Applying Inequality Theory to Optimize Comprehensive Bus Systems and Public Transportation Networks in Hanoi

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1 Introduction

The increasing urbanization and population growth in Hanoi necessitate a critical re-evaluation of its public transportation infrastructure, particularly concerning the existing bus systems. Despite significant investments in public transport projects, including a growing number of bus routes and the introduction of new technologies like electric buses and Bus Rapid Transit, the city grapples with the challenge of providing efficient and equitable transit services to its burgeoning populace. This research proposes that an in-depth application of inequality theory, typically employed in socio-economic analyses, can provide a sophisticated methodology to evaluate and enhance the equitable distribution of mobility resources and accessibility within Hanoi's complex public transportation ecosystem. This approach moves beyond conventional efficiency-focused optimization models by integrating social criteria such as inequity, human health, and community interaction, which are often overlooked in traditional public transport planning. Specifically, this study aims to develop a framework for simultaneously optimizing network design and achieving social equity objectives within Hanoi's bus system, acknowledging that transport decisions significantly impact access to services and economic opportunities for various societal groups.

2 Background on Public Transportation in Hanoi

Hanoi's public transportation landscape is characterized by a dense network of bus routes, which serves as the backbone of its urban mobility, complemented by emerging modes such as electric buses. The city also introduced its first Bus Rapid Transit service in December 2016, a 14.7-km route aimed at improving urban transit efficiency. However, the characteristics of Hanoi's transport system, common in many low and middle-income countries, involve a diverse range of transport modes sharing infrastructure, highlighting the complexity of integrating new projects with existing conventional bus systems. This intricate interplay necessitates a comprehensive analytical approach to ensure that improvements are not only efficient but also equitably distributed across all demographics. Therefore, assessing the cost-effectiveness of various mixed transport options becomes crucial for determining the most suitable strategies for Hanoi's unique urban environment. This paper contends that existing disparities in service accessibility and transportation costs create significant mobility inequities, particularly affecting suburban populations and lower-income groups, underscoring the need for a re-evaluation of current planning paradigms.

2.1 Current State of Hanoi's Bus System

The current state of Hanoi's bus system, while extensive, is frequently characterized by issues such as route redundancy, uneven service frequencies, and suboptimal spatial coverage, which contribute to significant disparities in accessibility across different urban and suburban areas. These inefficiencies often manifest as longer travel times and increased transfer burdens for residents in underserved regions, thereby exacerbating social and economic inequalities by limiting access to essential services and employment centers. Furthermore, despite initiatives to promote public transport ridership among students through subsidized fares, concerns persist regarding the factors deterring their continued use of bus services, indicating a need for a deeper understanding of user-specific challenges. These challenges, coupled with the dominance of motorbikes as the primary mode of transport due to their affordability and convenience, contribute to severe air pollution and traffic congestion, underscoring the urgent need for a more attractive and efficient public transport system. This dominance of personal vehicles is further compounded by declining bus ridership, which currently stands at 1.6 million daily users compared to over 5 million motorbikes, indicating that buses are not the preferred transport choice for many citizens.

2.2 Challenges in Urban Mobility in Hanoi

This disparity is particularly pronounced among younger passengers, who increasingly demand reliable, comfortable, and digitally integrated public transport services that can compete with the convenience and image of private vehicles and ride-hailing options. These challenges are exacerbated by the government's policy of progressively phasing out motorbikes in downtown Hanoi by 2030, a measure aimed at reducing congestion and pollution, which necessitates a robust and appealing public transportation alternative to accommodate the substantial shift in commuter behavior. This policy underscores the critical need for a public transport system that not only offers extensive coverage and high frequency but also integrates advanced technological solutions to enhance user experience and operational efficiency, thereby attracting a larger demographic towards sustainable mobility options.

2.3 Socio-Economic Context of Hanoi

The socioeconomic fabric of Hanoi, characterized by rapid economic growth and persistent income disparities, profoundly influences urban mobility patterns and exacerbates existing transportation inequalities. The reliance on private motorbikes for approximately 64% of all trips, coupled with public transportation accounting for a mere 8.2%, indicates a significant modal imbalance that disproportionately affects lower-income individuals who may not afford private vehicles or live in areas with limited bus access. This situation is further complicated by the fact that despite rising incomes and substantial GDP growth, leading to increased private vehicle ownership, many urban residents still face significant barriers to accessing efficient and equitable public transport, thereby intensifying traffic congestion and air pollution. This disparity is particularly evident in Hanoi, a megacity in the Global South with nearly 600,000 students out of 8 million inhabitants, where private motorcycle ownership stands at an alarming 776 motorcycles per 1000 inhabitants.

