An overview to structural seismic design optimisation frameworks

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Structural seismic design optimization frameworks: An overview

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Abstract

The application of the performance-based seismic design concept using alternative problem formulations is presented in this work. The formulations discussed, are implemented within an automated structural design framework using a metaheuristic optimisation algorithm. Such frameworks are able to accommodate any advanced -linear or nonlinear, static or dynamic- analysis procedure and thus replace, the conventional trial-and-error process. The formulations presented treat the seismic design problem in a deterministic or a probabilistic manner, with one or more objectives that represent the initial cost or the cost of future earthquake losses that may occur during the lifetime of a structural system. The implementations presented are all consistent with the performance-based design concept and take into consideration the structural response at a number of limit-states, from serviceability to collapse.

Keywords: Performance-based design, Reliability-based optimisation, Particle swarm optimization, Pushover analysis, Life-cycle cost, Single and multiple objectives.

Contents

1. Introduction

Performance-based earthquake engineering (PBEE) implies the design, evaluation, construction and maintenance of engineering facilities in order to meet the objectives set by the society and the owners/users of a facility. In the case of earthquakes, the aim is to make structures having a predictable and reliable performance, or in other words, the structures should be able to resist earthquakes with quantifiable confidence in order to assist design engineers to take decisions regarding the desired performance. Therefore, the modern conceptual approach of seismic structural design is that structures should meet multiple performance-based objectives defined for a number of different hazard levels ranging form earthquakes with small intensity and small return period, to more destructive events with large return periods. The current state of practice is defined by US guidelines [1, 2]. These guidelines do not differ conceptually and introduce procedures that can be considered as the first significant diversification from current design codes [3, 4] which are only partially performance-based, since they attempt to tie all design criteria to one performance level, usually to that of Life Safety or Collapse Prevention.

Towards this goal, the guidelines [1, 5] suggest higher-order analysis procedures for the design and assessment in seismic prone areas. A highly-efficient design framework can be offered by structural optimisation taking advantage of the benefits offered by nonlinear, static or dynamic analysis methods. Performance-based design formulated as a structural optimisation problem is a topic of growing interest and has been the subject of extensive research over the last years. Advancements in structural optimisation have made possible the move from traditional trial-and-error design procedures towards fully automated procedures using a structural optimisation-based search-engine. This is mostly attributed to the rapid development of metaheuristic optimizers, which are capable of handling complicated structural problems at the expense of more optimisation cycles.

In this study we present and compare alternative design formulations that can be nested in a structural optimisation framework, offering a powerfull environment for design. The formulations presented can accommodate any linear or nonlinear analysis method, while improved control through the direct calculation of probabilistic criteria is also offered. The designs are compared in terms of cost and/or performance with the aid of one or more objective functions that measure the initial or future cost and/or the variation of the demand. Although structural optimisation formulations are implemented for the design of steel structures, the formulations presented can be also applied on concrete or composite structures, as discussed in section **??**.

2. Seismic design problem formulations

In an optimum design problem the aim is to minimize an objective function under certain behavioural constraints. The objective function is often the cost of the structure or a quantity directly proportional to the cost. The constraints may refer to any Engineering Demand Parameter (EDP) (e.g. stresses, stress resultants, displacements, maximum interstorey drift, plastic rotation, maximum floor accelerations, etc). A design $\mathbf{s}$ is a vector of design variables, $\mathbf{s} = [s_1, s_2, \ldots, s_m]^T$, while the objective function is expressed as a linear or nonlinear combination of $\mathbf{s}$. For skeletal structures, the design variables are usually chosen to be the cross-sections of the members of the structure. Due to engineering practice demands, the members are divided into groups of design variables, thus providing a trade-off between the use of more material and the need for symmetry and uniformity. Therefore, the number of design variables m may be less than the total number of beams and columns of the frame. Such problems are also known as discrete optimization problems, since the design variables take values from a discrete set of values D^{n_d} such as tables of steel sections (e.g. AISC [6]).

2.1. Deterministic design

A discrete deterministic-based structural optimisation (DBO) problem is formulated as:

$$\begin{aligned} & \min_{\mathbf{s}\in\mathcal{F}} C(\mathbf{s}) & & \\ \text{subject to :} \quad & g_i(\mathbf{s}) \geq 0 & & i = 1, \ldots, \ell \\ & s_j \in D^{n_d} & & j = 1, \ldots, m \end{aligned} \tag{1}$$

where C is the objective function to be minimized, usually corresponding to the cost of the structural system that can be either the initial or the life-cycle cost of the structure. g_i are the ℓ deterministic constraints and D^{n_d} is a given set of discrete values from which the design variables s_j, j=1,...,m take values. $\mathcal{F}$ is the feasible region where all the constraint functions g_i are satisfied. The above formulation is the simplest possible and is sufficient for a wide range of structural design problems.

2.2. Reliability-based design

In real-world engineering uncertainties need to be taken into consideration during the design process. Uncertainties are always present and, depending on their source, they can be reduced but not avoided [7]. A discrete Reliability-Based Design optimisation (RBDO) problem introduces uncertainties as additional constraints. Thus the problem formulation, takes the form:

$$\begin{aligned} & \min_{\mathbf{s}\in\mathcal{F}} C(\mathbf{s}) & & \\ \text{subject to :} \quad & g_i(\mathbf{s}) \geq 0 & & i = 1, \ldots, \ell \\ & s_j \in D^{n_d} & & j = 1, \ldots, m \\ & h^k\left(p_f^k(\mathbf{s}) \leq p_f^{k,lim}(\mathbf{s})\right) & & k = 1, \ldots, n \end{aligned} \tag{2}$$

where h^k are n probabilistic constraints of the k^{th} limit-state. More specifically, p_f^k is the limit-state probability that has to be lower than a preset threshold $p_f^{k,lim}$. Alternatively, we may express the probabilistic constraints as function of the limit-state Mean Annual Frequency

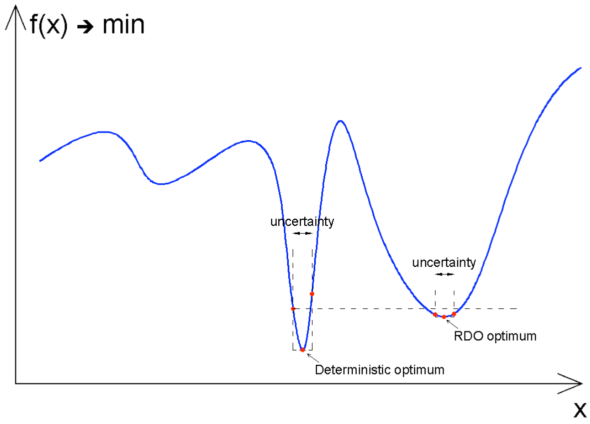

Figure 1: The concept of Robust Design optimisation

(MAF) of exceedance. MAF convolves the limit-state probabilities p_f with the hazard at the site allowing to set design criteria on the annual frequency that the k^{th} limit-state is exceeded. In both cases, the probabilistic constraints, h^k, are expressed in terms of an engineering demand parameter, used to identify the level of damage.

