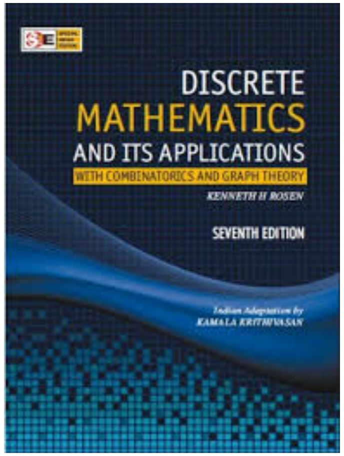
Vietnam National University of HCMC
International University
School of Computer Science and Engineering



Session 2: Logic and Propositions

Nguyen Van Sinh, Ph.D

nvsinh@hcmiu.edu.vn

Propositional Logic

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
- Are the following sentences propositions?
  - Hanoi is the capital of Vietnam. (Yes)
  - Read this carefully. (No)
  - 123 is divided by 3 (Yes)
  - x+1=2 (No)
  - How are you? (No)

Examples:

- Check the following sentences are propositions? If yes, it is True or False?
  - The discrete math is a required course to all IT students.
  - 97 is a prime number.
  - N is a prime number.

Basic operators:

<u>Formal Name</u>	<u>Operator</u>	<u>Notation</u>
Negation operator	NOT	$\neg$
Conjunction operator	AND	$\wedge$
Disjunction operator	OR	$\vee$
Exclusive-OR operator	XOR	$\oplus$
Implication operator	IMPLIES	$\rightarrow$
Biconditional operator	IFF	$\leftrightarrow$

Propositional Logic

- Propositional Logic – the area of logic that deals with propositions
- Propositional Variables – variables that represent propositions: *p*, *q*, *r*, *s*
  - E.g. Proposition *p* – "Today is Friday."
- Truth values – T, F (or 1,0)

Propositional Logic

DEFINITION 1

Let *p* be a proposition. The negation of *p*, denoted by ¬*p*, is the statement "It is not the case that *p*."

The proposition ¬*p* is read "not *p*." The truth value of the negation of *p*, ¬*p* is the opposite of the truth value of *p*.

- Examples
  - Find the negation of the proposition "Today is Friday." and express this in simple English.

    Solution: The negation is "It is not the case that *today is Friday*." In simple English, "Today is not Friday." or "It is not Friday today."
  - Find the negation of the proposition "At least 10 inches of rain fell today in Miami." and express this in simple English.

    Solution: The negation is "It is not the case that *at least 10 inches of rain fell today in Miami*."
    In simple English, "Less than 10 inches of rain fell today in Miami."

Propositional Logic

- Note: Always assume fixed times, fixed places, and particular people unless otherwise noted.
- Truth table:

The Truth Table for the Negation of a Proposition.	
p	$\neg p$
T	F
F	T

- Logical operators are used to form new propositions from two or more existing propositions. The logical operators are also called connectives.

Propositional Logic

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

- Examples
  - Find the conjunction of the propositions p and q where p is the proposition "Today is Friday." and q is the proposition "It is raining today.", and the truth value of the conjunction.

    Solution: The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

Propositional Logic

DEFINITION 3

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q". The conjunction $p \vee q$ is false when both p and q are false and is true otherwise.

- Note:
  inclusive or: The disjunction is true when at least one of the two propositions is true.
  - E.g. "Students who have taken calculus or computer science can take this class." – those who take one or both classes.

  exclusive or: The disjunction is true only when one of the proposition is true.
  - E.g. "Students who have taken calculus or computer science, but not both, can take this class." – only those who take one of them.
- Definition 3 uses *inclusive or*.

Propositional Logic

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The Truth Table for the Exclusive *Or* (*XOR*) of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Propositional Logic: conditional Statements

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \to q$, is the proposition "if p, then q." The conditional statement is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

- A conditional statement is also called an implication.
- Example: "If I am elected, then I will lower taxes." $p \to q$

implication:

elected, lower taxes.	T	T	T
not elected, lower taxes.	F	T	T
not elected, not lower taxes.	F	F	T
elected, not lower taxes.	T	F	F

Propositional Logic

- Example:
  - Let p be the statement "Nam learns discrete mathematics." and q the statement "Nam will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

  Solution: Any of the following -

  "If Nam learns discrete mathematics, then he will find a good job.

  "Nam will find a good job when he learns discrete mathematics."

  "For Nam to get a good job, it is sufficient for him to learn discrete mathematics."

  "Nam will find a good job unless he does not learn discrete mathematics."

More examples:

- "If this lecture ever ends, then the sun will rise tomorrow." *T* or *F* ?
- "If Tuesday is a day of the week, then I am a penguin." *T* or *F* ?
- "If 1+1=6, then Obama is President." *T* or *F* ?
- "If the Moon is made of green cheese then I am richer than Bill Gates." *T* or *F* ?