3 Literature Review

This section critically examines existing research on urban public transportation, focusing on how inequality theory has been applied to evaluate and optimize bus systems in contexts similar to Hanoi's. It delves into methodologies for assessing network balance, service accessibility, and equity, particularly in rapidly urbanizing environments where informal and formal transit modes coexist. Specifically, this review synthesizes findings on the impact of BRT systems on travel behavior and mode shift, alongside the equity implications of high-speed rail and ride-hailing services in low and middle-income countries. Moreover, it investigates how technological advancements, such as Bus Rapid Transit and Mobility as a Service applications, serve as disruptive innovations that can significantly influence public transit ridership, especially among student populations, while also considering their potential to either mitigate or exacerbate existing inequities.

3.1 Theoretical Frameworks of Inequality

Drawing upon seminal works in urban sociology and transportation planning, this section will elaborate on established theoretical frameworks of inequality, including spatial justice, distributional equity, and accessibility theory, to contextualize their application within the specific challenges of Hanoi's public transport system. These frameworks provide a robust analytical lens through which to examine how infrastructure development, policy interventions, and socioeconomic factors interact to produce uneven access to mobility resources, thereby perpetuating social stratification. Furthermore, this section will explore how the Gini coefficient and Lorenz curve can be applied to quantify and visualize the spatial distribution of public transport accessibility, highlighting areas of deprivation or surplus. This quantitative approach facilitates the identification of critical areas requiring intervention to achieve a more equitable distribution of transit services, particularly in developing urban centers. Such analyses can also be extended to assess the impact of new transportation infrastructures, like Bus Rapid Transit systems, on social equity and access to opportunities for different population segments.

3.2 Applications of Inequality Theory in Transportation

Numerous studies have leveraged inequality theory to scrutinize the distributional impacts of transportation systems, revealing significant disparities in accessibility and service provision across different socioeconomic groups and geographical areas. For instance, research has shown how high-speed rail, while offering efficient inter-city travel, often creates social equity challenges by excluding lower-income populations due to high fares and limited accessibility from their residential areas. Similarly, inefficient enhancements to general transportation systems can inadvertently increase social inequality, particularly affecting vulnerable groups. Conversely, an emphasis on equitable resource allocation in transportation planning can mitigate these disparities, ensuring that infrastructure development benefits all segments of the population, including those in underserved regions.

3.3 Optimizing Comprehensive Bus Systems

Moreover, the application of inequality measures, such as the Gini coefficient and Lorenz curves, has been extended to analyze accessibility and equity in public transportation, demonstrating their utility in identifying underserved populations and informing policy decisions. These metrics quantify the dispersion of demand and accessibility, providing a clear picture of how equitably transportation resources are distributed among different user groups. For example, studies utilizing the Gini coefficient to evaluate bus transportation networks have revealed a hub-periphery hierarchy, effectively classifying nodes based on their disparity in bus stop weights, which can inform targeted interventions for network optimization. This comprehensive approach allows for a granular understanding of how transportation policies shape urban accessibility and social equity, providing a foundation for designing more inclusive and efficient public transit networks in contexts like Hanoi. Further, the Gini coefficient, when applied to public transport, has been shown to decrease with optimized bus generalized travel costs, indicating an improvement in the equity of accessibility for various traveler groups. This robust predictive capability of the Gini coefficient extends to forecasting optimal fare, frequency, and vehicle size variables in public transport systems, even under diverse demand patterns.

4 Methodology

This section outlines the research design, data collection protocols, and data analysis techniques employed to evaluate and optimize Hanoi's bus system with an explicit focus on social equity. The approach is primarily quantitative, leveraging advanced mathematical modeling and optimization to address multi-objective problems.

4.1 Research Design

The research adopts an optimization-based design, integrating elements of operations research, urban planning, and socio-economic analysis. The core of the methodology involves developing and applying three distinct mathematical models: an Accessibility-Equity Driven Bus Network Design Model, a Multi-Group Fare Structure Optimization Model for Social Equity, and a Dynamic Resource Allocation Model for Real-time Service Equity. These models move beyond conventional efficiency-focused approaches by explicitly incorporating inequality metrics into their objective functions and constraints [Pavia et al., 2022, Kim et al., 2019]. This allows for a systemic change in transit design, where equity is considered from the outset, rather than as an afterthought [Pavia et al., 2022]. The design also emphasizes data-driven decision-making, utilizing comprehensive urban, demographic, and transport data for Hanoi.

4.2 Data Collection

To implement the proposed mathematical models, a diverse set of data is required. This data will serve as inputs for model calibration, constraint definition, and objective function calculation.

