Prescribing Closed-Loop Behavior Using Nonlinear Model Predictive Control

Masoud Kheradmandi, Prashant Mhaskar*

Dept. of Chemical Engineering, McMaster University, Hamilton, ON, Canada L8S 4L7

Abstract

In this work, we address the problem of control of nonlinear systems to deliver a prescribed closed-loop behavior. In particular, the framework allows for the practitioner to first specify the nature and specifics of the desired closed-loop behavior (e.g., first order with smallest time constant, second order with no more than a certain percentage overshoot, etc.). An optimization based formulation then computes the control action to deliver the best attainable closed loop behavior. To decouple the problems of determining the best attainable behavior and tracking it as closely as possible, the optimization problem is posed and solved in two tiers. In the first tier, the focus is on determining the best closed-loop behavior attainable, subject to stability and tracking constraints. In the second tier, the inputs are tweaked to possibly improve the tracking of the optimal output trajectories given by the first tier. The efficacy of the proposed method, and the various specific formulations needed are illustrated through implementation on a linear system subject to output feedback, a nonlinear CSTR subject to uncertainty and rate of change of input constraints, and a reactor separator system. The simulation results demonstrate significantly improved adherence to the prescribed performance criteria over a predictive controller representative of existing approaches.

*Correspondence concerning this article should be addressed to P. Mhaskar at mhaskar@mcmaster.ca

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Introduction

The operation of chemical plants faces numerous challenges such as inherent nonlinearity, complex variable interactions and process constraints. The presence of these complexities often prevents classical (Proportional Integral Derivative (PID)) controllers from readily achieving closed-loop behavior that the control practitioners are looking for (e.g., smooth first order response).

One well established control method that enables incorporating performance considerations more directly in computing control calculations is model predictive control (MPC). In the MPC approach, an open-loop optimal control problem is solved at each sampling instance over a finite time horizon, subject to the dynamic response of the plant and constraints. Contributing to, and benefiting from the industrial application of MPC, several studies have focused on the stability properties of MPC formulations. In one direction, this has led to the development of Lyapunov-based MPC which can explicitly characterize the region from where stability of the closed-loop system is guaranteed in the presence of input constraints [1,2] and uncertainty [3]. Another direction in exploring the properties of MPC formulations and their ability to handle plant model mismatch has led to the development offset-free MPC designs that focus on the disturbance rejection mechanism by augmenting the state variables with fictitious states that estimate and counteract the uncertainty [4,5]. In these MPC designs, the focus has primarily been on stabilization, with the objective function being used as a tuning mechanism.

The tuning of the objective function to achieve a good closed-loop performance, however, has remained a non-trivial task. In this direction, numerous MPC performance/tuning assessment methods are proposed to evaluate the closed-loop performance by comparing the controller with a benchmark [6,7]. Several excellent contributions have been made to address the challenging problem of controller tuning [8,9]. In multi-objective MPC (MOPC) [10] the notion of utilizing multi-tier optimization problem is utilized to decouple the tuning issues with the multi-objective optimization problem[11].

The desire to explicitly include economic considerations in the control calculations has fostered the recent development of economic MPC (EMPC) formulations where the controller determines the set-point internally to satisfy the prescribed economic objective[12], supported by a rigorous analysis that ensures that stability is preserved[13,14]. In recent contributions, EMPC capabilities for handling constraints, such as limited input rate-of-change[15], while improving economic performance and ensuring closed loop stability have also been addressed[16].

The nature of the predictive controller and existing performance assessment methodologies notwithstanding, controller performance assessment is often conducted by control practitioners in more simpler terms (such as, smoothness of the response or first-order response characteristics). In the direction of computing control laws to deliver specific desire behavior, internal model control (IMC)[17] has been proposed where the controller is designed to achieve a pre-specified closed-loop transfer function for linear, single-input, single-output (SISO) systems. However, it is not always guaranteed that the transfer function of the IMC-based control law will be consistent with the proportional-integral-derivative (PID) control structure. Moreover, the IMC approach does not generalize readily for multi-input, multi-output (MIMO) systems with constraints. In an effort to handle MIMO systems, the funnel control approach[18] is proposed where the time-varying output error feedback controller forces the tracking error to be within a bounded prescribed function. Although the funnel control approach is capable of handling nonlinear MIMO systems, the method does not explicitly consider input constraints. A related approach is model algorithmic control (MAC), where the desired close-loop trajectory is first order trajectory[19,20], with the time constant being a tuning parameter. The model algorithmic approach has also been developed for nonlinear systems where the delay free part of the system equations is unstable[21].The MAC approach only allows for prescribing a first order response, and does not explicitly account for the presence of input constraints.

An MPC framework has recently been proposed that enables specifying desired closed-loop behavior for linear MIMO systems subject to input constraints[22], which is then implemented in conjunction with offset-free model predictive control. The developed approach[22] considers systems that are invertible (i.e. the inputs can be explicitly computed). A similar approach was utilized in[23] for linear systems. There does not exist a formulation, however, that allows the ability to explicitly prescribe the nature of the closed-loop behavior and have the formulation determine the best achievable closed-loop behavior for nonlinear systems.

Motivated by the above considerations, in this work we address the problem of control design for nonlinear systems that allows prescribing and determining the best achievable closed-loop behavior of a desired nature. The rest of the manuscript is organized as follows: First, the general mathematical description for the types of nonlinear systems considered in this work, and a representative formulation for nonlinear model predictive control (NMPC) are presented. Then the proposed bi-layer performance specification based nominal MPC scheme for achieving desired trajectories is given. The proposed framework enables specifying a desired nature of the closed-loop behavior and then determining the optimal feasible implementation of such behavior. Rigorous feasibility and stability properties are established for a formulation to achieve the best first order trajectory. Other formulations are also presented that demonstrate how, say a second order trajectory and input rate of constraints can be accommodated. The efficacy of the proposed method is first illustrated through formulations and implementations for a linear system subject to output feedback and a nonlinear continuous stirred-tank reactor (CSTR) with input rate of change constraints and uncertainty and a reactor separator plant. Finally, concluding remarks are presented.

Preliminaries

In this section, we first describe the class of nonlinear systems considered in this work, followed by a representative existing nonlinear model predictive control formulation.

System Description

In this work, we consider a class of nonlinear systems with input constraints described as follows:

$$\dot{x} = f(x, u) \tag{1}$$

$$y = h(x, u) \tag{2}$$

where $x \in \mathbb{R}^n$ denotes the vector of state variables, $u \in \mathbb{R}^m$ denotes the vector of constrained control (manipulated) input variables, taking values in a nonempty convex subset $\mathcal{U} \subset \mathbb{R}^m$, where $\mathcal{U} = \left\{u \in \mathbb{R}^m \mid u_{\min} \leq u \leq u_{\max}\right\}$, $u_{\min} \in \mathbb{R}^m$ and $u_{\max} \in \mathbb{R}^m$ denote the lower and upper bounds of the input variables, and $y \in \mathbb{R}^p$ denotes the vector of measured output variables. It is assumed that the functions, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ are locally Lipschitz in their arguments and that the system is controllable. In keeping with the discrete implementation of MPC, u is piecewise constant and defined over an arbitrary sampling instance k as:

$$u(t) = u(k), \quad k\Delta t \leq t < (k+1)\Delta t$$

where Δt is the sampling time and x_k and y_k denote state and output at the kth sample time.

