# Model Predictive Control with Closed-loop Re-Identification

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## Abstract

In this work we address the problem of handling plant-model mismatch by designing a subspace identification based MPC framework that includes model monitoring and closed-loop identification components. In contrast to performance monitoring based approaches, the validity of the underlying model is monitored by proposing two indexes that compare model predictions with measured past output. In the event that the model monitoring threshold is breached, a new model is identified using an adapted closed-loop subspace identification method. To retain the knowledge of the nominal system dynamics, the proposed approach uses the past training data and current input, output and set-point as the training data for re-identification. A model validity mechanism then checks if the new model predictions are better than the existing model, and if they are then the new model is utilized within the MPC. The effectiveness of the proposed method is illustrated through simulations on a nonlinear polymerization reactor.

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# 1 Introduction

The operation of chemical plants faces numerous challenges such as inherent nonlinearity, complex variable interactions and process constraints. The most common control method that can handle these challenges is model predictive control (MPC). In several industrial applications of MPC, a linear model is used, in part due to the simplicity of developing linear models and in part due to the computational ease with using linear models. In order to handle the resultant plant-model mismatch, robust MPCs and offset-free MPC approaches have been developed.

In robust MPC approaches, the control action is computed to handle the worst case effect of the uncertainty [1, 2]. These include Lyapunov-based MPC which enables explicit characterization of the region from where stability of the closed loop system under MPC controller is achievable in the presence of constraints and uncertainty [3]. In another approach, the so-called offset-free MPC, the nominal model is integrated with augmented disturbance states to eliminate offset in set-point tracking [4, 5].

While these approaches are often able to eliminate uncertainty at steady state operation, the closed-loop performance certainly stands to improve if a better model is utilized in the control

design. To determine if the closed-loop system is behaving as expected, existing approaches have focused on the area of control performance monitoring. In this direction, numerous MPC performance assessment methods are proposed to monitor the closed-loop performance by comparing the controller with a benchmark [6, 7, 8]. Most of these methods focus on tuning of the controller parameters to remedy the performance degradation. In model predictive approaches, where the control action is more directly dependent on the underlying model, there exists the possibility of explicitly monitoring directly addressing model validity which is the key sources of performance degradation.

There exist some results on MPC with re-identification (IMPC) where model validity is accounted for by requiring excitation constraints to ensure that the model parameters remain identifiable [9]. In this approach, identification is performed at every time step. Furthermore, the approach requires finding the right trade-off between the inevitable performance deterioration (due to excitation conditions) and the possibility of loss of model validity. In [10] MPC Relevant Identification (MRI) was extended to Enhanced Multi-step Prediction Error Method (EMPEM). In [11] single input single output IMPC was extended to improve performance of the output regulation by not disturbing the plant when the model is deemed to have an acceptable precision. Acceptable precision is quantified through bounds on the variance of parameter estimates and parameter convergence rate in the MPC cost function.

In particular, in this method, autoregressive models with exogenous inputs are used and recursive weighted least-squares algorithm is utilized to estimate model parameters. In order to solve the trade-off between control performance and persistence of excitation, in a recent contribution [12, 13] maximizing the MPC objective function is used instead of minimization to maximize signal variance and address the feasibility and stability of MPC with re-identification. In another direction, based on the analysis of partial correlations between model residuals and the manipulated variables, location of model mismatch is detected and model updated, if necessary. These methods utilize transfer function models [14, 15] and 'refresh' the model entirely. Thus, training data is not retained in the new model identification, and these methods are well suited to address situations where the system is changed significantly and previous data are not at all representative of the plant in question. In situations where plant model mismatch arises due to change in operating condition (with the possibility of reverting back to the nominal plant operation), it becomes useful to merge old and new plant data in the re-identification step.

Motivated by the above considerations, in this work we address the problem of plant model mismatch by developing a model monitoring and closed-loop re-identification based MPC design. The rest of the manuscript is organized as follows: First, the general mathematical description for the systems considered in this work, and a representative formulation for linear model predictive control are presented. Then the proposed approach for closed-loop re-identification of plant is

explained. The efficacy of the proposed method is illustrated through formulations and implementations for a nonlinear polymerization continuous stirred-tank reactor (CSTR) with input rate of change constraints and measurement noise. Finally, concluding remarks are presented.

# 2 Preliminaries

In this section, a brief description of the general class of processes that are considered in this study is provided. Then, the orthogonal projection based subspace identification and a representative MPC formulation is presented.

## 2.1 Problem Statement

In this work, we consider a general multi-input multi-output (MIMO) controllable systems. For notational convenience, consider that $u \in \mathbb{R}^{n_u}$ denotes the vector of constrained control (manipulated) input variables, taking values in a nonempty convex subset $\mathcal{U} \subset \mathbb{R}^{n_u}$, where $\mathcal{U} = \{u \in \mathbb{R}^{n_u} \mid u_{\min} \leq u \leq u_{\max}\}$, $u_{\min} \in \mathbb{R}^{n_u}$ and $u_{\max} \in \mathbb{R}^{n_u}$ denote the lower and upper bounds of the input variables, and $y \in \mathbb{R}^{n_y}$ denotes the vector of measured output variables. In keeping with the discrete implementation of MPC, $u$ is piecewise constant and defined over an arbitrary sampling instance $k$ as:

$$u(t) = u(k), \;\; k\Delta t \leq t < (k+1)\Delta t$$

where $\Delta t$ is the sampling time and $x_k$ and $y_k$ denote state and output at the $k$th sample time.

## Subspace Identification

In this section the conventional state space subspace identification method is reviewed [16, 17, 18]. In the subspace identification approach, the goal is to determine the system matrices for a discrete linear time invariant model of the following form:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + Du_k + v_k \tag{2}$$

where $x \in \mathbb{R}^{n_x}$ denotes the vector of state variables, $y \in \mathbb{R}^{n_y}$ denotes the vector of measured outputs, $w \in \mathbb{R}^{n_x}$ and $v \in \mathbb{R}^{n_y}$ are zero mean, white vectors of process noise and measurement noise with the following covariance matrices:

$$E[\begin{pmatrix} w_i \\ v_j \end{pmatrix} \begin{pmatrix} w_i^T & v_j^T \end{pmatrix}] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{ij} \tag{3}$$

where $Q \in \mathbb{R}^{n_x \times n_x}$, $S \in \mathbb{R}^{n_x \times n_y}$ and $R \in \mathbb{R}^{n_y \times n_y}$ are covariance matrices, and, $\delta_{ij}$ is the Kronecker delta function. To identify the system matrices, Hankel matrices are first constructed by stacking the process variables as follows:

$$U_p = U_{1|i} = \begin{bmatrix} u_1 & u_2 & \dots & u_j \\ u_2 & u_3 & \dots & u_{j+1} \\ \dots & \dots & \dots & \dots \\ u_i & u_{i+1} & \dots & u_{i+j-1} \end{bmatrix} \tag{4}$$

$$U_p = U_{i+1|2i} = \begin{bmatrix} u_{i+1} & u_{i+2} & \dots & u_{i+j} \\ u_{i+2} & u_{i+3} & \dots & u_{i+j+1} \\ \dots & \dots & \dots & \dots \\ u_{2i} & u_{2i+1} & \dots & u_{2i+j-1} \end{bmatrix} \tag{5}$$

where $U_p$ and $U_p$ denote the past and future input Hankel matrices. $i$ is a user-specified parameter that limits the order of the system ($n$) (which in itself is a user-specified parameter). Similar block-Hankel matrices are made for output, process and measurement noises $Y_p, Y_f, V_p, V_f \in \mathbb{R}^{in_y \times j}$ and $W_p, W_f \in \mathbb{R}^{in_x \times j}$ are defined in the similar way. The state sequences are defined as follows:

$$X_p = \begin{bmatrix} x_1 & x_2 & \dots & x_j \end{bmatrix} \tag{6}$$

$$X_f = \begin{bmatrix} x_{i+1} & x_{i+2} & \dots & x_{i+j} \end{bmatrix} \tag{7}$$

furthermore with:

$$\Psi_p = \begin{bmatrix} Y_p \\ U_p \end{bmatrix} \tag{8}$$

$$\Psi_f = \begin{bmatrix} Y_f \\ U_f \end{bmatrix} \tag{9}$$

The orthogonal projection of row space of matrix $A$ onto row space of matrix $B$, $(A/B)$ is defined as:

$$A/B = AB^{\dagger}B \tag{10}$$

where the superscript † stands for pseudo-inverse. By recursive substitution into the state space model equations Eqs. (1,2), it is straightforward to show:

$$Y_f = \Gamma_i X_f + \Phi_i^d U_f + \Phi_i^s W_f + V_f \tag{11}$$

$$Y_p = \Gamma_i X_p + \Phi_i^d U_p + \Phi_i^s W_p + V_p \tag{12}$$

$$X_f = A^i X_p + \Delta_i^d U_p + \Delta_i^s W_p \tag{13}$$

where:

