a_{\max} = -A\omega^2 \tag{5}$$

where A is the amplitude of motion which in this case is half the total motion of the DC housing on the slider. Because the DC housing will move a total of 2.5 inches in each direction, the amplitude of its motion is 1.25 inches. The rotational velocity ω of the stepper motor is measured in radians per second and t is time.

Eq. (4c) can be solved for $a_{\max}$ using the maximum desired frequency for ω. In order to reproduce the data collected from [1], the stepper motor must reach a minimum of 2.3475 Hz which converts to 140.85 RPM. However the article also states the maximum rotation rate of the aluminum disk is 178 RPM [1] so the acceleration will be calculated with the higher RPM: 178 which is equivalent to 18.640 rad/sec. The 1.25 inch amplitude is equivalent to 0.03175 meters. Using Eq. (5) the acceleration is calculated to be 11.032 m/s^2.

The mass of the DC housing and all the moving pieces is 500.377 grams. Using this mass and the acceleration calculated in the paragraph above, the force is calculated using Eq. (3) to be 5.520 Newtons. The torque can then be calculated using the force and the distance from the scotch yoke pin to the stepper motor shaft since this will be the lever arm length where the latter is designed to be 0.03175 meters. The resulting maximum torque on the stepper motor is 0.1753 Nm. Using a 20% factor of safety, the stepper motor motor needs to have a torque of at least 0.2103 N.m or 210.3 mN.m. The stepper motor in the parts list has a torque of 230 mNm which is enough to withstand the maximum possible torque from the machine.

The maximum possible frequency this motor can withstand can be calculated using its max torque of 230 mNm. By dividing the max torque by the lever arm length of 0.03175 meters, leaves a force of 7.244 N. Then dividing this force by the mass of the assembly which is 500.377 grams leaves an acceleration of 14.477 m/s^2. Going back to Eq. (5), the angular velocity can be determined to be 21.35 rad/sec which converts to 3.40 Hz. The maximum possible frequency of this assembly with this motor is 3.40 Hz.

4 Pressure Fit Calculations

Pressure fitting is an easy way of securing the magnets into place. Whether the magnets are in a location where set screws are not plausible or because of the offset weight the set screws would provide to a spinning part, pressure fitting is used to secure the majority of the magnets into place. Whenever a pressure fit is used to secure a magnet, a calculation must be completed so be sure the resulting fit will provide enough force to hold the magnet into place during movement. If the force pulling the magnet out of place is larger than the pressure fit forces holding the magnet in place, the magnet will be removed from its location. The forces pulling the magnet out of its hole can be modeled by the centripetal force on the magnet. The centripetal

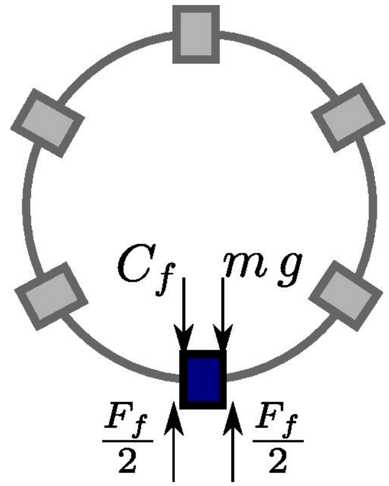


Figure 17: Free Body Diagram showing the forces acting on one of the rotating pendulums (see F in Fig 1).

force C_f can be calculated according to

$$C_f = \frac{mv^2}{r}, \tag{6}$$

where m is the mass of the magnet, v is its linear velocity, and r is the distance from the magnet's center of mass to the axis of rotation. However, because the magnets are spinning in vertical circles, the weight of the object must also be calculated. The maximum force occurs when the magnet is at the bottom of its rotation. At this point the magnet's weight is aligned with the force pulling out the magnet as the free body diagram in Fig. 17 shows, the maximum force pulling out the magnet is modeled by

$$C_f = \frac{mv^2}{r} + mg, \tag{7}$$

which includes the acceleration due to gravity g. In this figure C_f is the centripetal force caused by normal acceleration, mg is the weight of the magnet, and $\frac{F_f}{2}$ is the friction force on each side of the magnet.

The mass for each magnet in the assembly is 1.51 grams while the maximum angular velocity of all the magnets is 0.5809 m/s. This value was computed using the maximum angular velocity of the DC motor. Since the masses, gravity, and velocity are all the same for the magnets, only the distance r differentiates the centripetal forces of the magnets. Each r is measured from the center of mass of the magnet to the rotational axis. The centripetal force of magnet A in Fig. 1 can be calculated to be 0.02933 Newtons. Each spinning magnet in the right tower has a maximum centripetal force of 0.05705 Newtons.

