Transactions Letters

On Systematic Punctured Convolutional Codes

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***Abstract*— This paper investigates high rate systematic punctured convolutional codes based on Forney's results. High rate systematic punctured convolutional codes of rate-2/3 with constraint length $3 \leq K \leq 7$ and rate-3/4 through 15/16 with $K = 7$ are derived using the rate-1/2 best known nonsystematic convolutional codes. Weight spectra of newly discovered systematic punctured convolutional codes are provided and compared with the best known nonsystematic punctured codes. Simulation results on bit-error probabilities (BEP's) are also given.**

***Index Terms*— Punctured convolutional codes, Viterbi decoding.**

I. Introduction

Convolutional codes with Viterbi decoding have been widely used as an efficient and powerful class of error correcting codes. An important subclass of convolutional codes is the class of systematic codes, where one output sequence is exact replica of the input sequence. There are two types of systematic encoder: feedforward and feedback [1]. The traditional systematic encoders are feedforward, which have not been widely used with Viterbi decoding. This is due to, as shown by Forney and Costello, their smaller minimum free distance d_{free} than nonsystematic feedforward encoder for a given rate and encoder memory [1]–[4]. In [1], Forney also showed that one can construct a systematic feedback encoder from nonsystematic feedforward encoder with identical distance properties of nonsystematic feedforward encoders where feedback represents a division operation. Recently, recursive systematic convolutional (RSC) codes have been applied to turbo-codes by Berrou *et al*. [5]. RSC codes can be viewed as systematic codes with the feedback encoder given in [1]. Also, Couleaud *et al*. have proposed systematic convolutional codes with feedback encoder in [6].

For applications requiring high data transmission rate, powerful high rate convolutional codes are necessary. For efficient high rate codes, punctured convolutional codes have been widely used since they can significantly reduce the complexity of the Viterbi decoder. The punctured convolutional codes are obtained by periodically perforating code symbols from the output of a low rate-$1/n$ original code [7]–[9]. So far, most punctured convolutional codes use nonsystematic feedforward encoders for the well-known fact that the smallest number of information bit errors c_d along all incorrect paths of weight d increases when low rate-$1/n$ nonsystematic codes are converted to systematic codes [9]–[12]. However, in high rate punctured convolutional codes, this fact is not always true and systematic punctured codes appear to perform better than nonsystematic punctured convolutional codes.

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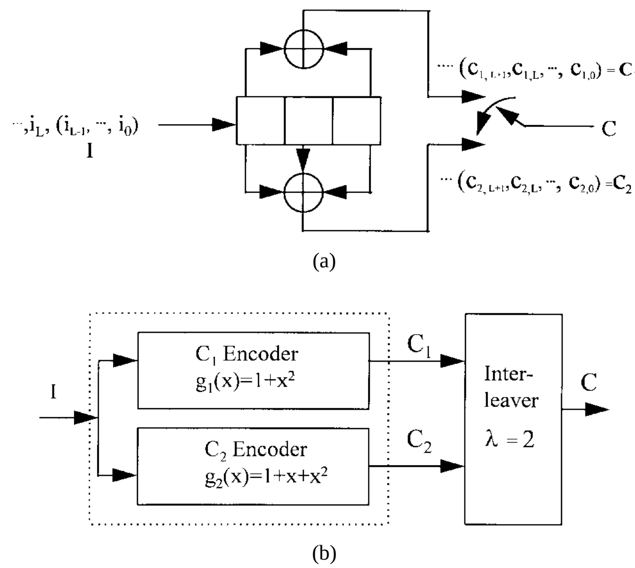


Fig. 1. Encoder for rate-1/2, $K = 3$, nonsystematic convolutional code with zero tail bits. (a) Typical encoder. (b) Equivalent encoder.