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{bmatrix} \tag{14}$$

$$\Phi_i^d = \begin{bmatrix} D & 0 & 0 & \dots & 0 \\ CB & D & 0 & \dots & 0 \\ CAB & CB & D & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \dots & D \end{bmatrix} \tag{15}$$

$$\Phi_i^s = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ C & 0 & 0 & \dots & 0 & 0 \\ CA & C & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 & 0 \\ CA^{i-2} & CA^{i-3} & CA^{i-4} & \dots & C & 0 \end{bmatrix} \tag{16}$$

$$\Delta_i^d = \begin{bmatrix} A^{i-1}B & A^{i-2}B & \dots & AB & B \end{bmatrix} \tag{17}$$

$$\Delta_i^s = \begin{bmatrix} A^{i-1} & A^{i-2} & \dots & A & I \end{bmatrix} \tag{18}$$

Eq. (11) can be rewritten in the following form to have the input and output data at the LHS of the equation([19]):

$$\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \begin{bmatrix} Y_f \\ U_f \end{bmatrix} = \Gamma_i X_f + \Phi_i^s W_f + V_f \tag{19}$$

By orthogonal projecting of Eq. (19) onto $\Psi_p$:

$$\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f / \Psi_p = \Gamma_i X_f / \Psi_p + \Phi_i^s W_f / \Psi_p + V_f / \Psi_p \tag{20}$$

The last two terms in Eq. 20 are orthogonal projection of the future noise onto the row space of $\Psi_p$, and since the noise terms are independent, these two term are equal to zero. Thus Eq. (20) is

simplified as follows:

$$\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f/\Psi_p = \Gamma_i X_f/\Psi_p \tag{21}$$

Equation (21) indicates that the column space of $\Gamma$ is equal to column space of $\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f/\Psi_p$, and the row space of $X_f/\Psi_p$ is the same as the row space of $\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f/\Psi_p$. This equation can be solved using singular value decomposition (SVD), and the system matrices can be calculated from the results. This is the essence of the open-loop subspace identification method which assumes that future inputs are independent of the future disturbances. Note that this assumption dose not hold for data under closed-loop control, and can result in biased results [20]. To deal with this, the subspace identification approach was adapted by utilizing a new variable, denoted as an instrument variable, as part of the identification procedure. The new instrument variable, that satisfies the independence requirement, is used to project both side of the Equation 20 and the result is used to determine LTI model matrices. In existing results, the innovation form of the LTI model is used, as follows:

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{22}$$

$$y_k = Cx_k + Du_k + e_k \tag{23}$$

where $e_k$ is the innovation term, and, $K$ is filter gain. In these methods, after determining the $A$ matrix, $B$ and $K$ are estimated using least squares. In contrast to the existing results, in this

work we first estimate the noise terms, then calculate noise covariance matrices and based on these covariances the observer gain is calculated.

The system identification procedures require input signal to be quasi-stationary and persistently exciting of order $2i$ [21, 22]. In the system identification step, only the observable part of a system is identified therefore the LTI model is always observable. Also note that, the order of the LTI system $n$ is selected in a way that the identified system is controllable and the prediction of validation data is acceptable.

## 2.2 Model Predictive Control

The MPC formulation uses the identified model to determine the optimum control action. In order to handle plant-model mismatch, often an offset-free mechanism is utilized. Thus the identified model is augmented with integrating disturbances, $d$ as follows.

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \tag{24}$$

$$y_k = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + Du_k \tag{25}$$

where $d \in \mathbb{R}^{n_d}$ is the vector of augmented disturbances, and $B_d$ and $C_d$ are matrices with appropriate dimensions. The MPC computations require estimation of the subspace and disturbance states. One way to estimated these is to use a Luenberger observer as follows:

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{d}_{k+1} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k - L(y_k - \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} - Du_k) \tag{26}$$

where $L$ is predictor gain matrix.

In the Offset-free MPC at each sample time $l$, the control action is computed as follows:

$$\min_{\tilde{u}_k,\ldots,\tilde{u}_{k+P}} \sum_{j=1}^{P} ||\tilde{y}_{k+j} - y_{k+j}^{\mathrm{SP}}||_{Q_y}^2 + ||\tilde{u}_{k+j} - \tilde{u}_{k+j-1}||_{R_{du}}^2$$

subject to:

$$\begin{aligned} \tilde{x}_{k+1} &= A\tilde{x}_k + B_d\tilde{d}_k + B\tilde{u}_k \\ \tilde{d}_{k+1} &= \tilde{d}_k \\ \tilde{y}_k &= C\tilde{x}_k + C\tilde{d}_k + D\tilde{u}_k \\ \tilde{u} &\in \mathcal{U}, \qquad \Delta\tilde{u} \in \mathcal{U}_\delta, \qquad \tilde{x}(k) = \hat{x}_l \end{aligned} \tag{27}$$

where $P$ denotes the prediction horizon, $y_k^{\mathrm{SP}}$ is the desired output (asymptotically constant reference signal), and, $\tilde{y}_k$ is the predicted output trajectory at the time $k\Delta t$. $Q_y \in R^{n\times n}$ is a positive definite matrix, and, $R_{du}$ is a positive semi-definite matrix, and they are chosen so the nominal closed-loop system is stable. For the conditions required to ensure offset free tracking, see, e.g, [23].

*Remark* 1. Note that there are different versions of the offset-free MPC approach, including those where stability constraints are utilized. We present here a generic offset-free MPC approach to simply illustrate our model monitoring and re-identification approach.

# 3 Model Monitoring based MPC Implementation

In this section, we present the model monitoring based MPC implementation. To this end, we first describe the model monitoring approach, followed by the trigger for re-identification. Subsequently, a variation of the closed-loop subspace identification method is presented.

## 3.1 Model Monitoring and Re-Identification Trigger

An MPC computes the control action by minimizing an objective function along a prediction horizon, by predicting the output trajectory using the current state of the system, and candidate input moves. The validity of the process model is therefore critical to, although not the only factor in achieving good closed-loop performance. Most of the existing approaches focus on control performance monitoring. In the present manuscript, we instead directly focus on the model prediction error as one of the root causes of performance degradation. In this section, we describe an approach that monitors the health of the process model, and propose a couple of triggers for re-identification.

The key idea behind model monitoring is to compare model predictions with observed behavior. Before we present the details of the proposed method, let us consider other possibilities that will not allow a model validation online. The first of these recognitions is that at any point in time, predicting ahead (either as part of the MPC or otherwise), will not provide any information about model validity, because future process variables are simply not yet available. The only recourse thus

is to 'go back', and predict ahead up to the current time. With this also several possibilities exist. The first depends on the nature of the model. If the model being utilized in the MPC is a nominal model (without augmented states), and state measurements are available, then starting from the measured states in the past, and utilizing the known input values, the predictions can be computed, and compared with process measurements. Furthermore, even in that case, the trajectory computed by the MPC at the past point in time can not be utilized as the basis for comparison, because the actual input trajectory implemented on the plant will likely be different due to the feedback and receding horizon implementation of MPC. Finally, when a subspace model is being utilized (along with augmented disturbance states), then further care should be taken to enable an appropriate model validity comparison, primarily by ensuring that the best known values of the state estimates (known at the point in time in the past) are utilized instead of recomputing those (as with moving horizon estimation approaches). See Remark 2 for further discussion on this.

A schematic is presented in Fig. 1 to illustrate the monitoring approach. In particular, model monitoring at a sample time $k$ is achieved by going back to the sample time $k-P$, where $P$ is the chosen 'lookback' horizon and using the state estimate generated by the state estimator at time step $k-P$ ($\hat{x}_{k-P}$) and inputs from $k-P$ to $k-1$ to predict the output trajectory from $\bar{y}_{k-P}$ to $\bar{y}_{k-1}$. Then the predicted past output trajectory is compared with the measured output of plant and based on this comparison, the model accuracy is evaluated.

