Answers

```
# install package
#install.packages(c('psych'))

library(psych)
```

Question 1

```
# reading covariance matrix into R
bears <- read.table('bears_covariance.txt', sep = ',', header = T)
```

a.

```
bears[2,3]; bears[3,2]
```

```
## [1] 324.25
```

```
## [1] 324.25
```

```
all.equal(bears[2,3], bears[3,2])
```

```
## [1] TRUE
```

Both are equal because they are mirror images of themselves or symmetric to each other. For example, within the rows, the second variable is body length while the third variable along the columns is neck circumference (bears[2,3]). Also, along the rows, the third variable is neck circumference while the second variable along the columns is body length.

b.

```
# verifying that it is a covariance matrix
isSymmetric.matrix(unname(as.matrix(bears)))
```

```
## [1] TRUE
```

The covariance above is a covariance matrix

c. Performing PCA

```r
# PCA
# Getting the eigen values and vectors for the covariance matrix
pca <- eigen(bears, symmetric = T)

# get eigen values and vectors
eigen_vectors <- pca$vectors
eigen_values <- pca$values

# get the scaled variances
scaled_variance <- eigen_values/ sum(eigen_values)

scaled_variance
```

```
## [1] 0.9582856531 0.0326211775 0.0069159101 0.0017366358 0.0003245417
## [6] 0.0001160817
```

The data can be summarized in a fewer number of variables. From the PCA result above, the first principal component has the highest variance of about 95%. Hence, the first principal component (weights) can be used as the only variable since about 96% of the variance comes from its direction.

d.

The first principal component is where the variability in the data is most. From the result in part c, it accounts for about 96% of the variance in the data.

e.

```r
eigen_vectors <- as.data.frame(eigen_vectors)
names(eigen_vectors)  <- paste0('PC', 1:6)
row.names(eigen_vectors) <- names(bears)


print.data.frame(eigen_vectors)
```

```
##                            PC1         PC2         PC3         PC4          PC5
## weight              0.84933911  0.47083174  0.22660616 -0.07426037  0.008692249
## body_length         0.36855162 -0.84607780  0.36813225 -0.01275430  0.110783709
## neck_circumference  0.19413158 -0.05812709 -0.30314314  0.92838758  0.012288753
## girth               0.31467769 -0.21674766 -0.84857608 -0.35506048  0.082353215
## head_length         0.04391809 -0.06035442 -0.00181524  0.06016158 -0.440118616
## head_width          0.06445825 -0.09202640 -0.03388032 -0.05226733 -0.887138069
##                             PC6
## weight               0.0002016017
## body_length          0.0191050346
## neck_circumference   0.0705967427
## girth               -0.0326656761
## head_length         -0.8928053752
## head_width           0.4432635475
```

From above, the variable with the most influence in the first principal component is weight, while the variable with the least influence is head_length

Question 2

a.

```
wine <- read.csv('wine.csv', )

# extract relevant data
data <- wine[c('chem3', 'chem5', 'chem6', 'chem7', 'chem8', 'chem9')]

# checking for missing values
sapply(data, function(x) sum(is.na(x)))
```

```
## chem3 chem5 chem6 chem7 chem8 chem9
##     0     0     0     0     0     0
```

No missing values in any of the extracted variables

b.

```
get_summary_stat <- function(x){
  mean <- mean(x)
  median <- median(x)
  sd <- sd(x)
  range <- max(x) - min(x)
  c(mean=mean, median=median, sd =sd, range=range)
}


# get summary data
sapply(data, get_summary_stat)
```

```
##           chem3    chem5    chem6     chem7     chem8     chem9
## mean   2.366517 99.74157 2.295112 2.0292697 0.3618539 1.5908989
## median 2.360000 98.00000 2.355000 2.1350000 0.3400000 1.5550000
## sd     0.274344 14.28248 0.625851 0.9988587 0.1244533 0.5723589
## range  1.870000 92.00000 2.900000 4.7400000 0.5300000 3.1700000
```