2.3. Robust design

In Robust Design optimisation (RDO) an additional objective function is considered related to the random nature of the problem parameters. Therefore, in the case of a structural variant of RDO the aim is to minimize both the initial cost and the coefficient of variation of the response when the structural properties and/or seismic loading are considered as random variables. A thorough literature review on Robust Design optimisation can be found in [8].

An RDO problem is stated as a multi-objective optimisation problem as follows:

$$\begin{array}{rll} & \min_{\mathbf{s}\in\mathcal{F}} & [C_{in}(\mathbf{s}), COV_{EDP}(\mathbf{s},\mathbf{x})] \\ \text{subject to :} & g_i(\mathbf{s},\mathbf{x}) \geq 0 & i = 1,\ldots,\ell \\ & s_j \in D^{n_d} & j = 1,\ldots,m \end{array} \tag{3}$$

where **s** and **x** represent the design and the random variables vectors, respectively. The objective functions considered are the initial construction cost, C_{in}, and the coefficient of variation of an EDP, COV_{EDP}. The conceptual difference between a DBO and an RDO optimization problem, is explained schematically in Fig. 1.

In a multi-objective optimisation problem there is no unique point that would represent the optimum in terms of either minimum C_{in} and COV_{EDP} and thus the solution of the problem presented in Eq. 3 will provide the design engineer with a set of optimum solutions to select.

2.4. Minimum life-cycle cost design

Another formulation for seismic design would be the one that minimizes the total cost during the structure's lifetime, typically defined as the sum of the initial and the life-cycle cost. Typically, life-cycle cost refers to the deterioration of the structural components' capacity over time due to phenomena such as corrosion or the deterioration of the joints or the bearings [9]. However, life-cycle cost may also refer to the risk related to natural hazards, such as wind or earthquake. In this case it is related to the possible losses due to the unsatisfactory performance of the structure under loading with random occurrence and intensity during its life. The design process should consider both direct or indirect economic and human life losses within a given social context [10].

The design for minimum life-cycle cost is preferably formulated as a two-obective problem and thus can be stated as:

$$\begin{array}{rll} & \min_{\mathbf{s}\in\mathcal{F}} & [C_{in}(\mathbf{s}), C_{lcc}(\mathbf{s})] \\ \text{subject to :} & g_i(\mathbf{s}) \geq 0 & i = 1,\ldots,\ell \\ & s_j \in D^{n_d} & j = 1,\ldots,m \end{array} \tag{4}$$

where C_{in} and C_{lcc} are the initial and the life-cycle cost objective functions, **s** represents the design vector that corresponds to the cross-sections of each member of the structure and $\mathcal{F}$, $g_i(\mathbf{s})$ have been previously defined. Since this is a multi-objective problem, again a set of optimum designs are obtained.

3. Design objectives

In structural optimisation terminology, "optimum" refers to the minimum construction cost compatible with a pre-established set of performance criteria. Usually the total weight of a steel frame is considered directly proportional to the total construction cost and thus it is the most common choice for the objective function that will be minimized. For a skeletal steel structure that consists of N members, the objective function, also known as cost function, will be:

$$C_{in} = \sum_i^N A_i L_i \tag{5}$$

where A_i and L_i are the cross section area and the length of member i, respectively. The capacity is strictly proportional to the section area only for axially loaded structures (e.g. trusses) and thus for more complicated structural systems Eq. 5 is not strict exact. However, for many practical applications, and especially steel structures, the total weight objective function has been found to be sufficient.

There may be design problems where Eq. 5 is not representative and thus the objective function needs to be modified accordingly. For reinforced concrete structures a more direct measure of the initial construction cost has to be adopted. Instead of the total weigh, the sum of the total cost of the concrete material and the total cost of the reinforcing steel material should be used. In this case, the total initial cost is given by the formula:

$$C_{in} = C_c + C_s = \sum_i^N w_c b_i h_i L_i + \sum_i^N w_s \left(A_{S1,i} + A_{S2,i}\right) L_i \tag{6}$$

where C_c and C_s is the cost of concrete and the cost of reinforcing steel, respectively. w_c, w_s are the unit cost coefficients of each material, b_i and h_i are the section dimensions of member i, L_i is its length and finally $A_{S1,i}$ $A_{S2,i}$ are the bottom and the top reinforcement areas, respectively. Previous studies have shown that the choice of the w parameter can influence the properties of the most cost-efficient design and also the extent that a higher reinforcement amount is preferable to larger section dimensions when attempting to satisfy efficiently the design requirements [11].

In earthquake engineering practice the term "optimum" has a broader meaning including both the direct construction cost and also the future maintenance and damage costs, usually referred as "life-cycle cost" [12, 13]. Life-cycle cost analysis is an important tool in economic analysis and constitutes the core of the investment decision-framework for structure and infrastructure assistant management where it is used to evaluate alternative investment options. The total expected cost C_{tot} of a structure, may refer either to the design life period of a new structure or to the remaining life period of a retrofitted structure and is expressed:

$$C_{tot}(t, \mathbf{s}) = C_{in}(\mathbf{s}) + C_{lcc}(t, \mathbf{s}) \tag{7}$$

where C_{in} is the initial cost of a new structure or the cost of retrofitting, C_{lcc} is the present value of the expected limit-state cost; $\mathbf{s}$ are the design variables and t is the corresponding time period. The initial cost is related to the material and the labor cost for the construction of the building, labor cost for placement as well as the nonstructural component cost. The initial cost of a new structure refers to the cost right after construction. The life-cycle cost, C_{lcc}, consists of the following components:

$$C_{lcc} = C_{dam} + C_{con} + C_{ren} + C_{inc} + C_{inj} + C_{fat} \tag{8}$$

where C_{dam} is the damage repair cost, C_{con} is the loss of contents cost, C_{ren} is the loss of rental cost, C_{inc} the income loss cost, C_{inj} is the cost of injuries and C_{fat} is the cost of human fatality. The quantification of losses in economical terms depends on several socio-economic parameters. Details about the calculation for each limit-state cost and representative cost values for each category can be found in [14, 15].

