BKU, VNU-HCM
FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** *(One personal A4-sheet allowed)*
Test date: **June 3, 2020**

(*There are **20** MCQs, each question is worth **0.5** points. Indicate your choice on the answer sheet.*)

Question 1. Which of the following statements is true for a pair of primal and dual problems?

(A) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(B) The dual problem of the dual problem is different from the primal problem.
(C) Variables in one program correspond to constraints in the other.
(D) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.

Question 2. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I only. (B) III only. (C) I and III. (D) I and II.

Question 3. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y(P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is a brother of w.
(B) v is a nephew of w.
(C) v is a grandfather of w.
(D) v is an uncle of w.

Question 4. In the branch and bound technique, what is the definition of the incumbent?

(A) The upper bound of the objective function.
(B) The best integer solution that we obtain at each step of branching and bounding.
(C) The lower bound of the objective function.
(D) None of the other choices is correct.

For questions 5–7, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 5. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

Question 6. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y (x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z-2x_2+3x_3 \le 0; z+2x_1-3x_4 \le 0; z-3x_1+4x_4 \le 0; z+3x_2-4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(B) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(C) max z subject to $\{2x_2 - 3x_3 \le 0; -2x_1 + 3x_4 \le 0; 3x_1 - 4x_4 \le 0; 3x_2 - 4x_3 \le 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

(D) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1, x_2, x_3, x_4 \ge 0\}$

Question 7. The optimal value of z is

(A) 0 (B) 1 (C) 3 (D) 2 or 4

Question 8. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 57.15% of the time hide 1 and guess 2.
(B) 42.85% of the time hide 1 and guess 1.
(C) 57.15% of the time hide 2 and guess 1.
(D) 42.85% of the time hide 2 and guess 2.

Question 9. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(B) From A infer $A \wedge B$.
(C) From A and $A \longrightarrow B$ infer B.
(D) From A infer $\neg\neg A$.

Question 10. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; G$. (B) $B; G$. (C) $B; E$. (D) $C; E$.

Question 11. Which of the following statements is true?

(A) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(B) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.
(C) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.
(D) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.

Question 12. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US\$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US\$ 1200 (B) US\$ 4500 (C) US\$ 5400 (D) US\$ 6500

Question 13. Consider the linear pogramming problem

$$\max 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
(B) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$
(C) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$
(D) $\min 5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \geq 6; 3x_1 + x_2 + x_3 \geq 4; x_2 - x_1 \geq 7\}$

Question 14. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Graph the problem.
(B) Change the objective function coefficients to whole integer numbers.
(C) Solve the original problem by allowing continuous noninteger solutions.
(D) Compare the lower bound to any upper bound of your choice.

Question 15. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $0; 0.$ (B) $1; 1.$ (C) $0; 1.$ (D) $1; 0.$

Question 16. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000),(100000),(110000),(111000),(111100),(111110),$$

$$(111111),(011111),(001111),(000111),(000011),(000001)\}.$$

For each $0 \leq i \leq 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k}\neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right)\wedge\left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5}\neg b_i\right)\right)$.

(B) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k}\neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right)\wedge\left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5}\neg b_i\right)\right)$.

(C) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k}\neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right)\vee\left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5}\neg b_i\right)\right)$.

(D) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k}\neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right)\vee\left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5}\neg b_i\right)\right)$.

Question 17. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.

(B) Brown is the killer.

(C) Clark is the killer.

(D) The given information is insufficient to discover the killer.

Question 18. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \wedge\}$.

(B) $\{\neg, \rightarrow\}$.

(C) $\{\rightarrow, \perp\}$.

(D) $\{\rightarrow, \wedge\}$.

Question 19. In this question, assume the following predicate and constant symbols:

$W(x,y)$: x wrote y	h : Hardy	p : Pride and Predjudice.
$L(x,y)$: x is longer than y	a : Austen	
$N(x)$: x is a novel	j : Jude the Obscure	

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x(W(h,x) \rightarrow L(x,a)))$.

(B) $\forall x\exists y(L(x,y) \rightarrow W(h,y) \wedge W(a,x))$.

(C) $\forall x\forall y(W(h,x) \wedge W(a,y) \rightarrow L(x,y)))$.

(D) $\exists x(N(x) \wedge W(h,x) \wedge \forall y(N(y) \wedge W(a,y) \rightarrow L(x,y)))$.

Question 20. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
    $r := r * m;$
    $i := i + 1$

is

(A) $(m \geq 0) \wedge (n \geq 0)$.

(B) $m \geq 0$.

(C) $n \geq 0$.

(D) $(m > 0) \wedge (n \geq 0)$.

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Question 2. Which of the following statements is true for a pair of primal and dual problems?

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(B) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
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(A) $\exists x(N(x) \wedge W(h,x) \wedge \forall y(N(y) \wedge W(a,y) \rightarrow L(x,y)))$.
(B) $\forall x(W(h,x) \rightarrow L(x,a)))$.
(C) $\forall x \exists y(L(x,y) \rightarrow W(h,y) \wedge W(a,x))$.
(D) $\forall x \forall y(W(h,x) \wedge W(a,y) \rightarrow L(x,y)))$.

Question 5. Suppose that $P(x,y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v,w)$ equals

$$M(v) \wedge \exists x \exists y(P(x,y) \wedge P(x,v) \wedge (y \neq v) \wedge P(y,w)),$$

then what is the meaning of the expression $F(v,w)$?

