

Lab 6 part 2 a solution

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This is a solution to the worksheet for the QDA lab 6 independent practice. Different maximal models can be selected and as such the resulting minimal adequate model will differ. The important part is to justify the approach and the steps taken.

```
library(ggplot2)
```

1. Load the *crime-analysis-data.csv* data into an R notebook.

```
crime<-read.csv("crime-analysis-data.csv")
```

2. Explore the data numerically and graphically. Confirm the variables that are categorical and numerical/continuous and that R has read them in appropriately

```
summary(crime)
```

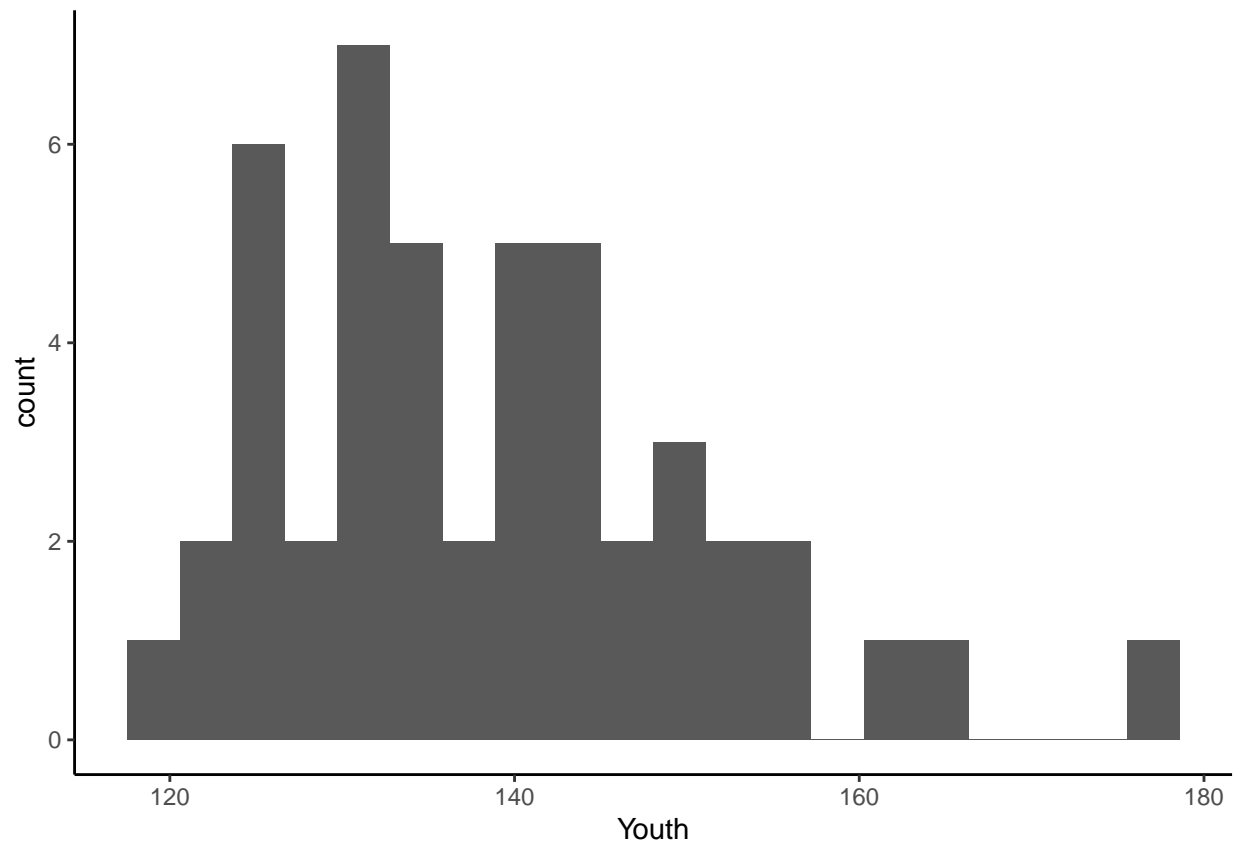
```
##      CrimeRate      Youth      Education      ExpenditureYear0
##  Min.       : 45.5    Min.       :119.0    Min.       :10.00    Min.       : 45.0
## 1st Qu.: 82.7    1st Qu.:130.0    1st Qu.:11.55    1st Qu.: 62.5
##  Median :103.0    Median :136.0    Median :12.40    Median : 78.0
##  Mean   :102.8    Mean   :138.6    Mean   :12.39    Mean   : 85.0
## 3rd Qu.:120.7    3rd Qu.:146.0    3rd Qu.:13.20    3rd Qu.:104.5
##  Max.   :161.8    Max.   :177.0    Max.   :15.10    Max.   :166.0
##  LabourForce  YouthUnemployment  MatureUnemployment  HighYouthUnemploy
##  Min.       :480.0    Min.       : 70.00    Min.       :20.00    Min.       :0.0000
## 1st Qu.:530.5    1st Qu.: 80.50    1st Qu.:27.50    1st Qu.:0.0000
##  Median :560.0    Median : 92.00    Median :34.00    Median :0.0000
##  Mean   :561.2    Mean   : 95.47    Mean   :33.98    Mean   :0.3191
## 3rd Qu.:593.0    3rd Qu.:104.00    3rd Qu.:38.50    3rd Qu.:1.0000
##  Max.   :641.0    Max.   :142.00    Max.   :58.00    Max.   :1.0000
##      Wage      StateSize
##  Min.       :288.0    Min.       : 3.00
## 1st Qu.:459.5    1st Qu.: 10.00
##  Median :537.0    Median : 25.00
##  Mean   :525.4    Mean   : 36.62
## 3rd Qu.:591.5    3rd Qu.: 41.50
##  Max.   :689.0    Max.   :168.00
```

The variable HighYouthUnemploy should be a categorical variable (actually binary as it only has two levels). R has read it in as numerical so this can be fixed by making it into a Factor.

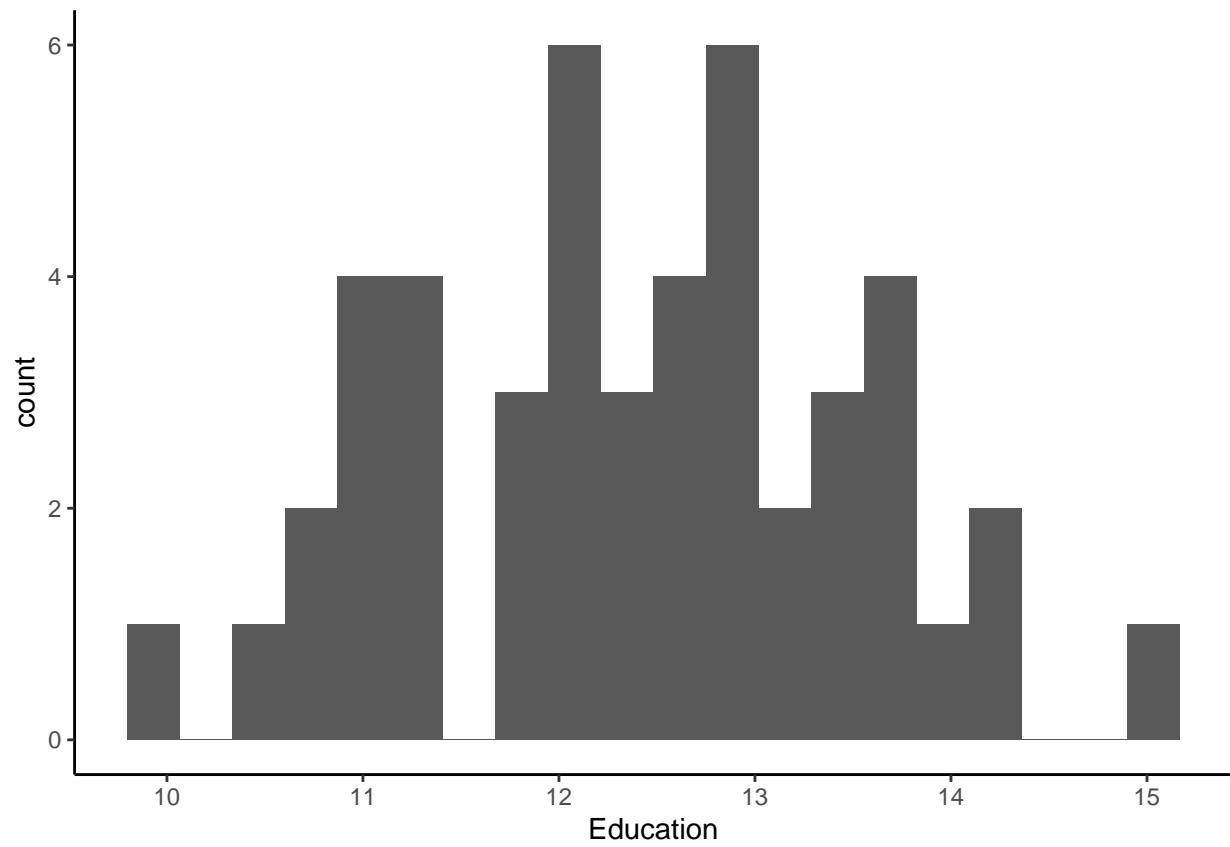
```
crime$HighYouthUnemploy<-as.factor(crime$HighYouthUnemploy)
```

Lets look at the distribution of the variables:

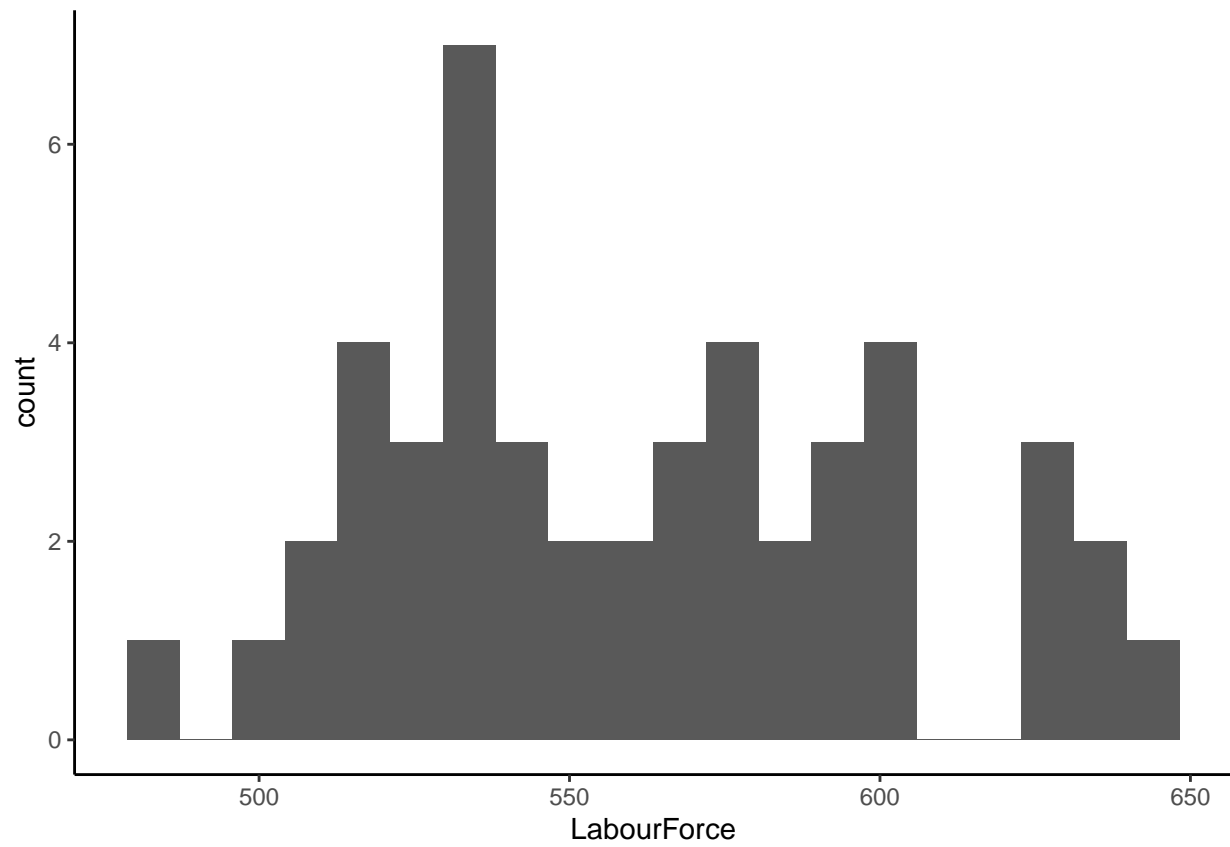
```
ggplot(data = crime, aes(x=Youth)) + geom_histogram(bins = 20) + theme_classic()
```



