EOR5432 – ENHANCED OIL RECOVERY (EOR)

"Buckley-leverett 1D displacement"

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Lectures #9

Buckley-leverett 1D displacement

- In 1942 Buckley and Leverett presented to describing immiscible displacement in one dimension.

- For water displacing oil, the equation determines the velocity of a plane of constant water saturation travelling through a linear system.

- Assuming the diffuse flow condition, the conservation of mass of water flowing through volume element $A\phi\, dx$, may be expressed as

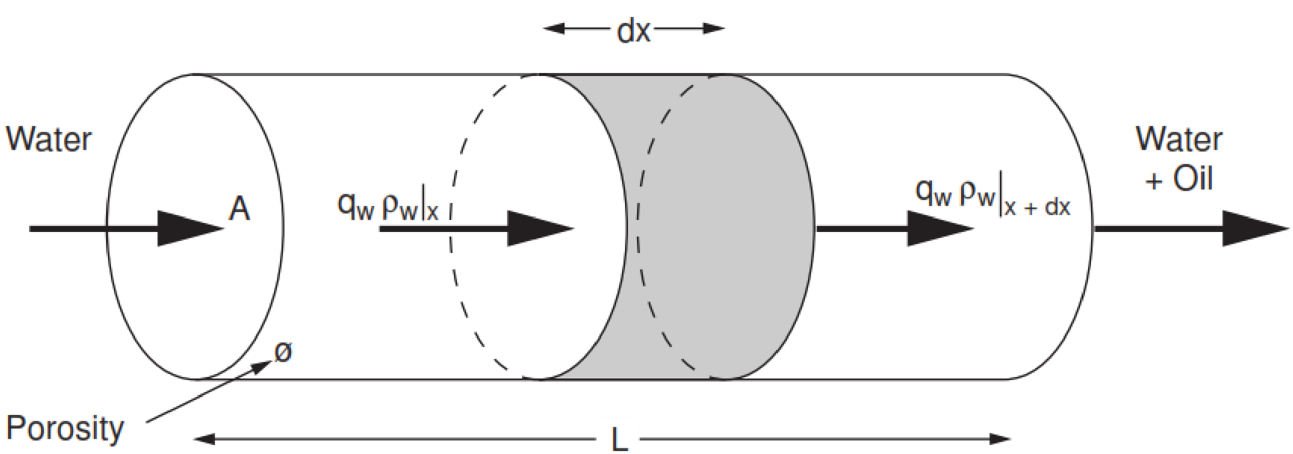
Buckley-leverett 1D displacement

Mass flow rate in –mass flow rate out =rate of increase of mass in the volume.

$$q_w\rho_w\big|_x - q_w\rho_w\big|_{X+dx} = A\phi dx\frac{\partial x}{\partial t}(\rho_w S_w)$$

or

$$q_w\rho_w\big|_x - \left(q_w\rho_w\big|_x + \frac{\partial}{\partial x}(q_w\rho_w)dx\right) = A\phi dx\frac{\partial}{\partial t}(\rho_w S_w)$$



Buckley-leverett 1D displacement

This becomes

$$\frac{\partial}{\partial x}\left(q_w \rho_w\right) = -A\phi \frac{\partial}{\partial t}\left(\rho_w S_w\right)$$

Since we are assuming incompressible flow, ρ_w is a constant. Therefore;

$$\left.\frac{\partial q_w}{\partial x}\right|_t = -A\phi \left.\frac{\partial S_w}{\partial t}\right|_x \qquad \textbf{Eq. (1)}$$

The differential of water saturation is

$$dS_w = \left.\frac{\partial S_w}{\partial x}\right|_t dx + \left.\frac{\partial S_w}{\partial t}\right|_x dt$$

Buckley-leverett 1D displacement

We are examining the advancement of a particular saturation value. Since S_w is constant $dS_w=0$.

Then

$$\left.\frac{\partial S_w}{\partial t}\right|_x = -\left.\frac{\partial S_w}{\partial x}\right|_t \left.\frac{dx}{dt}\right|_{sw}$$

Eq. (2)

Also

$$\left.\frac{\partial q_w}{\partial x}\right|_t = \left.\left(\frac{\partial q_w}{\partial S_w}\cdot\frac{\partial S_w}{\partial x}\right)\right|_t$$

Eq. (3)

Inserting equations 2 and 3 in equation 1 gives;

$$\left.\frac{\partial q_w}{\partial S_w}\right|_t = A\phi\left.\frac{dx}{dt}\right|_{S_w}$$

Buckley-leverett 1D displacement

For incompressible flow, the total injection rate, q_t is constant, and the water flow rate is the total rate times the fractional flow, $q_w = q_t \times f_w$. Rearranging equation [illegible] therefore gives:

$$\boxed{v_{S_w} = \left.\frac{dx}{dt}\right|_{S_w} = \frac{q_t}{A\phi} \left.\frac{\partial f_w}{\partial S_w}\right|_{S_w}}$$

where v_{Sw} is the velocity of the plane of saturation, S_w.

Buckley-leverett 1D displacement

- Buckley-Leverett equation implies that, for a constant rate of water injection (qt = qi),

- The velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation evaluated for that saturation.