(A) v is an uncle of w.
(B) v is a brother of w.
(C) v is a nephew of w.
(D) v is a grandfather of w.

Question 6. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

Ⓐ From A infer $\neg\neg A$. Ⓑ From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
Ⓒ From A infer $A \wedge B$. Ⓓ From A and $A \longrightarrow B$ infer B.

Question 7. In the branch and bound technique, what is the definition of the incumbent?

Ⓐ None of the other choices is correct.
Ⓑ The upper bound of the objective function.
Ⓒ The best integer solution that we obtain at each step of branching and bounding.
Ⓓ The lower bound of the objective function.

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II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

Ⓐ I and II. Ⓑ I only. Ⓒ III only. Ⓓ I and III.

Question 10. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

Ⓐ $1; 0$. Ⓑ $0; 0$. Ⓒ $1; 1$. Ⓓ $0; 1$.

Question 11. In the first step of a branch and bound approach to solving integer programming problems is to

Ⓐ Compare the lower bound to any upper bound of your choice.
Ⓑ Graph the problem.
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The farmer has a budget of US\$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

Ⓐ US\$ 6500 Ⓑ US\$ 1200 Ⓒ US\$ 4500 Ⓓ US\$ 5400

Question 13. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
  $r := r * m;$
  $i := i + 1$

is

(A) $(m > 0) \wedge (n \geq 0)$.
(B) $(m \geq 0) \wedge (n \geq 0)$.
(C) $m \geq 0$.
(D) $n \geq 0$.

Question 14. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; E$.
(B) $C; G$.
(C) $B; G$.
(D) $B; E$.

For questions 15–17, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

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H2G1	-3	0	0	4
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(B)

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H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
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Question 16. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1, x_2, x_3, x_4 \geq 0\}$
(B) max z subject to $\{z-2x_2+3x_3 \leq 0; z+2x_1-3x_4 \leq 0; z-3x_1+4x_4 \leq 0; z+3x_2-4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$
(C) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$
(D) max z subject to $\{2x_2-3x_3 \leq 0; -2x_1+3x_4 \leq 0; 3x_1-4x_4 \leq 0; 3x_2-4x_3 \leq 0; x_1, x_2, x_3, x_4 \in \{0,1\}\}$

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(A) 2 or 4 (B) 0 (C) 1 (D) 3

Question 18. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 42.85% of the time hide 2 and guess 2.
(B) 57.15% of the time hide 1 and guess 2.
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Question 19. Consider the linear pogramming problem

$$\max\ 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) min $5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \geq 6; 3x_1 + x_2 + x_3 \geq 4; x_2 - x_1 \geq 7\}$
(B) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
(C) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$
(D) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$

Question 20. Which of the following statements is true?

(A) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.
(B) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(C) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.
(D) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** ***(One personal A4-sheet allowed)***
Test date: **June 3, 2020**

(*There are* ***20*** *MCQs, each question is worth* ***0.5*** *points. Indicate your choice on the answer sheet.*)

Question 1. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.
(B) The given information is insufficient to discover the killer.
(C) Brown is the killer.
(D) Clark is the killer.

Question 2. In this question, assume the following predicate and constant symbols:

$W(x, y)$: x wrote y $\quad$ h : Hardy $\quad$ p : Pride and Predjudice.
$L(x, y)$: x is longer than y $\quad$ a : Austen
$N(x)$: x is a novel $\quad$ j : Jude the Obscure

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x(W(h, x) \to L(x, a)))$.
(B) $\exists x(N(x) \wedge W(h, x) \wedge \forall y(N(y) \wedge W(a, y) \to L(x, y)))$.
(C) $\forall x \exists y(L(x, y) \to W(h, y) \wedge W(a, x))$.
(D) $\forall x \forall y(W(h, x) \wedge W(a, y) \to L(x, y)))$.

Question 3. In the branch and bound technique, what is the definition of the incumbent?

(A) The upper bound of the objective function.
(B) None of the other choices is correct.
(C) The best integer solution that we obtain at each step of branching and bounding.
(D) The lower bound of the objective function.

Question 4. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; G$. (B) $C; E$. (C) $B; G$. (D) $B; E$.

Question 5. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \wedge\}$. (B) $\{\to, \wedge\}$. (C) $\{\neg, \to\}$. (D) $\{\to, \perp\}$.

For questions 6–8, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 6. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

Question 7. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y (x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z - 2x_2 + 3x_3 \le 0; z + 2x_1 - 3x_4 \le 0; z - 3x_1 + 4x_4 \le 0; z + 3x_2 - 4x_3 \le 0; x_1 + x_2 + x_3 + x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(B) max z subject to $\{z + 2x_2 - 3x_3 \le 0; z - 2x_1 + 3x_4 \le 0; z + 3x_1 - 4x_4 \le 0; z - 3x_2 + 4x_3 \le 0; x_1, x_2, x_3, x_4 \ge 0\}$

(C) max z subject to $\{z + 2x_2 - 3x_3 \le 0; z - 2x_1 + 3x_4 \le 0; z + 3x_1 - 4x_4 \le 0; z - 3x_2 + 4x_3 \le 0; x_1 + x_2 + x_3 + x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(D) max z subject to $\{2x_2 - 3x_3 \le 0; -2x_1 + 3x_4 \le 0; 3x_1 - 4x_4 \le 0; 3x_2 - 4x_3 \le 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

Question 8. The optimal value of z is

(A) 0 (B) 2 or 4 (C) 1 (D) 3

Question 9. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 57.15% of the time hide 1 and guess 2.
(B) 42.85% of the time hide 2 and guess 2.
(C) 42.85% of the time hide 1 and guess 1.
(D) 57.15% of the time hide 2 and guess 1.