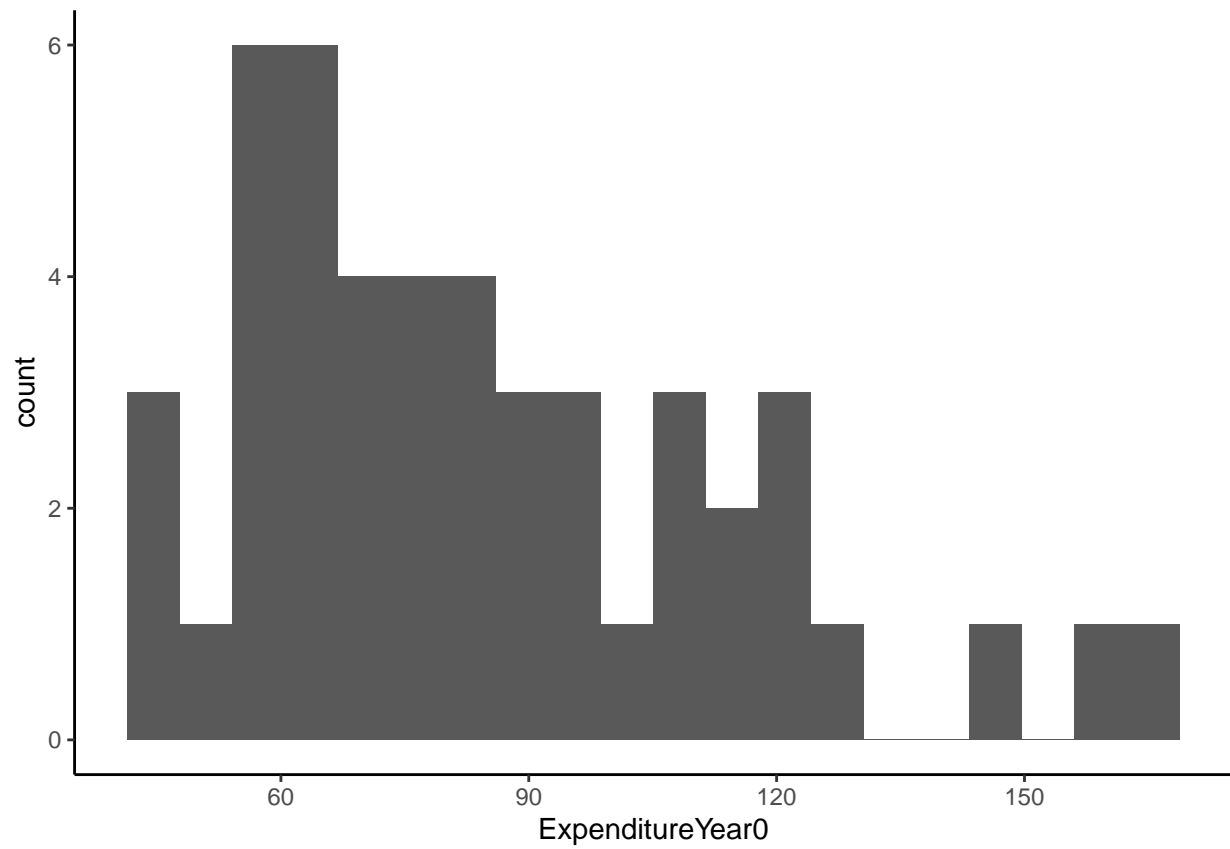
```
ggplot(data = crime, aes(x=Education)) + geom_histogram(bins = 20) + theme_classic()
```



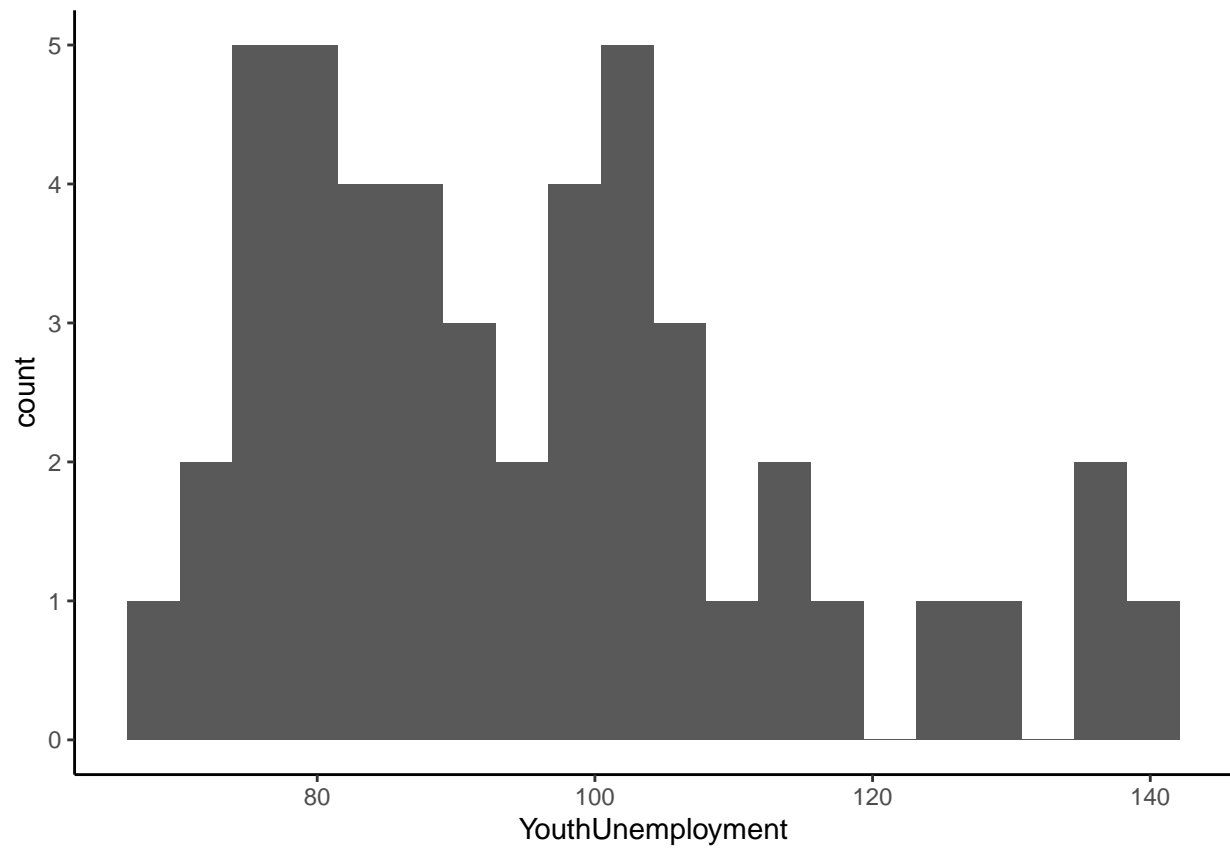
```
ggplot(data = crime, aes(x=LabourForce)) + geom_histogram(bins = 20) + theme_classic()
```



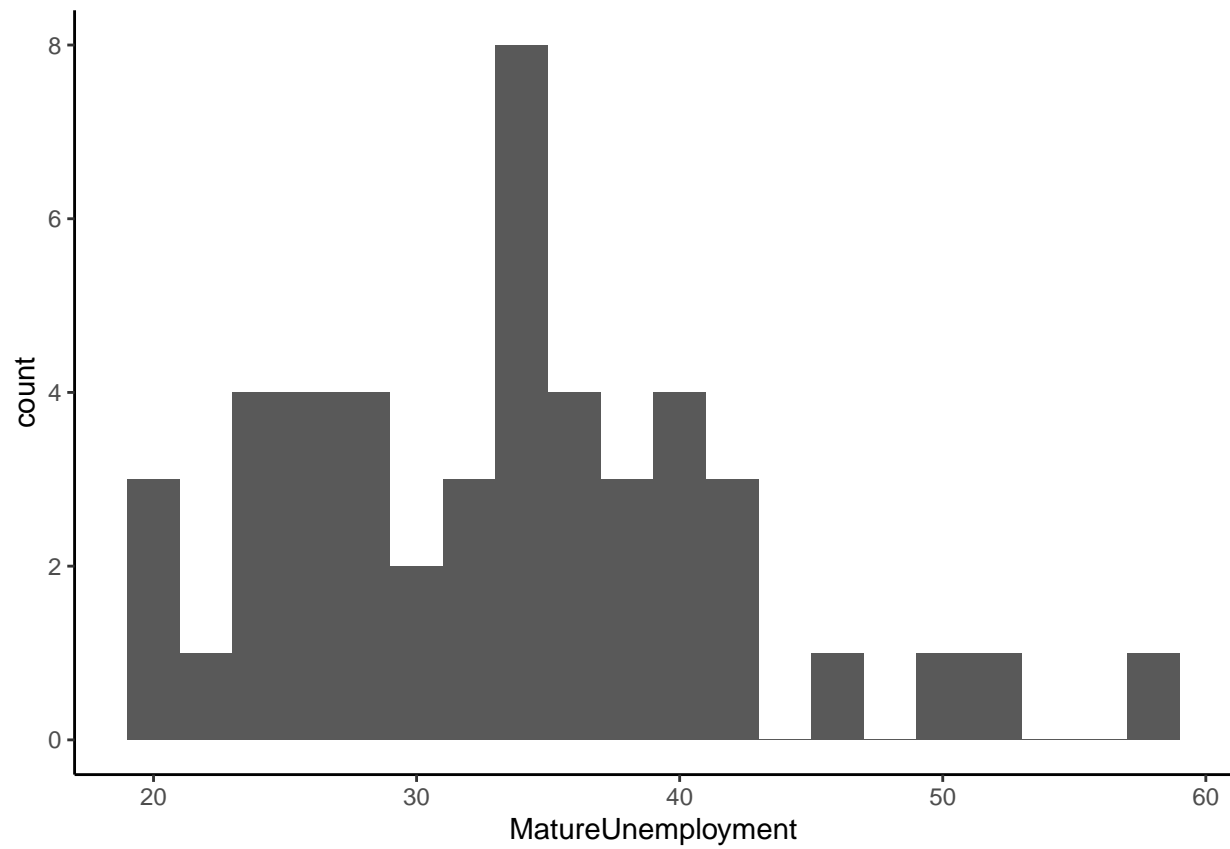
```
ggplot(data = crime, aes(x=ExpenditureYear0)) + geom_histogram(bins = 20) + theme_classic()
```