- If the capillary pressure gradient is neglected then fractional flow is strictly a function of the water saturation

Buckley-leverett 1D displacement

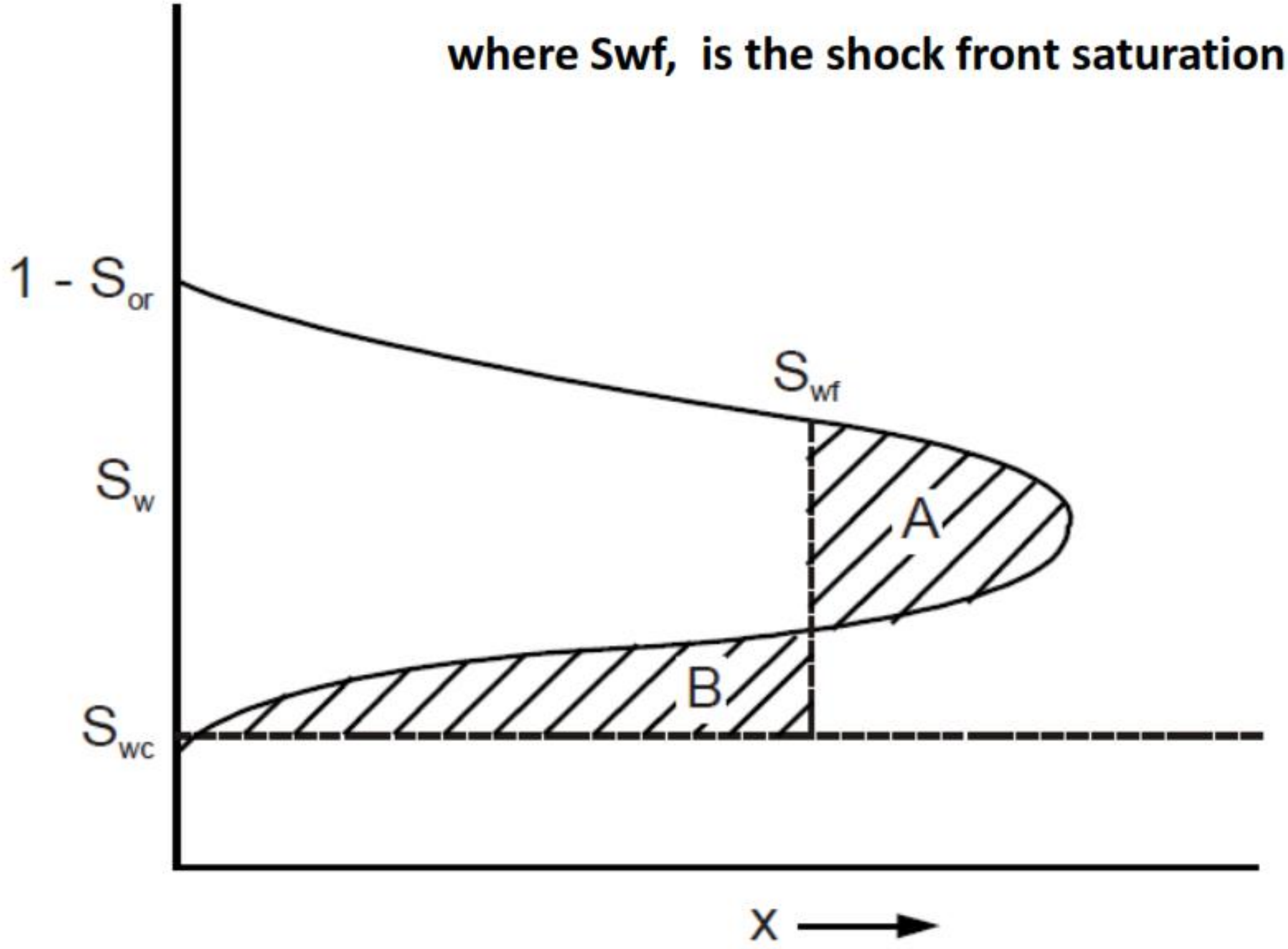
$$x_{S_w} = \frac{1}{A\phi} \frac{df_w}{dS_w} \int_0^t q_t dt$$

- Which can be written in the form below:

$$X = \frac{5.615 q_t\, t}{\varphi\, Ac} \left(\frac{\partial fw}{\partial Sw}\right) \qquad \textbf{Equation 3}$$

Buckley-leverett 1D displacement

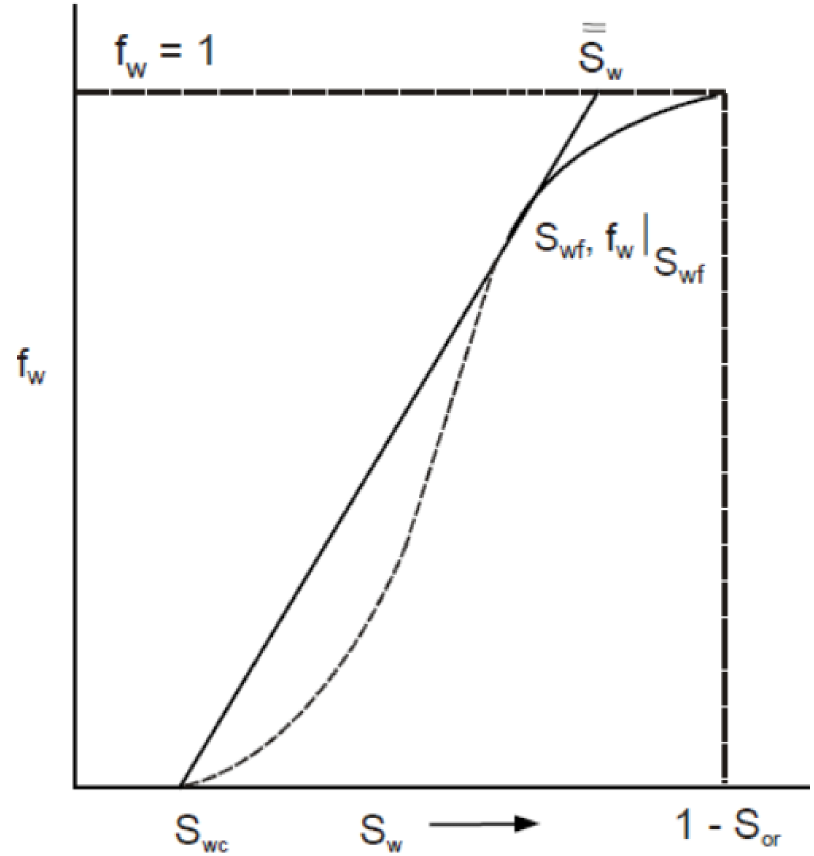
where Swf, is the shock front saturation