Question 10. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I only. (B) I and II. (C) III only. (D) I and III.

Question 11. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
    $r := r * m;$
    $i := i + 1$

is

(A) $(m \geq 0) \wedge (n \geq 0)$. (B) $(m > 0) \wedge (n \geq 0)$.
(C) $m \geq 0$. (D) $n \geq 0$.

Question 12. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$. (B) From A infer $\neg\neg A$.
(C) From A infer $A \wedge B$. (D) From A and $A \longrightarrow B$ infer B.

Question 13. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $0; 0$. (B) $1; 0$. (C) $1; 1$. (D) $0; 1$.

Question 14. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000), (100000), (110000), (111000), (111100), (111110),$$
$$(111111), (011111), (001111), (000111), (000011), (000001)\}.$$

For each $0 \leq i \leq 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(B) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(C) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(D) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

Question 15. Which of the following statements is true?

(A) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(B) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.
(C) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.
(D) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.

Question 16. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US$ 1200 (B) US$ 6500 (C) US$ 4500 (D) US$ 5400

Question 17. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is a brother of w.
(B) v is an uncle of w.
(C) v is a nephew of w.
(D) v is a grandfather of w.

Question 18. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Graph the problem.
(B) Compare the lower bound to any upper bound of your choice.
(C) Change the objective function coefficients to whole integer numbers.
(D) Solve the original problem by allowing continuous noninteger solutions.

Question 19. Which of the following statements is true for a pair of primal and dual problems?

(A) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(B) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.
(C) The dual problem of the dual problem is different from the primal problem.
(D) Variables in one program correspond to constraints in the other.

Question 20. Consider the linear pogramming problem

$$\max\ 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
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(C) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$
(D) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$

(*There are **20** MCQs, each question is worth **0.5** points. Indicate your choice on the answer sheet.*)

Question 1. Which of the following statements is true?

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while $i < n$ **do**
  $r := r * m;$
  $i := i + 1$

is

(A) $(m \geq 0) \wedge (n \geq 0)$.
(B) $n \geq 0$.
(C) $m \geq 0$.
(D) $(m > 0) \wedge (n \geq 0)$.

Question 3. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

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then what is the meaning of the expression $F(v, w)$?

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(D) v is an uncle of w.

Question 4. In the branch and bound technique, what is the definition of the incumbent?

(A) The upper bound of the objective function.
(B) The lower bound of the objective function.
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(B) $\bigvee_{k=0}^{5} \left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i \right) \vee \left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i \right) \right)$.
(C) $\bigwedge_{k=0}^{5} \left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i \right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i \right) \right)$.
(D) $\bigwedge_{k=0}^{5} \left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i \right) \vee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i \right) \right)$.

Question 6. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \wedge\}$. (B) $\{\to, \perp\}$. (C) $\{\neg, \to\}$. (D) $\{\to, \wedge\}$.

For questions 7–9, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 7. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

Question 8. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z - 2x_2 + 3x_3 \leq 0; z + 2x_1 - 3x_4 \leq 0; z - 3x_1 + 4x_4 \leq 0; z + 3x_2 - 4x_3 \leq 0; x_1 + x_2 + x_3 + x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

(B) max z subject to $\{2x_2 - 3x_3 \leq 0; -2x_1 + 3x_4 \leq 0; 3x_1 - 4x_4 \leq 0; 3x_2 - 4x_3 \leq 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

(C) max z subject to $\{z + 2x_2 - 3x_3 \leq 0; z - 2x_1 + 3x_4 \leq 0; z + 3x_1 - 4x_4 \leq 0; z - 3x_2 + 4x_3 \leq 0; x_1 + x_2 + x_3 + x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

(D) max z subject to $\{z + 2x_2 - 3x_3 \leq 0; z - 2x_1 + 3x_4 \leq 0; z + 3x_1 - 4x_4 \leq 0; z - 3x_2 + 4x_3 \leq 0; x_1, x_2, x_3, x_4 \geq 0\}$

Question 9. The optimal value of z is

(A) 0 (B) 3 (C) 1 (D) 2 or 4

Question 10. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 57.15% of the time hide 1 and guess 2.
(B) 57.15% of the time hide 2 and guess 1.
(C) 42.85% of the time hide 1 and guess 1.
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Question 11. In this question, assume the following predicate and constant symbols:

$W(x,y)$: x wrote y h : Hardy p : Pride and Predjudice.
$L(x,y)$: x is longer than y a : Austen
$N(x)$: x is a novel j : Jude the Obscure

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x(W(h,x) \to L(x,a)))$.
(B) $\forall x \forall y(W(h,x) \wedge W(a,y) \to L(x,y)))$.
(C) $\forall x \exists y(L(x,y) \to W(h,y) \wedge W(a,x))$.
(D) $\exists x(N(x) \wedge W(h,x) \wedge \forall y(N(y) \wedge W(a,y) \to L(x,y)))$.