4. Metaheuristic Algorithms

Many probabilistic-based search algorithms have been inspired by natural phenomena (e.g. Evolutionary Programming, Genetic Algorithms, Evolution Strategies). Recently, a family of optimization methods has been developed based on the simulation of social interactions among members of a specific species looking for food or resources in general. One of these methods is the Particle Swarm Optimization (PSO) [16] method that is based on the behavior reflected in flocks of birds, bees and fish that adjust their physical movements to avoid predators and seek for food. In this work PSO method is applied for solving the single objective optimization problems presented, while a PSO variant of the NSGA-II method is proposed for solving the multi-objective one.

4.1. Particle Swarm Optimization for single-objective problems

PSO has been found to be highly competitive for solving a wide variety of optimization problems. It can handle non linear, non convex design spaces with discontinuities. Compared to other non-deterministic optimization methods it is considered efficient in terms of number of function evaluations as well as robust since it usually leads to better or the same quality of results. Its easiness of implementation makes it more attractive as it does not require specific domain knowledge information, while being a population based algorithm, it can be straight forward implemented in parallel computing environments.

In a PSO formulation, multiple candidate solutions coexist and collaborate simultaneously. Each solution is called a "particle" that has a position and a velocity in the multi-dimensional design space. A particle "flies" in the problem search space looking for the optimal position. As "time" passes through its quest, a particle adjusts its velocity and position according to its own "experience" as well as the experience of other, neighboring, particles. Particle's experience is built by tracking and memorizing the best position encountered. As every particle remembers the best position it has visited during its "flight", the PSO possesses a memory. A PSO system combines local search method (through self experience) with global search method (through neighbouring experience), attempting to balance exploration and exploitation.

Each particle maintains two basic characteristics, velocity and position, that are updated as follows:

$$\begin{aligned} \boldsymbol{v}^j(t+1) = w\boldsymbol{v}^j(t) \quad &+c_1\boldsymbol{r}_1 \circ \left(\boldsymbol{s}^{Pb,j} - \boldsymbol{s}^j(t)\right) \\ &+c_2\boldsymbol{r}_2 \circ \left(\boldsymbol{s}^{Gb} - \boldsymbol{s}^j(t)\right) \end{aligned} \tag{9}$$

$$\boldsymbol{s}^j(t+1) = \boldsymbol{s}^j(t) + \boldsymbol{v}^j(t+1) \tag{10}$$

where $\boldsymbol{v}^j(t)$ denotes the velocity vector of particle j at time t, $\boldsymbol{s}^j(t)$ represents the position vector of particle j at time t, vector $\boldsymbol{s}^{Pb}$, j is the "best ever" position of the j^{th} particle, and vector $\boldsymbol{s}^{Gb}$ is the global best location found by the entire swarm. The acceleration coefficients c_1 and c_2 indicate the degree of confidence in the best solution found by the individual particle (*cognitive parameter*) and by the whole swarm (*social parameter*), respectively. $\mathbf{r}_1$ and $\mathbf{r}_2$ are random vectors uniformly distributed in the interval [0,1]. The symbol "$\circ$" of Eq. 9 denotes the Hadamard product, i.e. the element-wise vector or matrix multiplication.

The particle's current position $\mathbf{s}_j(t)$ at time t is represented by the dotted circle at the lower left of the drawing, while the new position $\mathbf{s}_j(t+1)$ at time $t+1$ is represented by the dotted bold circle at the upper right hand of the drawing. It can be seen how the particle's movement is affected by: (i) its velocity $\boldsymbol{v}_j(t)$; (ii) the personal best ever position of the particle, $\mathbf{s}^{Pb}$,

In the above formulation, the global best location found by the entire swarm up to the current iteration ($\mathbf{s}^{Gb}$) is used. This is called a fully connected topology (fully informed PSO), as all particles share information with each other about the best performer of the swarm. Other topologies have also been used in the past where instead of the global best location found by the entire swarm, a local best location of each particle's neighborhood is used. Thus, information is shared only among members of the same neighborhood.

Parameter w of Eq. 9 is the inertia weight, a scaling factor used to control the exploration abilities of the swarm, by scaling the current velocity value. The inertia weight was not part of the original PSO algorithm [16], as it was introduced later by Shi and Eberhart [17] who improved the convergence of the algorithm. Large inertia weights will force larger velocity updates allowing the algorithm to explore the design space globally. Similarly, small inertia values will force the velocity updates to concentrate in the nearby regions of the design space.

4.2. Particle Swarm Optimization for multi-objective problems

Several methods have been proposed for treating structural multi-objective optimisation problems [18]. In this work an algorithm based on NSGA-II is used in order to handle the two-objective optimisation problems at hand. NSGA-II has been developed by Deb *et al.* [19] as an improvement to the original NSGA algorithm employing the fast-non-dominated sort method for classifying a population into non-dominated fronts and calculating the crowding distance for preserving diversity in the population. A modified version of the NSGA-II algorithm, denoted as NSPSO-II, is used in this work where the Genetic Algorithm is replaced by the swarm optimization (PSO) algorithm.

The main part of the NSPSO-II algorithm is the fast-non-domination sort procedure according to which a population is sorted in non-dominated fronts. The complexity of the sorting procedure is $O(mn^2)$, where m is the number of objectives and n is the population size. The pseudocode of the procedure is given in Fig. 2. For each design $\mathbf{p}$, two entities are calculated (lines 2 to 10): (*i*) n_p represents the number of designs that dominate design $\mathbf{p}$ and (*i*) $\mathbf{S}_p$ corresponds to the set of designs that design $\mathbf{p}$ dominates. Every design $\mathbf{p}$ that belongs to the first non-dominated front is dominated by no other design and therefore $n_p = 0$. The criterion whether or not a population belongs to the first non-dominated front is given in lines 11-14, while in lines 17-28 the remaining fronts are created (Fig. 2).