Nonlinear MPC

In this subsection, we present a representative nonlinear MPC formulation where the control action at time t_k to driven the system to the origin is computed by solving the following optimization problem:

$$\min_{\tilde{u}} \sum_{j=1}^{P} ||\tilde{y}_{k+j}||_Q^2 + \sum_{j=0}^{N_u - 1} ||\tilde{u}_{k+j} - \tilde{u}_{k+j-1}||_R^2 \tag{3a}$$

subject to:

$$\dot{\tilde{x}} = f(\tilde{x}, \tilde{u}) \tag{3b}$$

$$\tilde{y} = h(\tilde{x}, \tilde{u}) \tag{3c}$$

$$\tilde{u} \in \mathcal{U}, \qquad \Delta\tilde{u} \in \mathcal{U}_\delta, \qquad \tilde{x}(k) = \hat{x}_k \tag{3d}$$

$$V(\tilde{x}_{k+1}) \leq V(\tilde{x}_k) \; \forall \; V(\tilde{x}_k) > \epsilon^* \tag{3e}$$

$$V(\tilde{x}_{k+1}) \leq \epsilon^* \; \forall \; V(\tilde{x}_k) \leq \epsilon^* \tag{3f}$$

where $\tilde{x}_{k+j}$, $\tilde{y}_{k+j}$ and $\tilde{u}_{k+j}$ denote predicted state and output and calculated manipulated input variables j time steps ahead computed at time step k, and $\hat{x}_k$ is the current estimation of state. The operator $||.||_Q^2$ denotes the weighted Euclidean norm defined for an arbitrary vector x and weighting matrix W as $||x||_W^2 = x^T W x$. Further, $Q > 0$ and $R \geq 0$ denote the positive definite and positive semi-definite weighting matrices for penalizing deviations in the output predictions and for the rate of change of the manipulated inputs, respectively. Moreover, P and N_u denote the prediction and control horizons, respectively, and the input rate of change, given by $\Delta\tilde{u}_{k+j} = \tilde{u}_{k+j} - \tilde{u}_{k+j-1}$, takes values in a nonempty convex subset $\mathcal{U}_\delta \subset \mathbb{R}^m$, where $\mathcal{U}_\delta = \left\{\Delta u \in \mathbb{R}^m \mid \Delta u_{\min} \leq \Delta u \leq \Delta u_{\max}\right\}$. Note finally, that while the system dynamics are described in continuous time, the objective function and constraints are defined in discrete time to be consistent with the discrete implementation of the control action.

Eqs. 3e and 3f are a representative example of stability constraints (various versions are used in in conventional nominal MPC formulations), and are the Lyapunov-based stability constraint[24,25], where $V(x_k)$ is a suitable control Lyapunov function, and $\epsilon^* > 0$ is a user-specified parameter. In the presented formulation, $\epsilon^* > 0$ enables practical stabilization to account for the discrete nature of the control implementation.

Note that regardless of the type of stability constraint involved, the objective function is used in conventional MPC formulations to 'tune' the tracking performance. While the tuning parameters can be varied (through an extensive trial and error exercise) to achieve a specific performance, existing MPC formulations do not allow for an explicit specification of the desired closed-loop behavior for general nonlinear systems. In the remainder of the manuscript, the MPC formulation of Eq. 3a-3f will be referred to as the 'nominal' MPC formulation, and utilized to compare with the proposed formulation.

MPC Formulation with Performance Specification (PSMPC)

In this section, the MPC formulation that enables specifying and optimizing a desired closed-loop behavior is presented, and is referred to as the performance specification MPC. This desired behavior could be for instance, fastest smooth first-order response, underdamped second-order response and/or a response that is cognizant of the rate of input change constraints. To handle the multi objective nature of the problem the proposed formulation utilizes a bi-layer approach. In the first layer, the best feasible trajectory that satisfies the prescribed desired criteria (subject to appropriate bounds around the desired trajectory) while considering the system dynamics and constraints. Then the second layer of the MPC computes the inputs to track the optimal trajectory satisfying the desired closed-loop behavior as obtained from the first tier (in the form of set-points for the controlled variables along the prediction horizon). Note that the notion of utilizing multi-layers to decouple the multi objective nature of the optimization problem is not the novel contribution of the present work. Existing results on, for instance, multi-objective MPC[10] have utilized this notion. The novel contribution of the present work is the posing of the control problem where the best achievable desired closed-loop trajectory is computed and implemented.

The two-tiered control structure is schematically presented in Fig. 1. We defer the presentation of the specifics regarding Tier 1 to a later section (note that the Tier 1 formulation changes depending on the various considerations pertaining to the desired response that need to be accounted for in the closed-loop). Instead, we first present the detailed formulation for Tier 2 in the following subsection, which implements the specified set-point profile computed by the first tier.

Tier 2: MPC formulation

The second layer of the MPC with explicit performance specification is formulated as follows:

$$\min_{\tilde{u}} \sum_{j=1}^{P} ||\tilde{y}_{k+j} - \bar{y}^{\mathrm{SP}}_{k+j}||^2_{Q_y}$$

subject to:

$$\begin{aligned}
&\dot{\tilde{x}} = f(\tilde{x}, \tilde{u}) \\
&\tilde{y} = h(\tilde{x}, \tilde{u}) \\
&\tilde{u} \in \mathcal{U}, \qquad \Delta\tilde{u} \in \mathcal{U}_\delta, \qquad \tilde{x}(k) = \hat{x}_k \\
&V(\tilde{x}_{k+1}) \leq V(\tilde{x}_k) \ \forall \ V(\tilde{x}_k) > \epsilon^* \\
&V(\tilde{x}_{k+1}) \leq \epsilon^* \ \forall \ V(\tilde{x}_k) \leq \epsilon^* \\
&|\tilde{y}_{k+j} - \bar{y}^{SP}_{k+j}| \leq \varepsilon, \text{ for } j = 1, \ldots, P
\end{aligned} \tag{4}$$

where P denotes the prediction horizon, $\bar{y}^{\mathrm{SP}}_k$ is the desired output trajectory (computed by Tier 1), and, $\tilde{y}_k$ is the predicted output trajectory at time $k\Delta t$ with Δt as the sampling time. $V(\cdot)$ is a control Lyapunov function, $Q_y \in R^{n \times n}$ is a positive definite matrix used to trade-off the relative importance among the controlled variables, and ε is a threshold for maintaining the outputs to be within an admissible neighborhood region of the desired trajectory.

In Fig. 2, a schematic presentation of the proposed bi-layer performance specification MPC is presented. The continuous line [−] is the best trajectory that minimizes the key characteristic of the desired closed-loop behavior (say time constant for a first order response), where the process is only required to be within a reasonable bounds of this best trajectory (specified via constraints) denote by the dashed lines [−−]. The corresponding outputs are shown by the dashed square line [−□−]. Thus, there exist a set of feasible input moves that would produce the trajectory denoted by the dashed square lines. The dashed plus line [− + −] is the predicted state trajectory to minimize the difference between the state trajectory and the ideal state trajectory [−]. Thus in the worst case scenario, the dashed plus line [− + −] would simply coincide with the dashed square line [−□−], or could be closer to the continuous line $[-]$, if possible. The lines indicated by $[-\cdot]$ and $[\cdot\cdot]$, indicate the Tier 1 and Tier 2 predicted inputs, respectively. Note that the inputs from Tier 1 are feasible for the Tier 2 optimization problem; Tier 2 inputs simply bring the trajectory closer to the ideal trajectory if possible.