The pressure holding on the magnets from a pressure fit is modeled by [2]

$$P = \frac{\delta}{\frac{r}{E_h}(\frac{r_h^2+R^2}{r_h^2-R^2} + \nu_h) + \frac{r}{E_p}(\frac{R^2+r_p^2}{R^2-r_p^2} - \nu_p)} \tag{8a}$$

$$\approx \frac{\delta}{\frac{r}{E_h}(1+\nu_h) + \frac{r}{E_p}(1-\nu_p)} \tag{8b}$$

where δ is the radial interference, r is the nominal radius, E_h is the Young's modulus and ν_h is the Poisson's Ratio for the hole, while E_p is the Young's modulus and ν_p is the Poisson's Ratio for the pin.

If we assume the radius of the hole r_h and the radius of the pin r_p are very close to the nominal radius, the equation can be simplified to Eq. (8b). This is accurate for our machine because we are dealing with an overlap in the hundredths of an inch between the magnet and its hole.

Using the pressure calculated in Eq. (8b), the normal force on the magnet is given by

$$F_N = P \times A = P(2\pi \times \text{radius} \times \text{height of magnet}), \tag{9a}$$

In the Eq. (9a), F_N is the normal force on the magnet and A is the surface area of the magnet in contact with its holder.

The friction force holding the magnet in the hole F_f is calculated using the normal force F_N from above and μ, the coefficient of friction between the magnet and its holder, according to

$$F_f = F_N\,\mu. \tag{10}$$

In order to solve Eq. (8b), the Poisson's ratio and Young's modulus of aluminum 6061, which is the material for the magnet holder are used: $\nu = 0.33$ and E = 69.0 GPA [3]. The Poisson's ratio and Young's modulus for nickel, which is the coating of the magnets, are 0.312 [4] and 205 GPa [5] respectively. The nominal radius of the magnet is 3.175 mm (0.125 inches) and using a radial interference of 0.1778 mm (0.007 inches), the resulting pressure from the pressure fit is calculated to be 2.39 GPa. The surface area of the sides of the magnet can be calculated using the diameter and height of the magnet. Since the diameter and height of the magnet are both equal to 6.35 mm (0.25 inches), the surface area of the sides of the magnet is 126.68 mm^2 (0.3927 inch^2). With the pressure and surface area, the normal force on the magnet can be calculated using Eq. (9a). The normal force is determined to be 302.6 KN. Using the coefficient of friction between the magnet and the aluminum holder, 0.33, the friction force holding the magnet in place can be calculated using Eq. (10). The friction force holding the magnet in place is determined to be 105.9 KN. Therefore, we conclude that the friction force of 105.9 kN is more than enough to hold the magnet in place from the centripetal force of 0.05705 N.

5 Controlling and Data Acquisition

Data is obtained from the setup in two different way: 1) using Arduino 2) using NI USB-6356. First, data has been collected directly from Arduino with the help of Processing. Processing is a Java based programming language that enables you to save the data read by Arduino. We are providing the codes for both Processing and Arduino inside of the repository. It has been noticed that sampling frequency of Arduino Leonardo is not enough to capture rapid changes in angle measurement coming from rotary encoder. Therefore, we have decided to use a data acquisition box that has high sampling rate feature. Then, NI USB-6356 with 1.25 MS/s per channel has been used to obtain data. In this section, we will explain both way of data collection and how to connect sensors and electric motors to run the setup.

5.1 Data acquisition using Arduino

The part list for data acquisition and controlling part of the setup is provided in Tab. 1. Two Arduino Leonardo have been used. One of them is for reading the output of the rotary encoder and photoelectric sensor, while other one is running the motors. The complete wiring diagram is given in Fig. 18.

Motor drives are one of the essential part of the setup. One L298N motor drive can run two DC motors or one step motor at the same time. Therefore, two L298N motor drives are connected to a stepper motor and a DC motor as shown in Fig. 18. L298N motor driver enables us to control the speed and direction of the motors (see Fig.19). Direction control pins of L298N represents the switches inside of H-Bridge circuit of L298N. DC motor will turn in clockwise or counter clockwise direction or will stop depending on the input given to these direction control pins. Tab. 2 summarizes the status of the DC motor based on the inputs. It also has 5V regulator which enables user to have 5V output from the pin at the right side of pin number 2 shown in Fig.19. We have disabled the 5V regulator in this setup. One breadboard is used to make parallel connections for sensor output pins and to transfer the power coming from DC motor. Both Arduino boards are powered with USB connections or power supply adapter. Constant voltage has been supplied to rest of the circuit using B&K Precision 1621A power supply.