The mean and standard deviation of the chemicals would be very relevant for the analysis asked by IWA. This is because, the mean will give information about the average concentration of chemicals in the wines grown within the same region in Italy. Similary, the standard deviation will provide information about how the concentrations of these chemicals deviate from themselves.

c.

```
par(mfrow=c(2,3))
for (col in names(data)) boxplot(wine[col], main=col, ylab='Concentration')
```

The boxplot (figure 1) above shows the concentration distributions for smell, taste and alcohol chemicals. From the plot, chem9, a smell chemical has two values that are outliers. Also, for the taste chemicals, both have outliers in them, while for alcohol chemicals, there are no outliers seen in them.

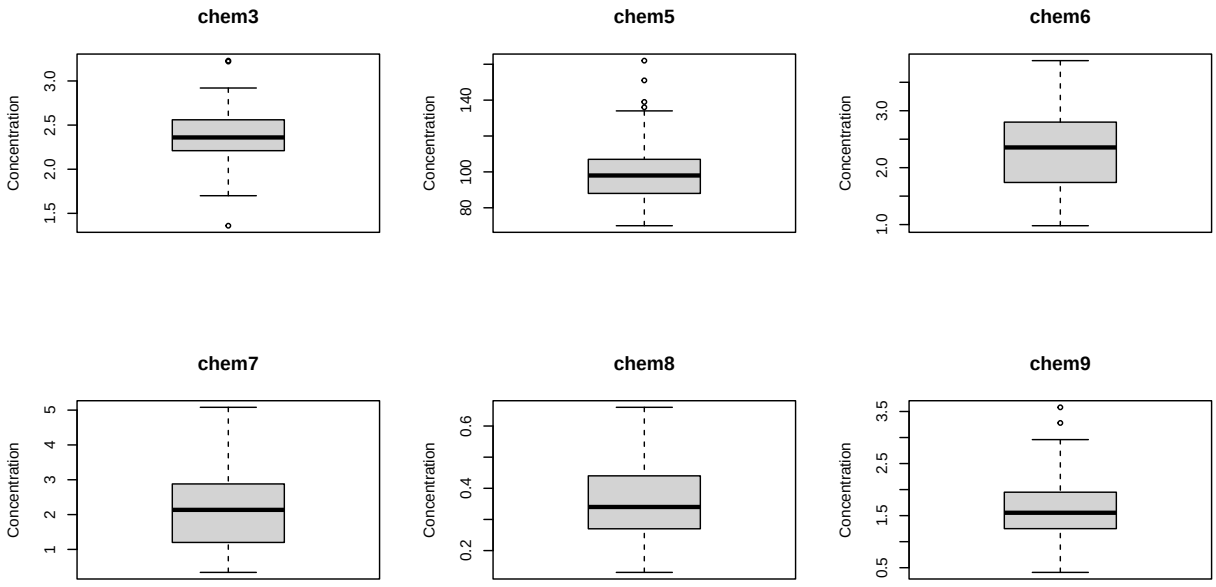


Figure 1: Figure 1: Checking for Outliers

d. Mahalanobis distance

```
m_obis <- mahalanobis(data, colMeans(data),  cov(data))
pvalues <- pchisq(m_obis, df=ncol(data)-1, lower.tail = F)

# outliers are considered as values < 0.001
cat('Number of observations with outliers is: ', sum(pvalues < 0.001), '\n\n')
```

```
## Number of observations with outliers is:  5
```

```
# print the data points
print.data.frame(data[which(pvalues < 0.001),])
```

```
##      chem3 chem5 chem6 chem7 chem8 chem9
## 60    1.36    88  1.98  0.57  0.28  0.42
## 70    1.75   151  1.85  1.28  0.14  2.50
## 96    2.20   162  2.50  2.27  0.32  3.28
## 111   1.82   107  3.18  2.58  0.24  3.58
## 122   3.23   119  3.18  5.08  0.47  1.87
```