The pseudocode of the NSPSO-II algorithm is shown in Fig. 3. The first eight lines of the code correspond to the PSO part of the algorithm, while the remaining lines to the NSPSO-II part of the algorithm. After the new *NP* particles are created (line 8), the combined group of particles is sorted by the fast-non-domination sort [19] (line 10). After the sorting is complete, the first non-dominated front $\mathbf{F}_1$ will contain the non-dominated designs. If the amount of designs in $\mathbf{B}_p^{(g+1)}$ is less than *NP*, $\mathbf{B}_p^{(g+1)}$ is populated with designs from the subsequent non-dominated fronts. If the number of designs in $\mathbf{B}_p^{(g+1)}$ is equal to *NP*, the algorithm continues with the next generation. Usually, $\mathbf{B}_p^{(g+1)}$ will contain more than *NP* solutions, so we must chose which of the designs of the last non-dominated front that entered $\mathbf{B}_p^{(g+1)}$,

```
For each p ∈ R^g
    S_p = ∅
    n_p = 0
    For each q ∈ R^g
        If p ≺ q Then
            S_p = S_p ∪ q
        Else if q ≺ p
            n_p = n_p + 1
        End
    End
    If n_p = 0 Then
        p_rank = 1
        F_1 = F_1 ∪ p
    End
    i = 1
    While F_i ≠ ∅ Do
        Q = ∅
        For each p ∈ F_i
            For each q ∈ S_p
                n_q = n_q − 1
                If n_q = 0 Then
                    q_rank = i + 1
                    Q = Q ∪ q
                End
            End
        End
        i = i + 1
        F_i = Q
    End
End
```

Figure 2: Pseudocode of the fast-non-dominated sort algorithm

```
1  Begin
2  g := 0
3  initialize(B_p^0 := s_m^0, f s_m^0 , m = 1,..., NP)
4      Repeat
5          For l := 1 To NP Do Begin
6              s̃_l := calc_new_particle s_l
7          End
8          B_o^g := s_l^g, f s_l^g , l = 1,..., NP
9          R^g = B_p^g ∪ B_o^g
10         S^g = fast−nondominated−sort R^g
11         B_p^{g+1} = ∅, i = 1
12         Until |B_p^{g+1}| + |S_i| ≤ μ Do
13             crowding−distance−assignment S_i
14             B_p^{g+1} = B_p^{g+1} ∪ S_i
15             i = i + 1
16         End
17         crowded−comparison−operator S_i
18         B_p^{g+1} = B_p^{g+1} ∪ S_i[1 : NP − |B_p^{g+1}|]
19         g := g + 1
20     Until termination_condition
21 End
```

Figure 3: Pseudocode of the NSPSO-II algorithm

will be kept. This procedure takes place in lines 17–18 where the last front is sorted according to the crowded comparison operator, and then an appropriate number of the least crowded designs are chosen so that the size of $\mathbf{B}_p^{(g+1)}$ is equal to *NP*.

5. The "design" phase

5.1. Outline

In the core of the optimisation-based design frameworks presented lies the "analysis" phase. The term "analysis", here does not literally refer to a single static or dynamic FE analysis, but rather to the whole process adopted in order to assess the frame capacity and determine whether a building design is acceptable. A number of checks, or "constraints" in the optimization terminology, have to be considered during optimisation in order to make sure that all candidate designs have acceptable performance, meeting the requirements set by the provisions of the design code or guideline that the structure is designed to.

A number of preliminary checks are applied on every candidate design, such as to ensure that the design complies with the "strong-column-weak-beam" philosophy. The next step is to check the structure against load combinations that do not contain seismic actions, e.g. gravity loads, live loads, etc. Therefore, all design code checks must be satisfied for the non-seismic load combinations [20, 21]. If all the constraints are satisfied, the capacity of the structure against seismic loads is subsequently assessed. The procedure followed to obtain the capacity and the corresponding constraints of this step depend on the chosen problem formulation followed.

5.2. Analysis procedures

Performance-based earthquake engineering (PBEE) brought forward the need for high-level analysis procedures. The guidelines for the seismic rehabilitation of buildings introduced a comprehensive framework for dynamic and nonlinear analysis procedures [1, 5]. The FEMA-356 [5] document specifies four alternative analysis procedures ranging from linear static to nonlinear dynamic analysis, while the ATC-40 [1] document emphasizes on the use of a nonlinear static analysis procedure to define the displacement capacity of reinforced concrete buildings. The choice of the analysis procedure to be adopted depends on several parameters such as the importance of the structure, the performance level (ie. expected level of nonlinearity), the structural characteristics (e.g., regularity, frequency properties), the amount of data available for developing a structural model, etc. The choice of the analysis method involves the decision whether the response will be considered as linear or nonlinear and also the form that the seismic actions will be applied on the model (e.g. response spectrum or ground motion record).

It is evident that a hierarchy among the procedures exists, since the accuracy of analysis is influenced by

Table 1: Analysis procedures for seismic design and assessment [22].

Category	Analysis procedure	Force-Deform. relationship	Seismic actions	Analysis method
Equilibrium	Plastic Analysis	Rigid-Plastic	Equivalent	Equilibrium lateral load
Linear	Linear-static	Linear	Equivalent	Linear-static lateral load
	Linear-dynamic procedure (I)	Linear	Response spectrum	Response spectrum
	Linear-dynamic procedure (II)	Linear	Ground motion record	Linear timehistory
Nonlinear	Nonlinear static	Nonlinear	Equivalent	Nonlinear-static lateral load
	Nonlinear dynamic	Nonlinear record	Ground motion	Nonlinear timehistory

the modeling characteristics, the complexity of the procedure and the computer resources available. Plastic analysis requires only the equilibrium relationships and gives the collapse load and the location of plastic hinges in members [22]. The linear static procedure and the linear dynamic procedure (I) refer to the structural analysis procedures suggested by regional seismic design codes (e.g. [3]) and require a response modification factor to account for the effect of ductile nonlinear response. The linear dynamic analysis method (I) is based on a linear-elastic model of the structure, which permits the use of vibration properties (frequencies and mode shapes), and in a simplified solution with a modal representation of the dynamic response. The linear dynamic procedure (II) is also linear-elastic but the seismic loading is more realistically applied with a ground motion record. A step towards increased accuracy is the use of nonlinear, static or dynamic, analysis procedures. Nonlinear static analysis, also known as pushover, offers significant advantages compared to elastic methods but still can not substitute dynamic response history analysis, especially for problems where modes other than the first contribute to the response.

5.2.1. Static pushover analysis

Static PushOver analysis (SPO) requires a mathematical model that directly incorporates the nonlinear load-deformation characteristics of the individual components and the elements of a building. The structure is subjected to monotonically increasing lateral forces that represent the seismic inertia forces. Compared to the more accurate nonlinear response history analysis, the static pushover offers its relative simplicity and its reduced computational effort. Despite its ease of use, this method can provide important information regarding the capacity of a structural system. More specifically, pushover analysis is able to predict realistically the force demands on potentially brittle elements, the deformation demands on inelastically deforming members as well as the interstorey drifts and their distribution along the height of a building [23]. Pushover analysis serves also as a powerful tool to identify the critical regions of a structural system and to determine the sequence of the elements yielding.