Question 12. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) 0; 0.
(B) 0; 1.
(C) 1; 1.
(D) 1; 0.

Question 13. Which of the following statements is true for a pair of primal and dual problems?

(A) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(B) Variables in one program correspond to constraints in the other.
(C) The dual problem of the dual problem is different from the primal problem.
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Question 14. Consider the linear pogramming problem

$$\max\ 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
(B) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$
(C) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$
(D) $\min\ 5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \geq 6; 3x_1 + x_2 + x_3 \geq 4; x_2 - x_1 \geq 7\}$

Question 15. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; G$.
(B) $B; E$.
(C) $B; G$.
(D) $C; E$.

Question 16. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I only. (B) I and III. (C) III only. (D) I and II.

Question 17. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(B) From A and $A \longrightarrow B$ infer B.
(C) From A infer $A \wedge B$.
(D) From A infer $\neg\neg A$.

Question 18. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.
(B) Clark is the killer.
(C) Brown is the killer.
(D) The given information is insufficient to discover the killer.

Question 19. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US$ 1200 (B) US$ 5400 (C) US$ 4500 (D) US$ 6500

Question 20. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Graph the problem.
(B) Solve the original problem by allowing continuous noninteger solutions.
(C) Change the objective function coefficients to whole integer numbers.
(D) Compare the lower bound to any upper bound of your choice.

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FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** ***(One personal A4-sheet allowed)***
Test date: **June 3, 2020**

(*There are* ***20*** *MCQs, each question is worth* ***0.5*** *points. Indicate your choice on the answer sheet.*)

Question 1. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\to, \wedge\}$. (B) $\{\neg, \wedge\}$. (C) $\{\to, \bot\}$. (D) $\{\neg, \to\}$.

Question 2. In the branch and bound technique, what is the definition of the incumbent?

(A) None of the other choices is correct.
(B) The upper bound of the objective function.
(C) The lower bound of the objective function.
(D) The best integer solution that we obtain at each step of branching and bounding.

Question 3. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000), (100000), (110000), (111000), (111100), (111110),$$

$$(111111), (011111), (001111), (000111), (000011), (000001)\}.$$

For each $0 \le i \le 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigwedge\limits_{k=0}^{5}\left(\left(\bigwedge\limits_{i=0}^{k}\neg b_i \wedge \bigwedge\limits_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge\limits_{i=0}^{k} b_i \wedge \bigwedge\limits_{i=k+1}^{5}\neg b_i\right)\right)$. (B) $\bigvee\limits_{k=0}^{5}\left(\left(\bigwedge\limits_{i=0}^{k}\neg b_i \wedge \bigwedge\limits_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge\limits_{i=0}^{k} b_i \wedge \bigwedge\limits_{i=k+1}^{5}\neg b_i\right)\right)$.

(C) $\bigvee\limits_{k=0}^{5}\left(\left(\bigwedge\limits_{i=0}^{k}\neg b_i \wedge \bigwedge\limits_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge\limits_{k=0}^{k} b_i \wedge \bigwedge\limits_{i=k+1}^{5}\neg b_i\right)\right)$. (D) $\bigwedge\limits_{k=0}^{5}\left(\left(\bigwedge\limits_{i=0}^{k}\neg b_i \wedge \bigwedge\limits_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge\limits_{i=0}^{k} b_i \wedge \bigwedge\limits_{i=k+1}^{5}\neg b_i\right)\right)$.

Question 4. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is an uncle of w. (B) v is a brother of w.
(C) v is a grandfather of w. (D) v is a nephew of w.

For questions 5–7, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 5. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

Question 6. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1, x_2, x_3, x_4 \geq 0\}$

(B) max z subject to $\{z-2x_2+3x_3 \leq 0; z+2x_1-3x_4 \leq 0; z-3x_1+4x_4 \leq 0; z+3x_2-4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

(C) max z subject to $\{2x_2-3x_3 \leq 0; -2x_1+3x_4 \leq 0; 3x_1-4x_4 \leq 0; 3x_2-4x_3 \leq 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

(D) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

Question 7. The optimal value of z is

(A) 2 or 4 (B) 0 (C) 3 (D) 1

Question 8. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 42.85% of the time hide 2 and guess 2.
(B) 57.15% of the time hide 1 and guess 2.
(C) 57.15% of the time hide 2 and guess 1.
(D) 42.85% of the time hide 1 and guess 1.

Question 9. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) The given information is insufficient to discover the killer.
(B) Adam is the killer.
(C) Clark is the killer.
(D) Brown is the killer.

Question 10. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $1; 0$. (B) $0; 0$. (C) $0; 1$. (D) $1; 1$.

Question 11. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From A infer $\neg\neg A$. (B) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(C) From A and $A \longrightarrow B$ infer B. (D) From A infer $A \wedge B$.

Question 12. In this question, assume the following predicate and constant symbols:

$W(x, y)$: x wrote y	h : Hardy	p : Pride and Predjudice.
$L(x, y)$: x is longer than y	a : Austen	
$N(x)$: x is a novel	j : Jude the Obscure	

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\exists x(N(x) \wedge W(h, x) \wedge \forall y(N(y) \wedge W(a, y) \rightarrow L(x, y)))$. (B) $\forall x(W(h, x) \rightarrow L(x, a)))$.
(C) $\forall x \forall y(W(h, x) \wedge W(a, y) \rightarrow L(x, y)))$. (D) $\forall x \exists y(L(x, y) \rightarrow W(h, y) \wedge W(a, x))$.