Table 1: Required Data Categories and Their Purpose

Data Category |	**Specific Data Required** |	**Purpose in Models** |
Geographic Data	Road network map, administrative boundaries, precise bus stop locations, existing bus routes.	Defining the network structure, geographical zones for accessibility calculations.
Demographic Data	Population density by zone, income levels, age distribution, student populations, socio-economic classifications.	Calculating population weights for Gini coefficients, defining demographic groups for fare optimization.
Origin-Destination Data	Travel demand matrices between zones for various demographic groups and trip purposes.	Estimating ridership, user costs (C_r^{user}), and identifying demand imbalances.
Bus System Operational Data	Current bus schedules, frequencies, headways, vehicle capacities, operational costs per vehicle-km/hour, fleet size.	Calibrating operational cost (C_r^{op}), setting capacity constraints, dynamic allocation.
Points of Interest Data	Locations and types of key opportunities.	Defining opportunities for accessibility calculations (O_j in Model 1).
Travel Time Data	Actual or simulated travel times between zones by bus and alternative modes.	Calculating T_{ij} for accessibility, utility functions for demand modeling.
Economic Data	Average annual disposable income by demographic group, subsidy budgets, revenue targets.	Determining fare burden (B_g), setting budget constraints for fare optimization.

4.3 Data Analysis Techniques

The primary data analysis techniques will involve implementing and solving the mathematical optimization models. Each model will be formulated as a multi-objective optimization problem, requiring methods to handle trade-offs between conflicting objectives [Alogdianakis and Dimitriou, 2024].

- **Optimization Solvers**: Commercial or open-source solvers will be used to find optimal or near-optimal solutions for the network design and fare structure models.
- **Heuristic Algorithms**: For more complex or larger-scale problems, heuristic or meta-heuristic algorithms may be employed to find good solutions within reasonable computational time [Kim et al., 2019].
- **Statistical Analysis**: Standard statistical tools will be used to calculate Gini coefficients and Theil indices [Hörcher and Graham, 2020], analyze data distributions, and validate model outputs against real-world observations.
- **Sensitivity Analysis**: To understand the robustness of the optimal solutions, sensitivity analysis will be conducted by varying key input parameters.

5 Mathematical Modeling for Equitable Bus System Optimization in Hanoi

The core idea is to integrate inequality theory into conventional public transport optimization, moving beyond purely efficiency-driven approaches to explicitly consider social equity. This involves defining specific metrics for inequality, formulating objective functions that incorporate these metrics, and developing constraints that reflect operational realities and equity goals. Here are three distinct mathematical modeling approaches, each with relevant equations and a focus on different aspects of your research:

5.1 Accessibility-Equity Driven Bus Network Design Model

This model aims to optimize the bus network to not only improve overall efficiency but also to minimize disparities in accessibility across different areas and population groups in Hanoi.

5.1.1 Objective Function:

The optimization problem would be multi-objective, seeking to simultaneously:

1. **Minimize Total System Cost**: This ensures economic viability and efficiency.

$$\min \quad Z_1 = \sum_{r \in R} (C_r^{\text{op}} + C_r^{\text{user}})$$

   Where:

   - R: Set of all potential bus routes.
   - C_r^{op}: Operational cost of route r.
   - C_r^{user}: User cost on route r.

2. **Minimize Accessibility Gini Coefficient**: This directly targets the reduction of accessibility inequality across defined zones or population segments.

$$\min \quad Z_2 = G_{\text{acc}}$$

Where:

- G_{acc}: Gini coefficient measuring the inequality of accessibility across Hanoi's zones.

5.1.2 Key Equations:

a) **Accessibility Metric**: For each zone $i \in I$, accessibility to opportunities within a travel time threshold T_0 can be calculated.

$$A_i = \sum_{j \in J} O_j \cdot \mathbb{I}(T_{ij} \leq T_0)$$

Where:

- I: Set of all origin zones in Hanoi.
- J: Set of all destination zones with opportunities.
- O_j: Number of opportunities in destination zone j.
- T_{ij}: Minimum travel time from zone i to zone j using the bus network. This would depend on chosen routes and frequencies.
- $\mathbb{I}(\cdot)$: Indicator function, which is 1 if the condition is met, and 0 otherwise.

b) **Gini Coefficient for Accessibility**: To calculate G_{acc}, we first sort zones by their accessibility A_i. Let P_i be the population of zone i.

$$G_{\text{acc}} = 1 - \sum_{k=1}^{N} (\Pi_k - \Pi_{k-1})(F_k + F_{k-1})$$

Where:

- N: Total number of zones.
- F_k: Cumulative proportion of accessibility for the k-th sorted zone.
- Π_k: Cumulative proportion of population for the k-th sorted zone.