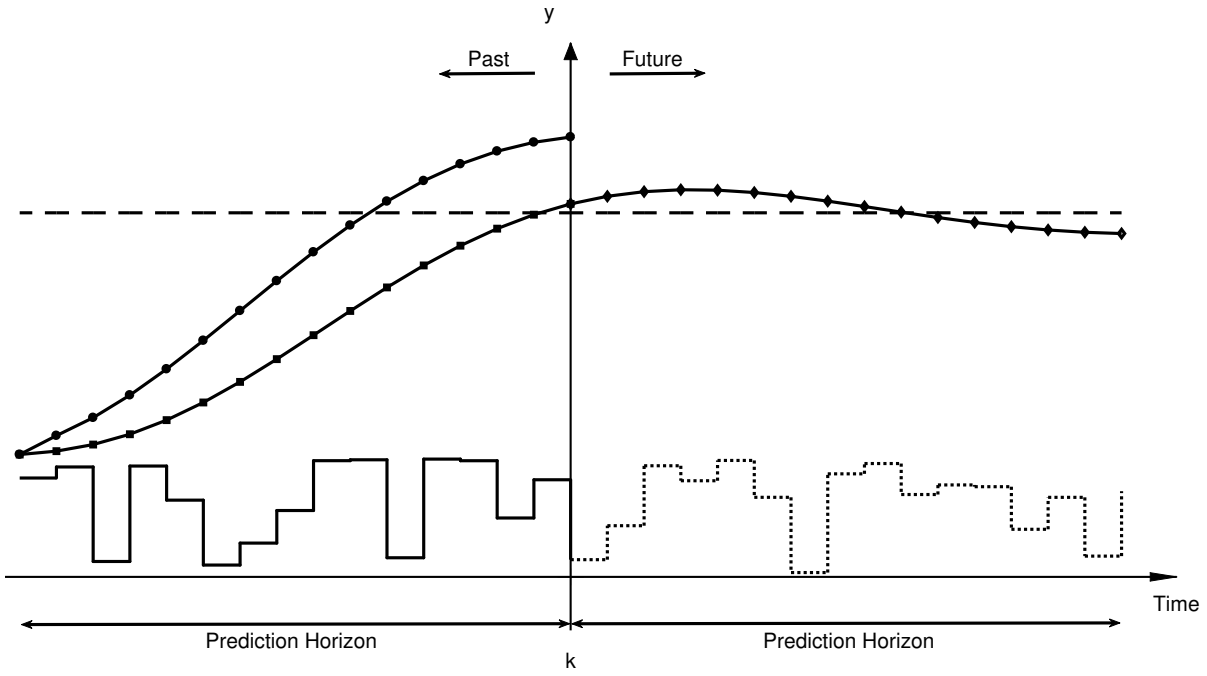


Figure 1: MPC with model prediction monitoring scheme([Measured Output:−■−], [Past Predicted Output:−●−], [Future Predicted Output:−◆−], [Setpoint:−−], [Past Input:−], [Future Predicted Input:..])

In the schematic in Figure 1, the line (−●−) is the predicted output trajectory that is calculated using the augmented identified LTI model with estimated state at the sample time $k-P$ (stored at each sample time for the purpose of model monitoring) and the implemented input from $k-P$ to $k-1$ (the continuous line(−)). The measured outputs of the system are shown by the line [−■−]. In the deterministic case with no model mismatch, the measured output and the predicted output by the MPC model would be exactly the same. Due to model mismatch, these two lines would be different. The dashed line(−) is the set-point. The doted line (..) is the future predicted input and the line (−◆−) is the future predicted output, calculated by the MPC at the current time step.

Note, that the MPC calculations at the current time step are not required for the model monitoring at the current time step. Finally, the model monitoring computations do not require the solution of any optimization problem.

In order to evaluate model accuracy at each sample time $l$, the updated state $x_{l-P}$ which was stored after calculation by state estimator at sample time $l-P$ and implemented input trajectory from $u_{l-P}$ to $u_{l-1}$ are utilized as follows:

$$\begin{aligned} \bar{y}_j &= C\bar{x}_j + Du_j \\ \bar{x}_{j+1} &= A\bar{x}_j + Bu_j \\ \text{where } j &= l-P, \ldots, l-1 \\ \bar{x}_l &= \hat{x}_{l-P} \end{aligned} \tag{28}$$

where $\bar{y}$ and $\bar{x}$ are predicted output and state. Having obtained the predictions for the process outputs, more than one indexes are possible to be formulated to represent model accuracy. Here, two illustrative indexes are proposed to evaluate MPC model prediction:

$$\begin{aligned} MPE_k &= \frac{1}{P}\sum_{j=k-P}^{k-1}(|y_j - \bar{y}_j|) \\ EPPE_k &= |y_{k-1} - \bar{y}_{k-1}| \end{aligned} \tag{29}$$

where $MPE_k \in R^{n_y}$ is the vector of mean prediction error, which is the mean value of prediction error along the past horizon at sample time $k$, $EPPE_k \in R^{n_y}$ is the vector of end-point prediction error at sample time $k$, and, $\bar{y}$ is the corresponding output prediction.

The re-identification triggers when the MPC model performance index exceeds a specific thresh-

old. This threshold can be set using training/validation data of the initial model by using Eq. (29) with the data. The index is calculated at each sample time and compared with the threshold to trigger re-identification. For re-identification, the recent data needs to be appropriately augmented with the past data to obtain the model. The details are presented in the next section.

After re-identification, a new observer gain is calculated. Then the new observer will utilize recent data to estimate current state to have the most up to date state to predict the plant behavior in MPC.

*Remark* 2. Note that the computation of the past output prediction is different from moving horizon estimation (MHE) method, which estimates past state using optimization techniques. The MHE approach can not be utilized to check for model applicability, because in the MHE approach, the state values at the past point in time are computed (the state values are one of the decision variables in the optimization problem), and done in a way that minimizes the error between the predictions and observed variables. While it is an excellent state estimation technique, the computation of the state estimate defeats the model monitoring objective. In contrast, in the proposed approach, the state estimates generated by the state estimator is used to focus on the model validity aspect.

## 3.2 Closed-Loop Identification

Having formulated the model monitoring and triggering approach, the next step is the utilization of an appropriate identification method that can handle the nature of data availability. As mentioned

earlier, most of the open-loop identification methods are based on the assumption that input and process disturbances are not correlated [24, 25, 26]. Under feedback control, the future input is correlated with noise signals, which may result in biased estimation of the model in the standard open-loop identification methods [22].

The following short-hand notation is used in closed-loop method:

$$\Psi_{pr} = \begin{bmatrix} R_f \\ \Psi_p \end{bmatrix} \tag{30}$$

where $R_f$ is data-Hankel matrix of set-point. Huang et al. in [20] showed that by using $\Psi_{pr}$ as an instrument subspace variable, the bias error of open-loop methods for closed-loop data can be avoided. Therefore by projecting Eq. (19) onto $\Psi_{pr}$:

$$\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f / \Psi_{pr} = \Gamma_i X_f / \Psi_{pr} + \Phi_i^s W_f / \Psi_{pr} + V_f / \Psi_{pr} \tag{31}$$

Since the future process and measurement noises are independent of the past input/output and future setpoint Eq. (20), the noise terms would be equal to zero, and the resultant equation would have the following form:

$$\begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f / \Psi_{pr} = \Gamma_i X_f / \Psi_{pr} \tag{32}$$

By multiplying Eq. (32) by the extended orthogonal observability $\Gamma_i^{\perp}$, the state term is eliminated:

$$(\Gamma_i^{\perp})^T \begin{bmatrix} I & -\Phi_i^d \end{bmatrix} \Psi_f / \Psi_{pr} = 0 \tag{33}$$

Therefore the column space of $\Psi_f/\Psi_{pr}$ is orthogonal to the row space of $\begin{bmatrix}(\Gamma_i^{\perp})^T & -(\Gamma_i^{\perp})^T\Phi_i^d\end{bmatrix}$. By performing singular value decomposition (SVD) of $\Psi_f/\Psi_{pr}$:

$$\Psi_f/\Psi_{pr} = U\Sigma V = \begin{bmatrix}U_1 & U_2\end{bmatrix}\begin{bmatrix}\Sigma_1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}{V_1}^T\\ {V_2}^T\end{bmatrix} \tag{34}$$

where $\Sigma_1$ contains dominant singular values of $\Psi_f/\Psi_{pr}$ and theoretically it has the order of the $n_u i+n$ and the order of the system can be determined by the number of the dominant singular values of the $\Psi_f/\Psi_{pr}$ [19]. The orthogonal column space of $\Psi_f/\Psi_{pr}$ is $U_2M$, where $M \in \mathbb{R}^{(n_y-n)i\times(n_y-n)i}$ is any constant nonsingular matrix and is typically chosen as an identity matrix. Huang et al. in [20], proposed the following steps to solve for the LTI model:

$$(\begin{bmatrix}\Gamma_i^{\perp} & -\Gamma_i^{\perp}\Phi_i^d\end{bmatrix})^T = U_2M \tag{35}$$