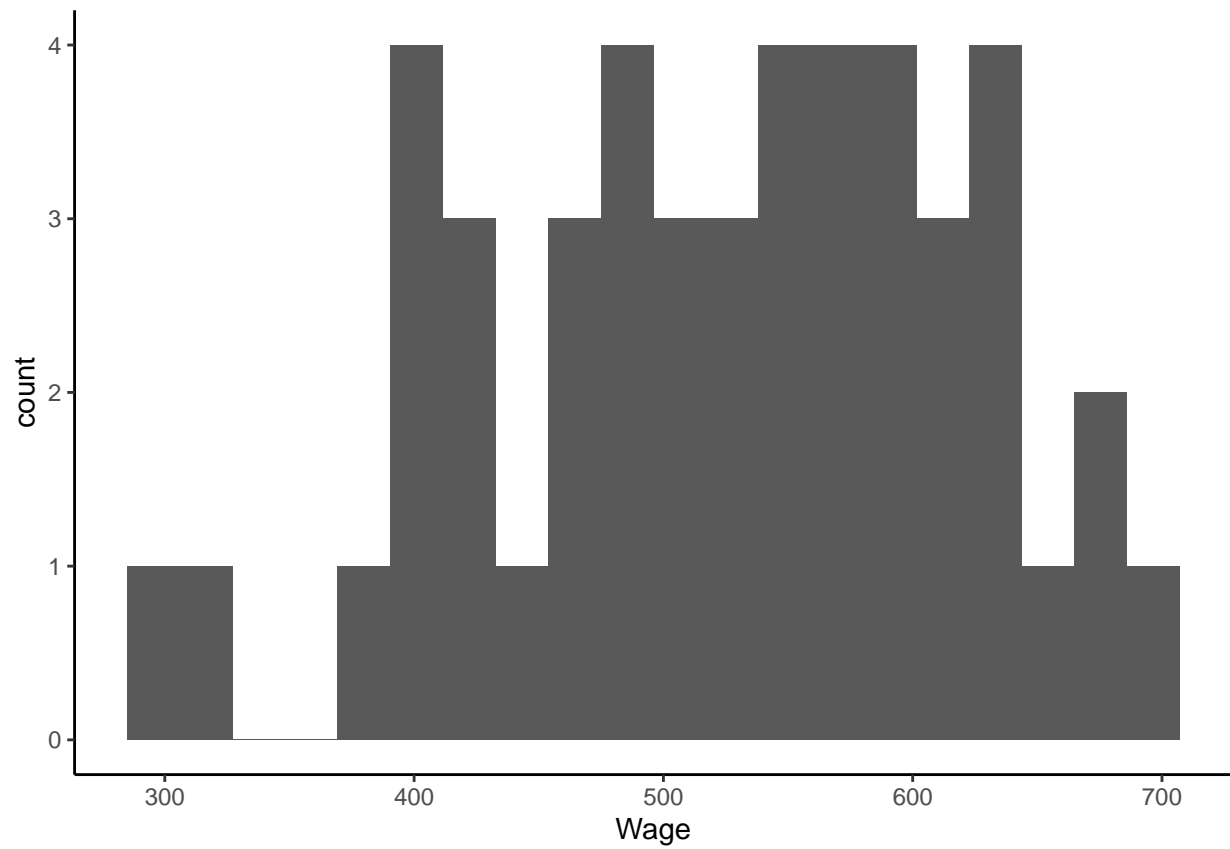
```
ggplot(data = crime, aes(x=YouthUnemployment)) + geom_histogram(bins = 20) + theme_classic()
```



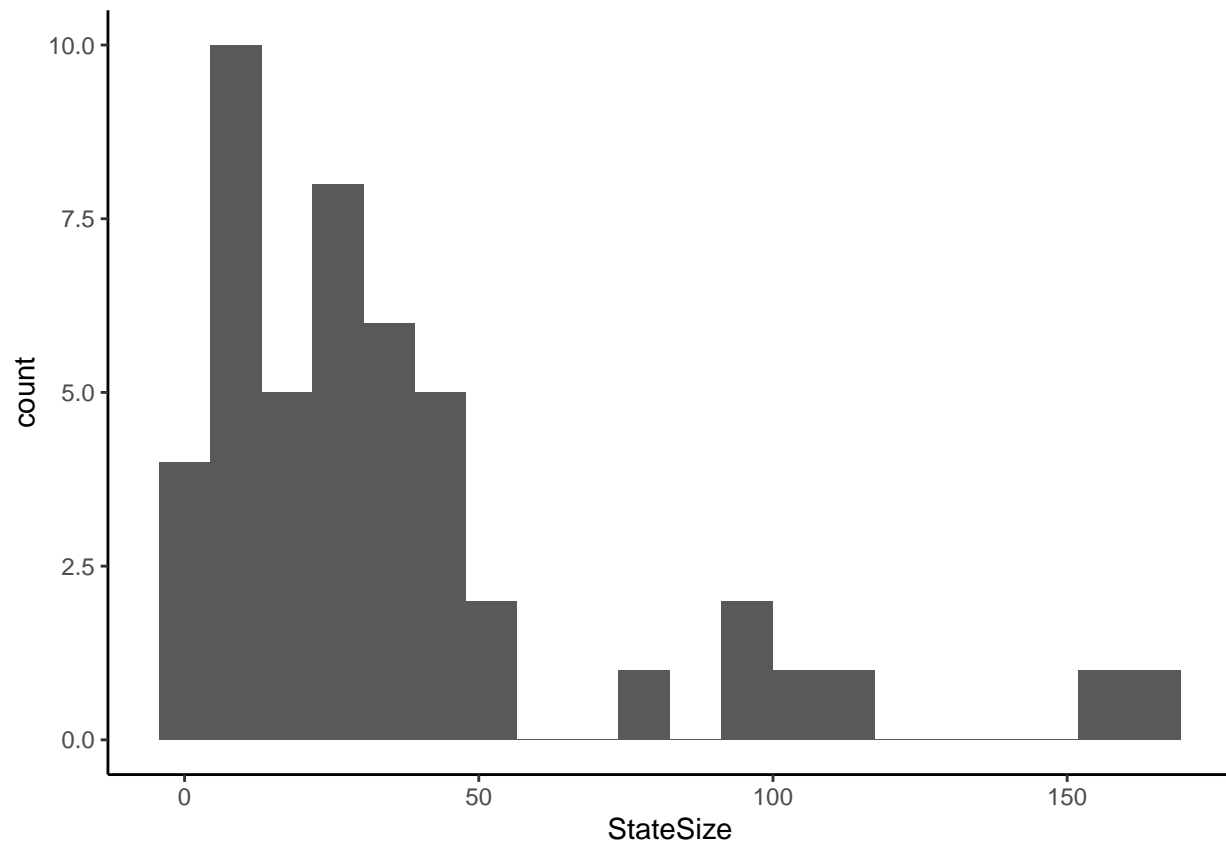
```
ggplot(data = crime, aes(x=MatureUnemployment)) + geom_histogram(bins = 20) + theme_classic()
```



```
ggplot(data = crime, aes(x=Wage)) + geom_histogram(bins = 20) + theme_classic()
```

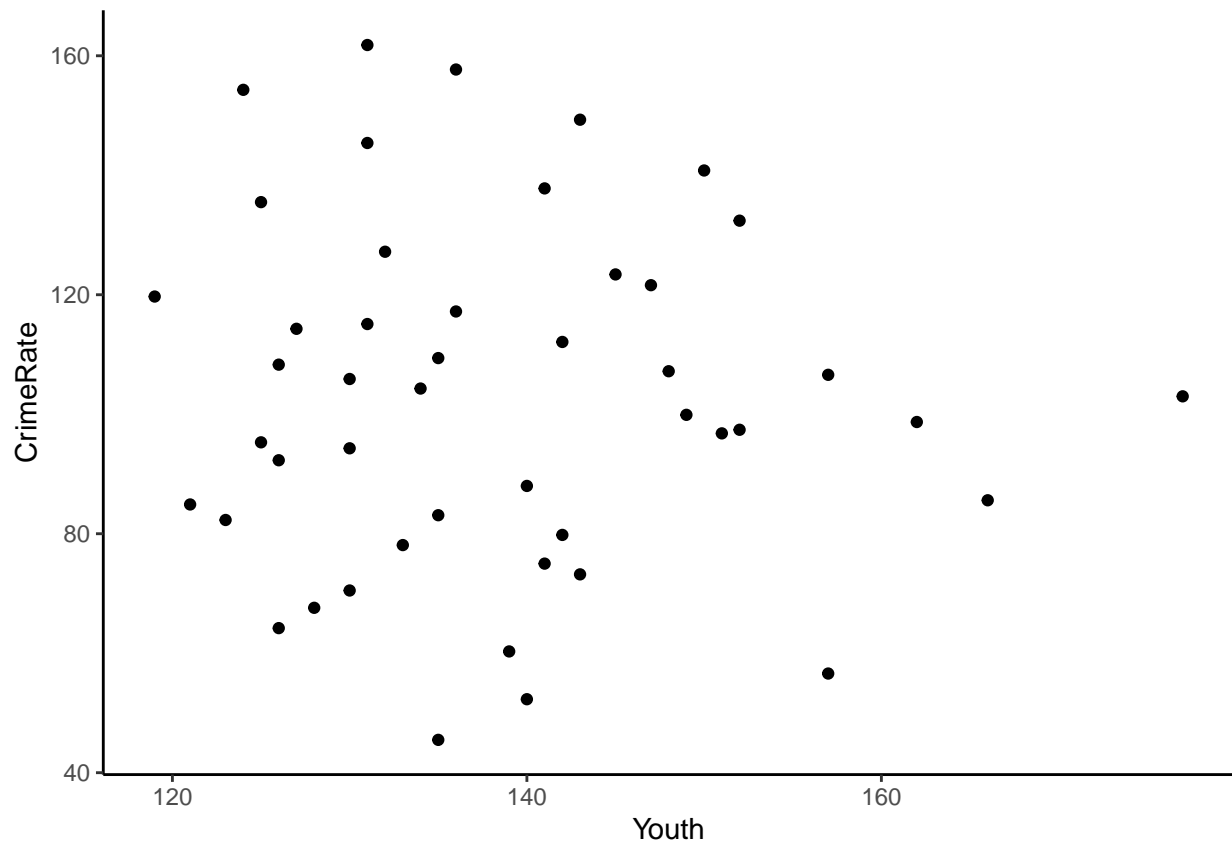


```
ggplot(data = crime, aes(x=Wage)) + geom_histogram(bins = 20) + theme_classic()
```