Table 1: Equipment list

Item Number	Equipment
1	TSINY TRS-775W 12000 rpm DC motor
2	Sparkfun Bipolar Stepper Motor (Hybrid Frame Size 17, 200 Step, 330mA 12VDC)
3	L298N Motor Drives
4	Arduino Leonardo
5	EM2-2-10000-I US Digital Rotary Encoder
6	PM-L45 - Photoelectric Sensor
7	B& K Precision 1621A Power Supply
8	Breadboard
9	NI USB-6356 data acquisition

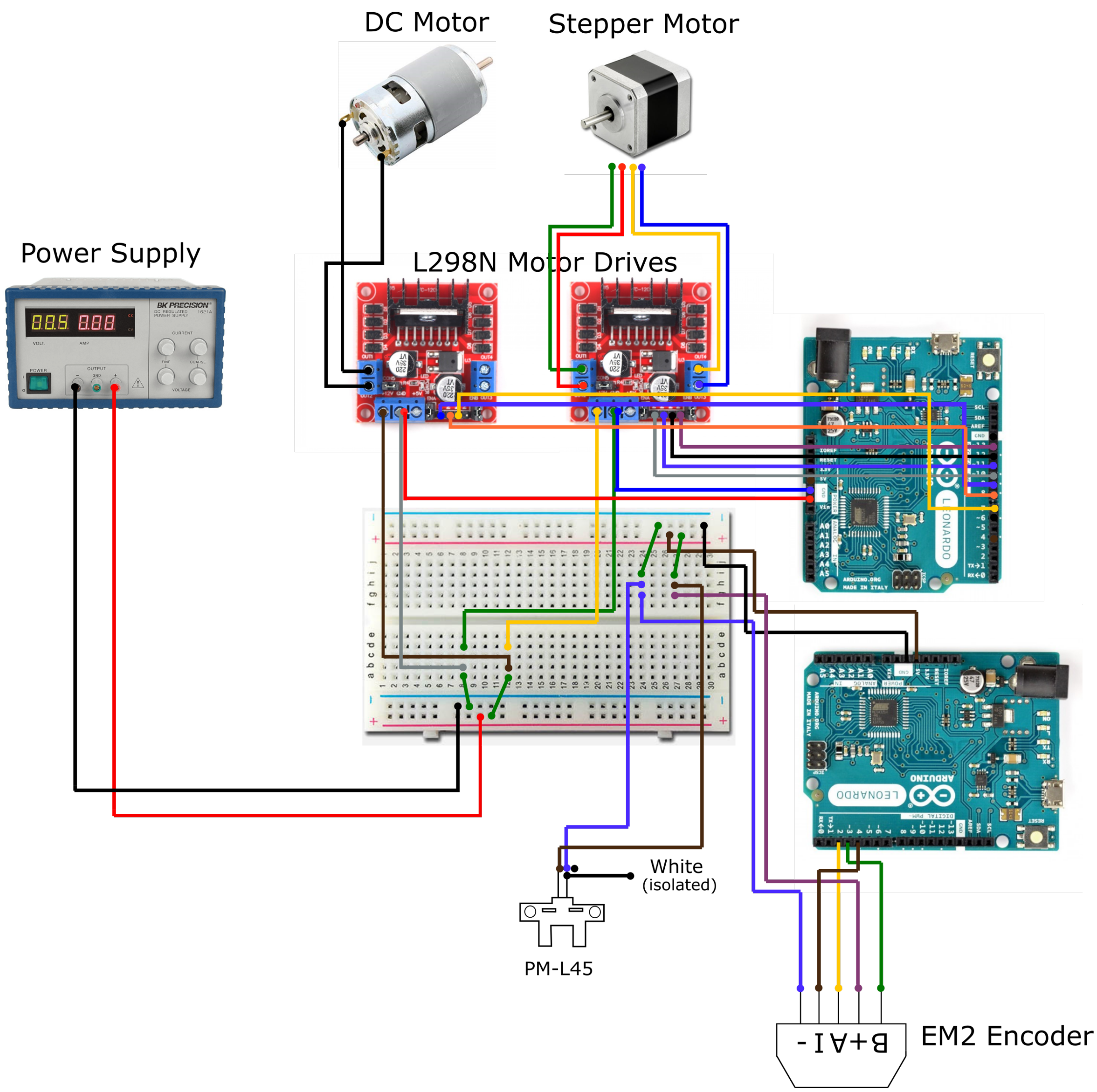
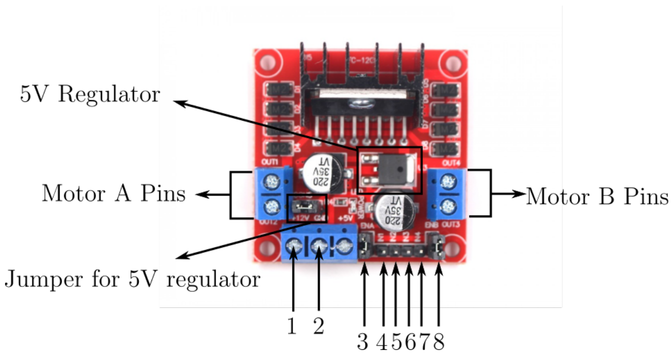