From Eq.(35), $\Gamma_i$ and $\Phi_i^d$ can be estimated.

$$\begin{bmatrix}{\Gamma_i}^{\perp}\\ -(\Phi_i^d)^T{\Gamma_i}^{\perp}\end{bmatrix} = U_2 \tag{36}$$

which results in (using MATLAB matrix index notation):

$$\begin{cases}\hat{\Gamma}_i = U_2(1:n_yi,:)^{\perp}\\ \hat{\Phi}_i^d = -(U_2(1:n_yi,:)^T)^{\dagger}U_2(n_yi+1:end,:)^T\end{cases} \tag{37}$$

The past state sequence can be calculated as follows:

$$\hat{X}_i = \hat{\Gamma}_i^{\dagger}\begin{bmatrix}I & -\hat{\Phi}_i^d\end{bmatrix}\Psi_f/\Psi_{pr} \tag{38}$$

The future state sequence can be calculated by changing data Hankel matrices as follows [20]:

$$R_f = R_{i+2|2i} \tag{39}$$

$$U_p = U_{1|i+1} \tag{40}$$

$$Y_p = Y_{1|i+1} \tag{41}$$

$$U_f = U_{i+2|2i} \tag{42}$$

$$Y_f = Y_{i+2|2i} \tag{43}$$

$$\Rightarrow \hat{X}_{i+1} = \underline{\hat{\Gamma}}_i^{\dagger} \begin{bmatrix} I & -\underline{\hat{H}}_i^d \end{bmatrix} \Psi_f / \Psi_{pr} \tag{44}$$

where $\underline{\hat{\Gamma}}_i$ is obtained by eliminating the last $n_y$ rows of $\Gamma_i$, and $\underline{H}_i^d$ is obtained by eliminating the last $n_y$ rows and the last $n_u$ columns of $H_i^d$. Then the model matrices can be estimated using least square:

$$\begin{bmatrix} X_{i+1} \\ Y_{i|i} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_i \\ U_{i|i} \end{bmatrix} + \begin{bmatrix} W_{i|i} \\ V_{i|i} \end{bmatrix} \tag{45}$$

The system matrices can be calculated as follows:

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \begin{bmatrix} X_{i+1} \\ Y_{i|i} \end{bmatrix} \begin{bmatrix} X_i \\ U_{i|i} \end{bmatrix}^{\dagger} \tag{46}$$

As mentioned before, the utilized closed-loop method is based on the recently presented subspace orthogonal projection identification method (CSOPIM) [20]. The difference between these two methods is that, in CSOPIM the innovation form of the LTI model is used to formulate the identification method. In CSOPIM, first the innovation term $E$ is estimated using residual of measured

output and estimated output ($CX_i + DU_{i|i}$), then the innovation gain ($K$) is calculated using least square. This method may cause a situation where $A - KC$ of the identified model ends up having eigenvalues out of the unit circle, therefore the LTI model with filter is not going to be stable. In order to avoid this problem, the presented approach uses a discrete-time linear time invariant state-space model for the closed-loop formulation and covariance matrices are calculated using the estimated residuals of state and output prediction instead of incorporating $K$ in the LTI model.

With the proposed approach, process and measurement noise Hankel matrices can be calculated as the residual of the least square of Eq. 45:

$$\begin{bmatrix} \hat{W}_{i|i} \\ \hat{V}_{i|i} \end{bmatrix} = \begin{bmatrix} X_{i+1} \\ Y_{i|i} \end{bmatrix} - \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} \begin{bmatrix} X_i \\ U_{i|i} \end{bmatrix} \tag{47}$$

Then the covariances of plant noises can be estimated as follows:

$$\begin{bmatrix} \hat{Q} & \hat{S} \\ \hat{S}^T & \hat{R} \end{bmatrix} = E(\begin{bmatrix} \hat{W}_{i|i} \\ \hat{V}_{i|i} \end{bmatrix} \begin{bmatrix} \hat{W}_{i|i}^T & \hat{V}_{i|i}^T \end{bmatrix}) \tag{48}$$

### 3.2.1 Formation of Batch Data Hankel Matrices

The proposed re-identification method intends to utilize past training data (used to create initial model) and recent plant data. In order to assimilate recent data with past training data, the different data sets are recognized as different batches of process data. Since these data are not continuous block of data, these data can not be handled as one block of data. The way these batches of data are handled is that the Hankel matrices for each single batch is concatenated horizontally and thus

the pertinent properties for subspace identification are retained [27]. For each batch of data $b$ with input variable define the Hankel sub-matrix as:

$$U_p^{(b)} = U_{1|i}^{(b)} = \begin{bmatrix} u_1^{(b)} & u_2^{(b)} & \dots & u_{j^{(b)}}^{(b)} \\ \vdots & \vdots & \vdots & \vdots \\ u_i^{(b)} & u_{i+1}^{(b)} & \dots & u_{i+j^{(b)}-1}^{(b)} \end{bmatrix} \tag{49}$$

The overall pseudo-Hankel matrix is formed by concatenating Hankel matrices horizontally:

$$U_p = \begin{bmatrix} U_p^{(1)} & U_p^{(2)} & \dots & U_p^{(B)} \end{bmatrix} \tag{50}$$

where $B$ is the total number of the data batches. The described system identification utilizes the concatenated data-Hankel matrices created as explained in Eq. 50 to identify an LTI model for MPC.

*Remark* 3. Note that data-Hankel matrices can not be used in the form of Equation 49 because it can only include one batch of data. Also, the other batch of data can not be added to the initial Hankel matrix because the identification method would treat the entire data as one single block of data, which would be incorrect. Therefore in order to create data-Hankel matrices without requiring that the end point of one batch of data is the beginning of next batch of data, Hankel matrices are created with horizontally concatenating data-Hankel matrices from the original data set and the recent data ($y$, $u$ and $r$).

*Remark* 4. Note, that in contrast to existing re-identification methods, this framework enables using prior training data, together with new data, in the identification of the new model. This allows

(and recognizes the fact), that the process may not have changed entirely, but is simply operating in a region where the control calculations will be better served by tweaking the original model (but not abandoning the original model altogether). One of the benefits of the proposed approach is that if the initial training data satisfies persistently excitation condition for a certain system order, then the new concatenated data will also be persistently exciting for that system order.

*Remark* 5. After re-identification the new identified model is augmented with disturbance states and state estimator is designed for the new augmented system. Since the MPC objective function and constraints only involves input and output terms, retuning of controller is not necessary.

# 4 Illustrative Simulation Results

In this section, we implement the proposed MPC with closed-loop re-identification on a polymerization reactor example [13]. The state space equation of the reactor is nonlinear with 7 states, 2 outputs and 2 inputs. The list of the reactions that occurs in the reactor are presented in Table 1.

The mathematical model of the dynamic system is as follows:

$$\begin{aligned}
\dot{C}_I &= \frac{Q_i C_{I_f} - Q_t C_I}{V} - k_d C_I \\
\dot{C}_M &= \frac{Q_M C_{M_f} - Q_t C_M}{V} - k_p C_M C_P \\
\dot{T} &= \frac{Q_t(T_f - T)}{V} + \frac{(-\Delta H)}{\rho c_p} k_p C_M C_P - \frac{hA}{\rho c_p V}(T - T_c) \\
\dot{T}_c &= \frac{Q_t(T_{cf} - T_c)}{V_c} + \frac{hA}{\rho_c c_{cp} V_c}(T - T_c) \\
\dot{D}_0 &= 0.5 k_t C_p^2 - \frac{Q_t D_0}{V} \\
\dot{D}_1 &= M_m k_p C_M C_p - \frac{Q_t D_1}{V} \\
\dot{D}_2 &= 5 M_m k_p C_M C_p + \frac{3 M_M k_p^2}{k_t} C_M^2 - \frac{Q_t D_2}{V}
\end{aligned} \tag{51}$$

where

$$\begin{aligned}
C_P &= (\frac{2 f_i k_d C_I}{k_t})^{0.5} \\
k_j &= A_j \exp(\frac{-E_j}{T}), j = d, p, t \\
Q_t &= Q_i + Q_s + Q_m
\end{aligned} \tag{52}$$

The measured outputs are reactor temperature ($T$) and the intrinsic viscosity ($\eta$), related to the process states as:

$$\eta = 0.0012(M_m \frac{D_2}{D_1})^{0.71} \tag{53}$$

The manipulated variables are the cooling liquid flow rate ($Q_c$) and monomer feed flow rate ($Q_m$). The process parameters and steady state condition for the CSTR are presented in Table (2) and Table (3).