This paper describes a method based on the Forney's results [1] for generating systematic convolutional codes from nonsystematic convolutional codes by transforming information bits prior to encoding with the division operation to produce a systematic codeword using a nonsystematic feedforward encoder. By this method, we derive a class of systematic punctured convolutional codes. In addition, weight spectra of newly discovered systematic punctured convolutional codes are provided and compared with the best known nonsystematic punctured codes. It will be shown that high rate systematic punctured convolutional codes perform better than nonsystematic punctured codes. In Section II, we will briefly describe the proposed systematic convolutional codes construction methods and derive the weight spectra of the proposed systematic punctured convolutional codes. In Section III, bit-error probabilities (BEP's) are analyzed on the additive white Gaussian noise (AWGN) channel for the proposed systematic punc-

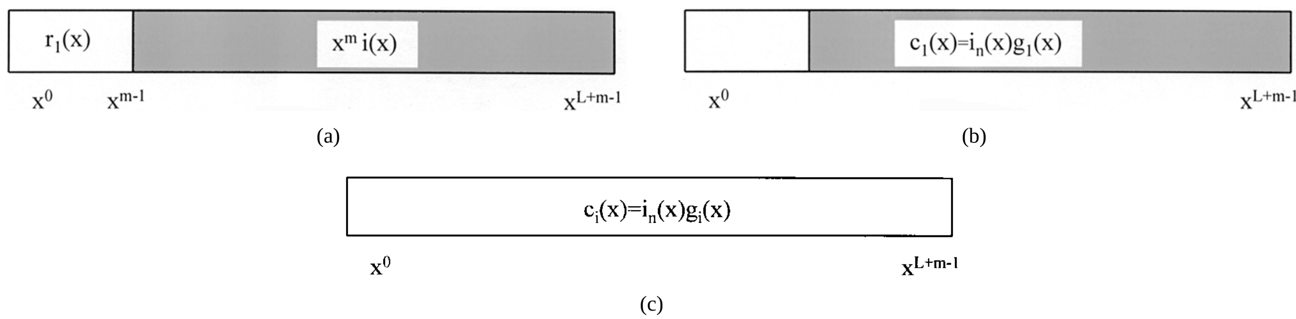


Fig. 2. Bit structures of codewords: (a) $c_s(x)$, (b) $c_1(x)$, and (c) $c_i(x)$.

tured convolutional codes and the best known nonsystematic convolutional codes.

II. Systematic Punctured Convolutional Codes

In this section, we briefly describe a method to generate rate-$1/n$ systematic convolutional codes based on the method given in [1]. Fig. 1 shows a feedforward encoder of rate-1/2, constraint length $K = 3$, and memory $m = 2$ nonsystematic convolutional code with generator polynomials $g_1(x)$ and $g_2(x)$ given by

$$g_1(x) = 1 + x^2 \tag{1}$$

$$g_2(x) = 1 + x + x^2. \tag{2}$$

The arbitrary L-input message bits $(i_0, i_1, \cdots, i_{L-1})$ are transformed into two output codewords $c_1(x)$ and $c_2(x)$ given by

$$c_1(x) = \sum_{j=0}^{L+1} c_{1,j} x^j = i(x) \cdot g_1(x) \tag{3}$$

$$c_2(x) = \sum_{j=0}^{L+1} c_{2,j} x^j = i(x) \cdot g_2(x) \tag{4}$$

where $i(x)$ is the message polynomial given by

$$i(x) = \sum_{j=0}^{L-1} i_j x^j. \tag{5}$$

In Fig. 1(a), one observes the output codeword C which is an interleaved $(2L + 4, L)$ linear code of C_1 and C_2 with an interleaving depth λ of 2 [3], [4]. Fig. 1(b) shows an equivalent encoder of (a). Since any linear nonsystematic code can be transformed into a systematic code with the same distance profile, these show that it is possible to construct systematic convolutional codes with the same distance profile of corresponding nonsystematic codes. In [1], it has been shown that systematic convolutional codes with a feedback encoder can be constructed from nonsystematic codes with a feedforward encoder.