Oil recovery calculations

- At the time of breakthrough the flood frontsaturation,Swf = Swbt ,

$$N_{pd_{bt}} = \left(\bar{\bar{S}}_{w_{bt}} - S_{wc}\right)$$



Oil recovery calculations

The procedure for the oil recovery calculations is summarised below.

1. Generate a fractional flow vs. water saturation curve for the system to be studied, using the appropriate relative permeability data.

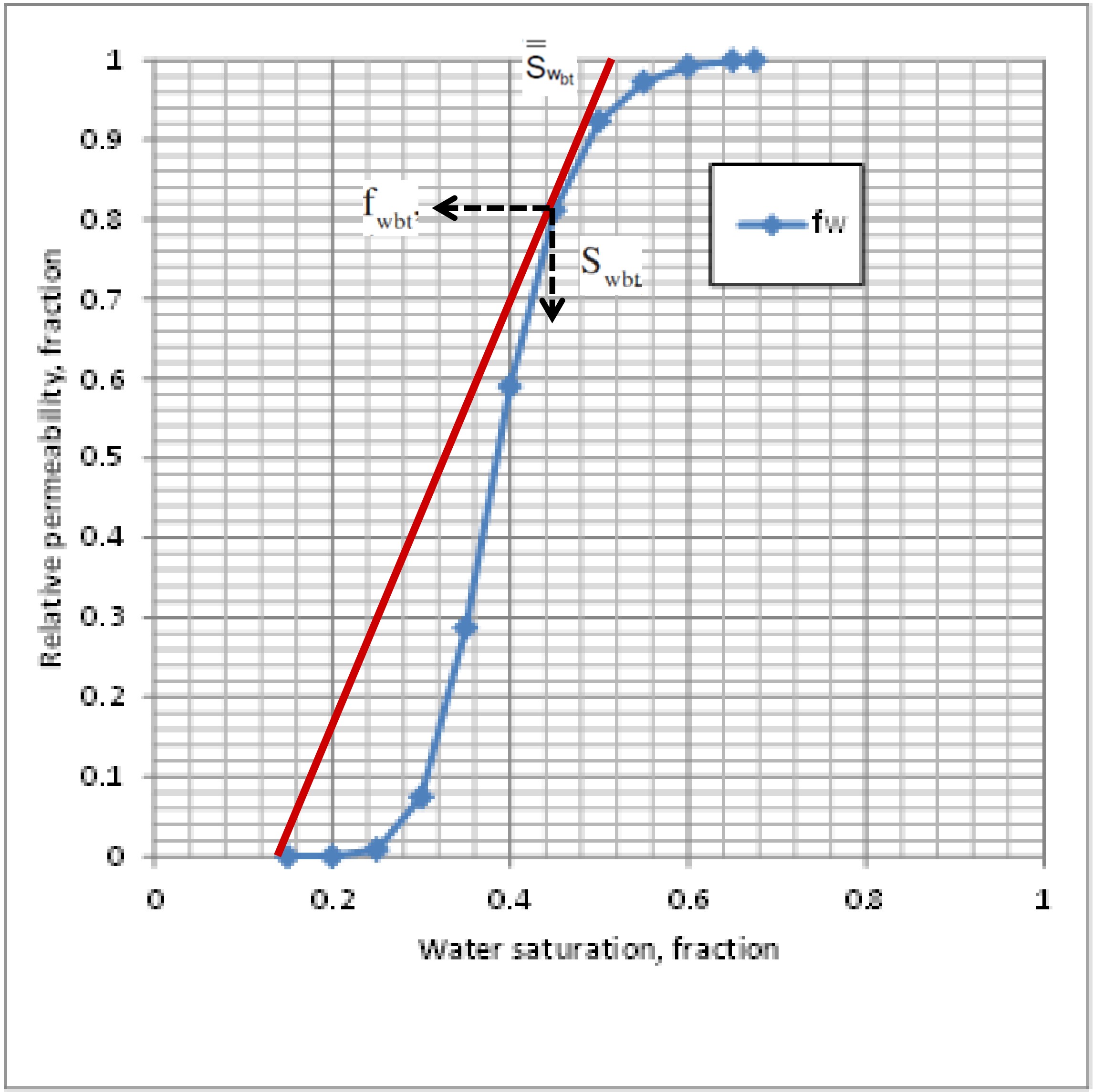
2. Draw a tangent to the fractional flow curve from the initial $S_w = S_{wi}$ position at $f_w=0$. At the point of tangency are the conditions of breakthrough ;

(i) $f_w=f_{wbt}$, $S_w=S_{wbt}$ and extrapolation of line to $f_w=1$ gives the average water Saturation value. $\overline{S}_{wbt}$

Also

$$N_{pd_{bt}} = \left(\overline{\overline{S}}_{w_{bt}} - S_{wc}\right)$$

Oil recovery calculations



Fractional Flow example

- Oil is being displaced by water in a horizontal, direct line drive under the diffuse flow condition.

- The rock relative permeability functions for water and oil are listed in below table.