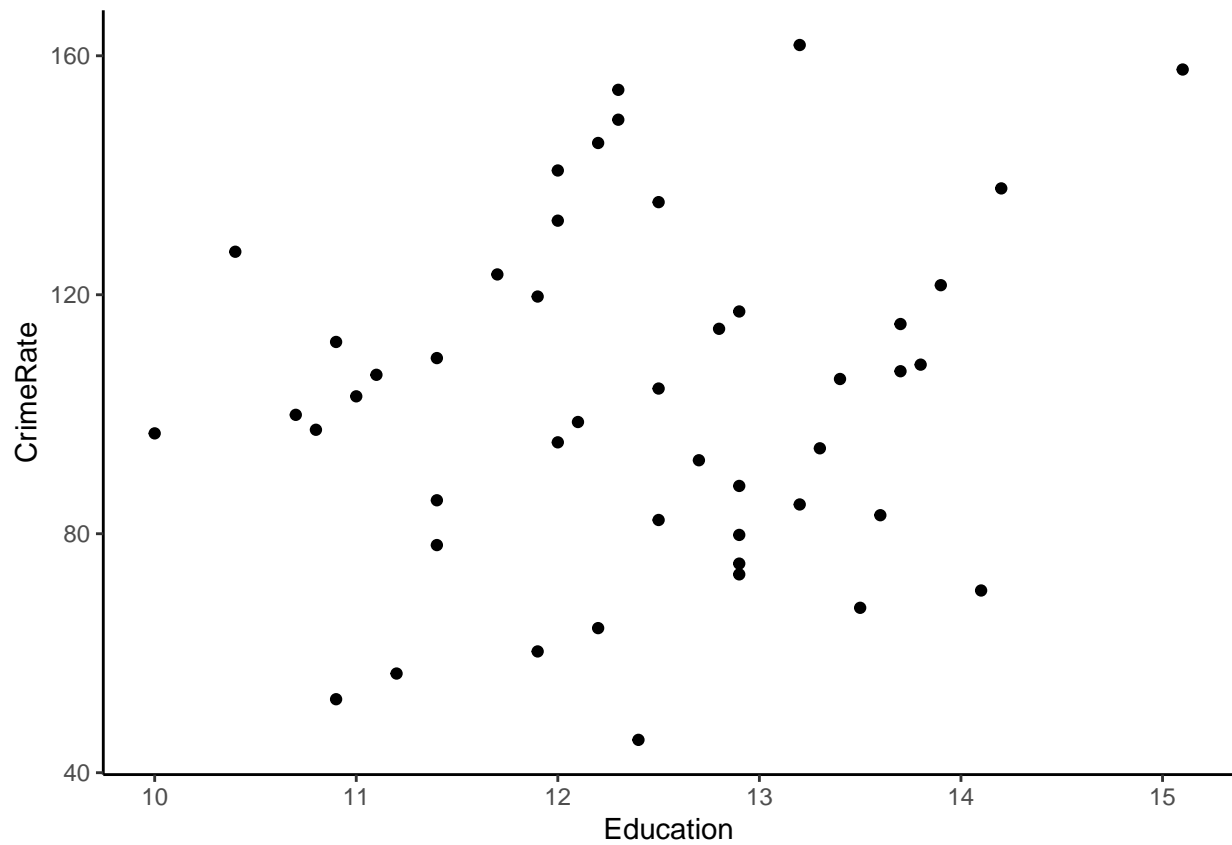



The distribution for ExpenditureYear0 seems skewed to the left, other variables look generally symmetric. This does not warrant any transformations at this stage.

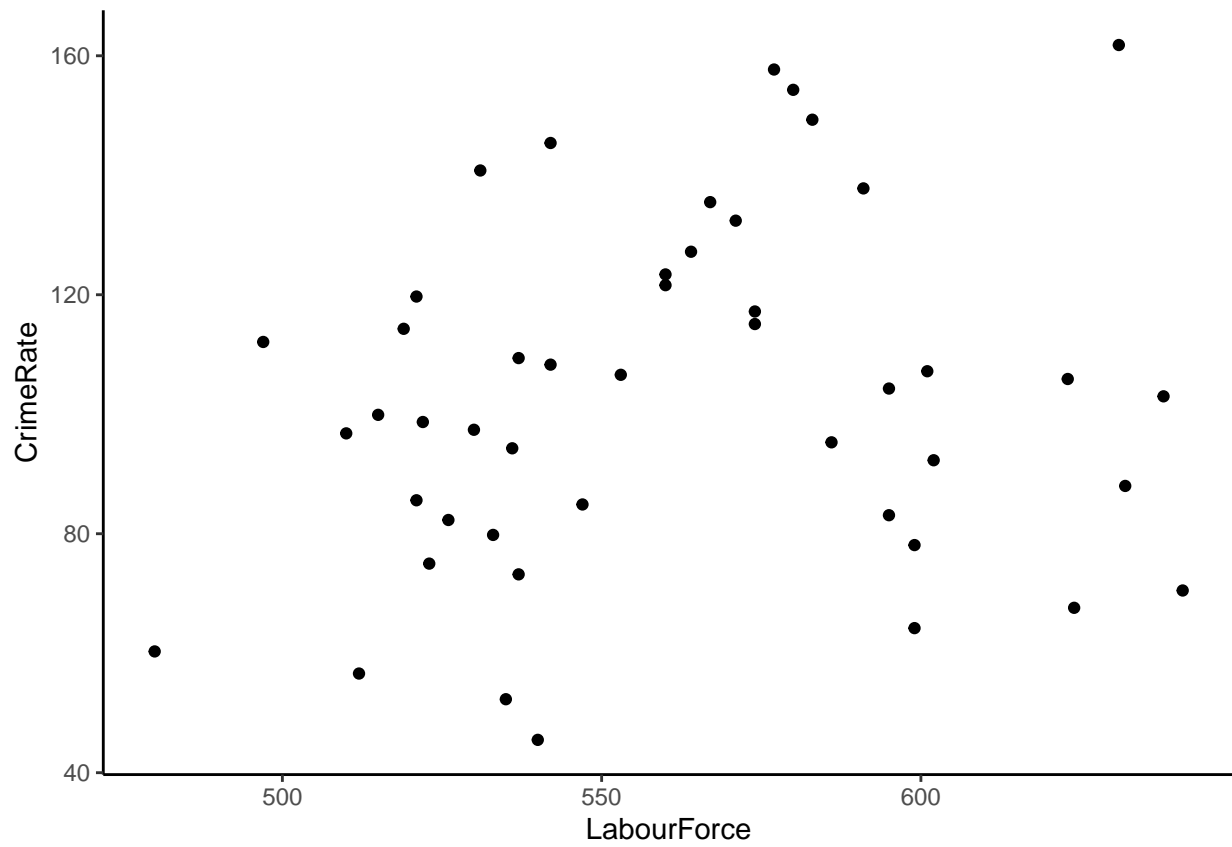
```
ggplot(data = crime, aes(x=Youth, y=CrimeRate)) + geom_point() + theme_classic()
```



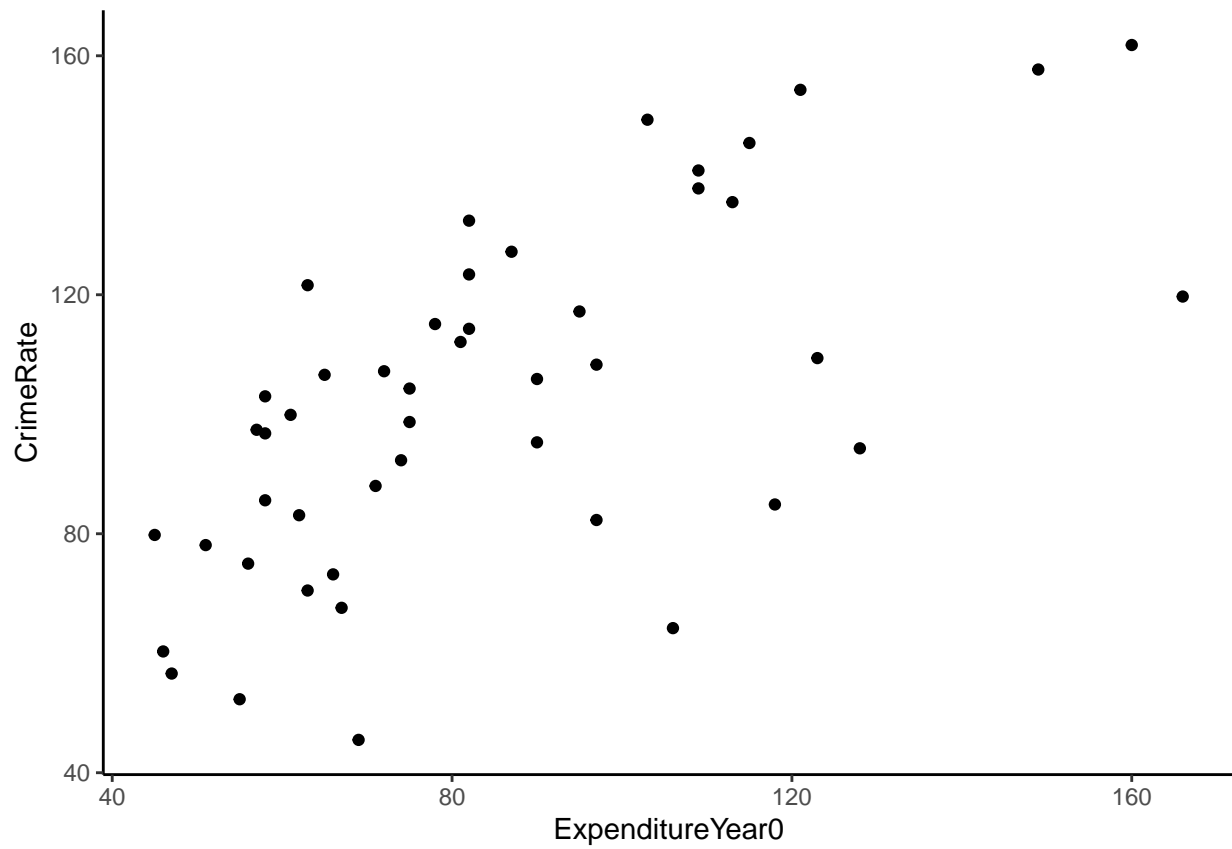
```
ggplot(data = crime, aes(x=Education, y=CrimeRate)) + geom_point() + theme_classic()
```



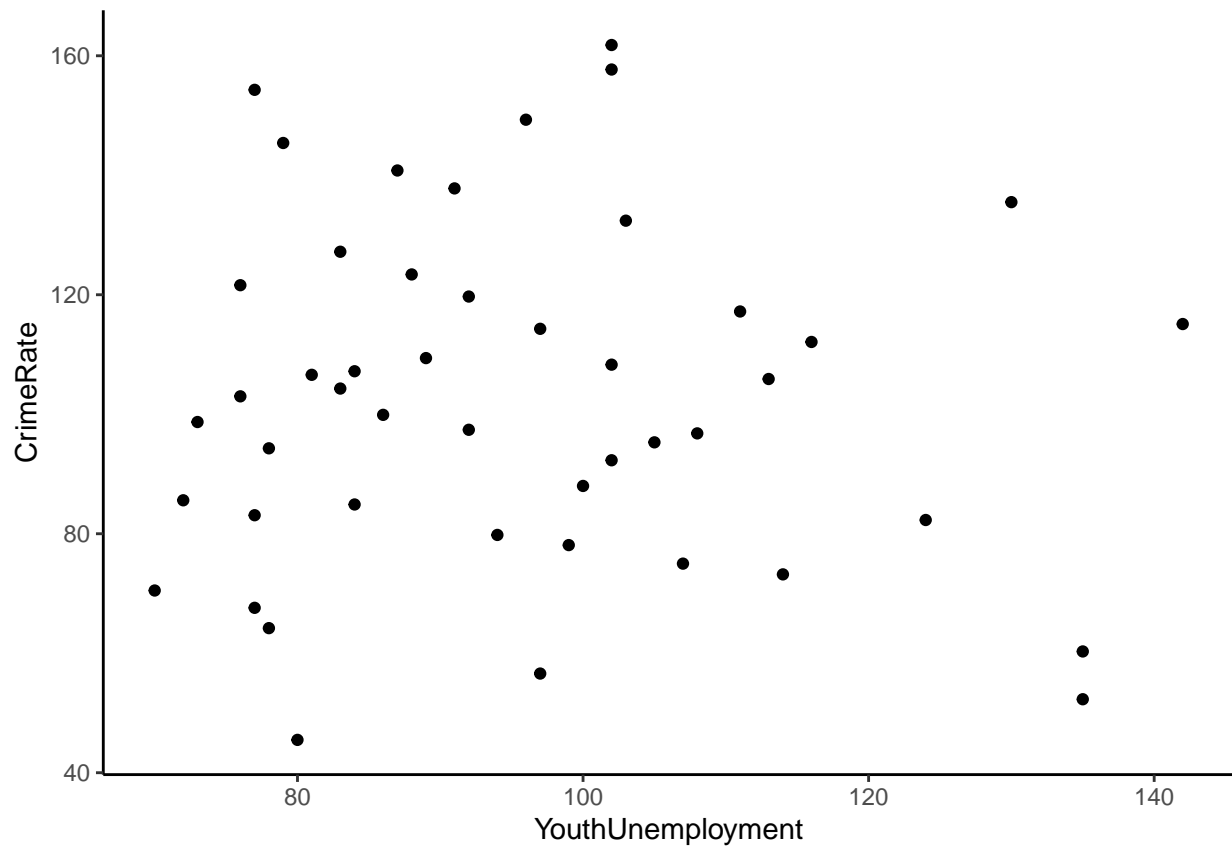
```
ggplot(data = crime, aes(x=LabourForce, y=CrimeRate)) + geom_point() + theme_classic()
```