Remark 1. *The tuning of the response behavior in existing MPC formulations is known to be a nontrivial task. In contrast, the tuning mechanism for the proposed performance specification MPC formulation is significantly less challenging. In particular, the weighting matrix (Q_y) can be readily chosen to scale the variables. Furthermore, since all the performance criteria are already accounted for in Tier 1, the Tier 2 formulation does not include additional penalty terms for the manipulated variables or the input rate of change. Therefore, compared to existing MPC formulations, the trade-off between the penalties is not a hindrance in achieving 'desired' performance in the proposed performance specification MPC.*

Remark 2. *Note that Tiers 1 and 2 are executed in series and at the same time, and the implementation does not require a time scale separation. The overall optimization is split into two tiers simply to enable easy tuning. In particular, the first tier computes the trajectory parameters and input sequence with the objective function only focusing on the 'best achievable' closed-loop response. Thus there is no need to trade off the penalty terms between the desired closed-loop response and the set-point trajectory tracking. Then the second tier is used to determine the input trajectory that enables closest tracking of the best achievable closed loop response as computed via the constraints of Tier 1.*

Remark 3. *For a class of linear systems that have an equal number of manipulated variables and controlled outputs, recent results*[22] *incorporated closed-loop performance specification utilizing equality constraints to exploit the fact that there exists a unique steady-state input for each set-point. In contrast the present work considers a general class of nonlinear systems, for which a given closed-loop behavior may simply be unachievable for all the controlled outputs. Thus, the proposed approach utilizes inequality constraints (Eq. 4) to compute and implement the desired closed-loop* *behavior to guarantee feasibility in such class of nonlinear systems. The choice of the threshold represents the trade-off between two conflicting objectives, feasibility and efficiency of the first tier. Note that, for a given value of the tolerance parameter ϵ, infeasibility of the optimization problem can be used by the practitioner to either relax the tolerance, or to ask for a different kind of response (e.g., a second order instead of a first order response).*

Remark 4. *Note that there are several key differences between the proposed approach and the MAC approach*[19,20]*. The first difference is the capability to handle input constraints. The second key difference is that the proposed formulation allows prescribing other kinds of responses, besides just the first order response. The third key difference is that the proposed approach is configured to not just prescribe a specific kind of response, but also prescribe the optimality criteria, for example, fastest first order response. Thus, the proposed formulation not only implements a first order response, but computes and determines the best first order response. Finally, we also show how the proposed approach is able to handle other kinds of constraints (such as rate of change constraints on the manipulated input).*

Achieving the Best First Order Trajectory

The key novelty of the proposed approach is that it allows to specify various desired response forms. To illustrate this, in the remainder, we show specific formulations to handle some instances of specific desired behavior. Note that the kinds of possible desired responses is not limited to the ones described next.

We first illustrate the formulation that enables implementing the best first order response, where in this context the best response is one with the smallest time constant. Note that the purpose of the formulation below is to illustrate how such a desired response can be incorporated; if the practitioner dictates other kinds of responses, they can be readily integrated in the approach (see later for other illustrations).

The first tier formulation of the proposed bi-layer control method for a desired first order trajectory takes the following form:

$$
\begin{aligned}
&\min_{\tau,\tilde{u}} ||\tau|| \\
\text{subject to:}\quad & \\
&\dot{\tilde{x}} = f(\tilde{x}, \tilde{u}) \\
&\tilde{y} = h(\tilde{x}, \tilde{u}) \\
&\tilde{u} \in \mathcal{U}, \qquad \Delta\tilde{u} \in \mathcal{U}_\delta, \qquad \tilde{x}(k) = \hat{x}_k \\
&V(\tilde{x}_{k+1}) \leq V(\tilde{x}_k) \ \forall \ V(\tilde{x}_k) > \epsilon^* \\
&V(\tilde{x}_{k+1}) \leq \epsilon^* \ \forall \ V(\tilde{x}_k) \leq \epsilon^* \\
&\bar{y}^{SP}_{i,k+j} = (1 - e^{\frac{-\Delta t}{\tau_i}}) y^{SP}_{i,k+j} + e^{\frac{-\Delta t}{\tau_i}} \bar{y}^{SP}_{i,k+j-1}, \text{ for } i = 1, ..., p \\
&|\tilde{y}_{k+j} - \bar{y}^{SP}_{k+j}| \leq \varepsilon, \text{ for } j = 1, ..., P, \text{ with } \bar{y}^{SP}_{i,k} = y_{i,k}
\end{aligned}
\tag{5}
$$

where $||\tau||$ is Euclidean norm of vector τ, p is the number of outputs, τ_i is time constant of the ith output. Further, $\bar{y}^{SP}_{i,k}$ is ith output set-point at sample time k based on the desired behavior, $y^{SP}_{i,k}$ is the set-point for ith output, $\hat{x}_k$ is the current estimation of state and y_k is the current output measurement.

Feasibility and closed-loop stability are next established. To this end, the next assumption simply states that the provided set point is indeed achievable in a first order fashion with practical stability (i.e., at the discrete sample times, the states are in a neighborhood of the equilibrium steady state after they get there). This assumption is similar to one of initial feasibility commonly employed in most existing formulations.

Assumption 1. Consider the constrained system of Eq.1-2 for an x_0 and w.l.o.g, that the origin is the equilibrium point of the nominal process. Then, there exist a Lyapunov function $V(\tilde{x}_k)$, $\tau \geq 0$, $u_j \in \mathcal{U}$, and $\Delta\tilde{u}_j \in \mathcal{U}_\delta$ for $j = 1, ..., \infty$, such that the constraints of Eq. 5 are satisfied.

The following theorem invokes Assumption 1 to guarantee that the proposed predictive controller achieves closed-loop stability while providing the desired closed-loop trajectory behavior in the 'best possible fashion'.

Theorem 1. Consider the constrained system of Eq.1-2 subject to Assumption 1 under the two tier MPC law of Eqs.4 & 5 or Eqs.4 & 7. The optimization problem of Eq. 4 is guaranteed to be feasible for all times, and $\lim_{j \to \inf} y_{k+j} = 0$, $\limsup_{j \to \inf} V(x_{k+j}) = \epsilon^*$.

Proof. Initial feasibility of the tier one optimization problem follows from Assumption 1, and since Tier 1 includes all the plant constraints, stability constraints and trajectory constraint of Tier 2, therefore, the optimization problem of Tier 2 will be feasible too. The solution from the previous time step, along with the assumption on practical stability guarantees that a feasible solution exists at subsequent time steps. The presence of the stability constraints in turn guarantee feasibility with the details of the proof following along similar lines as[26] and hence are not repeated here.

Remark 5. *In the formulation, the Euclidean norm is used in the objective function of the Tier 1. Note that if the practitioner chooses, a weighted norm for reflecting the relative importance of the controlled outputs could readily be utilized instead. The use of the weighted norm would not lead to additional tuning parameters, but simply enable the practitioner to specify a 'known'/desired relative importance of the outputs.*

Explicit Tuning Approach for an Underdamped Second Order Specification

As another illustration of the desired behavior, we consider a second order response, described (for each controlled variable) by:

$$\tau_i^2 \frac{\partial^2 \bar{y}_i^{SP}}{\partial t^2} + 2\zeta_i \tau_i \frac{\partial \bar{y}_i^{SP}}{\partial t} + \bar{y}_i^{SP} = y_i^{SP} \tag{6}$$

The solution of this ordinary differential equation with current measured output ($y_{i,0}$), initial derivative $\frac{\partial \bar{y}_i^{SP}}{\partial t}(0) = 0$ and y_i^{SP} as a step function, has the following form:

$$\bar{y}_{i,k}^{SP} = (y_i^{SP} - y_{i,0})(1 - e^{\frac{-\zeta k \Delta t}{\tau_i}}[cos(\frac{\sqrt{1-\zeta_i^2}}{\tau_i} k\Delta t) + \frac{\zeta_i}{\sqrt{1-\zeta_i{}^2}} sin(\frac{\sqrt{1-\zeta_i^2}}{\tau_i} k\Delta t)]) + y_{i,0} \tag{7}$$

where $0 \leq \zeta_i < 1$ is the damping factor. Thus the first tier formulated to achieved the best second order trajectory has the following form:

$$\min_{\tau,\zeta,\tilde{u}} ||\tau|| \tag{8a}$$

subject to: (8b)

$$\dot{\tilde{x}} = f(\tilde{x}, \tilde{u}) \tag{8c}$$

$$\tilde{y} = h(\tilde{x}, \tilde{u}) \tag{8d}$$

$$\tilde{u} \in \mathcal{U}, \qquad \Delta\tilde{u} \in \mathcal{U}_\delta, \qquad \tilde{x}(k) = \hat{x}_k \tag{8e}$$

$$V(\tilde{x}_{k+1}) \leq V(\tilde{x}_k) \; \forall \; V(\tilde{x}_k) > \epsilon^* \tag{8f}$$