Question 13. Consider the linear pogramming problem

$$\max 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lccccccc} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) $\min 5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \geq 6; 3x_1 + x_2 + x_3 \geq 4; x_2 - x_1 \geq 7\}$
(B) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
(C) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$
(D) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$

Question 14. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I and II. (B) I only. (C) I and III. (D) III only.

Question 15. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US\$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US\$ 6500 (B) US\$ 1200 (C) US\$ 5400 (D) US\$ 4500

Question 16. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither A nor F won, then B won
- if neither B nor G won, then C won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; E$. (B) $C; G$. (C) $B; E$. (D) $B; G$.

Question 17. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
  $r := r * m;$
  $i := i + 1$

is

(A) $(m > 0) \wedge (n \geq 0)$.
(B) $(m \geq 0) \wedge (n \geq 0)$.
(C) $n \geq 0$.
(D) $m \geq 0$.

Question 18. Which of the following statements is true for a pair of primal and dual problems?

(A) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.
(B) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(C) Variables in one program correspond to constraints in the other.
(D) The dual problem of the dual problem is different from the primal problem.

Question 19. Which of the following statements is true?

(A) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.
(B) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(C) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.
(D) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.

Question 20. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Compare the lower bound to any upper bound of your choice.
(B) Graph the problem.
(C) Solve the original problem by allowing continuous noninteger solutions.
(D) Change the objective function coefficients to whole integer numbers.

BKU, VNU-HCM
FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** *(One personal A4-sheet allowed)*
Test date: **June 3, 2020**

(*There are **20** MCQs, each question is worth **0.5** points. Indicate your choice on the answer sheet.*)

Question 1. In this question, assume the following predicate and constant symbols:

$W(x, y)$: x wrote y	h : Hardy	p : Pride and Predjudice.
$L(x, y)$: x is longer than y	a : Austen	
$N(x)$: x is a novel	j : Jude the Obscure	

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x(W(h, x) \to L(x, a)))$.
(B) $\exists x(N(x) \wedge W(h, x) \wedge \forall y(N(y) \wedge W(a, y) \to L(x, y)))$.
(C) $\forall x \forall y(W(h, x) \wedge W(a, y) \to L(x, y)))$.
(D) $\forall x \exists y(L(x, y) \to W(h, y) \wedge W(a, x))$.

Question 2. In the branch and bound technique, what is the definition of the incumbent?

(A) The upper bound of the objective function.
(B) None of the other choices is correct.
(C) The lower bound of the objective function.
(D) The best integer solution that we obtain at each step of branching and bounding.

Question 3. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y(P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is a brother of w.
(B) v is an uncle of w.
(C) v is a grandfather of w.
(D) v is a nephew of w.

For questions 4–6, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 4. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

Question 5. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z-2x_2+3x_3 \le 0; z+2x_1-3x_4 \le 0; z-3x_1+4x_4 \le 0; z+3x_2-4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(B) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1, x_2, x_3, x_4 \ge 0\}$

(C) max z subject to $\{2x_2-3x_3 \le 0; -2x_1+3x_4 \le 0; 3x_1-4x_4 \le 0; 3x_2-4x_3 \le 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

(D) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

Question 6. The optimal value of z is

(A) 0 (B) 2 or 4 (C) 3 (D) 1

Question 7. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 57.15% of the time hide 1 and guess 2.
(B) 42.85% of the time hide 2 and guess 2.
(C) 57.15% of the time hide 2 and guess 1.
(D) 42.85% of the time hide 1 and guess 1.

Question 8. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \wedge\}$. (B) $\{\rightarrow, \wedge\}$. (C) $\{\rightarrow, \perp\}$. (D) $\{\neg, \rightarrow\}$.

Question 9. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(B) From A infer $\neg\neg A$.
(C) From A and $A \longrightarrow B$ infer B.
(D) From A infer $A \wedge B$.

Question 10. Consider the linear pogramming problem

$$\max\ 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \le & 6 \\ & & & x_2 & + & x_3 & \le & 4 \\ & 3x_1 & + & x_2 & & & \le & 7 \end{array}$$

The dual problem is:

(A) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \ge 5; 3x_1 + x_2 + x_3 \ge 2; x_2 - x_1 \ge 1\}$
(B) $\min\ 5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \ge 6; 3x_1 + x_2 + x_3 \ge 4; x_2 - x_1 \ge 7\}$
(C) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \ge 5; x_2 + x_3 \ge 2; 3x_1 + x_2 \ge 1\}$
(D) $\min\ 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \ge 5; 3x_1 + x_2 + x_3 \ge 2; -x_2 + x_1 \ge 1\}$

Question 11. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; G$. (B) $C; E$. (C) $B; E$. (D) $B; G$.

Question 12. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Graph the problem.
(B) Compare the lower bound to any upper bound of your choice.
(C) Solve the original problem by allowing continuous noninteger solutions.
(D) Change the objective function coefficients to whole integer numbers.

Question 13. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US\$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US\$ 1200 (B) US\$ 6500 (C) US\$ 5400 (D) US\$ 4500

Question 14. Which of the following statements is true?

(A) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(B) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.
(C) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.
(D) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.

Question 15. Which of the following statements is true for a pair of primal and dual problems?