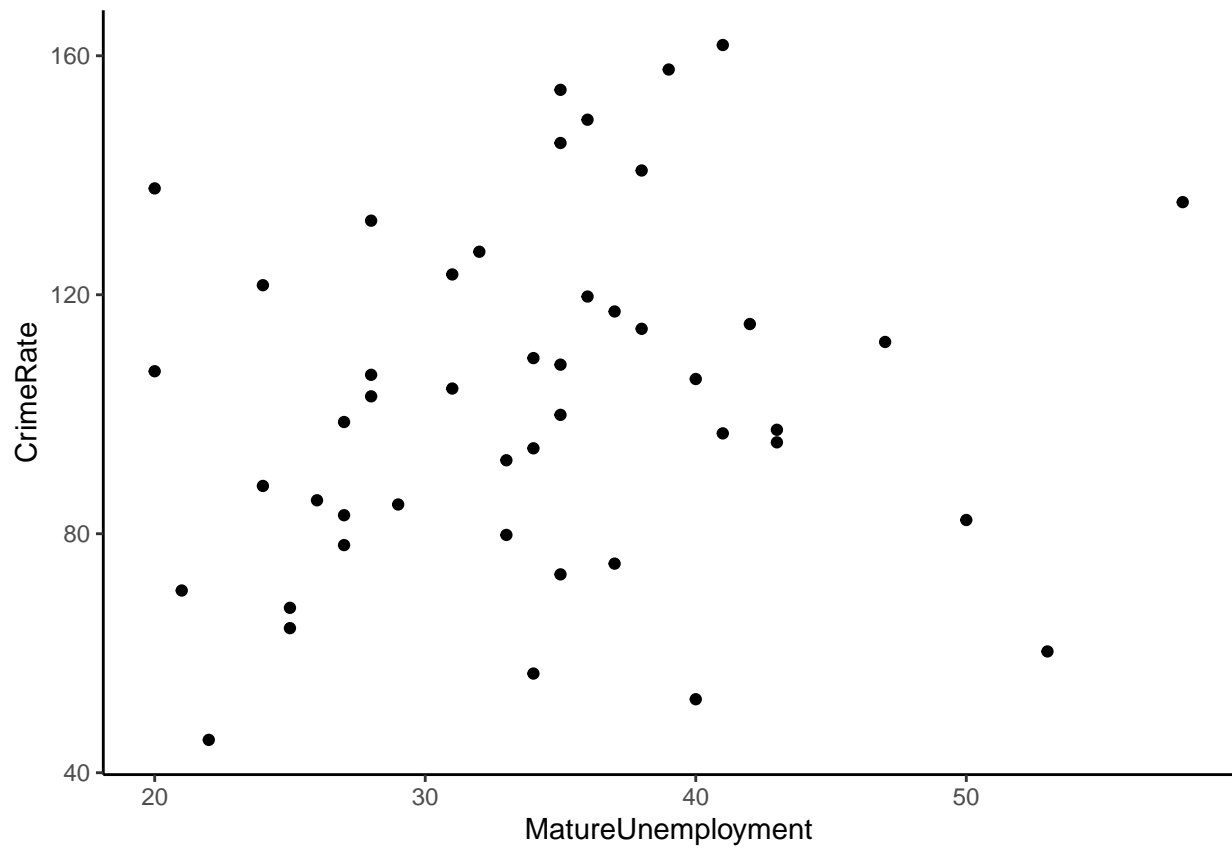
```
ggplot(data = crime, aes(x=ExpenditureYear0, y=CrimeRate)) + geom_point() + theme_classic()
```



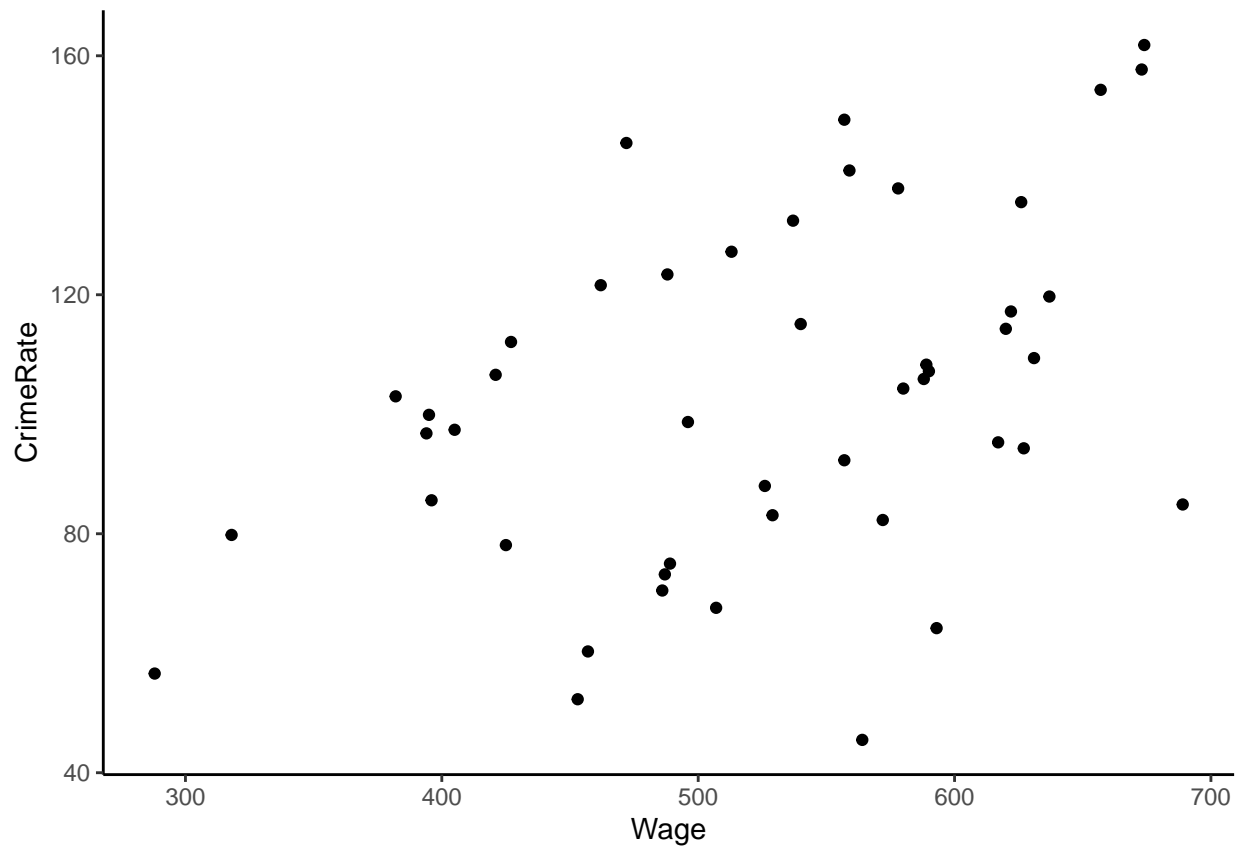
```
ggplot(data = crime, aes(x=YouthUnemployment, y=CrimeRate)) + geom_point() + theme_classic()
```



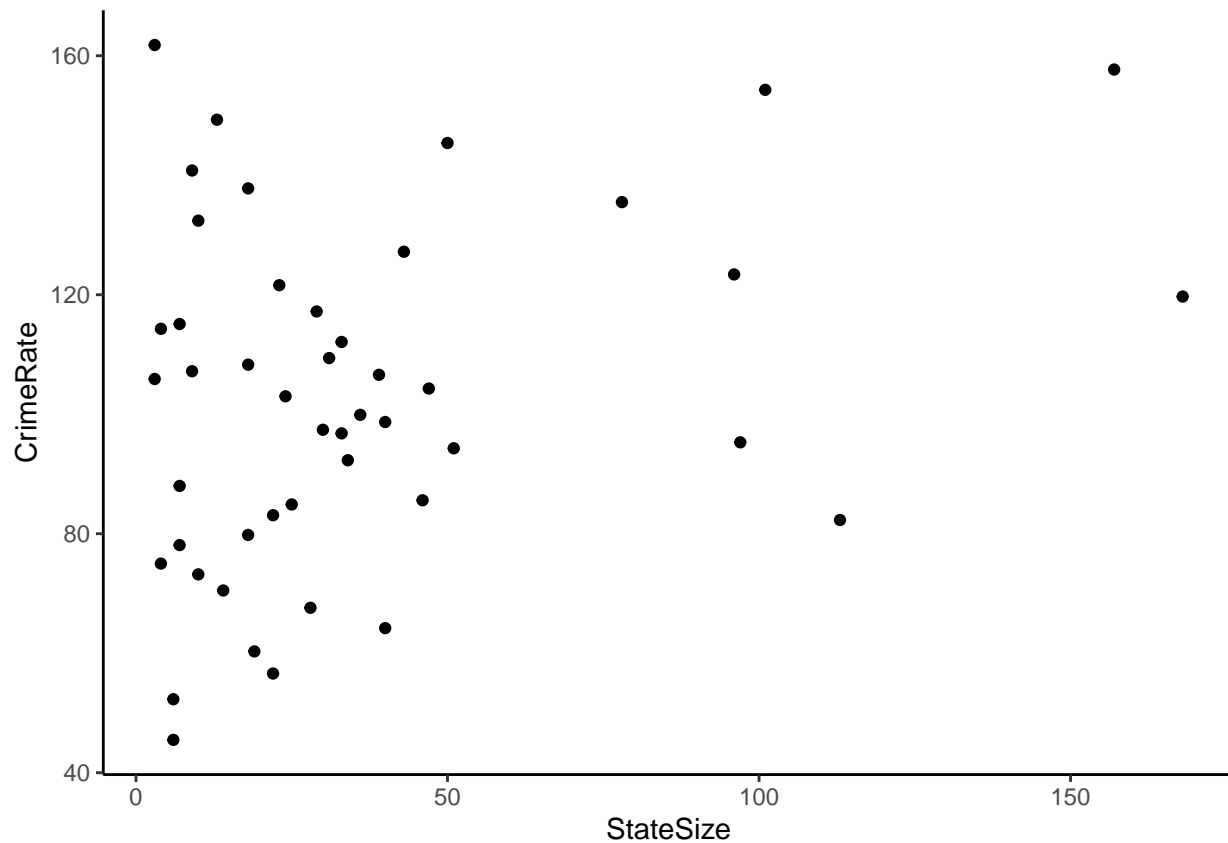
```
ggplot(data = crime, aes(x=MatureUnemployment, y=CrimeRate)) + geom_point() + theme_classic()
```



```
ggplot(data = crime, aes(x=Wage, y=CrimeRate)) + geom_point() + theme_classic()
```