$$V(\tilde{x}_{k+1}) \leq \epsilon^* \; \forall \; V(\tilde{x}_k) \leq \epsilon^* \tag{8g}$$

$$\bar{y}_{i,k}^{SP} = (y_i^{SP} - y_{i,0})(1 - e^{\frac{-\zeta k \Delta t}{\tau_i}}[cos(\frac{\sqrt{1-\zeta_i^2}}{\tau_i} k\Delta t) + \frac{\zeta_i}{\sqrt{1-\zeta_i{}^2}} sin(\frac{\sqrt{1-\zeta_i^2}}{\tau_i} k\Delta t)]) + y_{i,k} \tag{8h}$$

$$|\tilde{y}_{k+j} - \bar{y}_{k+j}^{SP}| \leq \varepsilon, \text{ for } j = 1, ..., P \tag{8i}$$

with (8j)

$$PO_{min} \leq PO \leq PO_{max} \tag{8k}$$

$$0 \leq \tau_i, \qquad 0 \leq \zeta_i < 1, \qquad \forall \; i = 1, \ldots, p \tag{8l}$$

where PO in the Eq. 8a is the percentage overshoot (PO) and PO_{min} and PO_{max} capture the ability to specify allowable ranges. This parameter can be computed as follows:

$$\begin{aligned} PO &= \begin{bmatrix} PO_1 \ldots PO_p \end{bmatrix}, \\ PO_i &= \frac{-\zeta_i \pi}{\sqrt{1-\zeta_i{}^2}}, \; \forall \; i = 1, \ldots, p \end{aligned} \tag{9}$$

Remark 6. *The focus in this work is different from economic model predictive controllers or two tier, integrated dynamic optimization and model predictive control problems with economic* *objectives*[27–30]. *It is not to come up with an economically optimal solution. It is to simply come up with a tool that allows the practitioner to directly prescribe the desired closed-loop behavior. Although economic model predictive controllers compute closed-loop trajectories that are guaranteed to be optimal with respect to the cost function, the closed-loop behavior of EMPC may not always be intuitive or admissible to the operator. To address this issue, the proposed two-tier performance specification MPC can be readily integrated with EMPC. In particular, the Tier 1 could also include a constraint that restricts the feasible region of the optimization to be within an allowable neighborhood of the optimal cost as computed by the EMPC. As such, the trajectory computed by the first layer of the performance specification MPC would simultaneously ensure economic optimality and be prescribed to be a good 'first-order' or 'second-order' like behavior.*

Formulations and Simulation Results Handling Specific Instances

In this section we show how various specific issues can be handled within the proposed framework, including implementation under output feedback problem and including rate constraints and uncertainty explicitly in the problem formulation.

Linear System under Output Feedback:

We first consider a the output feedback problem, and for the sake of illustration, consider a linear system of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{10}$$

with the system matrices given in Table 1. When implementing the formulation subject to output feedback, the formulation remains the same, except that the state estimates are used in computing the control action. For the purpose of the simulation, we utilize a Luenberger observer to generate state estimates (for nonlinear plants, nonlinear estimators such as moving horizon estimator can be used[31,32]). Thus state estimates $\hat{x}_k$ are given as follows:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - C\hat{x}_k) \tag{11}$$

where L is the observer gain and is computed using a pole placement method, and y_k is the vector of measured variables. The set point and desired behavior is specified for the measured outputs.

Note that the simulation example presented here is not the same linear system as in[22]. The key difference (beyond the system matrices being different), is that the example in the present manuscript is not a square system so the method proposed in[22] does not remain directly applicable. In particular, for the example in[22], the first order trajectory can be exactly followed and the solution is a unique input that can be computed analytically. In contrast, the input in the present example is non-unique and is computed using an optimization problem to follow the prescribed trajectory subject to an allowable threshold.

The simulation scenario comprises two step changes in the output variable set-points as indicated in Fig. 3, 4 and 5. The tuning parameters of the nominal MPC are varied through a trial and error exercise to achieve a reasonable closed-loop behavior and are reported in Table 1. The simulation results show that in both step changes, the proposed controller (with a prescribed desired first order and second order response) provides the prescribed closed-loop behavior as opposed to the nominal MPC (Eq. 3a). In particular, note that the nominal MPC results in significant overshoot. In principle, while it may have been possible to find the nominal MPC tuning parameters to give a similar behavior, there is simply no way to specify penalties on overshoot in the nominal MPC formulation. In contrast, the proposed approach enables specifying desired behavior explicitly in the control calculation. Note also that in this case, since the number of inputs is more than the number of outputs, each controller settles at a different steady state manipulated input value. The other key point to recognize is that proposed first and scened order trajectories CPU time required to solve the two-tired MPC optimization problem were only 75 and 95 percent more than nominal MPC, respectively, and could be further reduced via strategies where the first Tier optimization problem is solved infrequently.

Nonlinear System with Input Rate Constraints and Uncertainty:

We next illustrate the explicit handling of input rate constraints and uncertainty in the proposed approach using the nonlinear CSTR example also used in[22]. To enable a fair comparison, both the the nominal MPC and the proposed two Tier MPC are supplemented with an offset-free mechanism to handle plant-model mismatch.

The extended Kalman filter utilized to estimate the states takes the following form:

$$\begin{aligned}
\dot{\hat{x}}(t) &= f(\hat{x}(t), u(t)) + K(y(t) - h(\hat{x}(t))) \\
\dot{P}(t) &= F(t)P(t) + P(t)F(t)^T - K(t)H(t)P(t) + Q_K \\
K(t) &= P(t)H(t)^T R_K^{-1} \\
F(t) &= \frac{\partial f}{\partial x}|_{\hat{x}(t)} \\
H(t) &= \frac{\partial h}{\partial x}|_{\hat{x}(t)}
\end{aligned} \tag{12}$$

Where $\hat{x}(t)$ is the current state estimation, Q_K and R_K are state and measurement covariance matrices. The controller parameters are presented in Table 2.

The comparison of simulation results for second order and first order explicit trajectories and nominal MPC are presented in Fig. 6 and 7. The first and second order explicit trajectories for the performance specification controller are similar, and both the trajectories reach the set-point faster than the nominal MPC without any notable overshoot and with a smooth behavior. The implementation thus demonstrates the key point that the present formulation can be readily adapted to handle various forms of the desired characteristics (such as the additional requirement to handle rate of change of input constraints), and other practical considerations (such as handling uncertainty).

Remark 7. *One of the existing challenges with nonlinear MPC industrial implementation is the associated computational effort, where the difficulty is further compounded by the tuning effort to achieve a 'desirable' performance. By enabling prescription of the 'desirable' performance in the formulation itself (without increasing the computational complexity significantly), the proposed formulation is expected to alleviate the tuning issues, and thus make it easier to implement NMPC industrially. Note also that the proposed formulation does not comprise two entirely separate optimization problems because the second optimization problem (Tier II) is initialized with a feasible guess (provided by the solution from Tier I), thus limiting the increase in computational complexity.*

Application to a Reactor-Separator Plant

Finally we implement the proposed MPC formulation on a nonlinear reactor-separator process example with three unit operations. The plant includes two CSTRs and one flash tank separator. For details on the mathematical model and parameter values, see[33]. The manipulated variables are Q_1, Q_2, Q_3, F_{20}, and the nominal values of the plant are presented in Table. 3. The control objective is to track desired temperature of the three reactors. We assume that full state measurements are available, however, the desired behavior is only prescribed for some of the variables. The simulation results are presented in Fig. 8 and 9.

To provide a quantifiable comparison of the closed-loop performance in terms of the metric consistent with the prescribed behavior, the closed-loop trajectories of CSTR-Separator simulation example were fitted to a first order trajectory and time constants of these trajectories are reported in Table 4. It can be seen that the time constants of the proposed method are either the same or better than the nominal MPC. More importantly, the proposed MPC provides closed-loop trajectories without overshoot.

