Exercise for chapter 4 (Part 2)
Determinization, Optimization and Applications

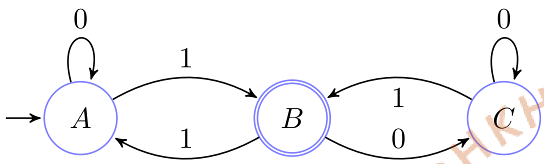
1 Introduction

In this exercise, we will pratice mainly on automata determinization - from NFA (nondeterministic finite automata) to DFA (deterministic finite automata). Students should review the slide and related theoretical documents before doing the exercises below.

2 Example

Question 1.

Give an execution of the following DFA on 0001, 01001, and 0110.



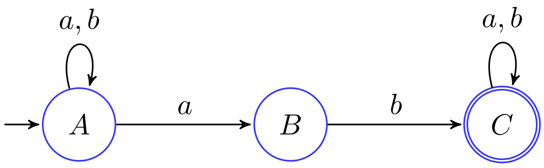
Solution.

- *$(A, 0001) \rightarrow (A, 001) \rightarrow (A, 01) \rightarrow (A, 1) \rightarrow (B, -) \Rightarrow 0001$ is a valid word.*
- *$(A, 01001) \rightarrow (A, 1001) \rightarrow (B, 001) \rightarrow (C, 01) \rightarrow (C, 1) \rightarrow (B, -)$. Then, 01001 is a valid word.*
- *$(A, 0110) \rightarrow (A, 110) \rightarrow (B, 10) \rightarrow (A, 0) \rightarrow (A, -)$. Since A is not an accepting state, then 0110 is an invalid word.*

□

Question 2.

Convert the following NFA into DFA.
Give an execution of the DFA on $aaba$, $bbabbbaa$, $bababaa$ and $bbabbbabbbabba$.

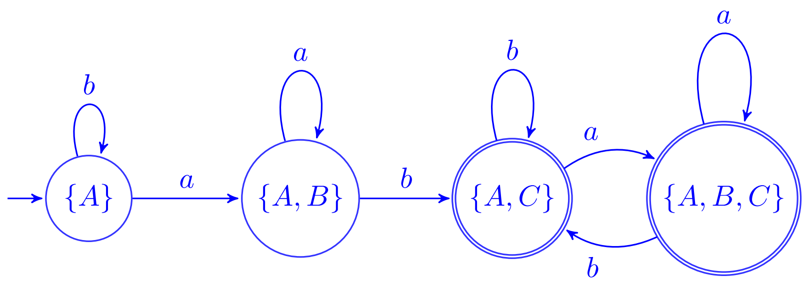


Solution.

First, we need define transition table containing useful sets of states as follows.

	a	b
$\rightarrow \{A\}$	$\{A, B\}$	$\{A\}$
$\{A, B\}$	$\{A, B\}$	$\{A, C\}$
$\{A, C\}^*$	$\{A, B, C\}$	$\{A, C\}$
$\{A, B, C\}^*$	$\{A, B, C\}$	$\{A, C\}$

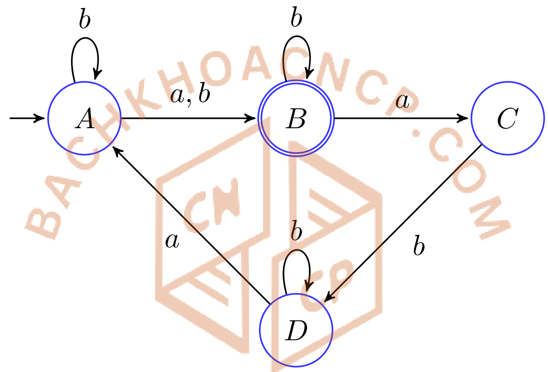
DFA could be determined in which each state refers a useful sets of NFA state.



3 Exercise

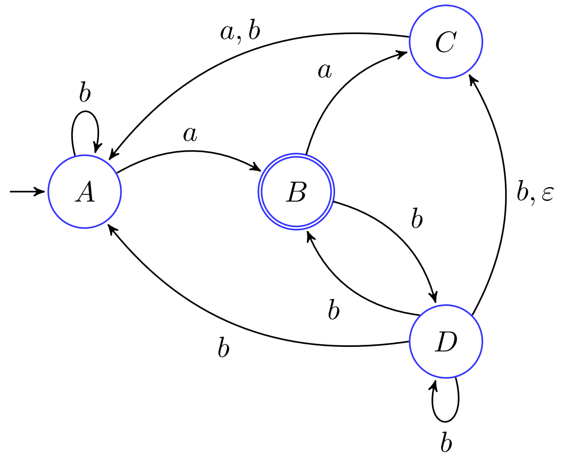
Question 3.

Convert the following NFA into DFA.



Question 4.

Convert the following NFA into DFA.



Question 5.