Table 1: List of Reactions in the Reactor [27]

| No. | Reaction | Description |
|---|---|---|
| 1 | $\mathrm{I} \xrightarrow{k_d} 2\,\mathrm{R}$ | Initiator Decomposition |
| 2 | $\mathrm{M} + \mathrm{R} \xrightarrow{k_i} \mathrm{P}_1$ | Chain Initiation |
| 3 | $\mathrm{P_n} + \mathrm{M} \xrightarrow{k_p} 2\,\mathrm{R}$ | Propagation |
| 4 | $\mathrm{P_n} + \mathrm{P_m} \xrightarrow{k_{td}} \mathrm{T_n} + \mathrm{T_m}$ | Termination by Disproportionation |
| 5 | $\mathrm{P_n} + \mathrm{P_m} \xrightarrow{k_{tc}} \mathrm{T_{n+m}}$ | Termination by Combination |

Table 2: List of Process Parameters for the Polymerization Process [27]

| Variable | Description | Units | Value |
|---|---|---|---|
| $A_d$ | Frequency factor for initiator decomposition | $\mathrm{h}^{-1}$ | $2.142 \times 10^{17}$ |
| $E_d$ | Activation energy for initiator decomposition | K | 14897 |
| $A_p$ | Frequency factor for propagation reaction | L/(mol.h) | $3.816 \times 10^{10}$ |
| $E_p$ | Activation energy for propagation reaction | K | 3557 |
| $A_t$ | Frequency factor for propagation reaction | L/(mol.h) | $3.816 \times 10^{10}$ |
| $E_t$ | Activation energy for termination reaction | K | 843 |
| $f_i$ | Initiator efficiency | – | 0.6 |
| $-\Delta H_r$ | Heat of polymerization | J/mol | $6.99 \times 10^4$ |
| $hA$ | Overall heat transfer coefficient | J/(K.h) | $1.05 \times 10^6$ |
| $\rho c_p$ | Heat capacity of reactor fluid | J/(K.L) | 1506 |
| $\rho_c c_{pc}$ | Heat capacity of cooling jacket fluid | J/(K.L) | 4043 |
| $M_m$ | Molecular weight of the monomer | g/mol | 104.14 |

In order to identify the process model, PI controllers (pairing $\eta$ with $Q_c$ and $T$ with $Q_m$) are implemented on the reactor. In particular, pseudo-random binary signals are used as set-point for PI controllers. The training data is shown in Figures (4 and 5).

The proposed variation on the closed-loop identification algorithm uses these data to identify an LTI model. The order of the identified LTI is selected as $n = 4$ and $i = 12$. Model validation results under a new set of set-point changes is presented in Figure 3.

In the model validation step, initially a steady state Kalman filter is used to update state estimate

Table 3: List of Steady-State Operational Condition for the Polymerization Process [27]

| Variable | Description | Units | Value |
|---|---|---|---|
| $C_{I_f}$ | Concentration of initiator in feed | mol/L | 0.5888 |
| $C_{M_f}$ | Concentration of monomer in feed | mol/L | 8.6981 |
| $C_I$ | Concentration of initiator in the reactor | mol/L | $6.6832 \times 10^{-}2$ |
| $C_M$ | Concentration of monomer in the reactor | mol/L | 3.3245 |
| $D_0$ | Molar concentration of dead polymer chains | mol/L | $6.7547 \times 10^{-}4$ |
| $D_1$ | Mass concentration of dead polymer chains | g/L | 16.110 |
| $Q_i$ | Flow rate of initiator | L/h | 108 |
| $Q_s$ | Flow rate of solvent | L/h | 459 |
| $Q_m$ | Flow rate of monomer | L/h | 378 |
| $Q_c$ | Flow rate of cooling jacket fluid | L/h | 471.6 |
| $T_f$ | Temperature of reactor feed | K | 330 |
| $T_{cf}$ | Inlet temperature of cooling jacket fluid | K | 295 |
| $T$ | Temperature of the reactor | K | 323.56 |
| $T_c$ | Temperature of cooling jacket fluid | K | 305.17 |
| $V$ | Reactor volume | L | 3000 |
| $V_c$ | Volume of cooling jacket fluid | L | 3312.4 |

until $t = 300hrs$ and after convergence of the states (gauged via convergence of the outputs), the model and the input trajectory (without the state estimator) is utilized to predict the future output. As shown in Figure 3 the model prediction is acceptable. This model is used as an initial model to design an MPC. In order to handle model mismatch, the model is augmented with disturbance states. The disturbance model in the simulation example is constructed with $B_d$ chosen as $B$ and $C_d$ chosen as the identity matrix [28].

The re-identification mechanism is triggered if the model prediction error exceeds specified threshold more than four times in a row. The simulation results are presented in Figures (6-9). At sample time $k = 57$ ($time = 28.5$) the model prediction error violated the specified $\epsilon_{MPE}$ threshold for model prediction accuracy (thresholds are reported in Table 4) and re-identification

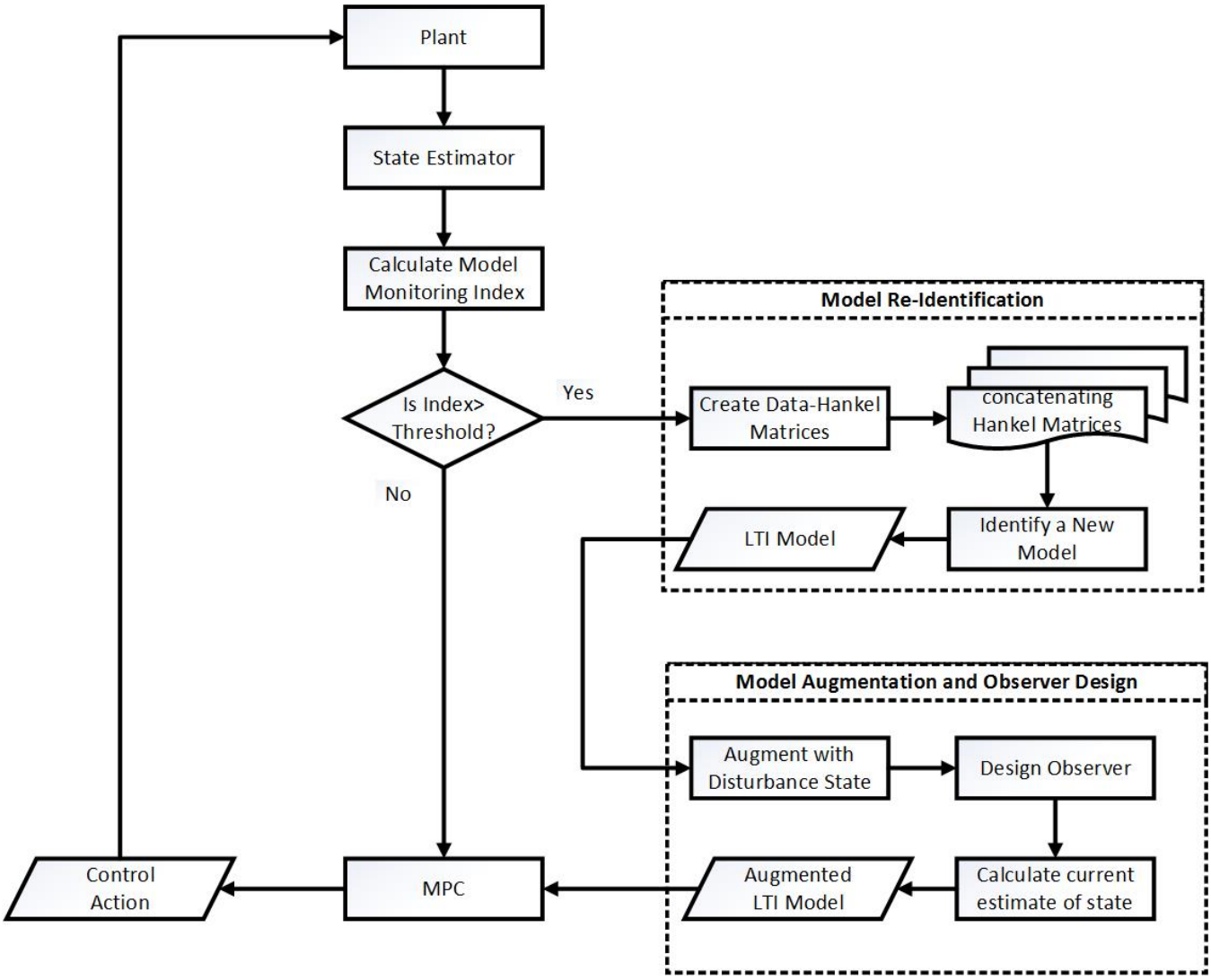


Figure 2: MPC with closed-loop re-identification

is triggered and the model is replaced with a new model with the same order. The new model also is augmented and the state of the new model is estimated using the recent data by utilizing the observer with recent data.