Note that some of the trajectories under the nominal MPC reach the set point faster compared to the first order or second order trajectories. The key is to recognize, however, that the objective with the control design was not to reach the set point fastest for one of the controlled variables, or all of the controlled variables, but rather was chosen to be the fastest first order behavior for all three controlled variables or the fastest second order response for all three controlled variables. From this standpoint, a faster (but not first order) response in one of the controlled variables with an overshoot in the other controlled variables would not qualify as a better (or to be more precise, close to the prescribed) closed loop behavior. Thus the simulation result demonstrate the significantly improved performance (adherence to the prescribed closed-loop behavior) under the proposed control design, and applicability to larger systems. Furthermore, the CPU time required to solve the two-tired MPC optimization problem were only 60 and 50 percent more than nominal MPC, respectively.

Conclusion

In this work, a novel MPC based approach is developed that allows specifying desired process behavior, subject to nonlinearity, constraints and uncertainty. The proposed approach is described and compared against traditional nominal MPC and shown to be able to provide desired closed-loop behavior through implementation on three examples that include a non-square linear model subject to output feedback, a CSTR reactor subject to input constraints and uncertainty and a reactor separator process example.

Acknowledgment

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List of Tables

List of Figures

Table 1: Parameters for the linear system

Variable	Value
A	$\begin{bmatrix} -6 & -1 & 1 \\ 3 & -5 & 3 \\ 4 & 1 & -2 \end{bmatrix}$
B	$\begin{bmatrix} -2 & -1 & -5 \\ 4 & 2 & 7 \\ -2 & -3 & 3 \end{bmatrix}$
C	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
$x(0)$	$\begin{bmatrix} -1.73 & 3.84 & 3.46 \end{bmatrix}$
u_{min}	$\begin{bmatrix} -100 & -110 & -120 \end{bmatrix}$
u_{max}	$\begin{bmatrix} 150 & 140 & 130 \end{bmatrix}$
PO_{min}	$\begin{bmatrix} 1.00 \times 10^{-2} & 1.00 \times 10^{-2} & 1.00 \times 10^{-2} \end{bmatrix}$
PO_{max}	$\begin{bmatrix} 0.25 & 0.25 & 0.25 \end{bmatrix}$
Δt(s)	5×10^{-2}
Q_y	diag $\{1, 1, 1\}$
τ_{max}(s)	1
ε	$0.1 \times \hat{x}_{Sp,i}$
$Q_{y,MPC}$	diag $\{1, 1, 1\}$
$R_{\Delta u,MPC}$	diag $\{100, 10, 100\}$
P	8
ϵ^*	1
$V(x)$	$(x - x_{sp})^T(x - x_{sp})$

Table 2: List of controllers parameters for the CSTR reactor

Variable	Value
Δt	$0.2 min$
Q_x	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$Q_{x,MPC}$	$10 \times diag([1/C_{A,s}, 1/T_{R,S}])$
$R_{\Delta u,MPC}$	$diag([1/C_{A0,max}, 1/Q_{max}])$
Q_K	$diag([10^3, 10^3])$
R_K	$diag([10^{-3}, 10^{-3}])$
τ_{min}	0
τ_{max}	5
ε_i	$10^{-3} \times x_{i,Sp}$
Δu_{min}	$\begin{bmatrix} -0.1 & -200 \end{bmatrix}$
Δu_{max}	$\begin{bmatrix} 0.1 & 200 \end{bmatrix}$
ϵ^*	1
$V(x)$	$(x - x_{sp})^T (x - x_{sp})$

Table 3: CSTR-Separator controller parameters

Parameter	Value
Δt	$25s$
u_{min}	$\begin{bmatrix} 2.9\times10^5 & 1\times10^5 & 2.9\times10^5 & 0.504 \end{bmatrix}$
u_{max}	$\begin{bmatrix} 5.8\times10^6 & 2\times10^6 & 5.8\times10^6 & 10.8 \end{bmatrix}$
PO_{min}	0
PO_{max}	0.5
$\Delta t(s)$	40
Q_y	$\text{diag}([1/T_{1,s}, 1/T_{2,s}, 1/T_{3,s}])$
τ_{max}(s)	100
τ_{min}(s)	10
ε	$\text{diag}([10^{-3}T_{1,s}, 10^{-3}T_{2,s}, 10^{-3}T_{3,s}])$
$Q_{y,MPC}$	$500\times\text{diag}([1/T_{1,s}, 1/T_{2,s}, 1/T_{3,s}])$
$R_{\Delta u,MPC}$	$\text{diag}([1/Q_{1,s}, 1/Q_{2,s}, 1/Q_{3,s}, 1/F_{20,s}])$
P	15
ϵ^*	1
$V(x)$	$(x-x_{sp})^T(x-x_{sp})$

Table 4: Closed-loop time constants (s) for the CSTR-Separator simulation example