There are 5 observations that are outliers. They should not be investigated further since the number of observations found to contain outliers is only a small number (~2.8%) out of the 178 records.

e.

```
psych::pairs.panels(data, lm=FALSE, smooth=F,density=F, pch=20, hist.col='steelblue',
                    jiggle=F, ellipses=F, breaks=25, cex.cor=0.7, stars=T)
```

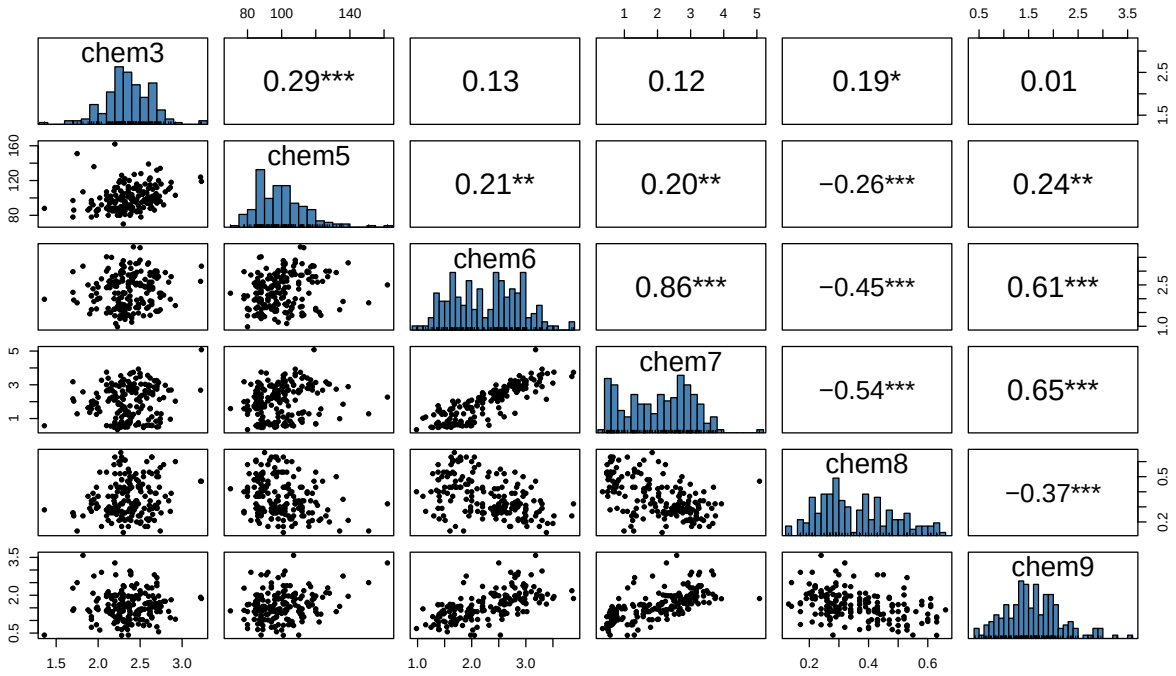


Figure 2: Figure 2: ScatterPlot Matrix

Figure 2, shows the scatterplot matrix of all the variables in the data. The lower matrix shows a scatterplot showing the relationship between one variable with the other. On the other side of the matrix is the correlation coefficient scores and their statistical significance (pvalues). The correlation coefficient is a numerical value that shows the linear relationship between one variable and another. Some of the values show that they are statistically significant at 5% threshold, while in some others, a relationship exists but it shows that these relationships are not statistically significant at 5% level. At the diagonal is the histogram chart showing the distribution of each of the variables (chemicals). From the above plot, there is a weak to strong relationship between one variable and another, except for chem 3 and chem 9.