S_w	k_{rw}	k_{ro}	S_w	k_{rw}	k_{ro}
.20	0	.800	.50	.075	.163
.25	.002	.610	.55	.100	.120
.30	.009	.470	.60	.132	.081
.35	.020	.370	.65	.170	.050
.40	.033	.285	.70	.208	.027
.45	.051	.220	.75	.251	.010
			.80	.300	0

Fractional Flow example

- $B_o = 1.3$ rb/stb and $B_w = 1.0$ rb/stb
- Compare the values of the producing watercut (at surface conditions) and the
- cumulative oil recovery at breakthrough for the following fluid combinations.

Case	oil viscosity	water viscosity
1	50 cp	.5 cp
2	5 "	.5 "
3	.4 "	1.0 "

Fractional Flow example solution

1. For horizontal flow the fractional flow in the reservoir is

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}}$$

- while the producing watercut at the surface, fws, is

$$f_{ws} = \frac{q_w / B_w}{q_w / B_w + q_o / B_o}$$

Fractional Flow example solution

- where the rates are expressed in rb/d. Combining the above two equations leads to an expression for the surface watercut as

$$f_{ws} = \frac{1}{1 + \frac{B_w}{B_o}\left(\frac{1}{f_w} - 1\right)}$$

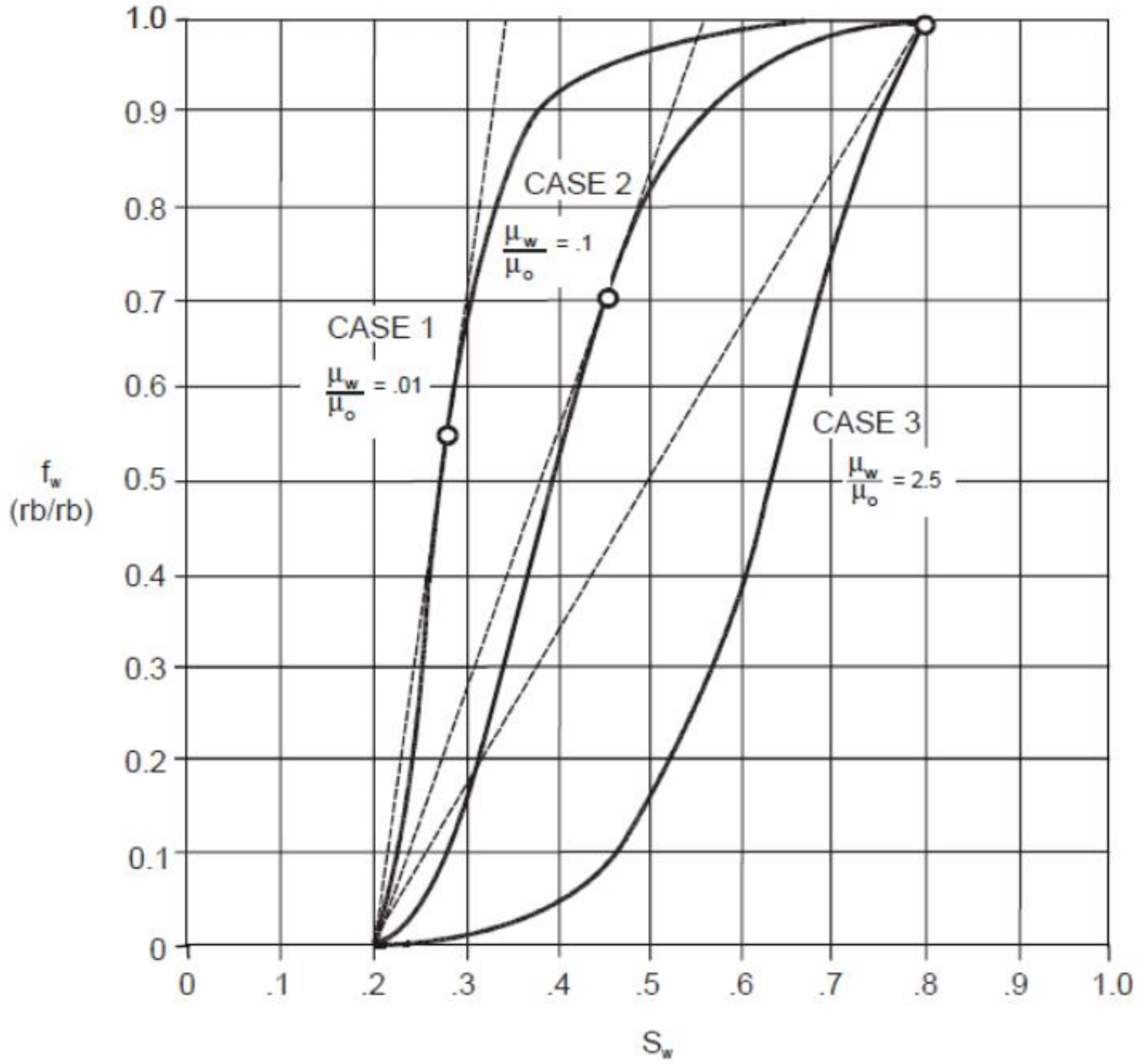
Fractional Flow example solution

				Fractional Flow (f_w)		
				Case 1	Case 2	Case3
S_w	k_{rw}	k_{ro}	k_{ro}/k_{rw}	$\mu_w/\mu_o = .01$	$\mu_w/\mu_o = .1$	$\mu_w/\mu_o = 2.5$
.2	0	.800	∞	0	0	0
.25	.002	.610	305.000	.247	.032	.001
.30	.009	.470	52.222	.657	.161	.008
.35	.020	.370	18.500	.844	.351	.021
.40	.033	.285	8.636	.921	.537	.044
.45	.051	.220	4.314	.959	.699	.085
.50	.075	.163	2.173	.979	.821	.155
.55	.100	.120	1.200	.988	.893	.250
.60	.132	.081	.614	.994	.942	.394
.65	.170	.050	.294	.997	.971	.576
.70	.208	.027	.130	.999	.987	.755
.75	.251	.010	.040	.999	.996	.909
.80	.300	0	0	1.000	1.000	1.000