A. Rate-$1/n$ Systematic Convolutional Codes

A method based on a canonical class of encoder for generating systematic codes from feedforward nonsystematic codes is proposed [1]. Consider a rate-$1/n$ nonsystematic convolutional code with constraint length K (or memory $m = K - 1$) and generator polynomials $g_1(x), g_2(x), \cdots$, and $g_n(x)$. Let $C_1, C_2, \cdots$, and C_n be nonsystematic codes generated by $g_1(x), g_2(x), \cdots$, and $g_n(x)$, respectively. Assume that L is the arbitrary length of input message bits. Let I and $i(x)$ be the original input message word and polynomial, respectively, and C_s be a message code containing unaltered input message bits. If C_s is generated by $g_1(x)$ then the codeword of C_s is given by

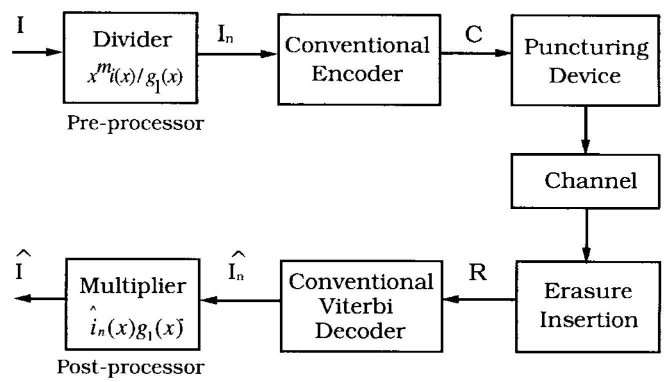


Fig. 3. Encoder and decoder of systematic punctured convolutional codes.

$$c_s(x) = r_1(x) + x^m i(x) \tag{6}$$

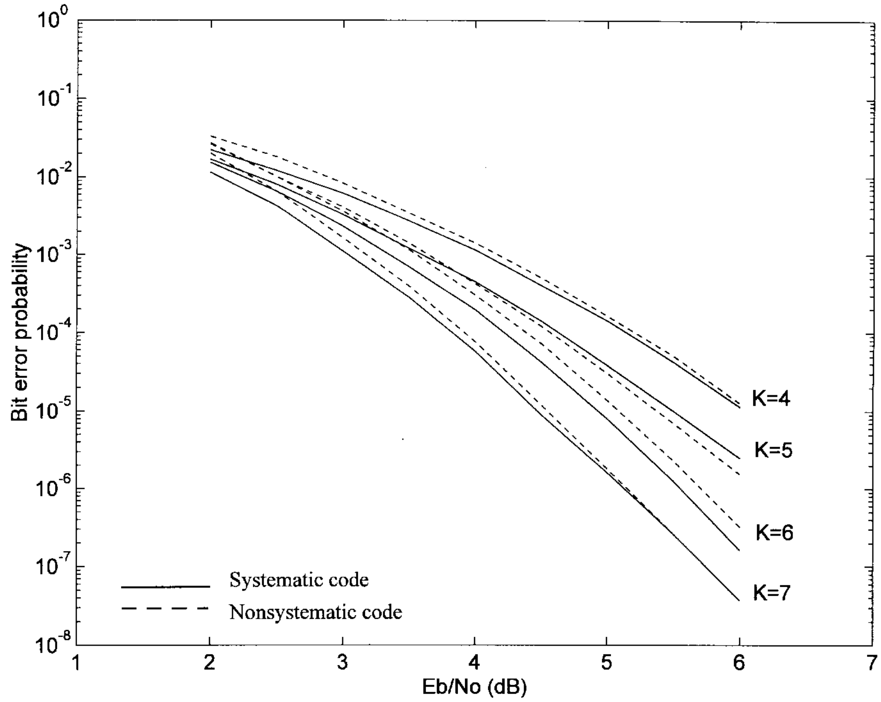
where $r_1(x)$ is the remainder of $x^m i(x)/g_1(x)$ [3], [4]. Due to linearity, C_1 and C_s have an identical set of codewords since both generator polynomials are $g_1(x)$. Futhermore, there exists a nonsystematic codeword $c_1(x)$ which is equal to $c_s(x)$ such that