Controller	τ_1	τ_2	τ_3
Proposed PSMPC	43.65	13.03	65.36
NMPC	84.64	48.67	94.60

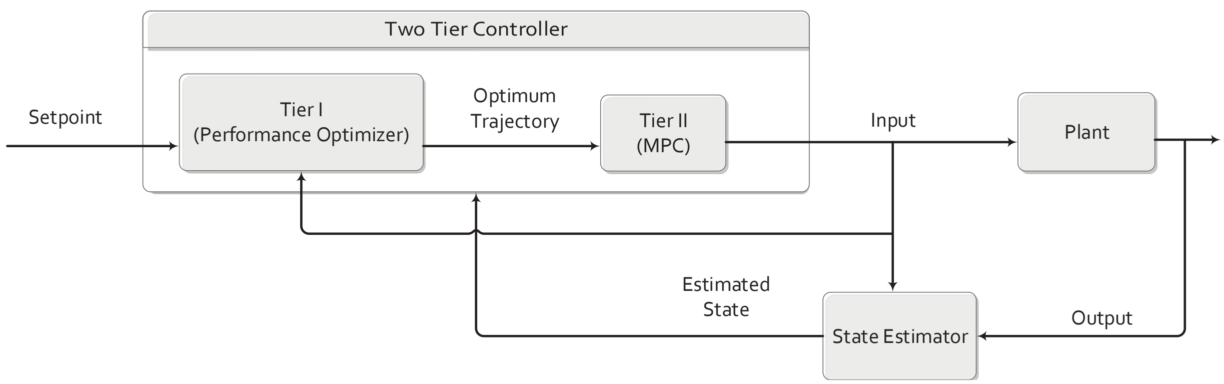

Figure 1: Two-tier control strategy

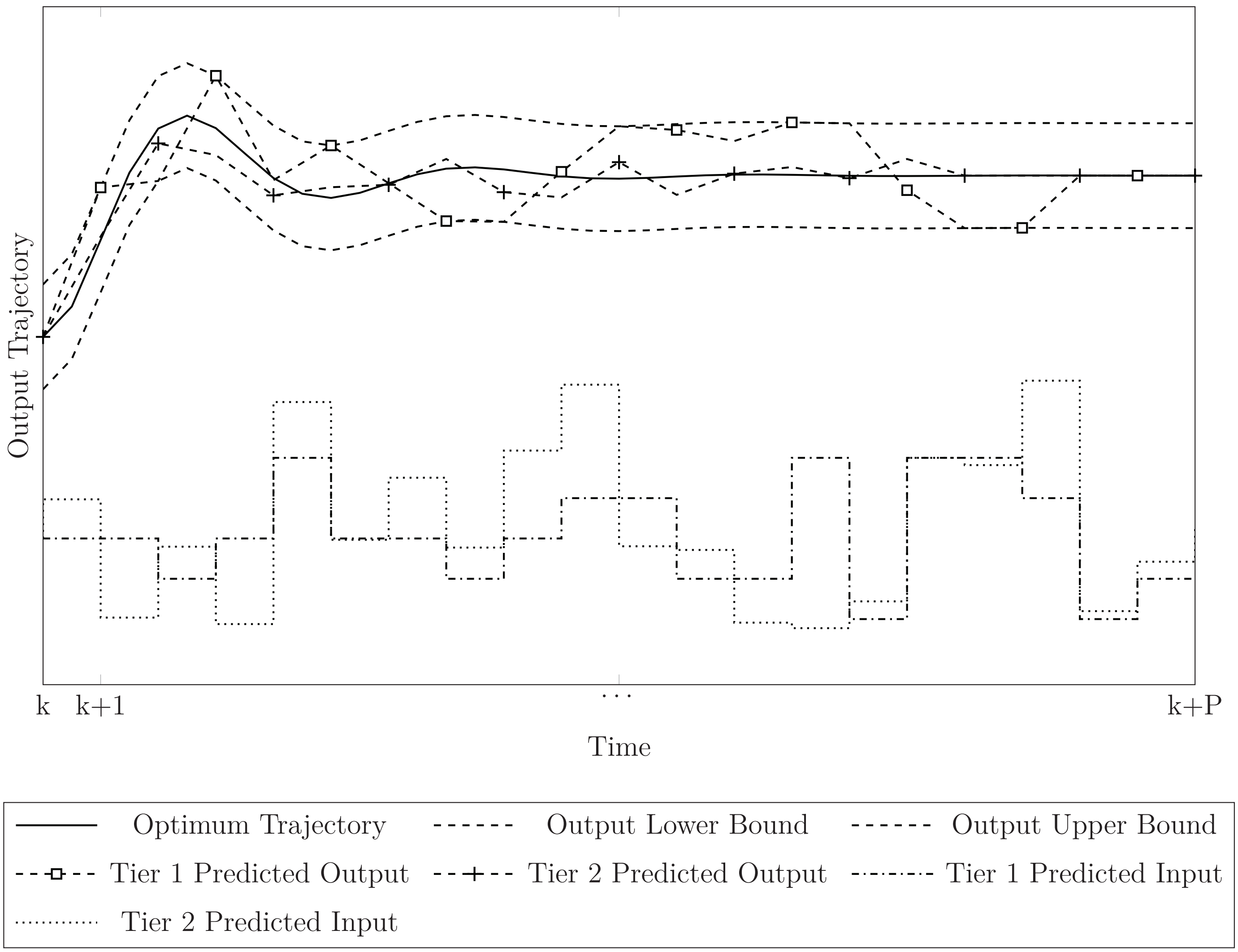

Figure 2: Two-tier MPC scheme

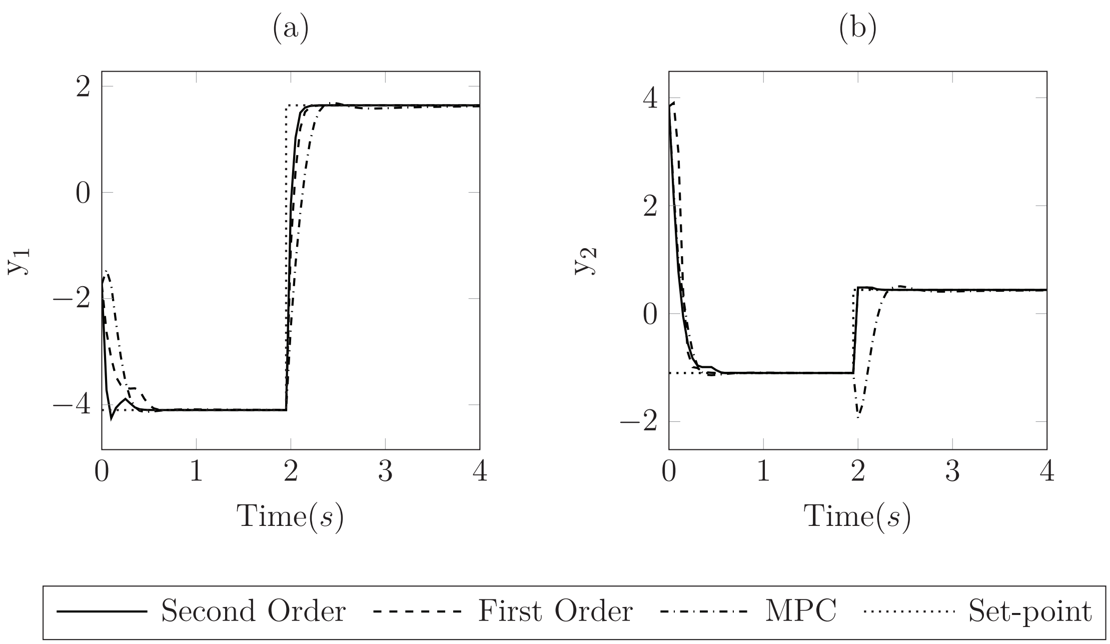

Figure 3: Comparison of the proposed MPC approach and Nominal MPC (output variables)

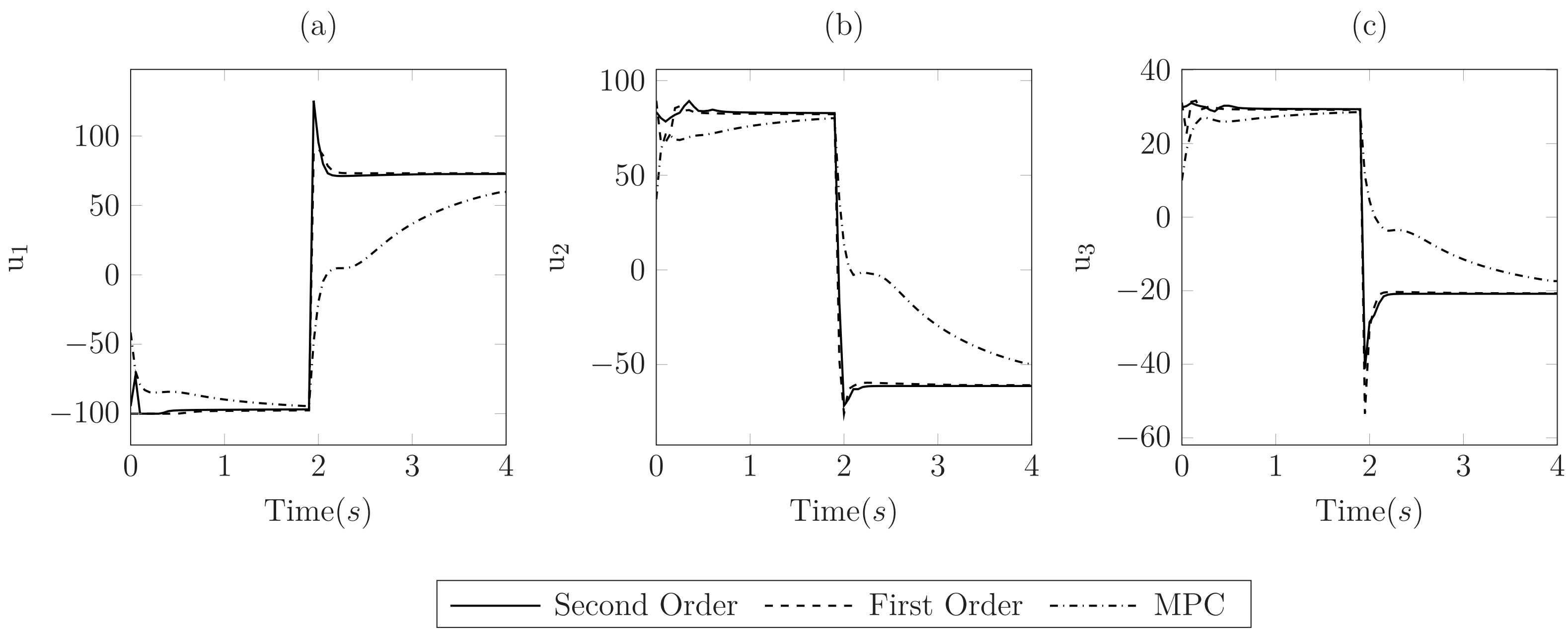

Figure 4: Comparison of the proposed MPC approach and Nominal MPC (input variables)