Fractional Flow example solution

Case	$S_{w_{bt}}$	$f_{w_{bt}}$ (reservoir)	$f_{ws_{bt}}$ (surface)	$\bar{\bar{S}}_{w_{bt}}$	$N_{pd_{bt}}$ (PV)
1	.28	.55	.61	.34	.14
2	.45	.70	.75	.55	.35
3	.80	1.00	1.00	.80	.60

Fractional Flow example solution



Fractional Flow example solution

- An important parameter in determining the effectiveness of a waterflood is the end point.

$$M = \frac{k'_{rw} / \mu_w}{k'_{ro} / \mu_o}$$

- More significant parameter for characterising the stability of Buckley Leverett displacement is the shock front mobility ratio, Ms, defined as:

$$M_s = \frac{k_{ro}(S_{wf}) / \mu_o + k_{rw}(S_{wf}) / \mu_w}{k'_{ro} / \mu_o}$$

Fractional Flow example solution

Case No. (exercise 10.1)	$\frac{\mu_o}{\mu_w}$	S_{wf}	$k_{rw}(S_{wf})$	$k_{ro}(S_{wf})$	M_s	M
1	100	.28	.006	.520	1.40	37.50
2	10	.45	.051	.220	.91	3.75
3	.4	.80	.300	0	.15	0.15

- Case 1 - this displacement is unstable due to the very high value of the oil/water viscosity ratio

- Case 2 - the oil/water viscosity ratio is an order of magnitude lower than in case 1 which leads to a much more favourable type of displacement ($M_s < 1$).

- Case 3 - for the displacement of this very low viscosity oil (μ_o = .4 cp) and piston-like occurs

Fractional Flow example (2)

For a certain reservoir the following k_{ro}/k_{rw}, s_w relationship are known, find the position of the interface after 60, 120, qnd 240 days? Where the flow rate 900 bbl/day, ϕ = 0.25, cross sectional area = 1320 ft x thickness 20 ft, μ_w/μ_o = 0.5

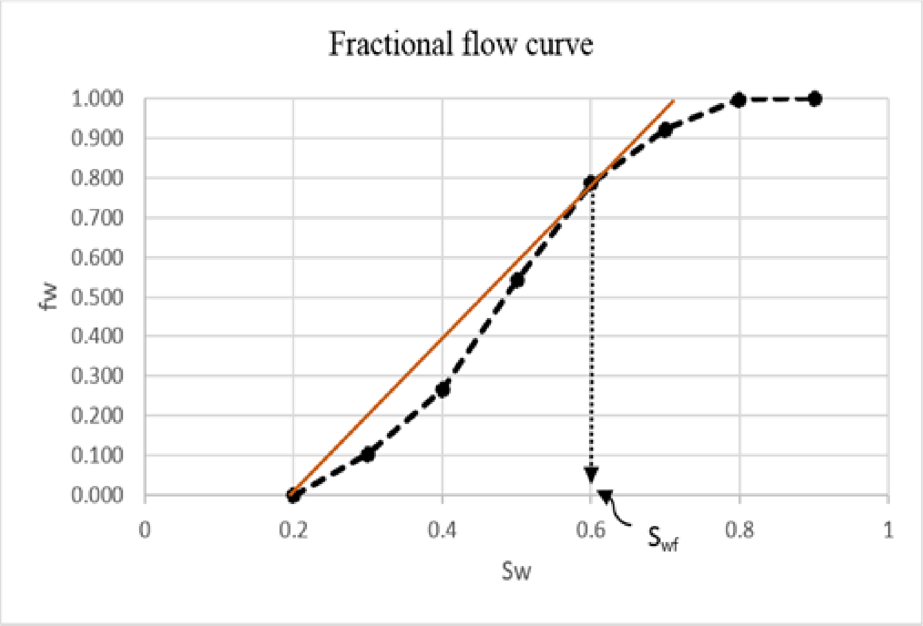
S_w	K_{ro}/k_{rw}
0.2	Inf
0.3	17
0.4	5.5
0.5	1.7
0.6	0.55
0.7	0.17
0.8	0.0055
0.9	0.000

Fractional Flow example solution (2)

1) Calculate f_w using equation

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o}\frac{K_{ro}}{K_{rw}}}$$

S_w	K_{ro}/k_{rw}	(1) f_w
0.2	Inf	0.000
0.3	17	0.105
0.4	5.5	0.267
0.5	1.7	0.541
0.6	0.55	0.784
0.7	0.17	0.922
0.8	0.0055	0.997
0.9	0.000	1.000



2) Calculate f_w using equation $\left(\frac{df_w}{ds_w}\right)$

$$\frac{K_{ro}}{K_{rw}} = a\, e^{-bS_w}$$

Fractional Flow example solution (2)

$$\frac{df_w}{ds_w} = \frac{(\mu_w/\mu_o)\ x\ b\ x\ ae^{-b\,S_w}}{[1+\ (\mu_w/\mu_o)\ x\ ae^{-b\,S_w}]^2} = \frac{\left(\mu_w/\mu_o\right)\ x\ b\ x\ \frac{K_{ro}}{K_{rw}}}{\left[1+\ (\mu_w/\mu_o)\ x\ \frac{K_{ro}}{K_{rw}}\right]^2}$$