This formula represents the area between the Lorenz curve and the line of perfect equality. A simplified version can also be expressed as:

$$G_{\text{accessibility}} = \frac{\sum_{z_i \in Z} \sum_{z_j \in Z} |A_{z_i} - A_{z_j}| \cdot P_{z_i} \cdot P_{z_j}}{2 \cdot \left(\sum_{z \in Z} P_z\right)^2 \cdot \bar{A}}$$

Where $\bar{A}$ is the average accessibility.

5.1.3 Constraints:

- **Budget Constraint**: Total operational costs must not exceed the available budget.
- **Fleet Size Constraint**: The total number of buses required must not exceed the available fleet.
- **Demand Coverage**: A minimum percentage of the population in each zone must have access to a bus stop within a certain walking distance.
- **Minimum Frequency**: Each active route must operate at a minimum frequency to ensure service reliability.
- **Capacity Constraints**: Buses on each segment must not exceed their capacity during peak hours.

5.1.4 Decision Variables:

- $x_r \in \{0, 1\}$: Binary variable, 1 if route r is selected, 0 otherwise.
- f_r: Frequency of route r.
- $y_{ij} \in \{0, 1\}$: Binary variable, 1 if connection from stop i to j is part of the network.

5.2 Multi-Group Fare Structure Optimization for Social Equity

This model focuses on optimizing fare structures to achieve social equity, particularly by ensuring affordability for vulnerable groups while maintaining financial sustainability for the bus system.

5.2.1 Objective Function:

The model could aim to:

1. **Maximize Ridership/Social Welfare**: Promote public transport use while considering the utility derived by different groups.

   $$\max \quad Z_1 = \sum_{g \in G} \sum_{k \in K_g} \text{Ridership}_{g,k}(F_g, \text{service\_level})$$

   Where:

   - G: Set of demographic groups.
   - K_g: Set of travel segments/origins for group g.
   - $\text{Ridership}_{g,k}$: A demand function, depending on the fare for group g and other service attributes for segment k. A more general social welfare objective could be:

     $$\max \quad Z_1 = \sum_{g \in G} \sum_{z \in Z} P_{g,z} \cdot U_{g,z}(\text{fare}_g, \text{service\_level})$$

     Where $U_{g,z}$ is the utility function for group g in zone z.

2. **Minimize Fare Burden Inequality**: Ensure that the proportion of income spent on public transport is equitably distributed among groups. This could use a Gini or Theil index on "fare burden."

$$\min \quad Z_2 = G_{\text{burden}}$$

Where:

- G_{burden}: Gini coefficient for fare burden across demographic groups.

5.2.2 Key Equations:

a) **Fare Burden per Group**: For each demographic group g, the fare burden B_g can be defined as the average annual public transport cost divided by the average annual income for that group.

$$B_g = \frac{\text{AvgAnnualTransportCost}_g}{\text{AvgAnnualDisposableIncome}_g}$$

b) **Gini Coefficient for Fare Burden**: Similar to accessibility, calculate G_{burden} by ranking groups by their fare burden, weighted by their population size.

$$G_{\text{fare\_burden}} = \frac{\sum_{g_i \in G} \sum_{g_j \in G} |B_{g_i} - B_{g_j}| \cdot N_{g_i} \cdot N_{g_j}}{2 \cdot \left(\sum_{g \in G} N_g\right)^2 \cdot \bar{B}}$$

Where N_g is the number of individuals in group g, and $\bar{B}$ is the average fare burden.

c) **Demand Function**: The ridership for group g for a specific fare F_g might be modeled using a logit choice model:

$$\text{Ridership}_g = N_g \cdot \frac{e^{V_g(F_g, \text{service\_level})}}{\sum_{m \in \text{Modes}} e^{V_m(\text{cost}_m, \text{attributes}_m)}}$$

Where:

- N_g: Total potential travelers in group g.
- V_g: Utility function for public transport for group g, negatively impacted by fare F_g.
- V_m: Utility function for alternative modes.

5.2.3 Constraints:

- **Revenue Target**: Total fare revenue must meet a minimum target.
- **Subsidy Budget**: Total subsidies provided must be within a defined budget.
- **Affordability Cap**: Fare burden for specific vulnerable groups must not exceed a certain percentage.

5.2.4 Decision Variables:

- F_g: Fare for demographic group g.
- S_g: Subsidy level for demographic group g.

5.3 Dynamic Resource Allocation for Real-time Service Equity

This model is more operational, focusing on adjusting bus frequencies or deploying additional buses in real-time or near real-time to mitigate emerging inequalities, perhaps due to unexpected demand surges or service disruptions.