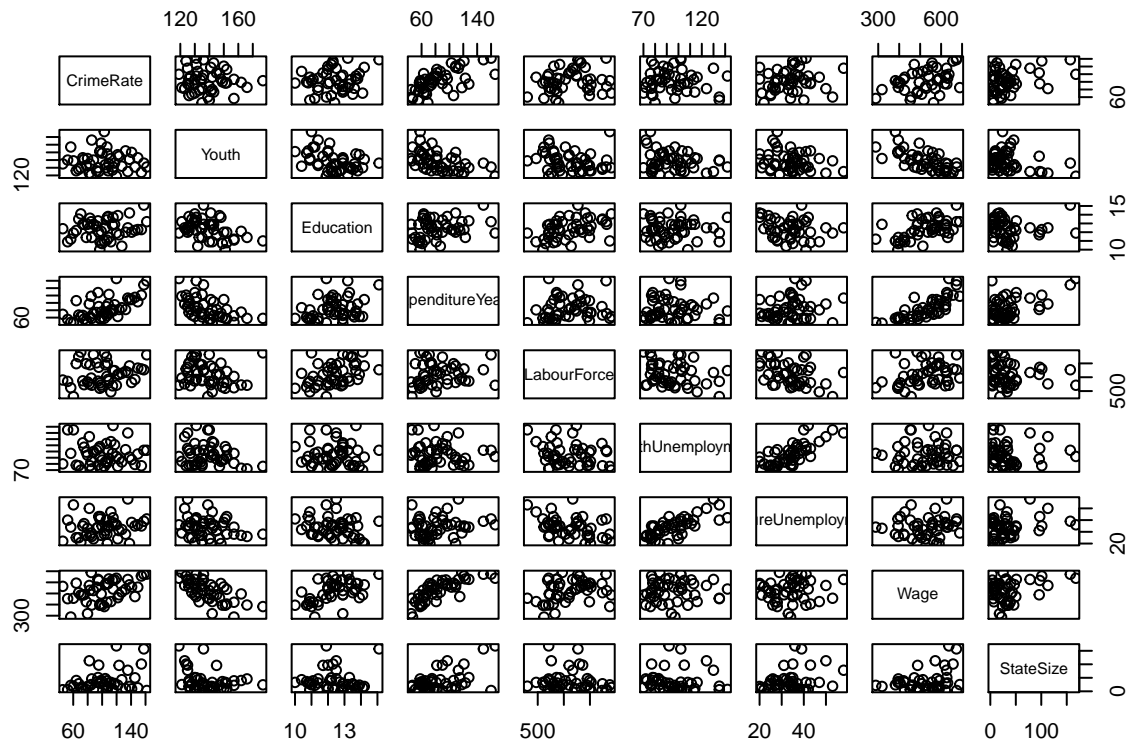
```
ggplot(data = crime, aes(x=Wage, y=CrimeRate)) + geom_point() + theme_classic()
```

The collection of scatter plots do not show that any of the variables is clearly linear, but some show a linear trend.

3. Focusing only on the continuous explanatory variables - check their correlations with the CrimeRate. We want to do this only for the continuous variables, so can look to remove the column that is binary from this plot. (This is done so that the pairs plot is legible and that we can run a corr function on the resulting dataframe)

```
crime.cont<-subset(crime, select=c("CrimeRate", "Youth", "Education", "ExpenditureYear0", "LabourForce"
                                   "Wage", "StateSize"))
pairs(crime.cont)
```



```
cor(crime.cont)
```

```
##           CrimeRate      Youth      Education ExpenditureYear0
## CrimeRate      1.00000000 -0.05500217  0.157004661      0.64621060
## Youth          -0.05500217  1.00000000 -0.404477055     -0.50573690
## Education       0.15700466 -0.40447706  1.000000000      0.30001793
## ExpenditureYear0 0.64621060 -0.50573690  0.300017928      1.00000000
## LabourForce     0.16930857 -0.16094882  0.427860495      0.12149320
## YouthUnemployment -0.05061320 -0.22438060 -0.026598090     -0.04369761
## MatureUnemployment 0.17183509 -0.24484339 -0.222656114      0.18509304
## Wage           0.42485301 -0.67005506  0.519186826      0.78722528
## StateSize       0.30794545 -0.28063762 -0.001403251      0.52628358
##
##           LabourForce YouthUnemployment MatureUnemployment      Wage
## CrimeRate      0.1693086      -0.05061320      0.17183509  0.42485301
## Youth          -0.1609488      -0.22438060     -0.24484339 -0.67005506
## Education       0.4278605      -0.02659809     -0.22265611  0.51918683
## ExpenditureYear0 0.1214932      -0.04369761      0.18509304  0.78722528
## LabourForce     1.0000000      -0.22939968     -0.42076249  0.29463231
## YouthUnemployment -0.2293997      1.00000000      0.74592482  0.04485720
## MatureUnemployment -0.4207625      0.74592482      1.00000000  0.09207166
## Wage           0.2946323      0.04485720      0.09207166  1.00000000
## StateSize      -0.1236722     -0.03811995      0.27042159  0.30826271
##
##           StateSize
## CrimeRate      0.307945450
## Youth          -0.280637618
## Education      -0.001403251
## ExpenditureYear0 0.526283581
## LabourForce    -0.123672219
## YouthUnemployment -0.038119948
## MatureUnemployment 0.270421586
## Wage           0.308262709
```

```
## StateSize          1.000000000
```

There do not seem to be any obvious multi collinearity (highly correlated explanatory variables) and a few of the plots above point to potential for a linear relationships, therefore at this stage I am not going to explore any transformations.

4. Using the continuous explanatory variables decide on a maximal model for CrimeRate and run it.

```
crime.lm<-lm(crime$CrimeRate~crime$Youth+crime$Education+crime$ExpenditureYear0+crime$MatureUnemployment+crime$LabourForce+crime$YouthUnemployment+crime$StateSize+crime$Wage)
summary(crime.lm)
```

```
##
## Call:
## lm(formula = crime$CrimeRate ~ crime$Youth + crime$Education +
##      crime$ExpenditureYear0 + crime$MatureUnemployment + crime$LabourForce +
##      crime$YouthUnemployment + crime$StateSize + crime$Wage)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -39.247 -14.381   0.036  10.666  34.043
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -245.73727    95.19350   -2.581 0.013821 *
## crime$Youth         1.03168     0.34474    2.993 0.004840 **
## crime$Education     2.47864     3.57762    0.693 0.492637
## crime$ExpenditureYear0 0.76387     0.19792    3.860 0.000427 ***
## crime$MatureUnemployment 1.29001     0.69947    1.844 0.072952 .
## crime$LabourForce    0.16571     0.09197    1.802 0.079515 .
## crime$YouthUnemployment -0.22757     0.29351   -0.775 0.442938
## crime$StateSize     -0.03859     0.10125   -0.381 0.705212
## crime$Wage         -0.00714     0.06657   -0.107 0.915155
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.51 on 38 degrees of freedom
## Multiple R-squared:  0.5836, Adjusted R-squared:  0.4959
## F-statistic: 6.656 on 8 and 38 DF,  p-value: 2.015e-05
```

(NOTE: it is possible to start with a model that has interactions, all interactions could be used or a Tree approach can help understand if the relationship between an explanatory variable and the target variable is different based on the value (or range) of the explanatory variable - page 199 Crawley)

5. Use a model selection approach to achieve a minimal adequate model

```
step(crime.lm)

## Start:  AIC=292
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
##      crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment +
##      crime$StateSize + crime$Wage
##
##              Df Sum of Sq  RSS    AIC
## - crime$Wage      1      4.8 15997 290.01
## - crime$StateSize  1     61.1 16053 290.17
## - crime$Education  1    202.0 16194 290.58
```

```

## - crime$YouthUnemployment 1 253.0 16245 290.73
## <none> 15992 292.00
## - crime$LabourForce 1 1366.2 17358 293.85
## - crime$MatureUnemployment 1 1431.4 17423 294.02
## - crime$Youth 1 3768.9 19761 299.94
## - crime$ExpenditureYear0 1 6268.9 22261 305.54
##
## Step: AIC=290.01
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
## crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment +
## crime$StateSize
##
## Df Sum of Sq RSS AIC
## - crime$StateSize 1 57.7 16054 288.18
## - crime$Education 1 199.8 16196 288.59
## - crime$YouthUnemployment 1 260.7 16257 288.77
## <none> 15997 290.01
## - crime$LabourForce 1 1371.3 17368 291.88
## - crime$MatureUnemployment 1 1446.3 17443 292.08
## - crime$Youth 1 4748.9 20746 300.23
## - crime$ExpenditureYear0 1 11682.9 27680 313.78
##
## Step: AIC=288.18
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
## crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment
##
## Df Sum of Sq RSS AIC
## - crime$Education 1 211.2 16266 286.79
## - crime$YouthUnemployment 1 216.8 16271 286.81
## <none> 16054 288.18
## - crime$MatureUnemployment 1 1391.9 17446 290.09
## - crime$LabourForce 1 1433.5 17488 290.20
## - crime$Youth 1 4868.6 20923 298.63
## - crime$ExpenditureYear0 1 12968.2 29022 314.01
##
## Step: AIC=286.79
## crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 + crime$MatureUnemployment +
## crime$LabourForce + crime$YouthUnemployment
##
## Df Sum of Sq RSS AIC
## - crime$YouthUnemployment 1 129.7 16395 285.17
## <none> 16266 286.79
## - crime$MatureUnemployment 1 1185.1 17451 288.10
## - crime$LabourForce 1 1796.2 18062 289.72
## - crime$Youth 1 4698.3 20964 296.72
## - crime$ExpenditureYear0 1 14469.2 30735 314.70
##
## Step: AIC=285.17
## crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 + crime$MatureUnemployment +
## crime$LabourForce
##
## Df Sum of Sq RSS AIC
## <none> 16395 285.17
## - crime$MatureUnemployment 1 1519.6 17915 287.33

```

```
## - crime$LabourForce      1    1684.0 18079 287.76
## - crime$Youth            1    5215.0 21610 296.15
## - crime$ExpenditureYear0 1   17961.5 34357 317.94

##
## Call:
## lm(formula = crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce)
##
## Coefficients:
##             (Intercept)              crime$Youth  crime$ExpenditureYear0
##                -230.0179                  1.0244                  0.7763
## crime$MatureUnemployment      crime$LabourForce
##                   0.8054                   0.1738
```

6. Once you have the minimal adequate model, explain its findings and test its residuals

```
mam.lm<-lm(formula = crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 +
  crime$MatureUnemployment + crime$LabourForce)
```