For subsequent comparison of the proposed methods a cost variable defined as follows:

$$j = \sum_{k=t_1}^{Nt} ||y_k - y_k^{\mathrm{SP}}||_{Q_y}^2 + ||u_k - \tilde{u}_{k-1}||_{R_{du}}^2 \tag{54}$$

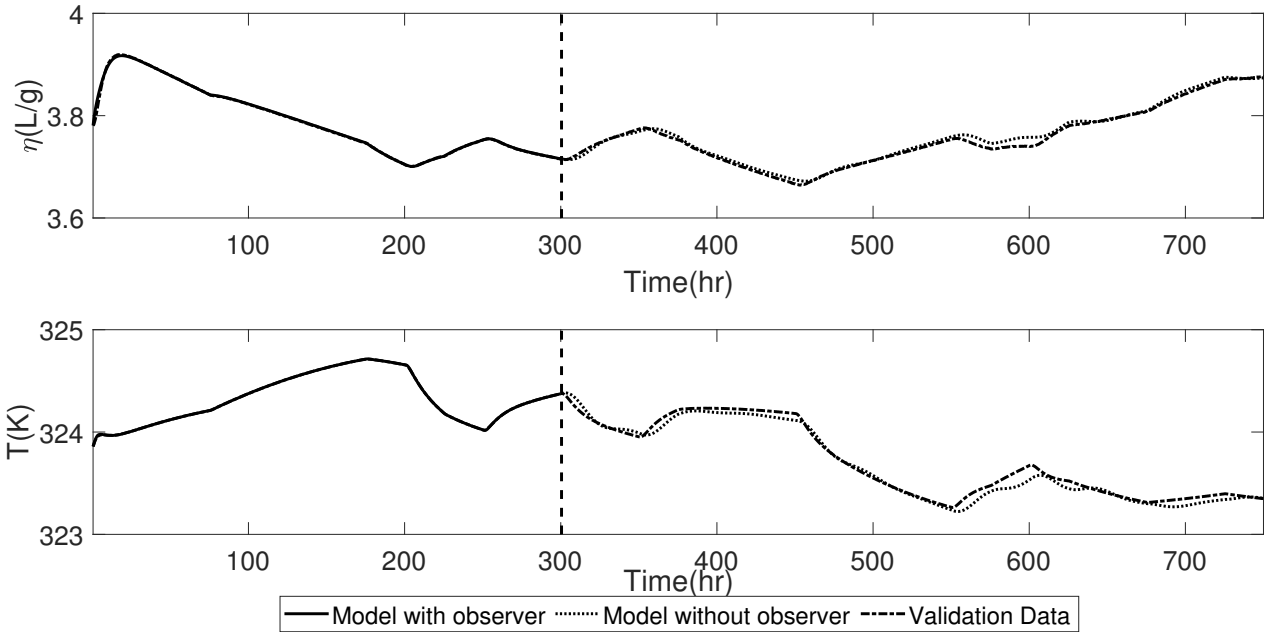


Figure 3: Model validation results under PI controller

where $t_1$ is the sample time the re-identification is triggered, and $N_t$ is the total number of sample times. This is the stage cost function of MPC, and its summation over the entire simulation time is used a a measure of closed-loop performance.

As can be seen, the closed-loop behavior is improved wherein the oscillation of the outputs are reduced significantly, and input changes are less aggressive. Furthermore, the realized MPC objective function, which used the implemented input, and realized output of the plant to calculate the objective function is less than nominal MPC.

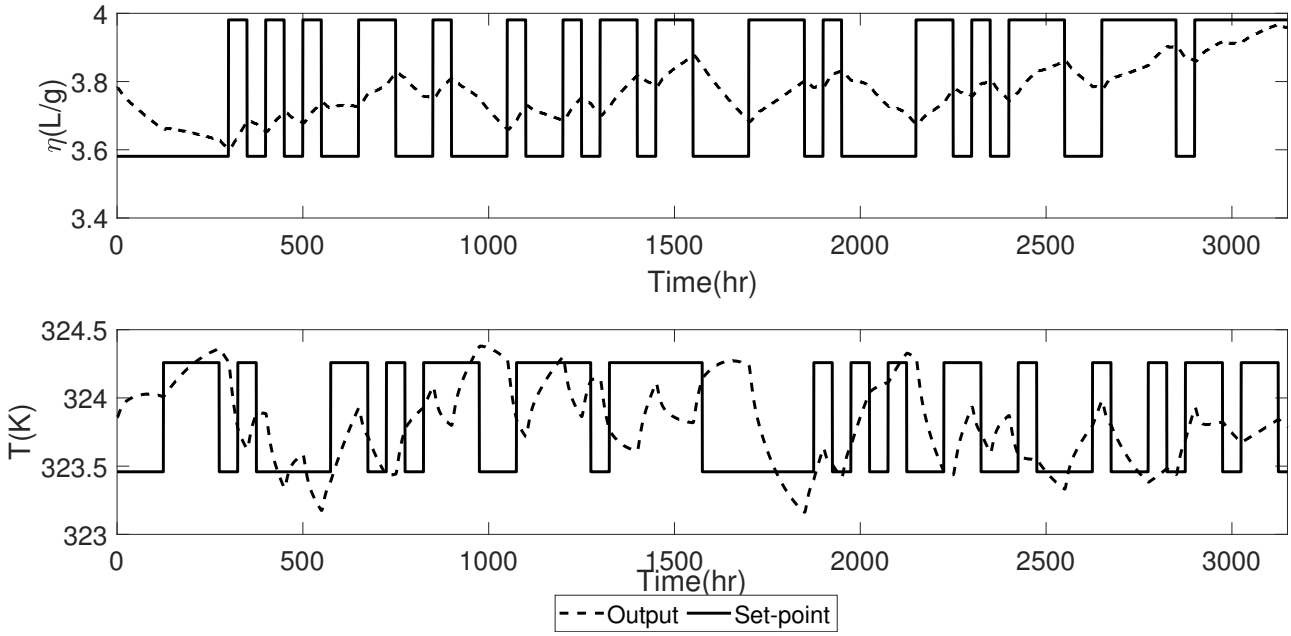


Figure 4: Model training data: measured outputs

# 5 Conclusions

In this study, a novel MPC with closed-loop re-identification approach is developed that enables monitoring and updating the model used in the MPC using both past training data and current data in system identification. The proposed approach is described and compared against traditional nominal MPC and shown to be able to provide improved closed-loop behavior through implementation on an example of a polymerization CSTR model subject to output feedback, input constraints and measurement noise.