f.

```
# performing PCA
pca <- prcomp(data, scale. = T)
summary(pca)
```

```
## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6
## Standard deviation     1.6994 1.1156 0.9260 0.73794 0.58473 0.35135
## Proportion of Variance 0.4813 0.2074 0.1429 0.09076 0.05699 0.02057
## Cumulative Proportion  0.4813 0.6887 0.8317 0.92244 0.97943 1.00000
```

```
pca$rotation
```

```
##              PC1          PC2        PC3        PC4        PC5         PC6
## chem3 -0.0683442 -0.799074970  0.3042731 -0.3363680  0.3840889  0.05965232
## chem5 -0.2376977 -0.518833910 -0.7046603  0.2829717 -0.3051573 -0.06760970
## chem6 -0.5241291  0.008255488  0.2788713 -0.1184179 -0.4972287  0.62144359
## chem7 -0.5415729  0.052491399  0.2382900 -0.1673048 -0.2044400 -0.75985178
## chem8  0.3945918 -0.292062832  0.5034399  0.5367731 -0.4440547 -0.14224943
## chem9 -0.4638035  0.064547416  0.1512238  0.6903944  0.5226261  0.08982632
```

From the cumulative variance proportion, 5 principal components is sufficient to be retained since they can account for a combined 98% variance in the data.

g.

The second largest eigenvalue is the second principal component. It is the direction where the variance is the most after the first component. It is usually the direction which is not accounted by the first principal component and is orthogonal to it. From the value, the second eigenvalue accounts for about 21% of the variance in the data.

h.

$$y = PC_1 + PC_2 + PC_3 + PC_4 + PC_5$$

where,

$$PC_1 = -0.07 * X_{chem3} - 0.24 * X_{chem5} - 0.52 * X_{chem6} - 0.54 * X_{chem7} + 0.39 * X_{chem8} - 0.46 * X_{chem9}$$

$$PC_2 = -0.80 * X_{chem3} - 0.51 * X_{chem5} + 0.01 * X_{chem6} + 0.05 * X_{chem7} - 0.29 * X_{chem8} + 0.06 * X_{chem9}$$

$$PC_3 = 0.30 * X_{chem3} - 0.70 * X_{chem5} + 0.28 * X_{chem6} + 0.24 * X_{chem7} + 0.50 * X_{chem8} + 0.15 * X_{chem9}$$

$$PC_4 = -0.34 * X_{chem3} + 0.28 * X_{chem5} - 0.12 * X_{chem6} - 0.17 * X_{chem7} + 0.54 * X_{chem8} + 0.69 * X_{chem9}$$

$$PC_5 = 0.38 * X_{chem3} - 0.31 * X_{chem5} - 0.50 * X_{chem6} - 0.20 * X_{chem7} - 0.44 * X_{chem8} + 0.52 * X_{chem9}$$

i.

From the cumulative variance proportion, the first two PCs measure about 69% different aspects in the data.

Question 3

a.

```r
households <- read.csv('household2.csv')

# selecting relevant data
households <- households[paste0('Q', 1:9)]
```

Gender should not be included because it is nominal data and does not fit conceptually since the aim is to identify the different aspects of housing that appeal much to individuals, irrespective of gender.

b.

An EFA will be preferred over PCA because the aim is to find out if there are any dimensions that underlie satisfaction with living conditions.

c.

```r
# Bartlett test for sphericity

cortest.bartlett(cor(households), n=nrow(households))
```

```
## $chisq
## [1] 11961.44
##
## $p.value
## [1] 0
##
## $df
## [1] 36
```

```r
bartlett.test(households)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  households
## Bartlett's K-squared = 22.343, df = 8, p-value = 0.004318
```

From the result of the Bartlett test for sphricity, the pvalue is less than 0.05, hence we will reject the null hypothesis that there's no correlation amongst variables and conclude that at 5% significance level there is evidence to show that there's correlation between variables.