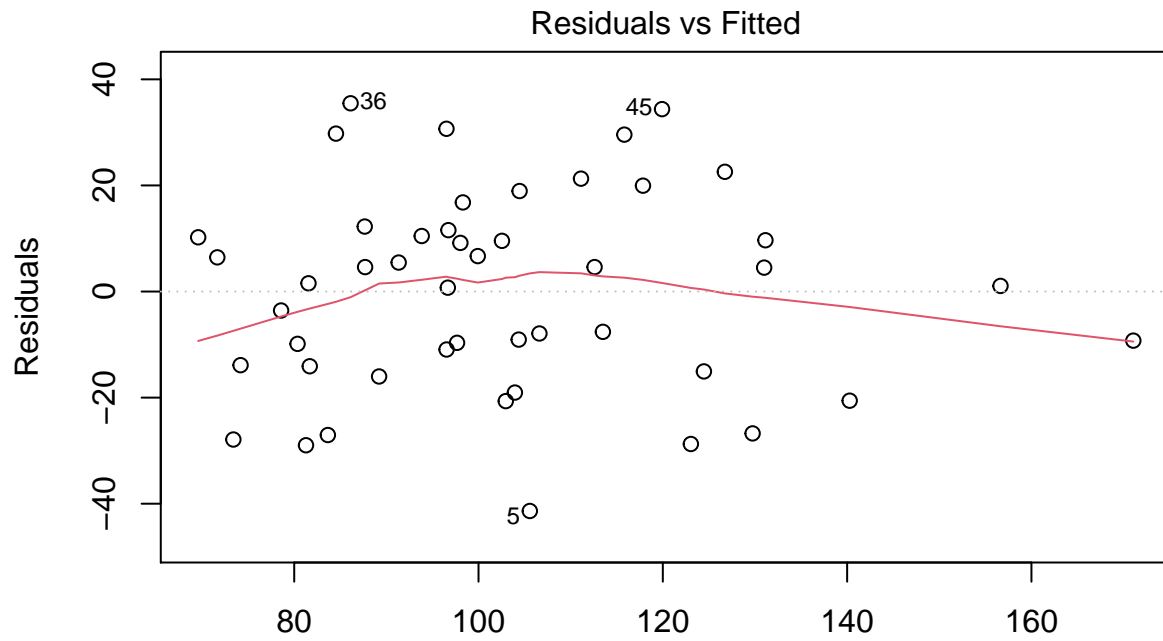
```
summary(mam.lm)
```

```
##
## Call:
## lm(formula = crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -41.368 -13.984   1.552  11.017  35.482
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -230.01795    77.24501  -2.978  0.004805 **
## crime$Youth      1.02436     0.28026   3.655  0.000709 ***
## crime$ExpenditureYear0  0.77626     0.11444   6.783  2.97e-08 ***
## crime$MatureUnemployment  0.80540     0.40821   1.973  0.055100 .
## crime$LabourForce    0.17379     0.08367   2.077  0.043956 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.76 on 42 degrees of freedom
## Multiple R-squared:  0.5731, Adjusted R-squared:  0.5324
## F-statistic: 14.09 on 4 and 42 DF,  p-value: 2.253e-07
```

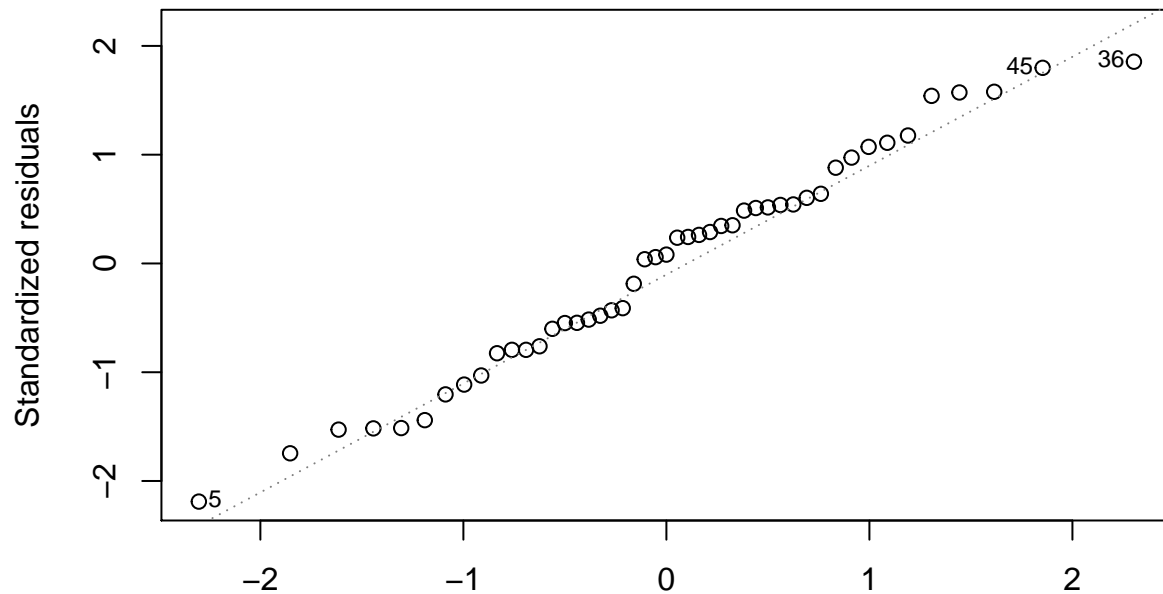
This model has acceptable goodness of fit, all the coefficients are significant (so there is no need to simplify further), r^2 is acceptable and the F statistic is significant.

Next the residuals should be scrutinised:

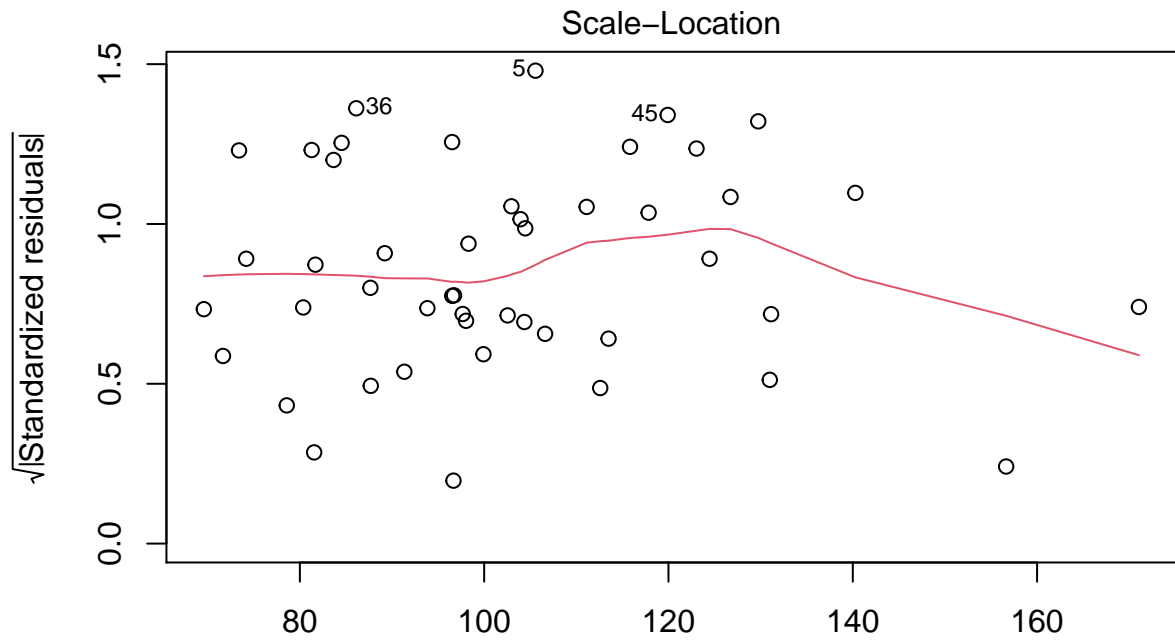
```
plot(mam.lm)
```



Fitted values
 $\text{lm}(\text{crime}\$CrimeRate \sim \text{crime}\$Youth + \text{crime}\$ExpenditureYear0 + \text{crime}\$MatureUne .$
 Normal Q-Q

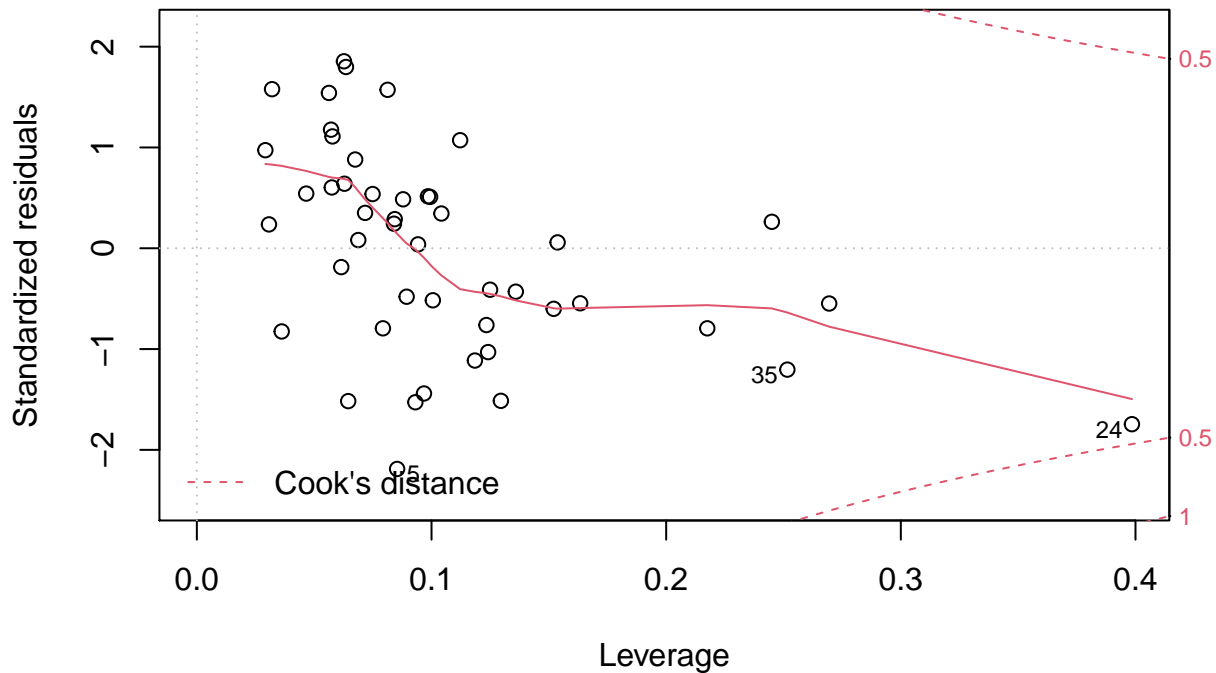


Theoretical Quantiles
 $\text{lm}(\text{crime}\$CrimeRate \sim \text{crime}\$Youth + \text{crime}\$ExpenditureYear0 + \text{crime}\$MatureUne .$



Im(crime\$CrimeRate ~ crime\$Youth + crime\$ExpenditureYear0 + crime\$MatureUnemplRate)

Residuals vs Leverage



Im(crime\$CrimeRate ~ crime\$Youth + crime\$ExpenditureYear0 + crime\$MatureUnemplRate)

In this case the residuals look ok, the variance is quite steady in the first plot - considering the data size. QQ plot also looks aligned.