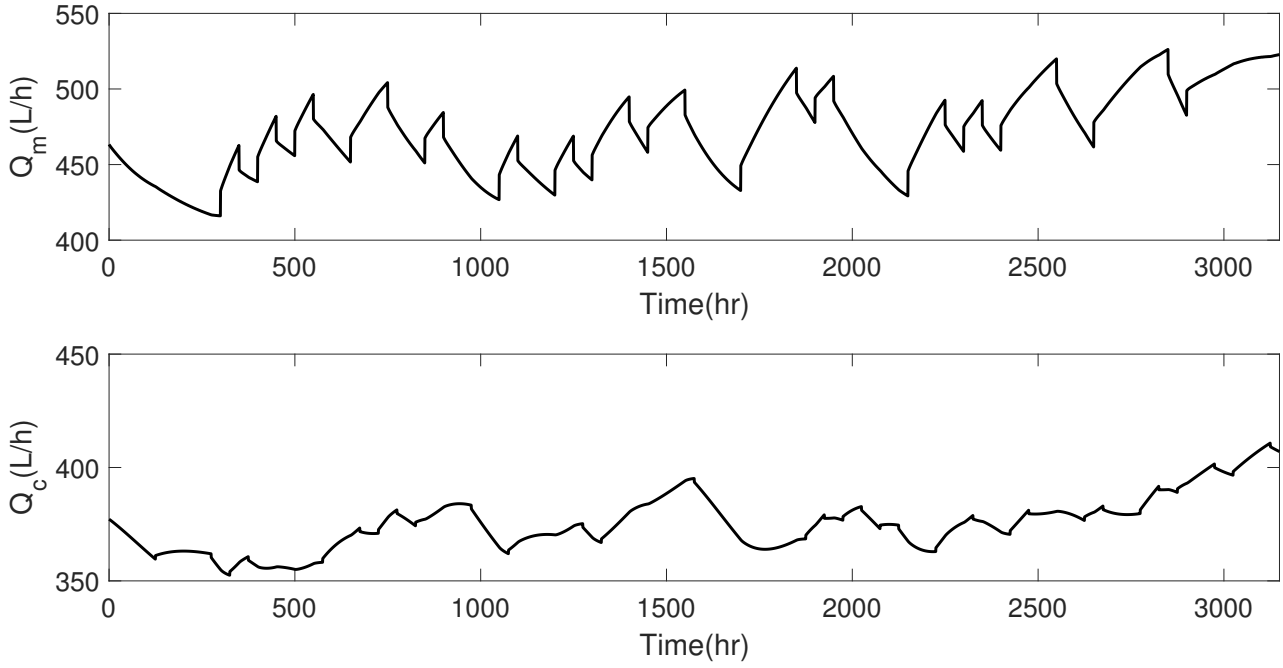


Figure 5: Model training data: manipulated inputs under PI controller

# ACKNOWLEDGMENTS

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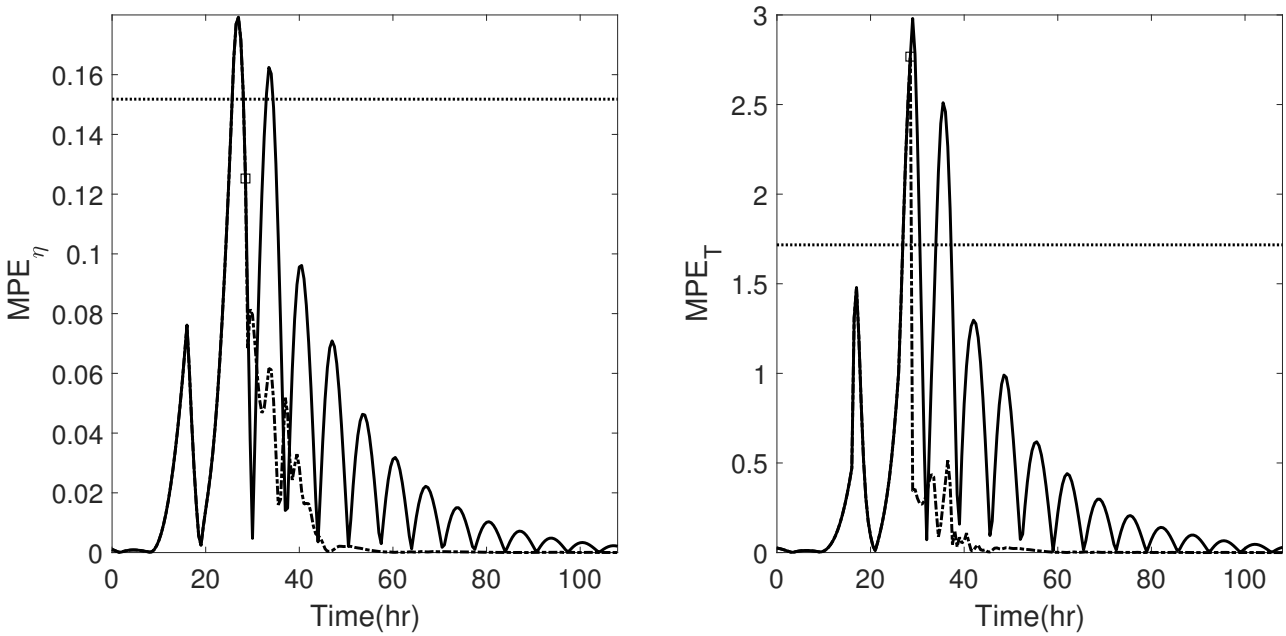


Figure 6: Model mean prediction error for the measurement variables with re-identification (continuous line) and without re-identification (dash dotted line), the square indicates the re-identification trigger point

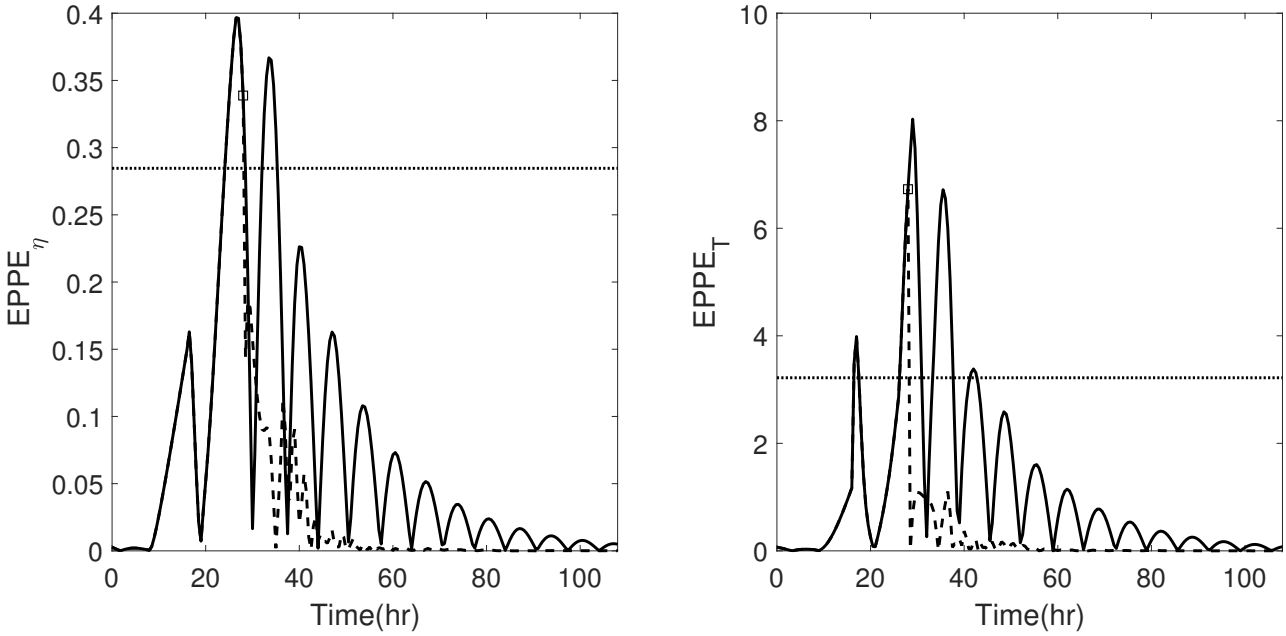


Figure 7: Model end-point mean prediction error for the measurement variables with re-identification (continuous line) and without re-identification (dashed line), the square indicates the re-identification trigger point

Table 4: Controller parameters

| Variable | Value |
|---|---|
| $k_p$ | $\begin{bmatrix}40.4 & 1.93\end{bmatrix}$ |
| $k_I$ | $\begin{bmatrix}1.96 & 0.258\end{bmatrix}$ |
| $Q_y$ | $\begin{bmatrix}50 & 0\\ 0 & 5\end{bmatrix}$ |
| $R_{du}$ | $\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}$ |
| $P$ | 15 |
| $u_{min}$ | [71.6,78] |
| $u_{max}$ | [870,670] |
| $\Delta u_{min}$ | [-20,-20] |
| $\Delta u_{max}$ | [20,20] |
| $\epsilon_{MPE}$ | [3.04,1.72] |
| $\epsilon_{EPPE}$ | [5.70,3.22] |

Table 5: Comparison of nominal MPC and MPC with closed-loop re-identification

| Controller | Cost |
|---|---|
| Nominal MPC with $t_1 = 140$ | 122.6 |
| Re-identification with $MPE$ | 97.85 |
| Nominal MPC with $t_1 = 139$ | 134.2 |
| Re-identification with $EPPE$ | 111.87 |