$$c_s(x) = c_1(x) \tag{7}$$

$$= i_n(x) g_1(x) \tag{8}$$

although the message bits are different in each of them [3], [4]. From (6) and (7), we see that the original L-input message bits $(i_0, i_1, \cdots, i_{L-1})$ lie between the $m+1'$th bit and $L+m'$th bit in a nonsystematic codeword $c_1(x)$. This is clearly depicted in Fig. 2(a) and (b) in which gray blocks represent the original input message bits. If $i_n(x) = c_s(x)/g_1(x)$, then Fig. 2(a) is equal to Fig. 2(b). Thus, without any loss of the input message bits, nonsystematic convolutional codes can be easily transformed into systematic convolutional codes.

Any C_i of $C_1, C_2, \cdots$, and C_n can be selected for the message code which determines its distance profile. Fig. 3 shows the encoder and decoder of proposed systematic convolutional codes with a message code C_1. The proposed



K=4: $\mathbf{A}=\begin{bmatrix}1&1\\0&1\end{bmatrix}$, K=5: $\mathbf{A}=\begin{bmatrix}1&1\\0&1\end{bmatrix}$, K=6: $\mathbf{A}=\begin{bmatrix}1&0\\1&1\end{bmatrix}$, K=7: $\mathbf{A}=\begin{bmatrix}1&1\\1&0\end{bmatrix}$.

Fig. 4. BEP's of rate-2/3 systematic and nonsystematic punctured convolutional codes.

encoder consists of a conventional encoder with a preprocessor performing $c_s(x)/g_1(x)$ and producing a new message word I_n (or polynomial $i_n(x)$). In Fig. 3, a preprocessor produces $i_n(x)$ by performing $x^m i(x)/g_1(x)$ and transferring only the quotient of $x^m i(x)/g_1(x)$ which is equal to $c_s(x)/g_1(x)$. This decoder is a standard nonsystematic Viterbi decoder. A postprocessor recovers the decoded sequence $\hat{i}(x)$ by performing $\hat{i}_n(x) \cdot g_1(x)$ and dropping the terms of degree $m-1$ *or less*. Encoding/decoding are performed continuously. Note that the proposed scheme can be easily implemented with existing Viterbi decoders. The proposed systematic codes are similar to RSC codes with combined feedback and feedforward encoders. The difference is that RSC codes use $x^m i(x)$ while the proposed systematic codes use $c_s(x)$ for a message code.

B. High-Rate Systematic Punctured Convolutional Codes

It is possible to find high rate systematic convolutional codes from nonsystematic convolutional codes directly through the same approach for generating rate-$1/n$ systematic convolutional codes. However, it is more attractive and effective to use puncturing methods to obtain high rate convolutional codes where inherent difficulties of encoding and decoding can be almost resolved [3], [7]–[9].

Consider rate-$1/n$ systematic convolutional codes with a constraint length of K. Assume that P is a puncturing period and A is an $n \times P$ puncturing matrix which is given by