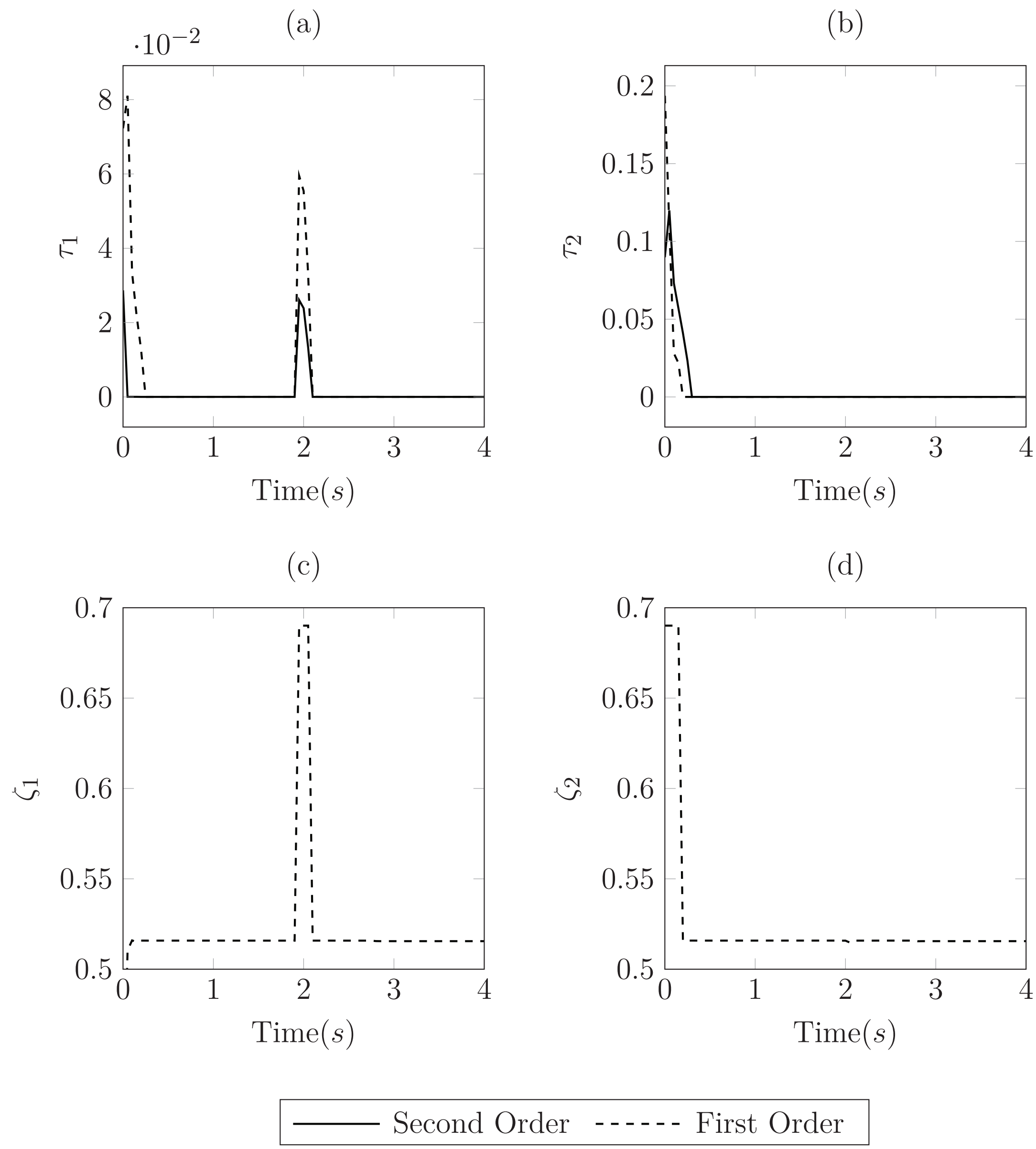

Figure 5: Trajectory parameters

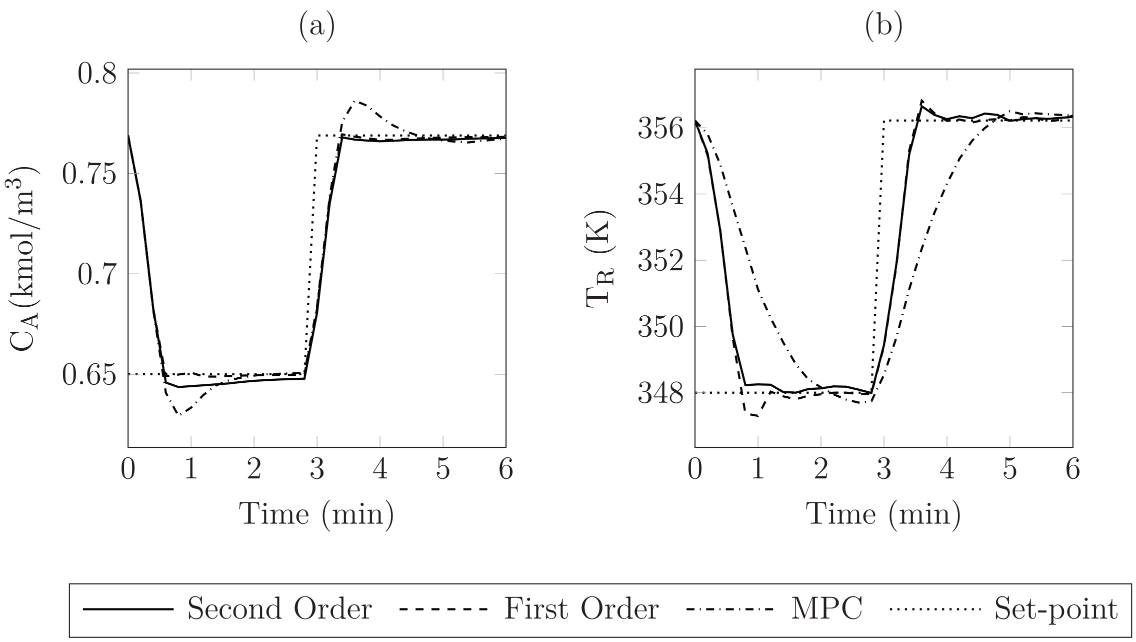

Figure 6: Illustrating the proposed approach with input rate constraints and uncertainty on a CSTR example (controlled variables)

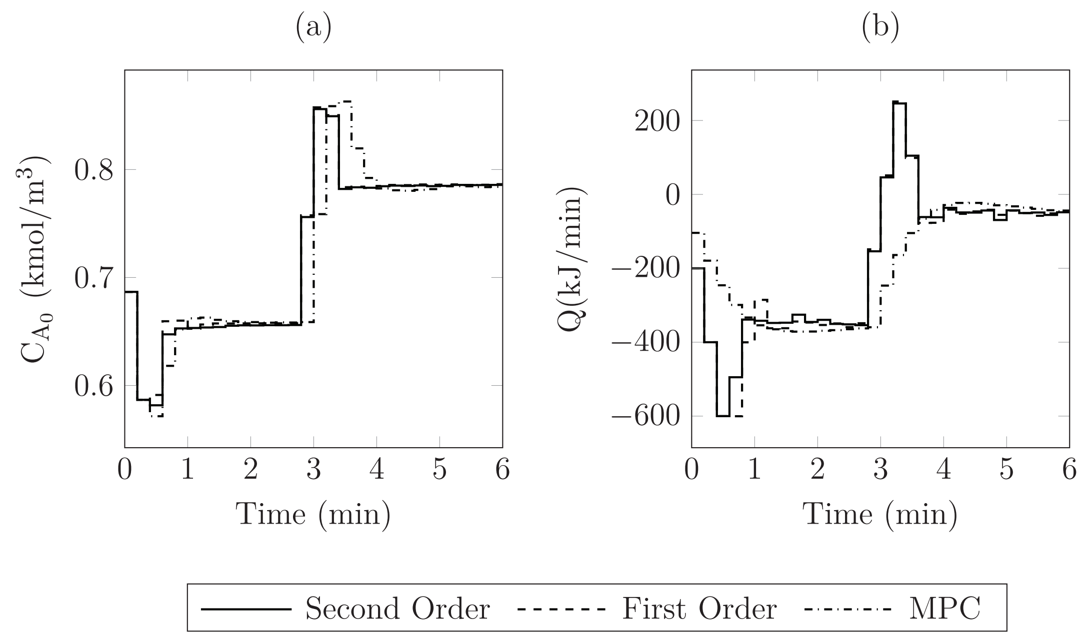

Figure 7: Illustrating the proposed approach with input rate constraints and uncertainty on a CSTR example (manipulating variables)