d.

```r
# Kaiser-Meyer-Olkin factor adequacy
KMO(households)
```

```
## Kaiser-Meyer-Olkin factor adequacy
## Call: KMO(r = households)
## Overall MSA =  0.96
## MSA for each item =
##   Q1   Q2   Q3   Q4   Q5   Q6   Q7   Q8   Q9
## 0.96 0.96 0.95 0.96 0.96 0.96 0.96 0.97 0.96
```

From the Kaiser=Meyer-Olkin factor adequacy result, the overall MSA score is 0.96 showing very excellent suitability for EFA. Also from their individual MSA scores, they all have scores above 0.9. Therefore, all factors should be investigated for EFA.

e.

```
# fitting an EFA model
efa_model <- fa(households, nfactors = ncol(households), fm='minres', rotate = 'none')

efa_model
```

```
## Factor Analysis using method =  minres
## Call: fa(r = households, nfactors = ncol(households), rotate = "none",
##     fm = "minres")
## Standardized loadings (pattern matrix) based upon correlation matrix
##     MR1   MR2   MR3   MR4   MR5   MR6   MR7   MR8 MR9   h2    u2 com
## Q1 0.93 -0.09 -0.12  0.08  0.00 -0.07  0.04 -0.01   0 0.90 0.095 1.1
## Q2 0.89 -0.16  0.18  0.09  0.02 -0.02 -0.01  0.03   0 0.87 0.133 1.2
## Q3 0.87  0.29 -0.03 -0.01  0.04  0.08  0.02  0.02   0 0.86 0.144 1.2
## Q4 0.86  0.25  0.15  0.07 -0.05 -0.02  0.00 -0.02   0 0.83 0.166 1.3
## Q5 0.92 -0.08 -0.15  0.01 -0.10  0.03  0.00  0.02   0 0.89 0.112 1.1
## Q6 0.94  0.08  0.00 -0.13 -0.05 -0.07 -0.02  0.00   0 0.92 0.082 1.1
## Q7 0.90 -0.15  0.12 -0.13  0.03  0.01  0.03  0.00   0 0.87 0.133 1.1
## Q8 0.90  0.06 -0.12  0.01  0.11 -0.04 -0.03  0.00   0 0.84 0.158 1.1
## Q9 0.91 -0.17 -0.01  0.01  0.00  0.09 -0.03 -0.03   0 0.87 0.131 1.1
##
##                        MR1  MR2  MR3  MR4  MR5  MR6  MR7  MR8  MR9
## SS loadings           7.36 0.25 0.12 0.06 0.03 0.03 0.01 0.00 0.00
## Proportion Var        0.82 0.03 0.01 0.01 0.00 0.00 0.00 0.00 0.00
## Cumulative Var        0.82 0.85 0.86 0.86 0.87 0.87 0.87 0.87 0.87
## Proportion Explained  0.94 0.03 0.02 0.01 0.00 0.00 0.00 0.00 0.00
## Cumulative Proportion 0.94 0.97 0.98 0.99 1.00 1.00 1.00 1.00 1.00
##
## Mean item complexity =  1.1
## Test of the hypothesis that 9 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  12.01 with Chi Sq
## The degrees of freedom for the model are -9  and the objective function was  0
##
## The root mean square of the residuals (RMSR) is  0
## The df corrected root mean square of the residuals is  NA
##
## The harmonic number of observations is  1001 with the empirical chi square  0  with prob <  NA
## The total number of observations was  1001  with Likelihood Chi Square =  0  with prob <  NA
##
```