Where **a** and **b** are constant

For instance, at Sw = 0.3 and 0.4 the $\frac{K_{ro}}{K_{rw}}$ =17 and 5.5 respectively

Therefor,

$$17 = a\ e^{-bx\ o.3}$$

$$5.5 = a\ e^{-b\ x\ 0.4}$$

Form the above two equations

$$\frac{5.5}{17} = \frac{e^{-b\ x\ 0.4}}{e^{-bx\ o.3}}$$

a = 540 and b = 11.5

Then by applying above equation we can determine the value of $\frac{df_w}{ds_w}$ for each value of Sw

Fractional Flow example solution (2)

S_w	K_{ro}/k_{rw}	(1)	(2)
		f_w	df_w/ds_w
0.2	Inf	0.000	0
0.3	17	0.105	1.08
0.4	5.5	0.267	2.25
0.5	1.7	0.541	2.86
0.6	0.55	0.784	1.95
0.7	0.17	0.922	0.83
0.8	0.0055	0.997	0.3
0.9	0.000	1.000	0

3) find the position of the interface after 60, 120, qnd 240 days by applying

$$X = \frac{5.615\, q_t\, t}{\emptyset\, A_c} \left(\frac{\partial f_w}{\partial S_w}\right)_{s_w}$$

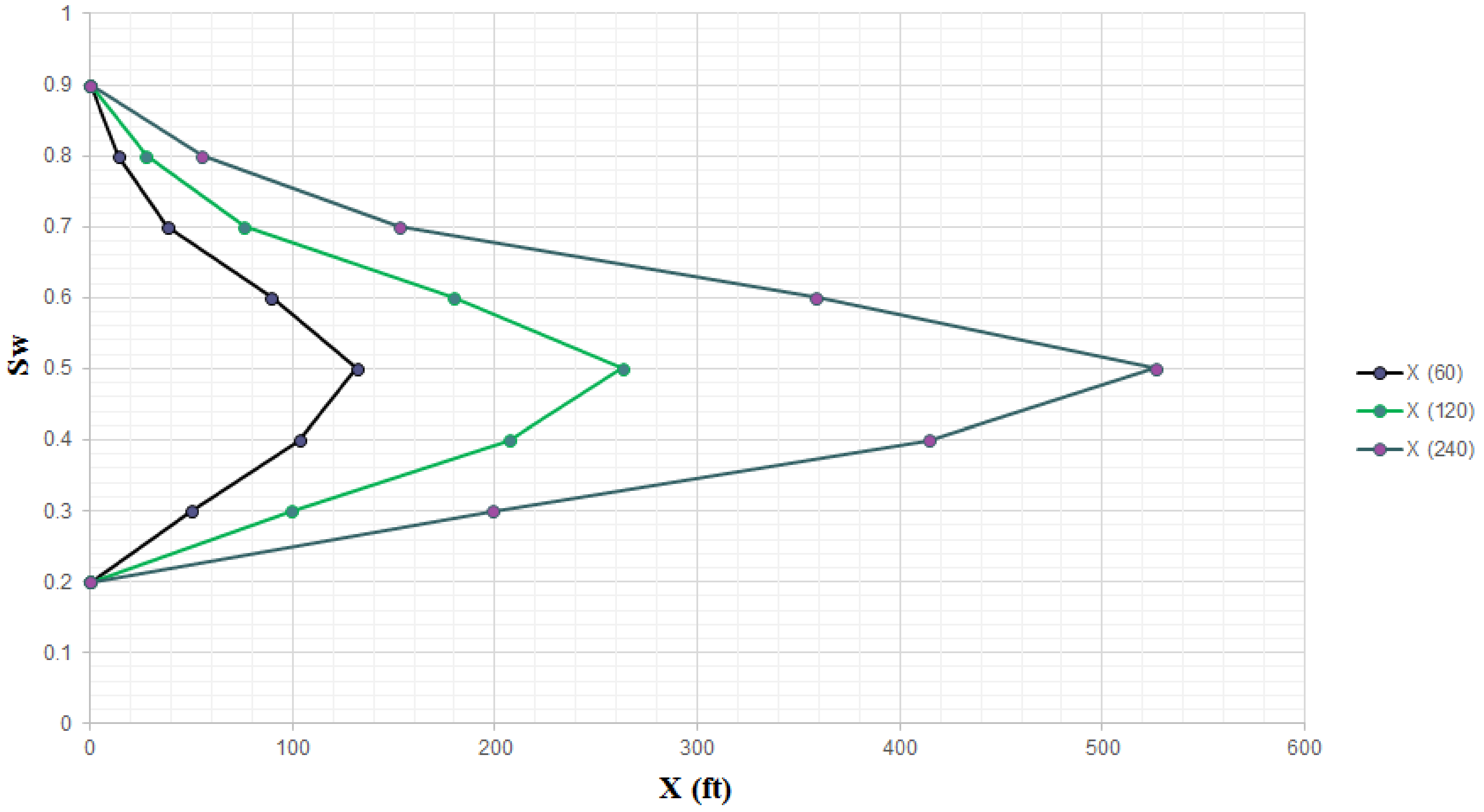
Fractional Flow example solution (2)

$$X = \frac{5.615\ x\ 900\ x\ t}{0.25\ x\ 1320\ x\ 20}\ (\frac{\partial f_w}{\partial S_w})_{S_w} = 0.7657\ t\ (\frac{\partial f_w}{\partial S_w})_{S_w}$$