7. OPTIONAL - model the relationship between the crime rate and the explanatory variables (including the ones that are not continuous).

```
model.all.lm<-lm(crime$CrimeRate~crime$Youth+crime$Education+crime$ExpenditureYear0
                +crime$MatureUnemployment + crime$LabourForce+crime$YouthUnemployment+crime$StateSize
summary(model.all.lm)
```

```
##
## Call:
## lm(formula = crime$CrimeRate ~ crime$Youth + crime$Education +
##     crime$ExpenditureYear0 + crime$MatureUnemployment + crime$LabourForce +
##     crime$YouthUnemployment + crime$StateSize + crime$Wage +
##     crime$HighYouthUnemploy)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -38.035 -13.917  -0.117   10.854   34.331
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -2.442e+02  9.659e+01  -2.528 0.015875 *
## crime$Youth      1.011e+00  3.587e-01   2.818 0.007720 **
## crime$Education   2.447e+00  3.625e+00   0.675 0.503810
## crime$ExpenditureYear0  7.588e-01  2.014e-01   3.768 0.000573 ***
## crime$MatureUnemployment  1.108e+00  1.005e+00   1.103 0.277285
## crime$LabourForce   1.704e-01  9.495e-02   1.795 0.080817 .
## crime$YouthUnemployment -1.595e-01  3.994e-01  -0.399 0.691987
## crime$StateSize   -3.932e-02  1.026e-01  -0.383 0.703596
## crime$Wage        -6.841e-03  6.741e-02  -0.101 0.919722
## crime$HighYouthUnemploy1 -2.962e+00  1.161e+01  -0.255 0.800053
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.77 on 37 degrees of freedom
## Multiple R-squared:  0.5843, Adjusted R-squared:  0.4832
## F-statistic: 5.778 on 9 and 37 DF,  p-value: 5.365e-05
```

The R^2 is higher but lets see what a step process would achieve in terms of simplifying the model:

```
step(model.all.lm)
```

```
## Start:  AIC=293.91
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment +
##     crime$StateSize + crime$Wage + crime$HighYouthUnemploy
##
##              Df Sum of Sq  RSS    AIC
## - crime$Wage      1      4.4 15968 291.93
## - crime$HighYouthUnemploy  1     28.1 15992 292.00
## - crime$StateSize    1     63.4 16027 292.10
## - crime$YouthUnemployment  1     68.8 16032 292.11
## - crime$Education     1    196.6 16160 292.49
## - crime$MatureUnemployment  1    524.6 16488 293.43
## <none>                  15964 293.91
## - crime$LabourForce     1   1390.2 17354 295.84
## - crime$Youth           1   3425.3 19389 301.05
## - crime$ExpenditureYear0  1   6126.5 22090 307.18
```



```

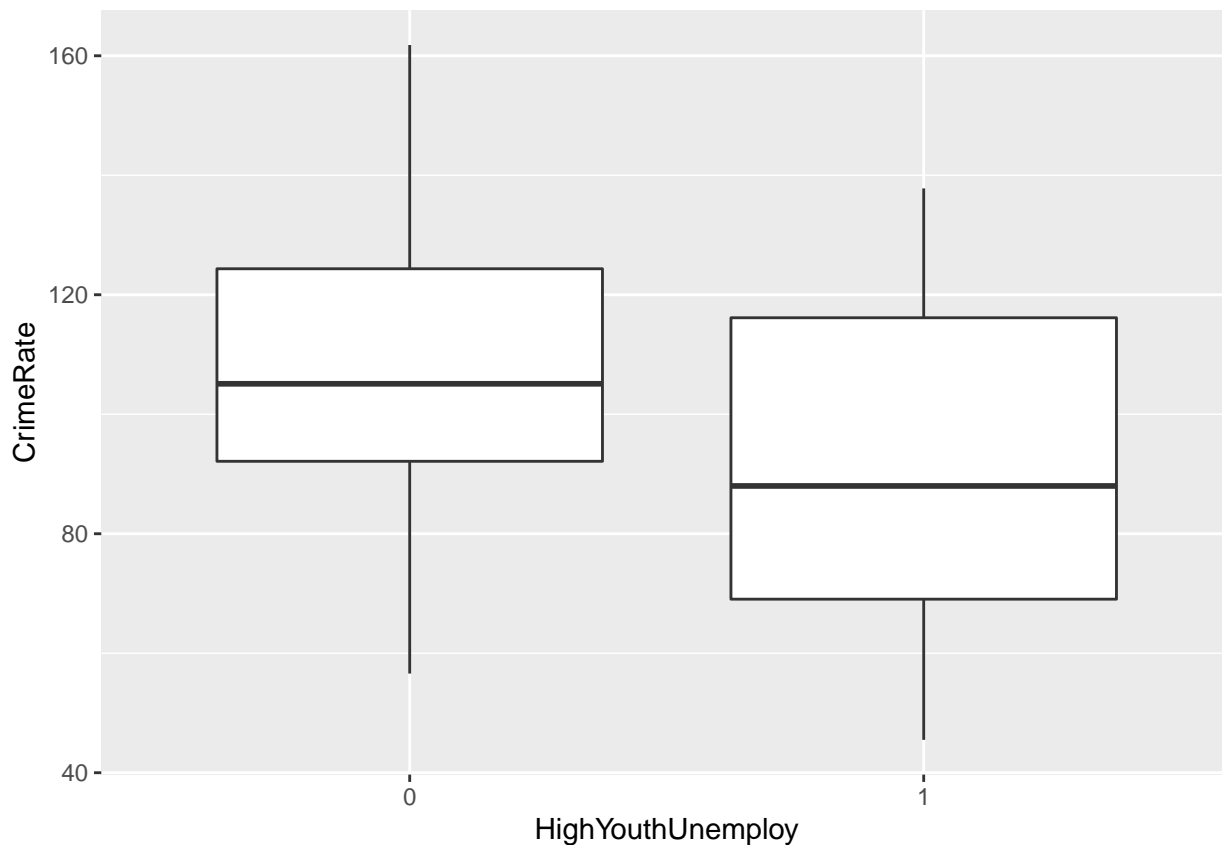
##
## Step: AIC=291.93
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment +
##     crime$StateSize + crime$HighYouthUnemploy
##
##           Df Sum of Sq  RSS    AIC
## - crime$HighYouthUnemploy  1      28.5 15997 290.01
## - crime$StateSize          1      60.1 16028 290.10
## - crime$YouthUnemployment  1      70.8 16039 290.13
## - crime$Education          1     195.0 16163 290.50
## - crime$MatureUnemployment  1     527.9 16496 291.45
## <none>                      15968 291.93
## - crime$LabourForce        1    1395.1 17363 293.86
## - crime$Youth              1    4244.4 20212 301.00
## - crime$ExpenditureYear0   1   11378.4 27346 315.21
##
## Step: AIC=290.01
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment +
##     crime$StateSize
##
##           Df Sum of Sq  RSS    AIC
## - crime$StateSize          1      57.7 16054 288.18
## - crime$Education          1     199.8 16196 288.59
## - crime$YouthUnemployment  1     260.7 16257 288.77
## <none>                      15997 290.01
## - crime$LabourForce        1    1371.3 17368 291.88
## - crime$MatureUnemployment  1    1446.3 17443 292.08
## - crime$Youth              1    4748.9 20746 300.23
## - crime$ExpenditureYear0   1   11682.9 27680 313.78
##
## Step: AIC=288.18
## crime$CrimeRate ~ crime$Youth + crime$Education + crime$ExpenditureYear0 +
##     crime$MatureUnemployment + crime$LabourForce + crime$YouthUnemployment
##
##           Df Sum of Sq  RSS    AIC
## - crime$Education          1     211.2 16266 286.79
## - crime$YouthUnemployment  1     216.8 16271 286.81
## <none>                      16054 288.18
## - crime$MatureUnemployment  1    1391.9 17446 290.09
## - crime$LabourForce        1    1433.5 17488 290.20
## - crime$Youth              1    4868.6 20923 298.63
## - crime$ExpenditureYear0   1   12968.2 29022 314.01
##
## Step: AIC=286.79
## crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 + crime$MatureUnemployment +
##     crime$LabourForce + crime$YouthUnemployment
##
##           Df Sum of Sq  RSS    AIC
## - crime$YouthUnemployment  1     129.7 16395 285.17
## <none>                      16266 286.79
## - crime$MatureUnemployment  1    1185.1 17451 288.10
## - crime$LabourForce        1    1796.2 18062 289.72

```

```
## - crime$Youth          1    4698.3 20964 296.72
## - crime$ExpenditureYear0 1   14469.2 30735 314.70
##
## Step: AIC=285.17
## crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 + crime$MatureUnemployment +
##      crime$LabourForce
##
##               Df Sum of Sq  RSS   AIC
## <none>                  16395 285.17
## - crime$MatureUnemployment 1    1519.6 17915 287.33
## - crime$LabourForce        1    1684.0 18079 287.76
## - crime$Youth              1    5215.0 21610 296.15
## - crime$ExpenditureYear0    1   17961.5 34357 317.94
##
## Call:
## lm(formula = crime$CrimeRate ~ crime$Youth + crime$ExpenditureYear0 +
##      crime$MatureUnemployment + crime$LabourForce)
##
## Coefficients:
##              (Intercept)              crime$Youth  crime$ExpenditureYear0
##                -230.0179                  1.0244                  0.7763
## crime$MatureUnemployment  crime$LabourForce
##                   0.8054                   0.1738
```

The binary variable we added as part of the explanatory variables does not add much and this is confirmed as the step process proposes a model that does not include it as an explanatory variable.

```
ggplot(crime, aes(x=HighYouthUnemploy, y=CrimeRate)) + geom_boxplot()
```



8. OPTIONAL - If the average education time in the population is 14 years. Compute the mean education time in this sample of 48 rows of data and test the hypothesis that the education time is significantly lower than the population education time.

```
education.mean<-mean(crime$Education)
education.mean
```

```
## [1] 12.39149
```

We will be testing the following hypotheses: $H_0 : \mu = 14$ and $H_1 : \mu < 14$. $n = 48$ so we can use the following test statistic:

$$\frac{\bar{x} - \mu}{S/\sqrt{n}}$$

```
education.s<-sd(crime$Education)
n<-length(crime$Education)
```

Computing the test statistic

```
den<-education.s/sqrt(n)
test.statistic<-(education.mean-14)/den
test.statistic
```

```
## [1] -9.842971
```

Now we should find the probability of such a test statistic or smaller to obtain the p-value:

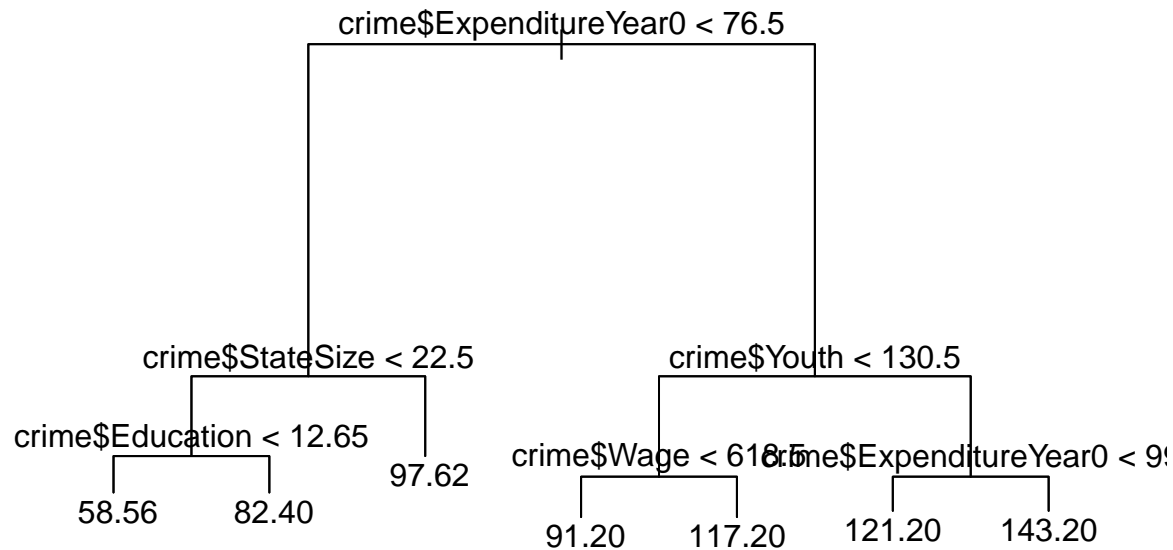
```
pnorm(test.statistic)
```

```
## [1] 3.675379e-23
```

This is a very small p-value, so we reject the Null hypothesis in this case.

OPTIONAL - If you want to use a Regression Tree to check for interactions:

```
library(tree)
crime.tree<-tree(crime$CrimeRate~crime$Youth+crime$Education+crime$ExpenditureYear0+crime$MatureUnemployo
plot(crime.tree)
text(crime.tree)
```



From this tree there are no “contradictions” in direction of the estimate. (see page 199 of Crawley for an example), therefore this does not point to the need for interactions in this case.