Find a regular expression for the set $\{a^n b^m : (n+m) \text{ is even } \}$.
Determine the corresponding DFA (or NFA and then convert NFA to DFA).

Question 6.

Give a regular expression for the language on $\Sigma = \{a, b, c\}$ containing no any sequence of a with length

greater than two.
Determine the corresponding DFA (or NFA and then convert NFA to DFA).

Question 7.
Give a regular expression for the language on $\Sigma = \{a, b\}$ containing all strings not ending in ab. Determine the corresponding DFA (or NFA and then convert NFA to DFA).

Question 8.
Let $\Sigma = \{a, b, c\}$. Give complet DFA's for the sets consisting of

a) all strings with exactly two 'a'.

b) all strings of odd length.

c) all strings which the number of appearances of both 'b' and 'c' is divisible by 3.

d) all strings ending with ca.

e) all strings not ending with 'a' and any aa appreared after bc.

f) all non-empty strings not ending with 'ca'.

g) all strings with at least one 'b'.

h) all strings with at most one 'a' and at least one 'b'.

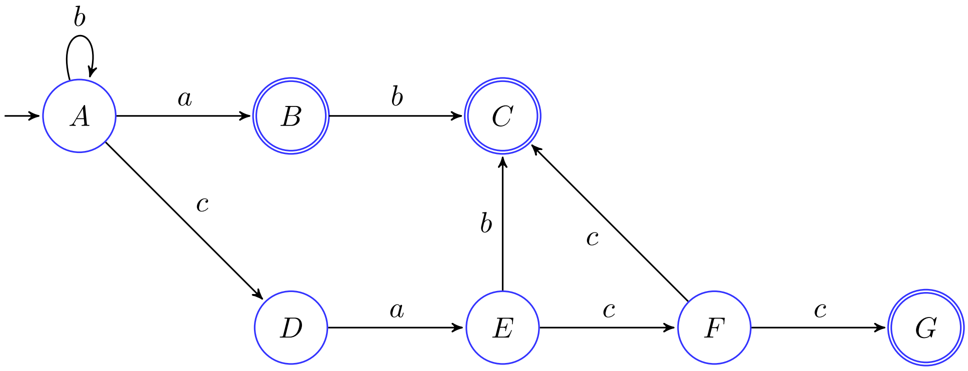
i) all strings without any 'a' and at most one 'b'.

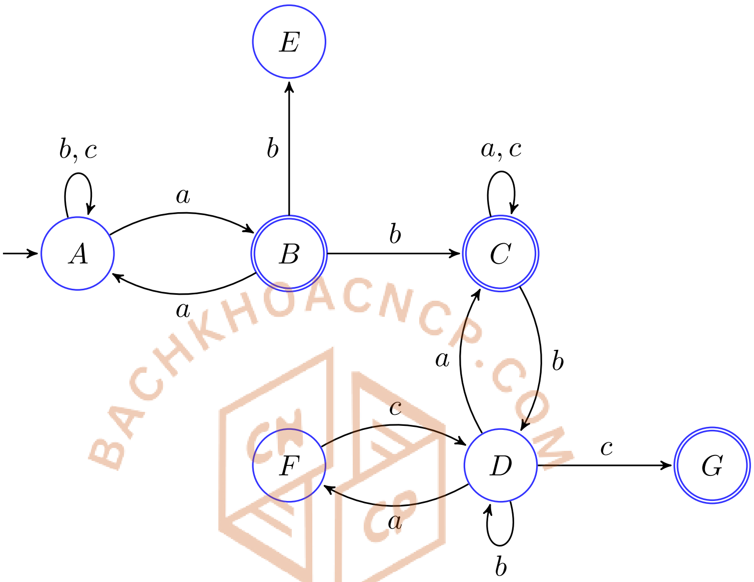
j) all strings including at least one a and whose the first appearance of 'a' is not followed by a 'b'.

Question 9.
Give a DFA that accepts language which represent by regular expression:

- $E_1 = ((a + b)^*b(a + ab)^*)$,
- $E_2 = b^*(a + b) + aa^*aba^*$,
- $E_3 = (aa + ca)b^*b + +cab^*a^*c$,
- $E_4 = b(ca + ac)(aa)^* + a^*(ca + ac)$,
- $E_5 = (ab)^{2*}c + (a + b)c^*$,
- $E_6 = b(b^* + a^*b)ac + a^*(b^* + a^*b)$,
- $E_7 = (b + c)ab + ba(c + ab)^*$,
- $E_8 = (b + c)^*ba + a(c + b)^*$,
- $E_9 = (a(b + c)^* + bc^*)^*$.

Question 10.
Minimize the following automatas.

1. 

2. 

Question 11.

Propose an automata to describe a vehicular multi-information display system with a given number of buttons.

For example, digital speedo meter of Honda Lead motor with only one button can display information about: petroleum level, speed, trip, date, time, engine oil life. (Hint: we distinguish two different actions: quickly press the button, press the button and hold-down over two seconds.)