English Phrases Meaning $p \rightarrow q$

- "*p* implies *q*"
- "if *p*, then *q*"
- "if *p*, *q*"
- "when *p*, *q*"
- "whenever *p*, *q*"
- "*q* if *p*"
- "*q* when *p*"
- "*q* whenever *p*"
- "*p* only if *q*"
- "*p* is sufficient for *q*"
- "*q* is necessary for *p*"
- "*q* follows from *p*"
- "*q* is implied by *p*"

We will see some equivalent logic expressions later.

Propositional Logic

- Other conditional statements:
  - Converse of $p \longrightarrow q$: $q \longrightarrow p$
  - Contrapositive of $p \longrightarrow q$: $\neg\, q \longrightarrow \neg\, p$
  - Inverse of $p \longrightarrow q$: $\neg\, p \longrightarrow \neg\, q$

- You can also use Truth Table to check and solve this problem.

Propositional Logic

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

- $p \Leftrightarrow q$ has the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$
- "*if and only if*" can be expressed by "*iff*"
- Example:
  - Let p be the statement "You can take the flight" and let q be the statement "You buy a ticket." Then $p \Leftrightarrow q$ is the statement

    "You can take the flight if and only if you buy a ticket."

    Implication:

    If you buy a ticket you can take the flight.

    If you don't buy a ticket you cannot take the flight.

Propositional Logic

The Truth Table for the Biconditional $p \leftrightarrow q$.		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example: translate the below sentence into a logical expression:
"you are not driven a motorbike without a helmet, unless driving a car"

Propositional Logic

Truth Tables of Compound Propositions

- We can use connectives to build up complicated compound propositions involving any number of propositional variables, then use truth tables to determine the truth value of these compound propositions.
- Example: Construct the truth table of the compound proposition

$$(p \vee \neg q) \rightarrow (p \wedge q).$$

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Propositional Logic

Precedence of Logical Operators

- We can use parentheses to specify the order in which logical operators in a compound proposition are to be applied.
- To reduce the number of parentheses, the precedence order is defined for logical operators.

Precedence of Logical Operators.	
Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

E.g. $\neg p \wedge q = (\neg p) \wedge q$

$p \wedge q \vee r = (p \wedge q) \vee r$

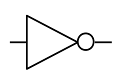
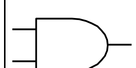
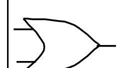
$p \vee q \wedge r = p \vee (q \wedge r)$

Boolean Operations Summary

- We have seen 1 unary operator (out of the 4 possible) and 5 binary operators (out of the 16 possible). Their truth tables are below.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	T	F	F	F	T	T
F	T	T	F	T	T	T	F
T	F	F	F	T	T	F	F
T	T	F	T	T	F	T	T

Some Alternative Notations

Name:	not	and	or	xor	implies	iff
Propositional logic:	$\neg$	$\wedge$	$\vee$	$\oplus$	$\rightarrow$	$\leftrightarrow$
Boolean algebra:	$\bar{p}$	pq	$+$	$\oplus$		
C/C++/Java (wordwise):	!	&&	$\vert\vert$	!=		==
C/C++/Java (bitwise):	~	&	$\vert$	^		
Logic gates:						

Propositional Logic:
Translating English Sentences

- English (and every other human language) is often ambiguous. Translating sentences into compound statements removes the ambiguity.
- Example: How can this English sentence be translated into a logical expression?

  "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Solution: Let q, r, and s represent "You can ride the roller coaster," "You are under 4 feet tall," and "You are older than 16 years old." The sentence can be translated into:

$$(r \land \neg s) \rightarrow \neg q.$$

Propositional Logic

- Example: How can this English sentence be translated into a logical expression?

  "You can access the Internet from campus if only if you are a computer science major or you are not a freshman."

Solution: Let a, c, and f represent "You can access the Internet from campus," "You are a computer science major," and "You are a freshman." The sentence can be translated into:

$$a \leftrightarrow (c \vee \neg f).$$

Propositional Logic:
Logic and Bit Operations

- Computers represent information using bits.
- A **bit** is a symbol with two possible values, 0 and 1.
- By convention, 1 represents T (true) and 0 represents F (false).
- A variable is called a Boolean variable if its value is either true or false.
- Bit operation – replace true by 1 and false by 0 in logical operations.

Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Propositional Logic

DEFINITION 7

A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

- Example: Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit string 01 1011 0110 and 11 0001 1101.

Solution:

01 1011 0110

11 0001 1101

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11 1011 1111 bitwise *OR*
01 0001 0100 bitwise *AND*
10 1010 1011 bitwise *XOR*