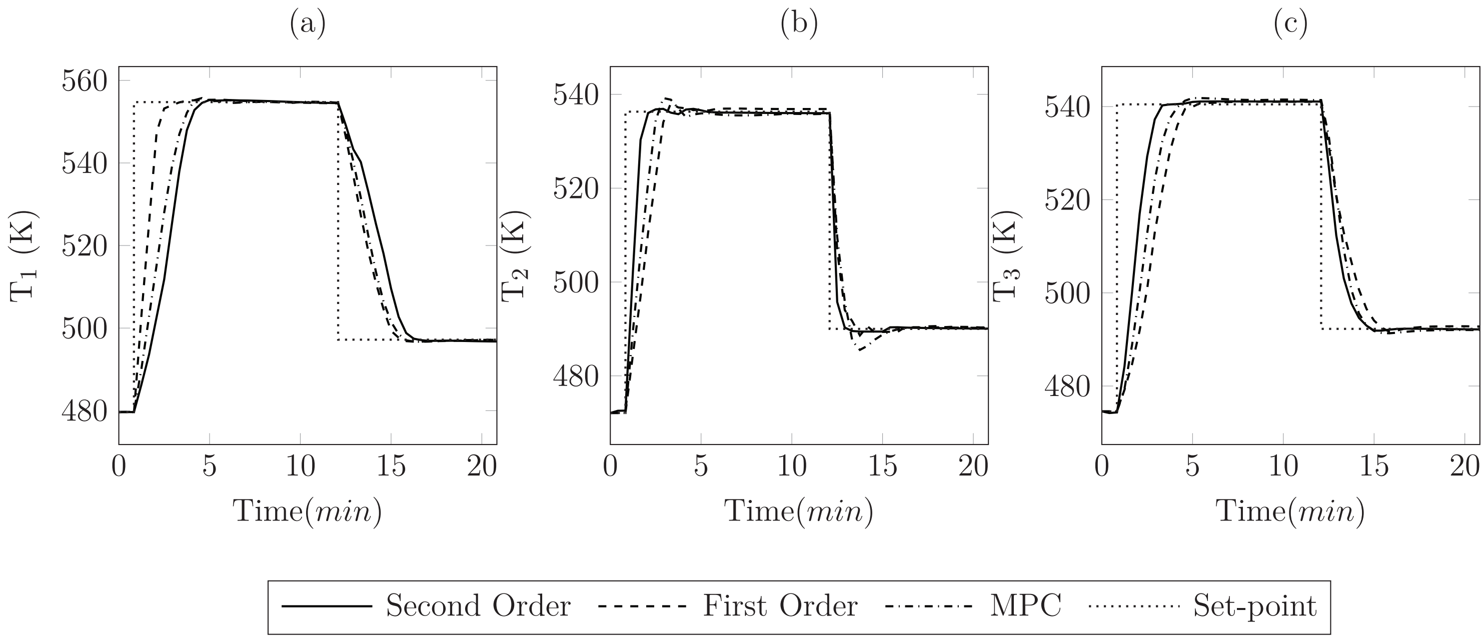

Figure 8: Comparison of proposed MPC approach and nominal MPC for CSTR-Separator plant outputs

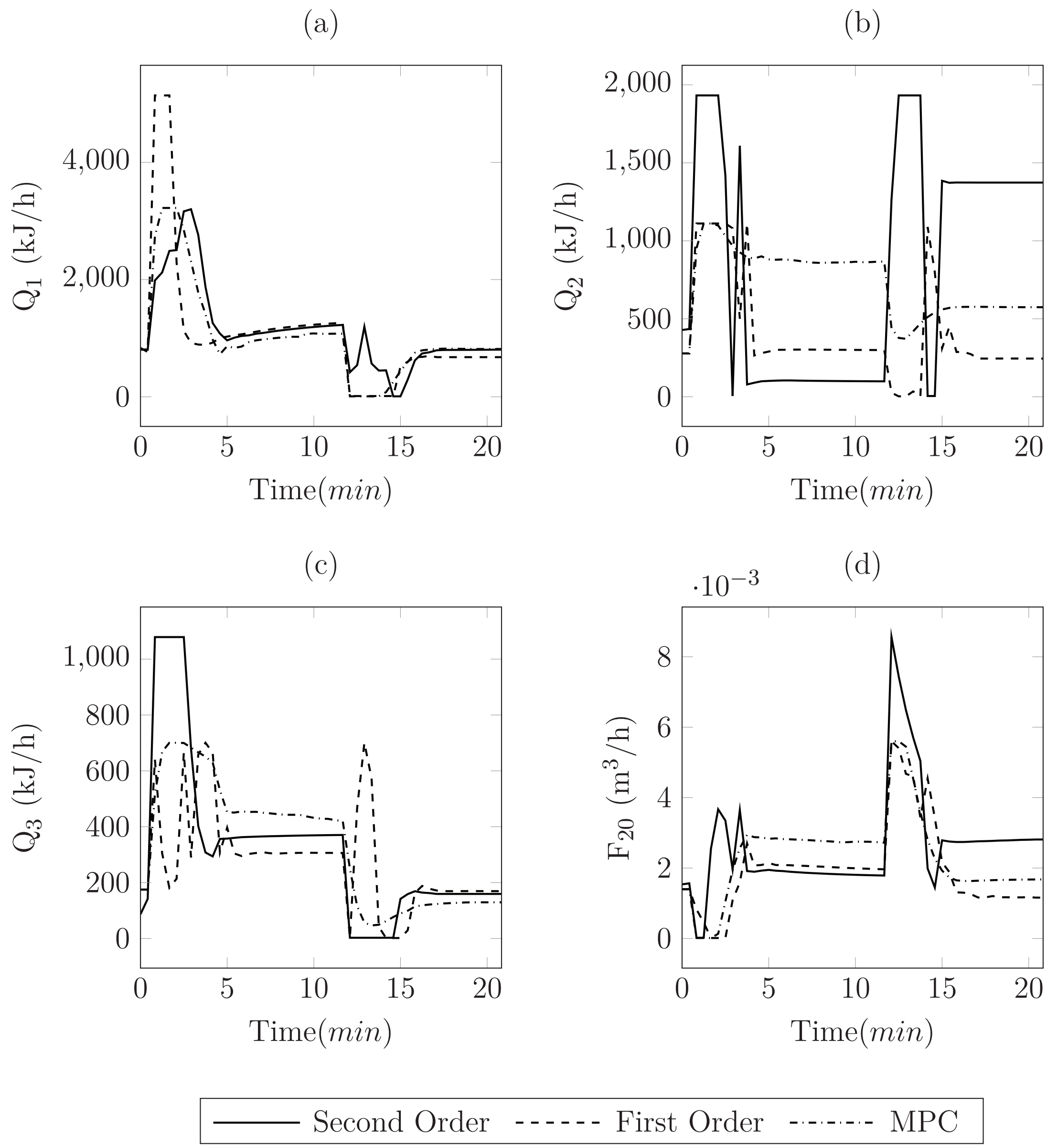

Figure 9: Comparison of proposed MPC approach and nominal MPC for CSTR-Separator plant inputs