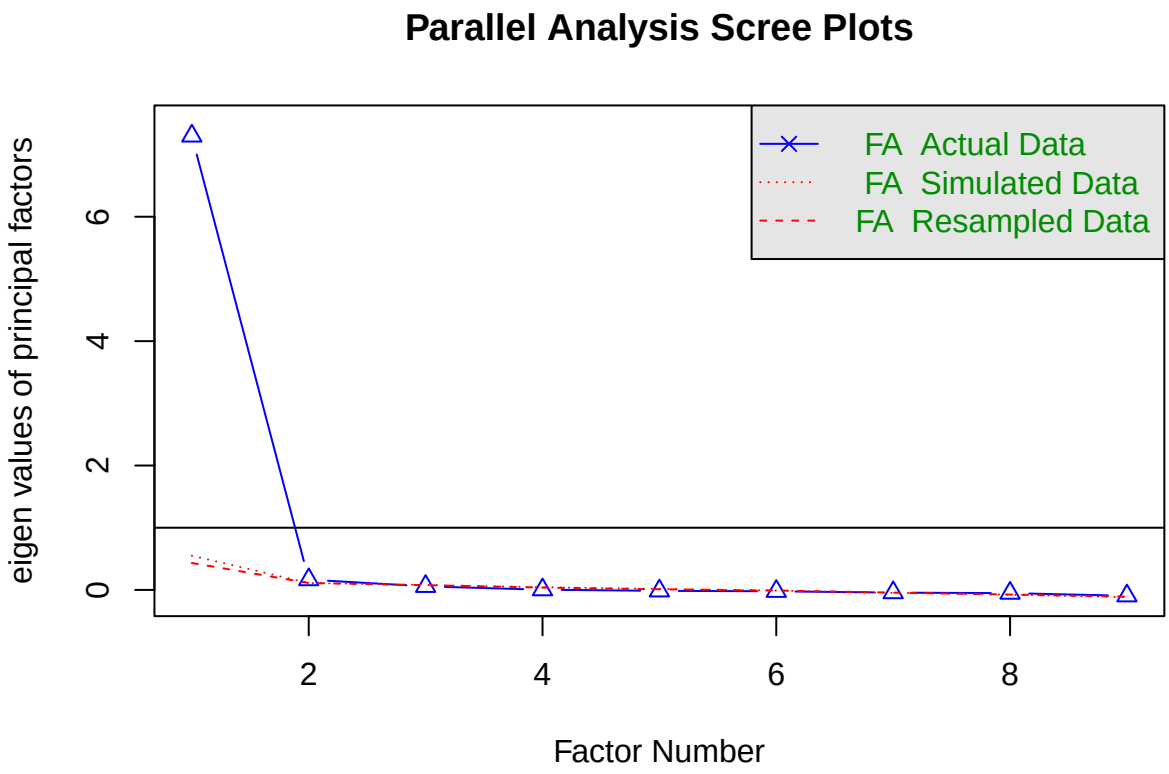
```
## Tucker Lewis Index of factoring reliability =  1.003
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                  MR1  MR2   MR3   MR4   MR5
## Correlation of (regression) scores with factors   0.99 0.80  0.69  0.58  0.42
## Multiple R square of scores with factors          0.98 0.64  0.48  0.34  0.18
## Minimum correlation of possible factor scores     0.97 0.28 -0.05 -0.32 -0.64
##                                                   MR6   MR7   MR8 MR9
## Correlation of (regression) scores with factors    0.44  0.21  0.15   0
## Multiple R square of scores with factors           0.19  0.05  0.02   0
## Minimum correlation of possible factor scores     -0.61 -0.91 -0.96  -1
```

```
efa_model$loadings
```

```
##
## Loadings:
##    MR1    MR2    MR3    MR4    MR5    MR6    MR7    MR8    MR9
## Q1  0.932        -0.117
## Q2  0.895 -0.159  0.176
## Q3  0.873  0.291
## Q4  0.862  0.246  0.148
## Q5  0.921        -0.153
## Q6  0.942               -0.133
## Q7  0.901 -0.153  0.115 -0.132
## Q8  0.900        -0.117         0.105
## Q9  0.910 -0.173
##
##                  MR1   MR2   MR3   MR4   MR5   MR6   MR7   MR8   MR9
## SS loadings    7.358 0.249 0.118 0.056 0.029 0.028 0.006 0.003 0.000
## Proportion Var 0.818 0.028 0.013 0.006 0.003 0.003 0.001 0.000 0.000
## Cumulative Var 0.818 0.845 0.858 0.865 0.868 0.871 0.872 0.872 0.872
```