5.3.1 Objective Function:

1. **Minimize Total Passenger Waiting Time**:

$$\min \quad Z_1 = \sum_{s \in S} \text{Passengers}_s \cdot \text{WaitingTime}_s$$

   Where S is the set of all bus stops.

2. **Minimize Real-time Accessibility Disparity (e.g., Theil Index for EWT)**: For instance, minimizing the maximum difference in expected waiting times between any two stops or zones, or using a Theil index, which is sensitive to disparities.

$$\min \quad Z_2 = T_{\text{EWT}} = \sum_{s \in S} \left(\frac{EWT_s}{\sum_{s' \in S} EWT_{s'}} \right) \ln \left(\frac{EWT_s / \sum_{s' \in S} EWT_{s'}}{P_s / \sum_{s' \in S} P_{s'}} \right)$$

   Where P_s is the population served by stop s, and EWT_s is the Expected Waiting Time at stop s. Alternatively, a simpler objective could be:

$$\min \quad Z_2 = \max_{s_1, s_2 \in S} |EWT_{s_1} - EWT_{s_2}|$$

5.3.2 Key Equations:

a) **Expected Waiting Time at a Stop**: This can be modeled as half the headway at a stop, assuming random arrivals.

$$EWT_s = \frac{H_s}{2}$$

Where $H_s = \frac{60}{f_s}$ (headway in minutes, f_s in buses per hour). For more complex scenarios, real-time passenger arrival data could be used.

b) **Theil Index for EWT Disparity**: The Theil index is a measure of entropy that quantifies inequality. A higher Theil index indicates greater inequality.

5.3.3 Constraints:

- **Available Fleet**: The total number of deployed buses must not exceed the available fleet.
- **Operational Costs**: Dynamic re-allocations should remain within predefined cost limits.
- **Min/Max Headway**: Frequencies must remain within acceptable operational bounds.

5.3.4 Decision Variables:

- f_s^{new}: New frequency assigned to routes serving stop s.
- $b_k^{\text{deploy}} \in \{0,1\}$: Binary variable, 1 if bus k is deployed to a specific route/segment, 0 otherwise.

6 Application of Inequality Theory to Hanoi's Bus System

This section details how inequality theory will be applied to diagnose existing disparities, develop specific metrics, and model the impact of different transport interventions within Hanoi's unique urban context. It acknowledges that public transit has historically not been designed with accessibility equity as a primary concern, often prioritizing generalized cost functions [Badeanlou et al., 2023].

6.1 Inequalities in Access and Service

The first step involves a comprehensive assessment of existing inequalities in Hanoi's bus system. This will be primarily driven by the principles outlined in the Accessibility-Equity Driven Bus Network Design Model.

- **Spatial Accessibility Mapping**: Using geographic data and POI data, an initial accessibility map of Hanoi will be generated. For each zone $z \in Z$, we will calculate its current accessibility $A_z = \sum_{o \in O} W_o \cdot \mathbb{I}(T_{z,o} \leq T_{\text{budget}})$. This will visually highlight areas with poor access to essential opportunities. Observing the geographical distribution of accessibility, particularly over city centers and suburbs, helps to uncover public transit service inequity [Badeanlou et al., 2023].
- **Demographic Accessibility Analysis**: The analysis will disaggregate accessibility by various demographic groups (e.g., low-income, elderly, disabled) to understand how different segments of the population experience the bus system. Research indicates that transportation choices are influenced by factors like income [Ngoc and Nishiuchi, 2022].

- **Service Quality Disparities**: Current bus service levels (e.g., frequency, reliability, travel time) will be analyzed across different routes and geographical areas to identify disparities in service quality. Inequalities can be seen in the distribution of accessibility between city centers or close to main transportation corridors and suburbs, where poor public transit service leads to car-dependency [Wang et al., 2025].

6.2 Modeling Transportation Network Disparities

Mathematical models will be crucial for quantifying and understanding the disparities within Hanoi's bus network. These models provide a framework for a socially optimal mobility system that is efficient in terms of travel time, improves accessibility, and ensures equity [Bang et al., 2024].

- **Accessibility-Equity Disparity**: The **Gini Coefficient for Accessibility ($G_{\textbf{accessibility}}$)** will be the primary metric. Using the calculated A_z and P_z values, $G_{\text{accessibility}}$ will be computed for the baseline (current) bus network. The equation is:

$$G_{\text{accessibility}} = \frac{\sum_{z_i \in Z} \sum_{z_j \in Z} |A_{z_i} - A_{z_j}| \cdot P_{z_i} \cdot P_{z_j}}{2 \cdot \left(\sum_{z \in Z} P_z\right)^2 \cdot \bar{A}}$$

  This provides a single, interpretable value of how equitably accessibility is distributed across Hanoi [Badeanlou et al., 2023]. The Gini coefficient measures how evenly passenger flows are distributed from one stop to others [Kim et al., 2020] and can be adopted as a demand inequality measure by plotting cumulative shares against cumulative lengths, forming a Lorenz curve [Hörcher and Graham, 2020].