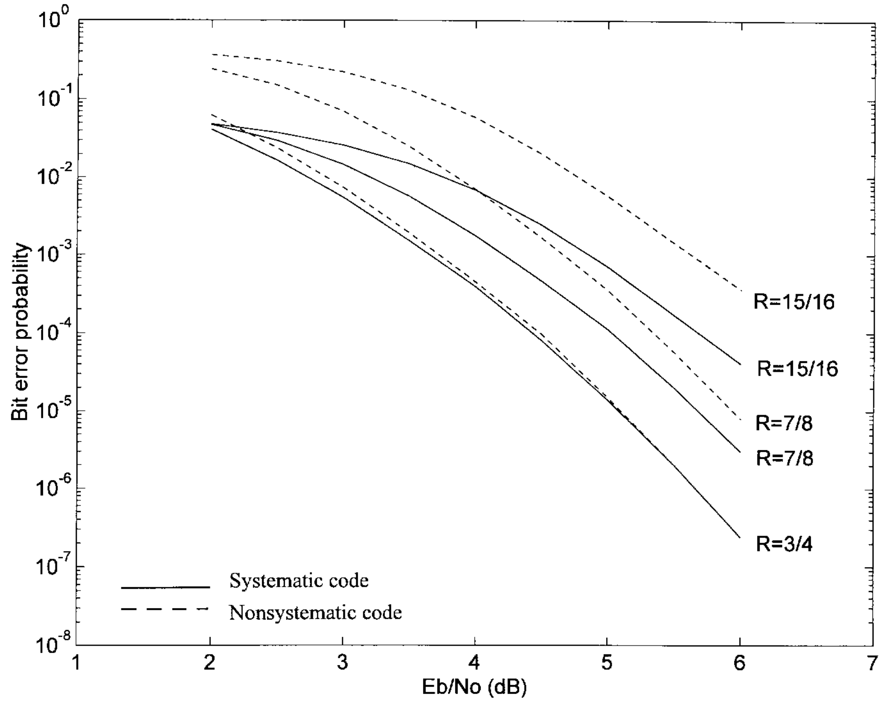
$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1P} \\ a_{21} & a_{22} & \cdots & a_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nP} \end{pmatrix} \tag{9}$$

where $a_{ij} \in \{0, 1\}$. Zero elements of A denote the positions of punctured bits. We can obtain various rate-k/n systematic convolutional codes from rate-$1/n$ systematic codes by selecting P and A. It has been well known that the conditions for choosing the puncturing matrix A is obvious: noncatastrophic property, maximum $d_{\rm free}$, and minimum a_d and c_d. In addition, one condition that one row of A must have all ones is required for systematic form. In a convolutional code, a_d is the number of incorrect paths of Hamming weight d $(d \geq d_{\rm free})$ that diverge from a correct path then remerge sometime later. c_d is the total number of information bit errors produced by incorrect paths [7]–[12]. In Fig. 3, the encoder and decoder of high rate punctured systematic convolutional codes are shown. The conventional encoder and decoder are modified by insertion of a puncturing device and an erasure-inserting device [7].

In the proposed systematic codes, error event probability P_E is the same as that of the corresponding nonsystematic codes since the two codes have the same set of codewords. However, each possess different BEP P_b because the weight spectra of information bits in the codes are different [2]–[4].

C. Weight Spectra of Systematic Punctured Convolutional Codes

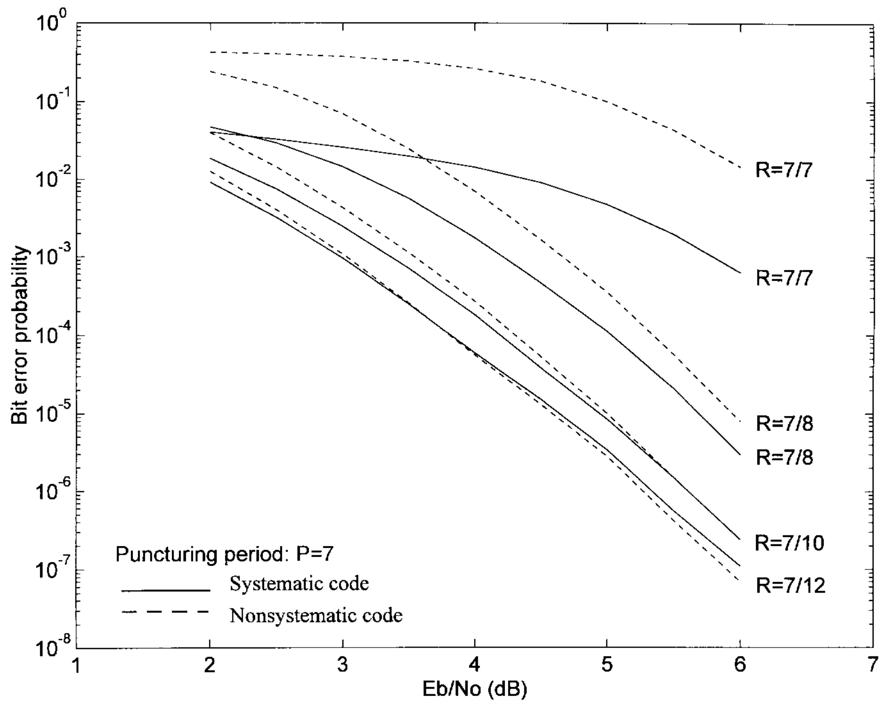
The proposed rate-$1/n$ systematic convolutional code have same distance profile of corresponding nonsystematic convolutional codes due to identical generator polynomials. When the fact that classical feedforward systematic convolutional codes have a smaller $d_{\rm free}$ than nonsystematic convolutional codes is considered, the proposed systematic codes show an