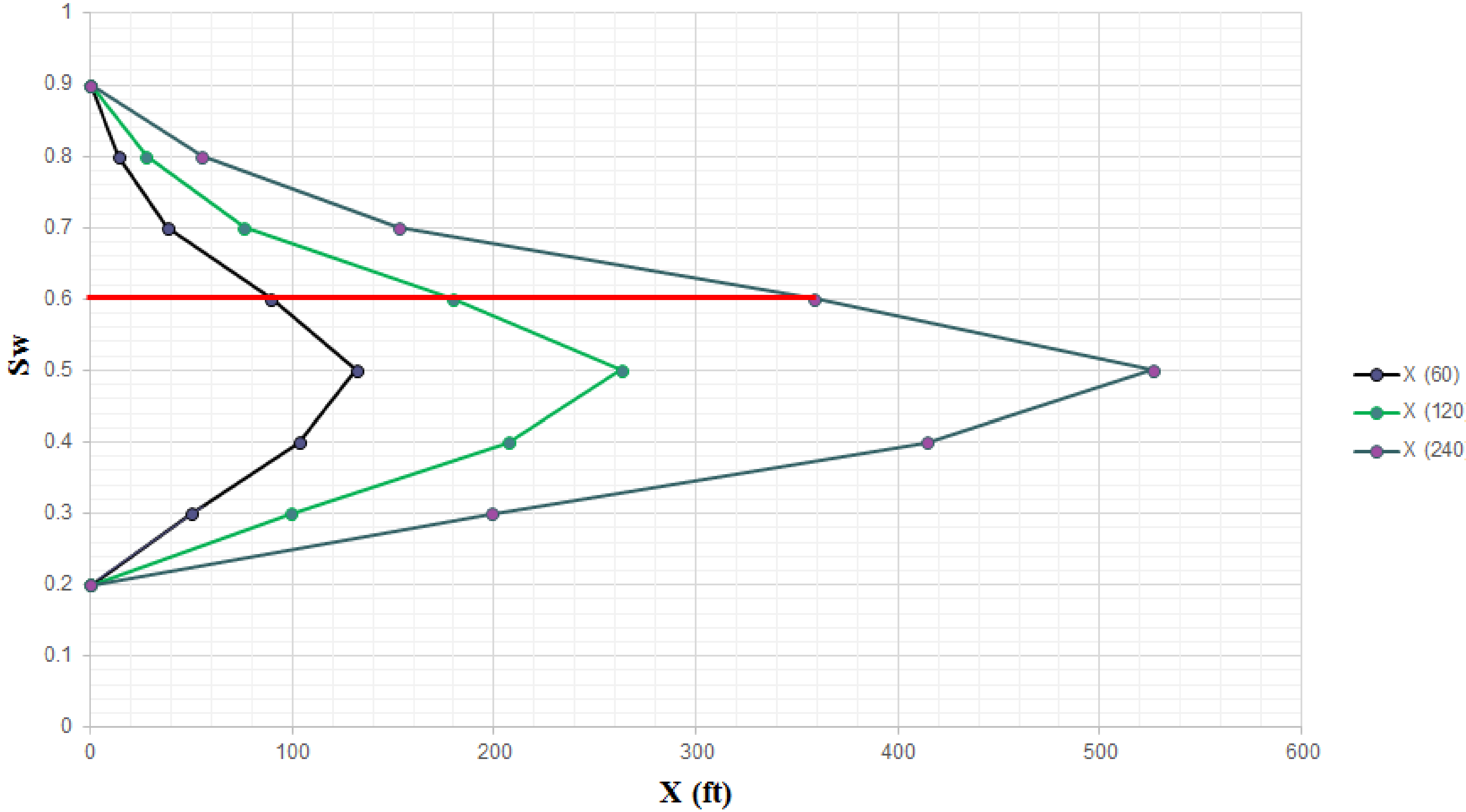
X (60) = 46$(\frac{\partial f_w}{\partial S_w})_{S_w}$, X (120) = 92$(\frac{\partial f_w}{\partial S_w})_{S_w}$, X (240) = 184 $(\frac{\partial f_w}{\partial S_w})_{S_w}$

S_w	K_{ro}/k_{rw}	(1) f_w	(2) df_w/ds_w	(3) X(ft) 60 days	(4) X(ft) 120 days	(5) X (ft) 240 days
0.2	Inf	0.000	0	0	0	0
0.3	17	0.105	1.08	50	100	200
0.4	5.5	0.267	2.25	104	208	416
0.5	1.7	0.541	2.86	131	262	524
0.6	0.55	0.784	1.95	89	179	358
0.7	0.17	0.922	0.83	38	76	153
0.8	0.0055	0.997	0.3	14	28	55
0.9	0.000	1.000	0	0	0	0

Fractional Flow example solution (2)



Fractional Flow example solution (2)



Recovery calculation example (3)

Water is being injected at a constant rate of 1200 bbl/d/well in a direct line drive in a reservoir whose rock and fluid properties, as well as the flood pattern geometry are listed in the data summary table below. The relative permeability data for oil and water are:

Data Summary

Injection rate	Q_i =	1,200	bbl/d/well
Water viscosity	μ_w =	0.5	cp
Oil viscosity	μ_o =	4.5	cp
Initial water saturation	Swc =	0.20	
Residual oil saturation	Sor =	0.20	
Porosity	ϕ =	0.22	
Dip angle	θ =	0°	
Reservoir thickness	h =	50	ft
Distance between injection wells	w =	800	ft
Distance - injectors and producers	L = 2,000	ft	

Recovery calculation example (3)

Sw	Krw	Kro
0.20	0.000	0.880
0.25	0.002	0.671
0.30	0.010	0.517
0.35	0.021	0.407
0.40	0.035	0.314
0.45	0.054	0.242
0.50	0.079	0.179
0.55	0.105	0.132
0.60	0.139	0.089
0.65	0.179	0.055
0.70	0.218	0.030
0.75	0.264	0.011
0.80	0.315	0.000