(A) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(B) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.
(C) Variables in one program correspond to constraints in the other.
(D) The dual problem of the dual problem is different from the primal problem.

Question 16. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I only. (B) I and II. (C) I and III. (D) III only.

Question 17. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $0; 0$. (B) $1; 0$. (C) $0; 1$. (D) $1; 1$.

Question 18. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000), (100000), (110000), (111000), (111100), (111110),$$

$$(111111), (011111), (001111), (000111), (000011), (000001)\}.$$

For each $0 \leq i \leq 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(B) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(C) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(D) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

Question 19. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.
(B) The given information is insufficient to discover the killer.
(C) Clark is the killer.
(D) Brown is the killer.

Question 20. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
  $r := r * m;$
  $i := i + 1$

is

(A) $(m \geq 0) \wedge (n \geq 0)$.
(B) $(m > 0) \wedge (n \geq 0)$.
(C) $n \geq 0$.
(D) $m \geq 0$.

BKU, VNU-HCM
FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** ***(One personal A4-sheet allowed)***
Test date: **June 3, 2020**

(*There are **20** MCQs, each question is worth **0.5** points. Indicate your choice on the answer sheet.*)

Question 1. In the branch and bound technique, what is the definition of the incumbent?

(A) The upper bound of the objective function.
(B) The lower bound of the objective function.
(C) None of the other choices is correct.
(D) The best integer solution that we obtain at each step of branching and bounding.

Question 2. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither A nor F won, then B won
- if neither B nor G won, then C won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $C; G$. (B) $B; E$. (C) $C; E$. (D) $B; G$.

For questions 3–5, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 3. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

Question 4. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z-2x_2+3x_3 \leq 0; z+2x_1-3x_4 \leq 0; z-3x_1+4x_4 \leq 0; z+3x_2-4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

(B) max z subject to $\{2x_2 - 3x_3 \leq 0; -2x_1 + 3x_4 \leq 0; 3x_1 - 4x_4 \leq 0; 3x_2 - 4x_3 \leq 0; x_1, x_2, x_3, x_4 \in \{0, 1\}\}$

(C) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1, x_2, x_3, x_4 \geq 0\}$

(D) max z subject to $\{z+2x_2-3x_3 \leq 0; z-2x_1+3x_4 \leq 0; z+3x_1-4x_4 \leq 0; z-3x_2+4x_3 \leq 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \geq 0\}$

Question 5. The optimal value of z is

(A) 0 (B) 3 (C) 2 or 4 (D) 1

Question 6. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 57.15% of the time hide 1 and guess 2.
(B) 57.15% of the time hide 2 and guess 1.
(C) 42.85% of the time hide 2 and guess 2.
(D) 42.85% of the time hide 1 and guess 1.

Question 7. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r); \ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $0; 0$. (B) $0; 1$. (C) $1; 0$. (D) $1; 1$.

Question 8. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(B) From A and $A \longrightarrow B$ infer B.
(C) From A infer $\neg\neg A$.
(D) From A infer $A \wedge B$.

Question 9. Which of the following statements is true?

(A) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(B) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.
(C) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.
(D) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.

Question 10. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.
(B) Clark is the killer.
(C) The given information is insufficient to discover the killer.
(D) Brown is the killer.

Question 11. Which of the following statements is true for a pair of primal and dual problems?

(A) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(B) Variables in one program correspond to constraints in the other.
(C) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.
(D) The dual problem of the dual problem is different from the primal problem.

Question 12. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x(P(x) \vee Q(x))$.

II. $\forall x(P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x(P(x) \vee Q(x))$.

(A) I only.
(B) I and III.
(C) I and II.
(D) III only.

Question 13. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Graph the problem.
(B) Solve the original problem by allowing continuous noninteger solutions.
(C) Compare the lower bound to any upper bound of your choice.
(D) Change the objective function coefficients to whole integer numbers.

Question 14. Consider the linear pogramming problem

$$\max 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \leq & 6 \\ & & & x_2 & + & x_3 & \leq & 4 \\ & 3x_1 & + & x_2 & & & \leq & 7 \end{array}$$

The dual problem is:

(A) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; x_2 - x_1 \geq 1\}$
(B) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \geq 5; x_2 + x_3 \geq 2; 3x_1 + x_2 \geq 1\}$
(C) $\min 5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \geq 6; 3x_1 + x_2 + x_3 \geq 4; x_2 - x_1 \geq 7\}$
(D) $\min 6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \geq 5; 3x_1 + x_2 + x_3 \geq 2; -x_2 + x_1 \geq 1\}$

Question 15. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \wedge\}$.
(B) $\{\rightarrow, \perp\}$.
(C) $\{\rightarrow, \wedge\}$.
(D) $\{\neg, \rightarrow\}$.

Question 16. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$$
\begin{array}{l}
r := 1; \\
i := 0; \\
\textbf{while } i < n \textbf{ do} \\
\qquad r := r * m; \\
\qquad i := i + 1
\end{array}
$$

is

(A) $(m \geq 0) \wedge (n \geq 0)$.

(B) $n \geq 0$.

(C) $(m > 0) \wedge (n \geq 0)$.

(D) $m \geq 0$.

Question 17. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000), (100000), (110000), (111000), (111100), (111110),$$

$$(111111), (011111), (001111), (000111), (000011), (000001)\}.$$

For each $0 \leq i \leq 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(B) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(C) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

(D) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

Question 18. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is a brother of w.