R=3/4: $\mathbf{A}=\begin{bmatrix}1&1&0\\1&0&1\end{bmatrix}$, R=7/8: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1\\1&0&0&0&0&0&0\end{bmatrix}$,

R=15/16: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$.

Fig. 5. BEP's of systematic and nonsystematic punctured convolutional codes. Code rate $R = p/(p+1), p = 3, 7, 15$. $K = 7$.



R=7/7: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1\\0&0&0&0&0&0&0\end{bmatrix}$, R=7/8: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1\\1&0&0&0&0&0&0\end{bmatrix}$,

R=7/10: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1\\1&0&0&0&1&1&0\end{bmatrix}$, R=7/12: $\mathbf{A}=\begin{bmatrix}1&1&1&1&1&1&1\\1&1&1&0&1&1&0\end{bmatrix}$.

Fig. 6. BEP's of systematic and nonsystematic punctured convolutional codes. Code rate $R = 7/(7+s), s = 0, 1, 3, 5$. $K = 7$.

TABLE I
WEIGHT SPECTRA OF RATE-2/3 SYSTEMATIC PUNCTURED CONVOLUTIONAL CODES GENERATED FROM THE BEST KNOWN RATE-1/2 CODE OF $3 \leq K \leq 7$ AND $P = 2$

Original codes				Punctured convolutional codes			
K	G_1	G_2	d_{free}	**A**	d_{free}	$a_d, c_d, c_{1d}, c_{2d}, d = d_{free}, d_{free}+1, \cdots$	
3	101	111	5	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	3	a_d	1,4,14,40,115,331,953,2744,7901,22750,65506
						c_d	1,10,54,226,853,3038,10423,34836,114197,368814,1177124
						c_{2d}	3,10,44,154,521,1724,5609,18008,57201,180106,562944
						$c_d - c_{2d}$	−2,0,10,72,332,1314,4814,16828,56996,188708,614180
4	1101	1111	6	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	4	a_d	3,11,35,114,378,1253,4147,13725,45428,150362
						c_d	10,43,200,826,3314,12857,48834,182373,672324,2452626
						c_{1d}	10,33,146,538,2046,7595,27914,101509,366222,1312170
						$c_d - c_{1d}$	0,10,54,288,1268,5262,20920,80864,306102,1140456
5	10011	11101	7	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	4	a_d	1,0,27,0,345,0,4515,0,59058,0,772627
						c_d	1,0,124,0,2721,0,50659,0,858436,0,13793381
						c_{1d}	3,0,106,0,1841,0,30027,0,471718,0,7201171
						$c_d - c_{1d}$	−2,0,18,0,880,0,20632,0,386718,0,6592210
6	101011	111101	8	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	6	a_d	19,0,220,0,3089,0,42725,0,586592
						c_d	96,0,1904,0,35936,0,637895,0,10640725
						c_{2d}	82,0,1260,0,21530,0,354931,0,5643947
						$c_d - c_{2d}$	14,0,644,0,14406,0,282964,0,4996778
7	1011011	1111001	10	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	6	a_d	1,16,48,158,642,2435,9174,34701,131533
						c_d	3,70,285,1276,6160,27128,117019,498835,2103480
						c_{1d}	3,76,269,960,4290,18034,74197,303431,1237276
						$c_d - c_{1d}$	0,−6,16,316,1870,9094,42822,195404,866204