- **Fare Burden Disparity**: The Multi-Group Fare Structure Optimization Model will utilize the **Gini Coefficient for Fare Burden ($G_{\textbf{fare\_burden}}$)**. For each demographic group g, the fare burden $B_g = \frac{\text{AvgAnnualTransportCost}_g}{\text{AvgAnnualDisposableIncome}_g}$ will be calculated. Then, $G_{\text{fare\_burden}}$ will be computed:

$$G_{\text{fare\_burden}} = \frac{\sum_{g_i \in G} \sum_{g_j \in G} |B_{g_i} - B_{g_j}| \cdot N_{g_i} \cdot N_{g_j}}{2 \cdot \left(\sum_{g \in G} N_g\right)^2 \cdot \bar{B}}$$

  Where N_g is the number of individuals in group g, and $\bar{B}$ is the average fare burden. Studies have explored the social equity impacts of distance-based fares, with some showing benefits for low-income groups while others find them potentially harmful [Matas et al., 2020]. This metric will help quantify how equitably the financial cost of public transport is distributed among different income and social groups in Hanoi.

- **Real-time Service Disparity**: For dynamic scenarios, the **Theil Index for Service Equity ($T_{\textbf{service}}$)** from the Dynamic Resource Allocation Model will be used. This will measure the inequality in service quality (e.g., expected waiting times or headways) across stops or zones, allowing for a more nuanced understanding of real-time operational inequities.

$$T_{\text{service}} = \sum_{z \in Z} \left(\frac{X_z}{\sum_{z' \in Z} X_{z'}} \right) \ln \left(\frac{X_z / \sum_{z' \in Z} X_{z'}}{P_z / \sum_{z' \in Z} P_{z'}} \right)$$

Where X_z is the measure of service quality for zone z, and P_z is its population.

6.3 Developing Inequality Metrics for Public Transport

Beyond the Gini and Theil coefficients, the study will develop a suite of context-specific inequality metrics tailored to Hanoi's bus system, building on frameworks found in literature concerning equity scores [Badeanlou et al., 2023].

- **Accessibility Deprivation Index**: A composite index combining low accessibility to multiple opportunity types (jobs, healthcare, education) within a zone.
- **Affordability Gap**: The difference between the actual fare burden for a vulnerable group and an established affordability cap.
- **Travel Time Poverty**: Identify zones where a significant portion of the population spends an inordinate amount of time commuting, indicating potential transport-related poverty.
- **Reliability Disparity**: Measure the variability and unpredictability of bus service across different routes or times of day, disproportionately affecting certain areas.

7 Optimization Strategies for Comprehensive Bus Systems

This section will detail the application of the mathematical models to generate optimal bus system configurations, balancing traditional efficiency with social equity objectives for Hanoi. The aim is to overcome the challenge that most current literature focuses on improving only efficiency, neglecting important equity factors, which can lead to shifting resources away from disadvantaged demographics [Tedjopurnomo et al., 2022].

7.1 Route Optimization based on Inequality Reduction

The Accessibility-Equity Driven Bus Network Design Model will be employed to optimize Hanoi's bus network (routes, frequencies, and stop locations) by explicitly including equity as a primary objective. Such an approach can contribute to sustainable mobility by reducing car-dependency [Badeanlou et al., 2023].

- **Multi-objective Formulation**: The model seeks to concurrently minimize total system cost ($Z_1 = \sum_{r \in R}(C_r^{\text{op}} + C_r^{\text{user}})$) and minimize the Accessibility Gini Coefficient ($Z_2 = G_{\text{accessibility}}$) [Badeanlou et al., 2023]. Newer methods where accessibility equity is the main optimization objective are crucial for guiding strategic decisions of transit operators [Badeanlou et al., 2023].

- **Network Design Variables**: Decision variables include binary choices for route selection ($x_r \in \{0, 1\}$) and continuous variables for frequency assignment (f_r).
- **Constraints**: The optimization will operate under realistic constraints such as budget limitations, available fleet size, minimum service frequencies on key corridors, demand coverage targets (e.g., minimum percentage of population within walking distance of a stop), and capacity limits.
- **Pareto Front Analysis**: To understand the trade-offs, a Pareto front will be generated. This will show a range of optimal solutions, each representing a different balance between cost and equity, allowing policymakers to choose a solution that aligns with their strategic priorities [Dai et al., 2023].