(B) v is a grandfather of w.

(C) v is an uncle of w.

(D) v is a nephew of w.

Question 19. In this question, assume the following predicate and constant symbols:

$W(x, y)$: x wrote y	h : Hardy	p : Pride and Predjudice.
$L(x, y)$: x is longer than y	a : Austen	
$N(x)$: x is a novel	j : Jude the Obscure	

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x (W(h, x) \rightarrow L(x, a)))$.

(B) $\forall x \forall y (W(h, x) \wedge W(a, y) \rightarrow L(x, y)))$.

(C) $\exists x (N(x) \wedge W(h, x) \wedge \forall y (N(y) \wedge W(a, y) \rightarrow L(x, y)))$.

(D) $\forall x \exists y (L(x, y) \rightarrow W(h, y) \wedge W(a, x))$.

Question 20. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US\$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US\$ 1200

(B) US\$ 5400

(C) US\$ 6500

(D) US\$ 4500

BKU, VNU-HCM
FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Maths Modelling** (CO2011)
Class: **CC18KHMT** Groups: **CC01, CC02**
Time: **60 minutes** *(**One personal A4-sheet allowed**)*
Test date: **June 3, 2020**

(*There are **20** MCQs, each question is worth **0.5** points. Indicate your choice on the answer sheet.*)

Question 1. A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and Barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US$ 10000 and availability of 1200 man-days during the planning horizon. Find the maximum profit that he can attain.

(A) US$ 4500 (B) US$ 1200 (C) US$ 5400 (D) US$ 6500

Question 2. A *precondition* (a condition specifies what should be true upon entering the program (i.e., under what inputs the program is expected to work).) of the following program

$r := 1;$
$i := 0;$
while $i < n$ **do**
  $r := r * m;$
  $i := i + 1$

is

(A) $m \geq 0$.
(B) $(m \geq 0) \wedge (n \geq 0)$.
(C) $n \geq 0$.
(D) $(m > 0) \wedge (n \geq 0)$.

Question 3. In the branch and bound technique, what is the definition of the incumbent?

(A) The best integer solution that we obtain at each step of branching and bounding.
(B) The upper bound of the objective function.
(C) The lower bound of the objective function.
(D) None of the other choices is correct.

Question 4. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

- if neither A nor E won, then G won
- if neither B nor G won, then C won
- if neither A nor F won, then B won
- if neither C nor F won, then E won.

Who were the two people elected?

(A) $B; G$. (B) $C; G$. (C) $B; E$. (D) $C; E$.

Question 5. By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $1; 1$. (B) $0; 0$. (C) $0; 1$. (D) $1; 0$.

Question 6. An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. Which of the following is an adequate set?

(A) $\{\neg, \rightarrow\}$. (B) $\{\neg, \wedge\}$. (C) $\{\rightarrow, \perp\}$. (D) $\{\rightarrow, \wedge\}$.

Question 7. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

(A) From A infer $A \wedge B$. (B) From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
(C) From A and $A \longrightarrow B$ infer B. (D) From A infer $\neg\neg A$.

Question 8. In this question, assume the following predicate and constant symbols:

$W(x, y)$: x wrote y	h : Hardy	p : Pride and Predjudice.
$L(x, y)$: x is longer than y	a : Austen	
$N(x)$: x is a novel	j : Jude the Obscure	

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

(A) $\forall x \exists y (L(x, y) \rightarrow W(h, y) \wedge W(a, x))$. (B) $\forall x (W(h, x) \rightarrow L(x, a)))$.
(C) $\forall x \forall y (W(h, x) \wedge W(a, y) \rightarrow L(x, y)))$. (D) $\exists x (N(x) \wedge W(h, x) \wedge \forall y (N(y) \wedge W(a, y) \rightarrow L(x, y)))$.

For questions 9–11, use the following information.

Payoff matrix. Consider a game for 2 players. The game proceeds in a series of identical rounds, in each round the 2 players make a move and depending on the moves either a tie is declared or one player is declared the loser and has to pay the winner a certain amount. A payoff matrix for the player one is defined as the matrix $A = (a_{ij})$ such that $a_{ij} > 0$ if the player I wins, $a_{ij} < 0$ if the player II wins, and $a_{ij} = 0$ if the round is a tie.

Morra game. In each round each player hides either one or two coins and also guesses how many coins the other player hid (they guess in secret and reveal their guesses simultaneously). If either both players guess incorrectly or both players guess correctly the round is a tie and no money changes hands. But if exactly one player guesses correctly, then that player gets to keep the coins hidden by both players.

Question 9. If $HmGn$ denotes a player hides m coins and guesses n coins for $m, n \in \{1, 2\}$. Which of the following is the payoff matrix of the Morra game for the player I?