From the proportion of variance, the optimal number of factor is 1, since it contributes about 82% of the variance. We will use the fa.parallel function to determine the optimal number of factors

```
# selecting optimal number of factor
set.seed(4)
fa.parallel(households, fa='fa')
```




```
## Parallel analysis suggests that the number of factors =  2  and the number of components =  NA
```

Parallel analysis suggests that the number of factors is 2.

```
# refitting with 2 factors
# fitting an EFA model
efa_model <- fa(households, nfactors = 2, fm='minres', rotate = 'none')

efa_model$loadings
```

```
##
## Loadings:
##    MR1    MR2
## Q1  0.930
## Q2  0.890 -0.140
## Q3  0.876  0.317
## Q4  0.856  0.191
## Q5  0.918
## Q6  0.940
## Q7  0.897 -0.136
## Q8  0.899
## Q9  0.911 -0.178
##
##                  MR1   MR2
## SS loadings    7.326 0.228
```

```
## Proportion Var 0.814 0.025
## Cumulative Var 0.814 0.839
```

From the loadings, a simple structure seems to have been achieved in the unrotated form, since all items load mostly on only one factor. For the second factor, the loadings of the items are typically less than 0.3, except in one item with less value.

f.

Based on the eigen values, the eigen values with the second largest value is 0.228. It gives the amount of variance contributed by each item to the second factor. From the loadings in e, we see that the variables contributing to the second factor include Q2, Q3, Q4, Q7 and Q9.

g.

```
# FA with varimax rotation

efa_model.varimax <- fa(cor(households), nfactors = 2, fm='minres', rotate = 'varimax')

print(efa_model.varimax)
```

```
## Factor Analysis using method =  minres
## Call: fa(r = cor(households), nfactors = 2, rotate = "varimax", fm = "minres")
## Standardized loadings (pattern matrix) based upon correlation matrix
##     MR1  MR2   h2   u2 com
## Q1 0.76 0.54 0.87 0.13 1.8
## Q2 0.76 0.48 0.81 0.19 1.7
## Q3 0.45 0.81 0.87 0.13 1.6
## Q4 0.52 0.70 0.77 0.23 1.8
## Q5 0.74 0.55 0.85 0.15 1.8
## Q6 0.67 0.67 0.89 0.11 2.0
## Q7 0.77 0.48 0.82 0.18 1.7
## Q8 0.64 0.63 0.81 0.19 2.0
## Q9 0.81 0.46 0.86 0.14 1.6
##
##                        MR1  MR2
## SS loadings           4.29 3.27
## Proportion Var        0.48 0.36
## Cumulative Var        0.48 0.84
## Proportion Explained  0.57 0.43
## Cumulative Proportion 0.57 1.00
##
## Mean item complexity =  1.8
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  12.01
## The degrees of freedom for the model are 19  and the objective function was  0.16
##
## The root mean square of the residuals (RMSR) is  0.01
## The df corrected root mean square of the residuals is  0.02
##
```

```
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                  MR1  MR2
## Correlation of (regression) scores with factors   0.90 0.87
## Multiple R square of scores with factors          0.82 0.77
## Minimum correlation of possible factor scores     0.64 0.53
```

```
efa_model.varimax$loadings
```

```
##
## Loadings:
##    MR1   MR2
## Q1 0.762 0.542
## Q2 0.765 0.477
## Q3 0.455 0.813
## Q4 0.522 0.705
## Q5 0.740 0.547
## Q6 0.666 0.667
## Q7 0.768 0.484
## Q8 0.644 0.630
## Q9 0.805 0.462
##
##                  MR1   MR2
## SS loadings    4.288 3.266
## Proportion Var 0.476 0.363
## Cumulative Var 0.476 0.839
```