To perform pushover analysis we need to know the expected target displacement, δ_t, for the specific loading conditions, i.e. response spectrum. The target displacement can be calculated either with one of the variations of the capacity spectrum method [1] or with the displacement coefficient method [5]. The latter method is usually more convenient since it can be easily programmed in a computer algorithm, while in terms of accuracy there is no evidence indicating which method is preferable.

The displacement coefficient method predicts the target displacement using the formula [5]:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \tag{11}$$

where C_0, C_1, C_2, C_3, are modification factors. C_0 relates the spectral displacement to the likely building roof displacement. C_1 relates the expected maximum inelastic displacements to the displacements calculated for linear elastic response. C_2 represents the effect of the hysteresis shape on the maximum displacement response and C_3 accounts for second order effects. T_e is the effective fundamental period of the building in the direction under consideration. S_a is the response spectrum acceleration corresponding to T_e.

5.2.2. Nonlinear response history analysis

Nonlinear response history analysis offers information regarding the cyclic response, the stiffness and strength degradation and also the amount of energy absorbed due to the hysteretic behavior. Moreover, it is the only procedure that takes into consideration the duration of the ground motion which directly correlates with the magnitude of the shaking. Similarly to the force-deformation capacity curve of the SPO procedure, the concept of the "dynamic capacity curve" has been introduced through the Incremental Dynamic Analysis (IDA) method [24]. Contrary to SPO, the latter procedure is not single-run, since it involves performing a series of nonlinear response history analyses in which the ground motion record is incrementally scaled up. The collapse capacity is considered to be reached when the capacity curve becomes flat. Nonlinear response history analysis has to be repeated using different ground motion records in order to obtain meaningful statistical averages of the response, usually expressed as the median curve.

5.3. Acceptance criteria

For nonlinear structural analysis it is essential to formulate complete system models, which incorporate all the features of the problem to be examined and therefore can "accurately" calculate the structural demand. In order to evaluate the relationship between demand and capacity, appropriate Engineering Demand Parameters (EDP) are necessary. EDP's are used to replace (or complement) the use of stress resultants or maximum allowable stresses, which are traditionally suggested by design codes. According to [5] the actions can be either force- or deformation-controlled depending on the capacity of the member to deform inelastically. The capacity of force-controlled members can be assessed using formulas based on stress resultants (e.g. [20, 21]), while for deformation controlled actions an appropriate EDP chosen instead.

According to performance-based design, pairs of performance levels and corresponding hazard levels have to be set, depending on the type of the structure. Thus, the structural response is evaluated for a number of objectives, following the FEMA-356 [5] terminology: immediate occupancy (IO), life safety (LS) and collapse prevention (CP). Each objective corresponds to a given probability of being exceed during the life-span of the structure, typically considered equal to 50 years. A common assumption is that the immediate occupancy level corresponds to a 50% probability of exceedance, the life safety level to a 10% probability and the collapse prevention to 2% probability of being exceeded in 50 years.

6. Performance-based earthquake engineering

In every reliability analysis problem the purpose is to calculate the limit-state probability of failure or the limit-state mean annual frequency of exceedance. For earthquake engineering problems where the performance-based design concept is implemented the probability has to be determined for every performance level considered. Therefore, the term "failure probability" is replaced by "probability of exceedance conditional on the limit-state", or simply by "limit-state probability of exceedance". The probability is calculated by applying the total probability theorem and conditioning the probabilities on one parameter that expresses the intensity of the seismic action *IM*.

The mean annual frequency of exceeding a limit-state refers to the annual rate that an engineering demand parameter (EDP) exceeds a given demand level (*edp*). The MAF of a limit-state is denoted as ν and is calculated using the total probability theorem:

$$\nu(EDP > edp) = \int_0^\infty \left[1 - P(EDP > edp \mid IM = im)\right] \left|\frac{\nu(IM)}{dIM}\right| dIM \tag{12}$$

where $P(EDP > edp/IM = im)$ is the limit-state probability that an engineering demand parameter exceeds a threshold value, conditional on a given intensity value im; the second term of the integral of Eq. 12 is the slope of the hazard curve or, in other words, it is the mean annual rate of ground motion intensity, IM. The absolute value is used because the slope has a negative value. Eq. 12 allows the integration of the results of structural analysis with data produced by seismologists. The first term of the integral of Eq. 12 is also know as 'fragility' or 'vulnerability' curve.

6.1. Direct calculation of the limit-state probabilities

The calculation of Eq. 12 requires first to determine the limit-state fragilities, while the slope $d\nu(IM)/dIM$ is extracted from the site hazard curve. In order to calculate analytically the fragilities it is assumed that the maximum interstorey drift, at a given intensity $S_a(T_1, 5\%)$ level, follow the lognormal distribution [25]. The probabilities are thus calculated as follows:

$$p_f = P(EDP > edp \mid IM = im) = \Phi\left[\frac{\ln(edp|im) - \ln(\mu_{edp|im})}{\ln(\sigma_{edp|im})}\right] \tag{13}$$

where $\ln(\mu_{edp|im})$ and $\ln(\sigma_{edp|im})$ are the logarithmic mean and the standard deviation of the EDP, respectively, conditional on the intensity.

6.2. Monte Carlo Simulation

The Monte Carlo Simulation (MCS) method is often employed when the analytical solution is not attainable and the failure domain can not be expressed or approximated analytically. This is mainly the case in problems of complex nature with a large number of basic variables where other reliability methods are not applicable. Expressing the limit-state function as $G(\mathbf{x})<0$, where $\mathbf{x} = [x_1, x_2, \ldots, x_M]^T$ is the vector of the random variables, the limit-state probability can be obtained as:

$$p_f = \int_{G(x)\geq 0} f_x(\mathbf{x})\, dx \tag{14}$$

where $f_x(\mathbf{x})$ is the joint probability of failure. If N_∞ is a large number, an unbiased estimator of the probability of failure is given by:

$$p_f = \frac{1}{N_\infty} \sum_{j=1}^{N_\infty} I\left(\mathbf{x}_j\right) \tag{15}$$

where $I(\mathbf{x}_j)$ is a Boolean vector indicating successful or unsuccessful simulations. For the calculation of p_f, a sufficient number of N_{sim} independent random samples is produced using a specific probability density function for each component of the array $\mathbf{x}$. The failure function is computed for every random sample and the estimation of p_f will be:

$$p_f \approx \frac{N_H}{N_{sim}} \tag{16}$$

where N_H is the number of simulations where the limit-state function has been exceeded, or in other words the number of simulations where the demand exceeded the capacity, while N_{sim} is the total number of simulations necessary to obtain an accurate estimation of the probability p_f. If a given accuracy δ_0 is required, the sample size can be obtained from the following expression:

$$N_{sim} = \frac{p_f}{\delta_0^2} \tag{17}$$

Therefore if the desired accuracy is $\delta_0 = 10\%$ and the probability sought is of the order of 0.01, the required sample size N_{sim} is 1000 simulations. Since a separate Monte Carlo Simulation is performed at each intensity level, significant computational effort that depends on both the limit-state examined and on the order of the probability sought, is required.