Determine oil recovery at breakthrough time

Recovery calculation solution (3)

The fractional flow in the reservoir (for horizontal flow) is calculated from:

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \frac{k_{ro}}{\mu_o}}$$

For this case, the water / oil viscosity ratio is:

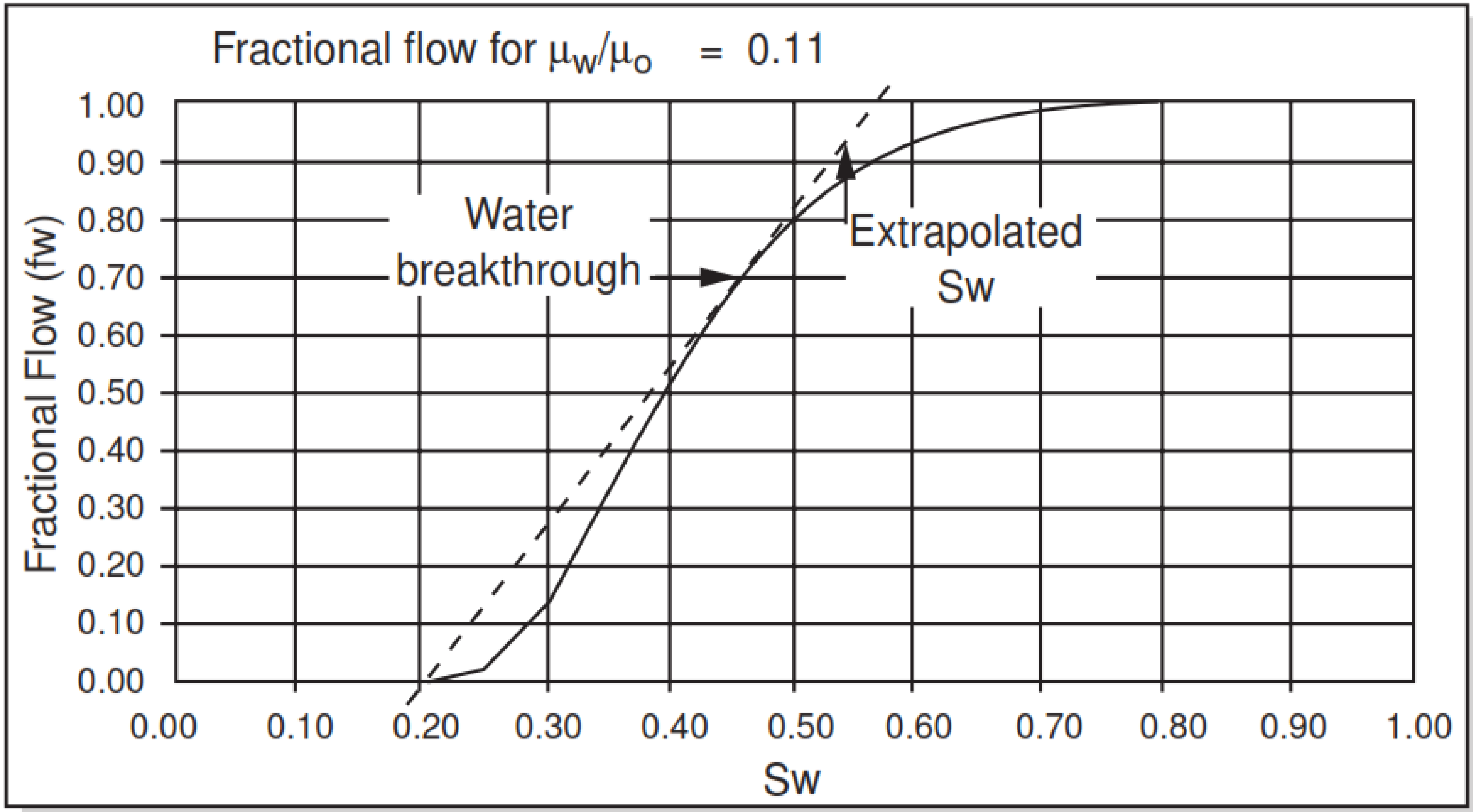
$$\mu_w / \mu_o = 0.11$$

Results of the fractional flow calculations for this case are summarised in table

Recovery calculation solution (3)

Sw	K_{rw}	K_{ro}	K_{ro}/K_{rw}	fractional flow **fw**
0.20	0.000	0.880	Infinite	0.000
0.25	0.002	0.671	319.5238	0.027
0.30	0.010	0.517	54.4211	0.142
0.35	0.021	0.407	19.3810	0.317
0.40	0.035	0.314	9.0346	0.499
0.45	0.054	0.242	4.5149	0.666
0.50	0.079	0.179	2.2754	0.798
0.55	0.105	0.132	1.2571	0.877
0.60	0.139	0.089	0.6429	0.933
0.65	0.179	0.055	0.3081	0.967
0.70	0.218	0.030	0.1360	0.985
0.75	0.264	0.011	0.0417	0.995
0.80	0.315	0.000	0.0000	1.000

Recovery calculation solution (3)



Recovery calculation solution (3)

From the above figure, breakthrough occurs when:

Water saturation at breakthrough	Swbt = 0.45
Fractional flow at breakthrough	fwbt = 0.665
Extrapolated Savg at fw = 1	Savg = 0.572

$$N_{pd_{bt}} = \bar{\bar{S}}_{wbt} - S_{wc}$$

$$N_{pd_{bt}} = 0.572 - 0.20$$

$$N_{pd_{bt}} = 0.372$$

Thank you

Any Question??