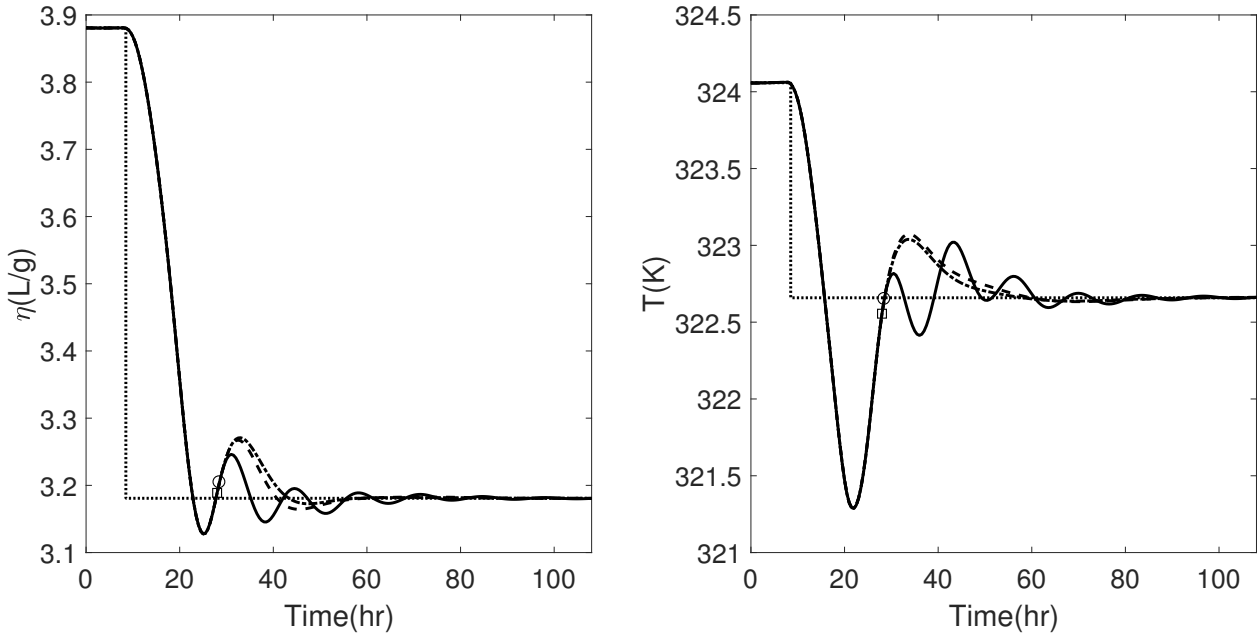


Figure 8: Comparison of the trajectories for the measured variables obtained from the proposed MPC with re-identification (dash dotted line with MPE, dashed line with EPPE) and conventional method (continuous line) and set-point (dotted line)

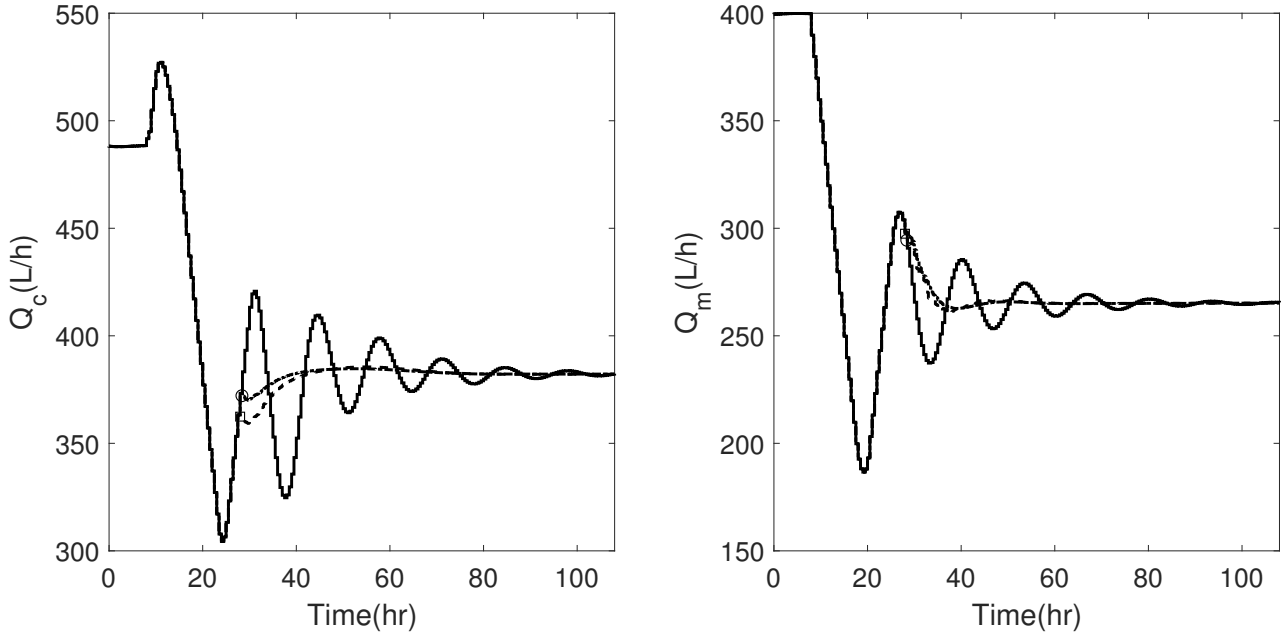


Figure 9: Closed-loop profiles of the manipulated variables obtained from the proposed MPC with re-identification (dash dotted line with MPE, dashed line with EPPE) and conventional method (continuous line)

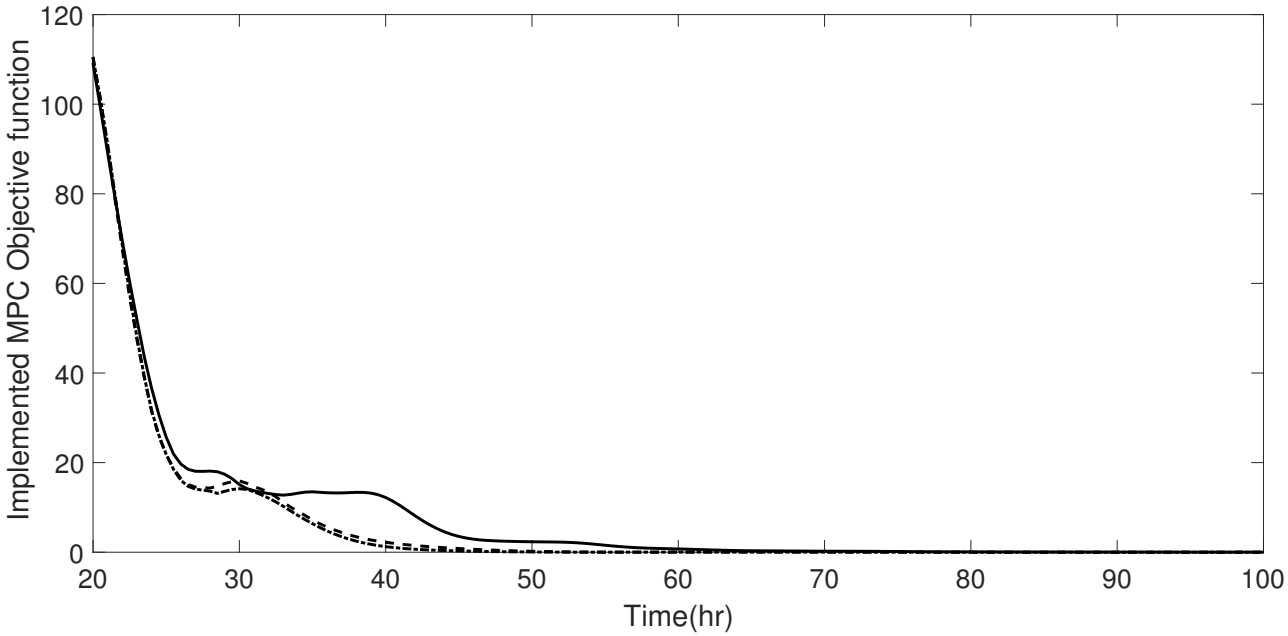


Figure 10: Closed-loop trajectories of the realized MPC objective function obtained from the proposed MPC with re-identification (dash dotted line with MPE, dashed line with EPPE) and conventional method (continuous line)

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