6.3. First Order Reliability Method (FORM)

In the general case of a nonlinear limit-state function, the main objective of the First Order Reliability Method (FORM) is to calculate the reliability index β. The Hasofer-Lind reliability index β is calculated by a process of minimization, and the probability of violation is:

$$p_f = \Phi(-\beta) \tag{18}$$

where Φ is the standard normal cumulative distribution function. This equation is exact when the failure criterion is linear and all random variables have normal distributions. Given a vector of basic variables $\mathbf{x}$, a failure surface on which the failure criterion $g(\mathbf{x}) = 0$ is satisfied and a safe region denoted by $g(\mathbf{x}) > 0$, the vector of the reduced variables $\mathbf{z}$ is defined as follows:

$$\mathbf{z} = \boldsymbol{S}_{\mathbf{x}}^{-1}(\mathbf{x} - \boldsymbol{\eta}_x) \tag{19}$$

where $\mathbf{S}_x$, is a diagonal matrix of the standard deviations and $\boldsymbol{\mu}_x$ is the vector of mean values. Then the Hasofer-Lind reliability index β is defined as:

$$\beta = min \sqrt{\mathbf{z}^T \mathbf{z}} \tag{20}$$

The point on the failure surface $g(\mathbf{x}) = 0$, where its transformation to the $\mathbf{z}$ space satisfies Eq. 19, is called design point and is denoted as $\mathbf{z}_D$. The design point $\mathbf{z}_D$ is located on the limit-state surface, and is the point of the minimum distance from the origin in the standard normal space. For applying either first or second-order methods to complex structural models it is necessary to have an explicit expression or an approximation of either the entire limit-state function $g(\mathbf{x})$ or of the limit-state surface $g(\mathbf{x}) = 0$ in the space of the random variables $\mathbf{x}$. This is because these methods require not only knowledge of the function but also of its gradient in the vicinity of its limit-state surface. In the case of unknown expression, the limit-state function is usually approximated by the response surface method.

When applying the RS method to calculate the failure probability two important issues should be considered to reduce the computational effort and have acceptable accuracy: (i) the definition of experimental points for defining the approximation of the limit-state function, and (ii) the analytical expression of the Response Surface function [26]. Usually, a quadratic function is assumed:

$$\bar{g}(\mathbf{x}) = a + \sum_{i=1}^{m} b_i x_i \sum_{i=1}^{m} c_i x_i^2 \tag{21}$$

defined in m-dimensional random variable space where the constants a, b_i and c_i are determined by evaluating $g(\mathbf{x})$ at given points.

Bucher and Bourgund [27] proposed an interpolation scheme for the solution of structural reliability problems, where the quadratic RS function of Eq. 21 is defined with $2m + 1$ experimental points. It was suggested the experimental points x_i, to be taken as $x_i = \mu_i \pm f\sigma_i$, where μ_i and σ_i are the mean value and standard deviation and f is an arbitrary factor taken equal to 3. Figure 4 shows the sampling method for defining the $2m + 1$ experimental points where $x_{i,low} = \mu_i - 6\sigma_i$ and $x_{i,up} = \mu_i + 6\sigma_i$. In order to avoid the undesirable case of Figure 4a, where unrealistic values of the random variables are generated (i.e. a negative value of the modulus of elasticity), a correction of the experimental points is performed by moving all trial points to an acceptable region as shown in Figure 4b.

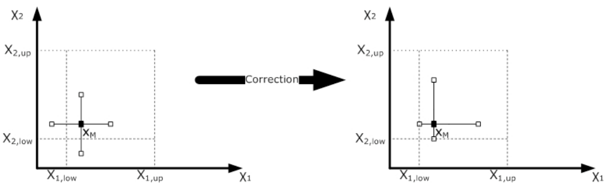

Figure 4: Standard sampling method [27]: (a) before, and (b) after correction.

6.4. The FEMA/SAC method

Following severe economic losses caused by earthquakes in the United States and Japan, a reliability-based and performance-oriented design procedure has been developed as part of the SAC/FEMA Joint Venture for Steel Buildings project [28]. The theoretical basis of this method is provided in reference [29]. The basic assumptions of this approach are briefly summarized below:

- Instead of the "load" and "resistance" terms, widely used in probabilistic assessment, the more generic terms "demand" and "capacity" are adopted. For the i-th limit-state both capacity C_i and demand D_i are assumed to be lognormally distributed.

- The relationship between the intensity and the demand can be obtained either with IDA or with regression in the $EDP-IM$ plane. In the FEMA/SAC method, this relationship can be approximated as $EDP = \alpha IM^b$, or simply $D_i = \alpha(s_a)^b$.

- The hazard curve is expressed in the form: $\nu(S_a) = Pr(S_a \geq s_a) = k_0(s_a)^{-k}$. If this is not possible, a linear local fit of the hazard curve around s_a has to be performed.

Following the above assumptions, Eq. 12 can be calculated analytically:

$$P_{LS,i} = P_{D_i \geq C_i} = \nu(s_a(C_i))exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_C^2 + \beta_D^2)\right] \quad (22)$$

where β_C and β_D denote the dispersion, which is approximately equal to the coefficient of variation of the capacity and the demand, respectively. The limitations of the method in essence depend on the validity of the above assumptions, while this approach offers a reasonable approximation of the limit-state probability with a relatively small number of simulations.