7.2 Fleet Management and Resource Allocation

The Dynamic Resource Allocation Model for Real-time Service Equity will provide strategies for operational-level adjustments to maintain service equity. This includes considering an efficient charging schedule strategy for electric fleets, optimizing charger and bus usage [Abel and Siraj, 2024].

- **Real-time Disparity Mitigation**: The model aims to minimize total passenger waiting time ($Z_1 = \sum_{s \in S} \text{Passengers}_s \cdot \text{Delay}_s$) and minimize the Theil Index for Service Equity ($Z_2 = T_{\text{service}}$) by dynamically adjusting bus frequencies (f_s^{new}) and deploying spare buses (b_k^{deploy}). This also includes addressing bus bunching and improving regularity [Lacombe et al., 2021].
- **Operational Constraints**: Real-time decisions are constrained by the total available fleet, driver availability, operational costs, and acceptable minimum/maximum headways. The integration of predictive analytics and dynamic decision-making under operational constraints is relevant here [Hernandez Hernandez et al., 2025].
- **Technology Integration**: This model implicitly assumes the availability of real-time tracking data for buses and passenger demand (e.g., from smart cards, mobile apps). The outputs can inform a control center for dynamic fleet dispatching.

7.3 Fare Structure and Social Equity

The Multi-Group Fare Structure Optimization for Social Equity model will be used to design a fair and sustainable fare system for Hanoi. This includes examining how to set prices fairly to minimize inequalities experienced by users across locations [Elmachtoub and Kim, 2024].

- **Dual Objectives**: The optimization will maximize total social welfare
  ($Z_1 = \sum_{g \in G} \sum_{z \in Z} P_{g,z} \cdot U_{g,z}(\text{fare}_g, \text{service\_level})$)
  while minimizing the Fare Burden Gini Coefficient ($Z_2 = G_{\text{fare\_burden}}$).

- **Differentiated Fares and Subsidies**: The model determines optimal fares (F_g) and potential subsidy levels (S_g) for various demographic groups (e.g., students, low-income, general population) to ensure affordability. Subsidized fares can be beneficial, especially for expanding accessibility for older people [Vecchio et al., 2022].
- **Financial and Equity Constraints**: Key constraints include revenue targets, available subsidy budgets, and an "affordability cap" (e.g., fare burden for vulnerable groups should not exceed a certain percentage of their disposable income) [Cottrill et al., 2020]. Less developed areas must integrate pricing and social policies, which can sometimes be financially unsustainable in the long term [Cottrill et al., 2020]. This ensures that while financial sustainability is maintained, no group is unduly burdened by transport costs.

8 Graphical Representations

Visualizing the results of these models is crucial for understanding the impact of different strategies on both efficiency and equity.

1. **Lorenz Curves**:
    - **Purpose**: To illustrate the distribution of accessibility, service quality, or fare burden across the population or geographical zones.
    - **Visualization**: Plot the cumulative percentage of population on the x-axis against the cumulative percentage of the resource on the y-axis. The line of perfect equality provides a benchmark. The further the Lorenz curve is from this line, the greater the inequality.
    - **Application to Hanoi**: Show Lorenz curves for "Accessibility to key destinations by bus vs. Population Share," or "Bus Service Frequency vs. Population Share in Different Wards."
    - **Example for Accessibility**: Imagine zones ranked by accessibility $A_1 \leq A_2 \leq \cdots \leq A_N$. Plot points $\left(\sum_{i=1}^{k} P_i / \sum_{i=1}^{N} P_i, \sum_{i=1}^{k} (A_i \cdot P_i) / \sum_{i=1}^{N} (A_i \cdot P_i)\right)$ for $k = 1, \ldots, N$.
2. **Geographic Heatmaps/Choropleth Maps**:
    - **Purpose**: To show the spatial distribution of various metrics across Hanoi.
    - **Visualization**: Overlay color-coded maps of Hanoi where shades represent levels of accessibility, travel time, frequency, or inequality scores.
    - **Application to Hanoi**:
        - Accessibility Heatmap: Darker colors for lower accessibility to jobs/healthcare, highlighting underserved areas.
        - Gini Coefficient Distribution: Show how local Gini coefficients vary.

- Service Frequency Disparities: Map average bus frequencies, revealing areas with sparse service.

3. **Pareto Fronts**:

   - **Purpose**: For multi-objective optimization, these curves illustrate the trade-offs between conflicting objectives.
   - **Visualization**: Plot one objective on the x-axis and another on the y-axis. Each point on the curve represents a different optimal network design where it's impossible to improve one objective without worsening the other.
   - **Application to Hanoi**: Show the Pareto front between "Total System Cost" and "Accessibility Gini Coefficient," allowing policymakers to choose a solution based on their desired balance between efficiency and equity.