attractive advantage [2]–[4]. Next, consider the weight spectra of punctured convolutional codes. The results in Tables I and II have been derived using the assumption that a decoder works on the low rate trellis of the original code as in [12]. Table I lists rate-2/3 systematic punctured convolutional codes generated from the best known rate-1/2 codes. G_1 and G_2 denote the generator polynomial coefficients of the original rate-1/2 codes. In Table I, $c_{id}, i = 1, 2,$ denotes the total number of information error bits when one of $C_i, i = 1, 2,$ is selected for a message code. Table I shows that the proposed systematic punctured codes have lower c_d than nonsystematic punctured codes except for three cases: c_{23} for $K = 3, c_{14}$ for $K = 5$, and c_{17} for $K = 7$. Table I also shows that the differences of $(c_d - c_{1d})$ (or $(c_d - c_{2d})$) are positive although they are small.

Table II lists the weight spectra of systematic punctured convolutional codes generated from the best known rate-1/2 code of $K = 7$ for various puncturing period P. For $P = 3, 5, 6,$ and 7, Table II lists the two different weight spectra according to puncturing matrices such that one is the proposed systematic puncturing matrix and the other is the best known nonsystematic puncturing matrix. It is shown that c_{1d} (or c_{2d}) becomes smaller than c_d as P increases or d increases for a fixed P. In fact, for $P = 15$, differences between c_d and c_{1d} are large and all positive. This implies that P_b of the systematic punctured code with $K = 7$ and rate-15/16 is significantly lower than nonsystematic code at both low and high E_b/N_o. Also, Table II shows that, at low E_b/N_o, all systematic punctured codes perform better than nonsystematic codes. Another interesting result is that, for $P = 4, 5, 6, 7,$ and 15, c_{1d} (or c_{2d}) is smaller than c_d of nonsystematic codes with the best known nonsystematic puncturing matrices denoted by "*."

III. SIMULATION RESULTS AND DISCUSSION

We simulated performances of a number of punctured convolutional codes on the AWGN channel with binary phase shift keying modulation. A Viterbi decoder is used with unquantized matched filter soft decision inputs. Fig. 4 shows BEP's P_b of $R = 2/3$ proposed systematic codes and nonsystematic codes. A gain of 0.2 dB has been obtained for $P_b = 10^{-5}$ for $K = 6$, and that simulation results and the weight spectra of Table I are in good agreement. For $R = 2/3$, the performance improvement of proposed systematic codes is minimal. To investigate the possible gains for high rate systematic punctured convolutional codes, BEP's for $K = 7, R = p/(p+1), p = 3, 7,$ and 15 were examined through simulation. Fig. 5 shows that systematic codes perform better than nonsystematic codes at both low and high E_b/N_o. The improvement increases as the code rate increases. For $R = 7/8$ and $15/16$, gains of 0.3 and 0.6 dB are obtained respectively at $P_b = 10^{-4}$. Fig. 6 shows P_b of the systematic and nonsystematic punctured codes with $K = 7$ for fixed $P = 7$ and various $R = 7/(7+s), s = 0, 1, 3, 5$. It also shows that,

TABLE II
WEIGHT SPECTRA OF SYSTEMATIC PUNCTURED CONVOLUTIONAL CODES GENERATED FROM THE BEST KNOWN RATE-1/2 CODE OF $K = 7$. $G_1 = [1011011], G_2 = [1111001]$