7. Life-Cycle Cost calculation

Based on the Poisson model of earthquake occurrence and the assumption that after a major damage-inducing seismic event, the building is immediately retrofitted to its original intact conditions, Wen and Kang [14] proposed the following formula for the expected life-cycle cost considering N damage states:

$$C_{lcc}(t, \mathbf{s}) = \frac{\bar{\nu}}{\bar{\lambda}}(1 - \exp^{-\bar{\lambda}t})\sum_{i=1}^{N} C_{lcc}^i P^i \quad (23)$$

where

$$P(\theta_{max} > \theta_{max}^i) = (-1/t)\ln[1 - \bar{P}_i(\theta_{max} - \theta_{max}^i)] \quad (24)$$
$$P^i = P(\theta_{max} > \theta_{max}^i) - P(\theta_{max} > \theta_{max}^{i+1})$$

According to Eq. 24 the limit-state costs C_{lcc}^i of Eq. 8 are used to calculate $C_{lcc}(t, \mathbf{s})$, where P_i is the probability of the i^{th} damage state being violated given the occurrence of an earthquake. Eq. 24 assumes the maximum interstorey drift θ_{max} as the characteristic EDP, while $P(\theta_{max} > \theta_{max}^i)$ is the exceedance probability given occurrence. θ_{max}^i, θ_{max}^{i+1} are the drift ratios defining the lower and upper bounds of the i^{th} damage state and $\bar{P}_i(\theta_{max} - \theta_{max}^i)$ is the annual exceedance probability of the maximum interstorey drift value θ_{max}^i. Finally, $\bar{\nu}$ is the annual occurrence rate of significant earthquakes modeled by a Poisson process and t is the service life of a new or the remaining life of a retrofitted structure. The exponential component of Eq. 23 is used to express C_{lcc} in present value, where $\bar{\lambda}$ is the annual momentary discount rate. The annual monentary discount rate is typically taken to be constant and equal to 5%, while its value lies in the range of 3 to 6%. Each damage state is defined by appropriate drift ratio limits depending on the type of structure, e.g. steel or RC structure. Typical values can be found in references [13, 30]. When one of the limit-state drifts is reached, the corresponding limit-state is assumed to be exceeded. The annual exceedance probability $\bar{P}^i$ is obtained from a relationship of the form:

$$\bar{P}^i(\theta_{max} - \theta^i_{max}) = \alpha_1 \left(\theta^i_{max}\right)^{-\alpha_2} \quad (25)$$

Parameters α_1 and α_2 are obtained by best fit of known $\bar{P}^i - \theta^i_{max}$. These pairs correspond to the 2, 10 and 50 percent in 50 years earthquakes that have known probabilities of exceedance. The maximum interstorey drift, θ^i_{max}, is obtained through analysis of the three intensity levels, while $\bar{P}^i$ is calculated using a Poisson model, e.g. the seismic event with 2% probability of being exceeded in 50 years has annual probability of exceedance equal to $\bar{P}_{2/50} = -ln(1 - 0.02)/50 = 4.04 \times 10^{-4}$.

8. Modelling and Finite Element analysis

Nonlinear static or dynamic analysis needs a detailed and accurate simulation of the structure in the regions where inelastic deformations are expected to occur. Given that the plastic hinge approach has limitations in terms of accuracy, especially under dynamic loading [7], fiber beam-column elements are more preferable. According to this finite element modelling, each structural element is discretized into a number of sections, and each section is further divided into a number of fibers, which are restrained to the beam kinematics. The sections are located either at the centre of the element or at its Gaussian integration points. The main advantage of the fiber approach is that every fiber has a simple uniaxial material model allowing an easy and efficient implementation of the inelastic behaviour. This approach is considered to be suitable for inelastic beam-column elements under dynamic loading and provides a reliable solution compared to other formulations. However, it results to higher computational demands in terms of memory storage and CPU time. When a displacement-based formulation is adopted the discretization should be adaptive with a dense mesh at the joints and a single elastic element for the remaining part of the member. On the other hand, force-based fiber elements allow modeling a member with a single beam-column element. A thorough discussion on the pros and cons of different variations of the fiber formulation can be found in reference [7].

In the case study that follows all analyses have been performed using the OpenSEES [31] platform. Each member is modeled with force-based beam-column elements. A bilinear material model with pure kinematic hardening is adopted, while +++++=.

9. Numerical Example

Selected seismic design frameworks are compared on a four-storey steel braced frame. The structure and its geometry are shown in Fig. 5. The simplified loading cases considered for all optimum design frameworks, are [32]:

$$\begin{aligned} S_G &= 1.35G + 1.50Q \\ S_E &= G + \psi Q + E_d \end{aligned} \quad (26)$$

where G, Q and E_d are the permanent, live and seismic actions, respectively and ψ is the combination coefficient for a quasi-permanent variable action, here taken equal to 0.30. Therefore, the beams are loaded with factored distributed load equal to q_G=43.8kN/m for the gravity load combination and q_E=20kN/m for the seismic combination (Eq. 26).

The frame consists of 20 steel columns, 16 beams and 16 bracing diagonals, grouped into four design variables. The cross-sections are selected among three databases. More specifically the beams are chosen from a database of 18 IPE sections, the external and the internal columns from 24 HEB sections and the diagonal braces are L-shaped from a database of 50 members.

The modulus of elasticity is assumed equal to 210GPa and the yield stress is 235MPa. The constitutive law is bilinear with pure strain hardening slope equal to 1% of the elastic modulus. The frame is assumed to have rigid connections and fixed supports. The base shear is obtained from the EC8 response spectrum ($\gamma_I = 1$, $T_A = 0.15s$, $T_B = 0.6s$). The damping correction factor is equal to 1.19, since a damping ratio of 2% is assumed for the frame.

Three distinct limit-states are considered, i.e. immediate occupancy (IO), life safety (LS) and collapse prevention (CP). The corresponding peak ground acceleration (PGA) values are 0.345g, 0.23g and 0.10g and the maximum interstory drift (θ_{max}) threshold values are set to 0.5, 1 and 2%. A first mode-based lateral load distribution is adopted. The target displacements of Eq. 11 are calculated assuming $C_0 = 1.35$, $C_1 = 1.13$ and $C_2 = C_3 = 1$, since the frame T_1 was found equal to 0.47 sec.