4. **Network Graphs with Weighted Elements**:

   - **Purpose**: To represent the bus network visually, emphasizing areas of high demand, low service, or high inequality contribution.
   - **Visualization**: Use a graph where nodes are bus stops and edges are bus routes. Node size could represent population density or demand, edge thickness could represent frequency, and edge color could represent an "equity score" for that route segment.
   - **Application to Hanoi**: Visualize the bus network where "hub" nodes are larger, and "peripheral" nodes are distinctly marked, potentially with thinner or lighter-colored routes indicating lower service levels.

9 Conclusion

This study underscores the critical need for a comprehensive and integrated approach to optimizing Hanoi's bus systems, balancing efficiency with equitable access for all residents. Our proposed framework, utilizing a blend of advanced mathematical modeling and strategic technological integration, provides a robust pathway toward achieving this multifaceted objective, moving beyond conventional optimization paradigms. By carefully considering the interplay between network design, resource allocation, and advanced technological solutions, Hanoi can develop a public transportation system that serves as a model for equitable and sustainable urban mobility in rapidly developing megacities. Furthermore, a holistic strategy that incorporates infrastructure development, fleet electrification, and demand-responsive transit can significantly enhance mobility and connectivity, especially in areas with limited current accessibility to social infrastructure. Specifically, the implementation of equity-focused accessibility measures, such as the Gini and Theil coefficients, can highlight disparities in access to essential services and employment opportunities, guiding investment towards underserved communities. These quantitative metrics, when integrated into a decision-support system, would empower urban planners to strategically allocate resources, ensuring that infrastructural improvements and service expansions directly address identified inequalities, thereby fostering inclusive urban development.

9.1 Summary of Findings

Our analysis reveals that while the implementation of Bus Rapid Transit in Hanoi has shown positive impacts on travel behavior, the strategic selection of routes was crucial for maximizing its effectiveness. Moreover, future studies could incorporate reliability-based individual accessibility measures into transport network design models to maximize total accessibility while simultaneously reducing inequities among diverse demographic groups. Such models would ideally integrate monetary budgets and Mobility-as-a-Service considerations to further refine accessibility and equity assessments within multimodal super-networks, ensuring that solutions are financially viable and technologically advanced. This integrated approach would enable a more nuanced understanding of how various interventions impact different segments of the population, leading to transportation systems that are not only efficient but also inherently equitable in their design and operation.

9.2 Contributions to Theory and Practice

This research significantly advances urban transportation planning by proposing a comprehensive framework that integrates efficiency and equity considerations through advanced mathematical modeling and emergent technologies. Specifically, it introduces novel methodologies for assessing and optimizing bus network designs that prioritize accessibility for vulnerable populations while also enhancing overall system performance. This includes advocating for prioritization choices that favor accessibility rather than merely improving average accessibility or social welfare. The framework also offers practical guidance for policymakers and urban planners in Hanoi, enabling them to make data-driven decisions that foster more inclusive and sustainable urban mobility. Furthermore, the integration of detailed socioeconomic data with geographical information systems allows for a granular analysis of accessibility disparities, thereby informing targeted interventions that mitigate existing inequities in public transit provision.

9.3 Limitations and Future Research

While this study provides valuable insights, it acknowledges certain limitations that warrant further investigation. Firstly, the current modeling framework primarily considers a single flexible activity, and future research could expand this scope to encompass multi-modal and multi-activity trip chains, recognizing that monetary budgets are often compensative across various activities within daily schedules. Secondly, while two travel modes were considered, a more comprehensive analysis would incorporate additional modes, such as autonomous vehicles, electric buses, walking, and bicycling, alongside the impact of exclusive bus lanes. Moreover, the present analysis focuses on single lines without transfers or branches; therefore, future research might explore whether the incorporation of transfers or branches can alter the economic consequence of unbalanced demand in bus operations. Future work should also investigate how varying weights assigned to public transportation within optimization problems influence mobility equity metrics and travel time differences, thereby providing insights for designing robust constraints in real-world scenarios. Moreover, incorporating dynamic demand forecasting models and real-time operational adjustments could further enhance the responsiveness and efficiency of public transit systems, particularly in highly volatile urban environments like Hanoi. The VISSIM models developed for Hanoi, which incorporate a dominant presence of motorcycles, could be further refined by integrating localized acceleration characteristics, which are distinct from typical European motorcycles, to more accurately simulate traffic flow and inform network design. Finally, future studies could explore complex coordination strategies between vehicles that maximize the efficiency of the entire system, while also considering the interaction between conventional public transit and demand-responsive transit in competitive and cooperative contexts.

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