Figure 18: Wiring diagram



(1) Motor VCC 12V

(2) Ground

(3) Speed Control Pin - Motor A

(4) (5) Direction Control Pins - Motor A

(6) (7) Direction Control Pins - Motor B

(8) Speed Control Pin - Motor B

Figure 19: Pins of L298N motor driver

Table 2: Status of DC motor based on values of the direction control pins

IN1 (Pin 4)	IN2 (Pin 5)	Status
0	0	Braking
0	1	Clockwise rotation
1	0	Counterclockwise rotation
1	1	Braking

The outputs of the sensors can be read from the *serial plotter* or *serial monitor* of Arduino. However, saving these outputs is not straight forward. There are several ways to save data using Arduino. For this setup, Processing which is a Java based programming language is used. Related Processing code is available in Github page. Mainly, we generate a connection between Arduino board and Processing code and it copies the Arduino reading into a table. It also tracks the time and data is saved into a Excel sheet. We have put a limitation to the number of readings, and the data is saved when this limit is reached. We can summarize the steps required to run the experiment and saving the data as follows

- Complete the circuit as shown in Fig. 18
- Adjust the initial conditions using lab jack and XY translation stage
- Check the speed value in line 44 of Arduino code that controls the motors. This value can be changed within a range between 0 and 255.
- Place the protective polycarbonate cage and wear safety glasses
- Switch on the power supply and start to increase the voltage to 12 V
- Connect the Arduino board that reads sensor outputs to your computer. Other board can be powered using USB or adapter
- Make sure that the lights on Arduino boards and L298N is lit
- Open Processing code
- Check if the port number in line 15 of the code matches with the USB port number where you connected the Arduino board that reads the sensor output. Change the number if there is a mismatch
- Click run button of the Processing code
- Wait until the specified reading has been reached and it will save the data as Excel sheet into the folder where you keep the processing code

5.2 Data acquisition using NI-6356

Wiring diagram for this way of data acquisition is slightly different than shown in Fig. 18. We replace one of the Arduino boards with NI-6356 and motor drivers are still connected to the other Arduino board. Analog input channels have been used to collect the data. For rotary encoder, we have collected A and B outputs, and the direction of the rotation and position can be tracked using these two square wave signals. These square waves are post-processed to find angular position in degrees or radians (See *Encoder_Post_Process.m* in the repository.) For photoelectric sensor, the output is again a square wave, and we are applying the method described in Ref.[6] to determine the rotational speed of the stepper motor (See *Step_Motor_Rpm_Reading.py* in the repository). In addition, required codes to be able to use *Step_Motor_Rpm_Reading.py* are available in this repository.

6 Conclusion

The machine modeled in this report is used to predict the behavior of a chaotic pendulum with variable interaction potential. Using the calculations and characteristics of the machine from one drive frequency, it is possible to predict the behavior of the machine at a new drive frequency. The majority of the parts for the machine are made out of Delrin plastic and aluminum although several of the smallest pieces are 3D printed. CAD files of all parts and the assembly can be found in the link provided in the repository. Calculations were also completed in order to be sure the stepper motor is also guaranteed to provide enough torque to move the DC motor.

References

[1] V. Tran, E. Brost, M. Johnston, and J. Jalkio, "Predicting the behavior of a chaotic pendulum with a variable interaction potential," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 23, p. 033103, sep 2013.

[2] R. Budynas and J. K. Nisbett, *Shigley's Mechanical Engineering Design (McGraw-Hill Series in Mechanical Engineering)*. McGraw-Hill Science/Engineering/Math, 2006.

[3] D. R. Askeland, *The Science and Engineering of Materials, 3rd Edition (PWS Series in Engineering)*. CL Engineering, 1993.

[4] H. Gercek, "Poisson's ratio values for rocks," *International Journal of Rock Mechanics and Mining Sciences*, 2007.

[5] T. Fritz, M. Griepentrog, W. Mokwa, and U. Schnakenberg, "Determination of young's modulus of electroplated nickel," *Journal of the International Society of Electrochemistry*, 2003.

[6] F. A. Khasawneh and E. Munch, "Topological data analysis for true step detection in periodic piecewise constant signals," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 474, p. 20180027, oct 2018.

List of Figures