For the solution of the single objective optimisation problems the PSO algorithm is adopted, while for the case of the two objective optimization problem the NSPSO-II method is used. In the implementatios of PSO for solving the single objective optimization problems, the number of particles NP were taken equal to 20, while for the case of the objective optimization problem it was taken equal to 100, the cognitive and social pa-

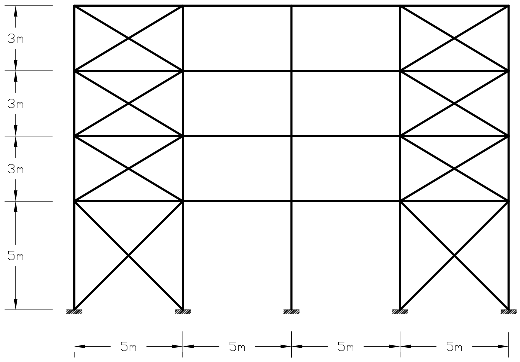

Figure 5: Four-storey steel braced frame

Table 2: Optimum designs and their properties

External Columns	Internal Columns	Beams	Braces	C_{LC} (€)	C_{in} (€)
Deterministic optimization (Initial cost):					
HEB120	HEB120	IPE100	L70×6	766354	327062
Deterministic optimization(Life-cycle cost):					
HEB240	HEB450	IPE120	L100×12	374810	354844
Reliability-based optimization (RBO) (Initial cost):					
HEB500	HEB240	IPE100	L100×12	568986	349020
Reliability-based optimization (RBO) (Life-cycle cost):					
HEB200	HEB550	IPE100	L90×7	352055	357304
Multi-objective optimization (Design A):					
HEB120	HEB120	IPE100	L70×6	766354	327062
Multi-objective optimization (Design B):					
HEB220	HEB360	IPE120	L90×10	371435	353925

rameters c_1 and c_2, respectively, were taken equal to 0.5 for both type of problems.

The frame is designed following the deterministic (DBO) and the reliability (RBO) design frameworks, as discussed in Section 2. More specifically, the following optimization frameworks are considered:

1. Deterministic design, where $C = C_{in}$ (Eq. 1),
2. Deterministic design, where $C = C_{lcc}$ (Eq. 1),
3. Reliability-based design, where $C = C_{in}$ (Eq. 2),
4. Reliability-based design, where $C = C_{lcc}$ (Eq. 2),
5. Minimum life-cycle cost design $C = [C_{in}, C_{lcc}]$ (Eq. 4).

Details on the calculation formula for each limit state along with the values of the basic cost for each category can be found in Table 4.

The optimum designs obtained with the above frameworks are summarized in Table 2. Following the minimum life-cycle cost design framework, the Pareto front curve with respect to the life-cycle and the initial cost is shown in Fig. 6. It is clear that a well distributed Pareto front curve has been obtained, demonstrating the efficiency of the NSPSO-II algorithm.

In order to compare the behaviour of the different optimum designs of the Pareto front curve, two characteristic designs were selected. These designs are marked as Design A and Design B on Fig. 6 and their properties are listed in Table 5. Extreme designs A and B correspond to the single-objective optima when the initial material cost or the life-cycle cost is the objective function, respectively.

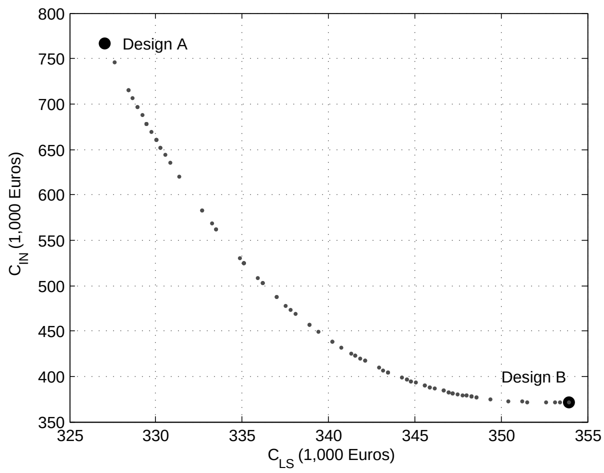

Figure 6: Optimum solutions of the multi-objective optimization problem (Pareto front).

Table 3: Damage states and limit-state cost values for the calculation of life-cycle cost [13]

Performance level	Damage state	Interstorey Drift	Cost (% of C_{in})
1	None	$\theta_{max} < 0.2$	0
2	Slight	$0.2 \leq \theta_{max} < 0.5$	0.5
3	Light	$0.5 \leq \theta_{max} < 0.7$	5
4	Moderate	$0.7 \leq \theta_{max} < 1.5$	20
5	Heavy	$1.5 \leq \theta_{max} < 2.5$	45
6	Major	$2.5 \leq \theta_{max} < 5.0$	80
7	Destroyed	$5.0 \leq \theta_{max}$	100

10. Conclusions

Alternative frameworks for the performance-based optimal seismic design have been discussed in this work. The procedures are applied for the design of a four-storey moment resisting steel braced frame. It is has been demonstrated that structural code design checks can be implemented in a straightforward manner when nonlinear analysis procedures are adopted and designs that meet the specified performance objectives with the desired confidence can be easily obtained. The Particle Swarm Optimization algorithm was implemented for the solution of the optimisation problem. Therefore, the trial-and-error design procedure that is performed on a heuristic fashion by structural engineers can be replaced by an automatic procedure based on a structural optimisation search engine while increased control of structural performance can be obtained since more elaborate analysis procedures are adopted for the design. The proposed design procedures are expressed both in deterministic and in probabilistic formats, the former applying directly the performance-based design concept as suggested by recent U.S. guidelines (e.g. FEMA-356) and the latter being a more elaborate performance-based design concept where the engineer can set his preference on the limit-state failure probabilities or on the mean annual frequencies of exceedance. Compared to the current design practice, all formulations lead to structures of improved seismic performance and reduced total cost.

Table 4: Limit state costs - Calculation formulas [14]

Cost Category	Calculation Formula	Basic Cost
Damage/repair (C_{dam})	Replacement × floor area × damage index	1500 (€/m^2)
Loss of contents (C_{con})	Unit contents × floor area × damage index	500 (€/m^2)
Rental (C_{ren})	Rental rate × gross leasable area × loss of function	10 (€/month/m^2)
Income (C_{inc})	Rental rate × gross leasable area × down time	2000 (€/year/m^2)
Minor Injury ($C_{inj,m}$)	Minor injury cost per person × floor area × occupancy rate* × expected minor injury rate	2000 (€/person)
Serious Injury ($C_{inj,s}$)	Serious injury cost per person × floor area × occupancy rate* × expected serious injury rate	2×10^4 (€/person)
Human fatality (C_{fat})	Human fatality cost per person × floor area × occupancy rate* × expected death rate	2.8×10^6 (€/person)

*Occupancy rate 2 persons/100 m^2

Table 5: Properties of selected designs of the Pareto front (**TO BE UPDATED**)

	Weight (m^3)	Life-cycle cost (m^3)	Total cost (m^3)	T_1 (sec)	V_b (kN)
A	23.2	73.2	96.5	1.23	5.200
B	29.2	22.9	51.9	1.03	7.667
C	38.8	19.8	58.6	0.91	9.834

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