Using the varimax rotation seems not to improve the solution. A simple structure seems not to be achieved since all the variables load on more than one factor.

h.

```
# comparison
 # fitting an EFA model with Oblimin rotation
efa_model.oblimin <- fa(households, nfactors = 2, fm='minres', rotate = 'oblimin',
                        n.obs = nrow(households))
```

```
## Loading required namespace: GPArotation
```

```
efa_model.oblimin
```

```
## Factor Analysis using method =  minres
## Call: fa(r = households, nfactors = 2, n.obs = nrow(households), rotate = "oblimin",
##     fm = "minres")
## Standardized loadings (pattern matrix) based upon correlation matrix
##     MR1   MR2   h2   u2 com
## Q1 0.95 -0.06 0.87 0.13 1.0
## Q2 0.92 -0.11 0.81 0.19 1.0
## Q3 0.78  0.36 0.87 0.13 1.4
## Q4 0.79  0.23 0.77 0.23 1.2
```

```
## Q5 0.93 -0.04 0.85 0.15 1.0
## Q6 0.91  0.11 0.89 0.11 1.0
## Q7 0.93 -0.11 0.82 0.18 1.0
## Q8 0.87  0.09 0.81 0.19 1.0
## Q9 0.95 -0.15 0.86 0.14 1.0
##
##                        MR1  MR2
## SS loadings           7.25 0.30
## Proportion Var        0.81 0.03
## Cumulative Var        0.81 0.84
## Proportion Explained  0.96 0.04
## Cumulative Proportion 0.96 1.00
##
##  With factor correlations of
##      MR1  MR2
## MR1 1.00 0.24
## MR2 0.24 1.00
##
## Mean item complexity =  1.1
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  36  and the objective function was  12.01 with Chi Sq
## The degrees of freedom for the model are 19  and the objective function was  0.16
##
## The root mean square of the residuals (RMSR) is  0.01
## The df corrected root mean square of the residuals is  0.02
##
## The harmonic number of observations is  1001 with the empirical chi square  9.04  with prob <  0.97
## The total number of observations was  1001  with Likelihood Chi Square =  155.61  with prob <  1.8e-1
##
## Tucker Lewis Index of factoring reliability =  0.978
## RMSEA index =  0.085  and the 90 % confidence intervals are  0.073 0.097
## BIC =  24.34
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                MR1  MR2
## Correlation of (regression) scores with factors   0.99 0.80
## Multiple R square of scores with factors          0.98 0.63
## Minimum correlation of possible factor scores     0.96 0.27
```

```
efa_model.oblimin$loadings
```

```
##
## Loadings:
##    MR1    MR2
## Q1  0.947
## Q2  0.921 -0.111
## Q3  0.777  0.359
## Q4  0.793  0.229
## Q5  0.929
## Q6  0.911  0.106
## Q7  0.927 -0.107
## Q8  0.874
## Q9  0.953 -0.150
```

```
##
##                   MR1   MR2
## SS loadings     7.205 0.252
## Proportion Var  0.801 0.028
## Cumulative Var  0.801 0.828
```

Based on the loadings, the oblimin rotation and the unrotated form seem to be the same. Most of the items load on the first factor, with items having less than 3 loading value loading on the second factor. The oblimin rotation technique and the unrotated technique seem to have simple structures.

i.

To achieve a simple structure, a one factor should be chosen and any of the oblimin rotation or unrotated techniques may be chosen.

j.

By investigating the communality results, EFA seems to be the best technique to use over PCA because, from the communality scores, we are able to understand the variance of each item that can be explained by the factors. For example, about 87% of the variance in Q1 can be explained by all the factors, 81% in Q2, 87% in Q3 and so on.