P	R	$\mathbf{\Lambda}$	d_{free}		$a_d,\ c_d,\ c_{1d},\ c_{2d},\ d = d_{free},\ d_{free}+1, \cdots$
3	3/4	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^*$	5	a_d	8,31,160,892,4512,23297,120976,624304
				c_d	42,201,1492,10469,62935,379546,2252394,13064540
				c_{1d}	46,203,1256,8301,47527,274684,1582618,8968812
				c_{2d}	44,217,1274,8123,47151,273196,1570834,8893944
		$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	4	a_d	3,0,149,0,3535,0,93326,0,2423904
				c_d	18,0,1581,0,53181,0,1819459,0,58117217
				c_{2d}	10,0,731,0,22405,0,730817,0,22568801
4	4/5	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^*$	4	a_d	3,24,172,1158,7408,48706,319563,2094852
				c_d	12,188,1732,15256,121367,945395,7167584,53348314
				c_{1d}	10,96,848,6552,47789,353697,2575710,18559164
5	5/6	$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}^*$	4	a_d	14,69,654,4996,39677,314973,2503576,19875546
				c_d	92,528,8694,79453,791795,7369828,67809347,609896348
				c_{1d}	88,508,6514,57913,541275,4893312,43802869,386252770
				c_{2d}	86,502,6678,58497,553047,4992962,44803141,395367510
		$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$	3	a_d	1,17,136,1143,8717,69488,550980,4362319
				c_d	3,187,1797,19202,180275,1703292,15555517,139587843
				c_{1d}	3,61,591,5900,52293,474356,4213541,36953475
6	6/7	$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}^*$	3	a_d	1,20,223,1961,18084,168982,1573256,14620204
				c_d	5,169,2725,32233,370771,4166922,45364308,482335989
				c_{1d}	5,149,1923,21239,231225,2502704,26506364,275916511
				c_{2d}	7,163,2261,25365,279653,3048934,32481236,339725101
		$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	3	a_d	2,32,310,2767,25617,235749,2170952,19982803
				c_d	10,349,4165,49523,567541,6194830,66146888,692510940
				c_{1d}	6,115,1383,14653,157453,1650726,17044626,173880868
7	7/8	$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}^*$	3	a_d	2,46,499,5291,56137,598557,6371293,67889502
				c_d	9,500,7437,105707,1402089,17888043,221889258,2699950506
				c_{1d}	7,310,4329,56939,721833,8917389,108152014,1292733888
				c_{2d}	15,464,6265,83887,1072601,13344173,162427398,1948489894
		$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	3	a_d	2,42,468,4939,52778,567636,6073705,65082861
				c_d	14,389,6792,97243,1317172,17162804,215514708,2651898998
				c_{1d}	6,149,2084,26181,326066,4000330,48097368,572029038
15	15/16	$\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$	2	a_d	1,77,1671,36222,774079,16547926
				c_d	32,1633,51514,1487671,39743233,1019689517
				c_{1d}	2,229,6440,173219,4426927,110110665

* The best known puncturing matrix for nonsystematic convolutional codes.

at increasing code rate, the systematic punctured convolutional codes perform better than the nonsystematic codes.

IV. Summary and Conclusions

In this paper, we have derived good high rate systematic punctured convolutional codes and analyzed their weight spectra and BEP performance. It was shown that high rate systematic punctured convolutional codes perform better than nonsystematic punctured convolutional codes as the code rate increases for constraint length $3 \leq K \leq 7$. For $K = 7, 3/4 \leq R \leq 7/8$, and $15/16$, the proposed systematic punctured convolutional codes provided better performance than the best known nonsystematic punctured convolutional codes. Simulation results of BEP showed good agreement with weight spectra results. In comparing nonsystematic punctured codes with $K = 7$, rate-7/8 and 15/16, corresponding systematic punctured codes demonstrated 0.3 and 0.6 dB improvements, respectively, at $P_b = 10^{-4}$.

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