(A)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	2	-3	0
H1G2	-2	1	0	3
H2G1	3	0	1	-4
H2G2	0	-3	4	1

(B)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	2	-3	0
H1G2	-2	0	0	3
H2G1	3	0	0	-4
H2G2	0	-3	4	0

(C)

	H1G1	H1G2	H2G1	H2G2
H1G1	1	-2	3	0
H1G2	2	1	0	-3
H2G1	-3	0	1	4
H2G2	0	3	-4	1

(D)

	H1G1	H1G2	H2G1	H2G2
H1G1	0	-2	3	0
H1G2	2	0	0	-3
H2G1	-3	0	0	4
H2G2	0	3	-4	0

Question 10. Each row and column of the payoff matrix A for the player I respectively is a strategy of the player I and the player II. If $y := (y_1, y_2, y_3, y_4)$ is the probability of the 4 strategies that the player II uses, the expected winnings of the player I will be $x^T Ay$, where $x := (x_1, x_2, x_3, x_4)$ is the probability of the 4 strategies that the player I plays. The player II thus wants $\min_y(x^T Ay)$, and the player I can win the game if he can find x maximizing $\min_y(x^T Ay)$. Let

$$z := \min_y(x^T Ay).$$

Which of the following can be a model for finding an optimal strategy for the player I in the Morra game?

(A) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(B) max z subject to $\{z-2x_2+3x_3 \le 0; z+2x_1-3x_4 \le 0; z-3x_1+4x_4 \le 0; z+3x_2-4x_3 \le 0; x_1+x_2+x_3+x_4 = 1; x_1, x_2, x_3, x_4 \ge 0\}$

(C) max z subject to $\{2x_2-3x_3 \le 0; -2x_1+3x_4 \le 0; 3x_1-4x_4 \le 0; 3x_2-4x_3 \le 0; x_1, x_2, x_3, x_4 \in \{0,1\}\}$

(D) max z subject to $\{z+2x_2-3x_3 \le 0; z-2x_1+3x_4 \le 0; z+3x_1-4x_4 \le 0; z-3x_2+4x_3 \le 0; x_1, x_2, x_3, x_4 \ge 0\}$

Question 11. The optimal value of z is

(A) 1 (B) 0 (C) 3 (D) 2 or 4

Question 12. An optimal solution for the player II is approximately $(y_1, y_2, y_3, y_4) = (0, 0.5715, 0.4285, 0)$. Thus an optimal strategy for the player II is:

(A) 42.85% of the time hide 1 and guess 1.
(B) 57.15% of the time hide 1 and guess 2.
(C) 57.15% of the time hide 2 and guess 1.
(D) 42.85% of the time hide 2 and guess 2.

Question 13. Consider the linear pogramming problem

$$\max\ 5x_1 + 2x_2 + x_3$$

$$\begin{array}{lrcrcrcl} \text{subject to} & x_1 & + & 3x_2 & - & x_3 & \le & 6 \\ & & & x_2 & + & x_3 & \le & 4 \\ & 3x_1 & + & x_2 & & & \le & 7 \end{array}$$

The dual problem is:

(A) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \ge 5; 3x_1 + x_2 + x_3 \ge 2; -x_2 + x_1 \ge 1\}$
(B) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_3 \ge 5; 3x_1 + x_2 + x_3 \ge 2; x_2 - x_1 \ge 1\}$
(C) min $6x_1 + 4x_2 + 7x_3$ subject to $\{x_1 + 3x_2 - x_3 \ge 5; x_2 + x_3 \ge 2; 3x_1 + x_2 \ge 1\}$
(D) min $5x_1 + 2x_2 + x_3$ subject to $\{x_1 + 3x_3 \ge 6; 3x_1 + x_2 + x_3 \ge 4; x_2 - x_1 \ge 7\}$

Question 14. Consider the set of 12 bit string of length 6 as follow:

$$\{(000000), (100000), (110000), (111000), (111100), (111110),$$

$$(111111), (011111), (001111), (000111), (000011), (000001)\}.$$

For each $0 \le i \le 5$, let's denote by b_i the proposition "the i-th bit in the string is 1." Which of the following formula cab be used for modeling the given set?

(A) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.
(B) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \wedge \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.
(C) $\bigvee_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{k=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.
(D) $\bigwedge_{k=0}^{5}\left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i\right) \vee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{5} \neg b_i\right)\right)$.

Question 15. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

(A) v is a nephew of w.
(B) v is a brother of w.
(C) v is a grandfather of w.
(D) v is an uncle of w.

Question 16. Which of the following statements is true?

(A) We begin the simplex algorithm and the interior-point methods at a vertex of the feasible polyhedron.
(B) It is always possible to obtain an optimal solution to a linear programming problem by using the simplex method while it is not the case where we use the interior-point methods.
(C) In the interior-point methods, the simplex tableaux must be established in order to obtain valid cuts.
(D) If we can obtain an optimal solution to a linear programming problem by using interior-point methods, strong duality holds for the problem.

Question 17. In the first step of a branch and bound approach to solving integer programming problems is to

(A) Change the objective function coefficients to whole integer numbers.
(B) Graph the problem.
(C) Solve the original problem by allowing continuous noninteger solutions.
(D) Compare the lower bound to any upper bound of your choice.

Question 18. Which of the following statements is true for a pair of primal and dual problems?

(A) The dual problem of the dual problem is different from the primal problem.
(B) If the primal problem is infeasible, it is possible that the dual problem still has an optimal solution.
(C) Variables in one program correspond to constraints in the other.
(D) There is no guarantee that the optimal solution to one problem will exist if the optimal solution to the other problem exists.

Question 19. There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Brown is the killer.
(B) Adam is the killer.
(C) Clark is the killer.
(D) The given information is insufficient to discover the killer.

Question 20. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x (P(x) \vee Q(x))$.

II. $\forall x (P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.

III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x (P(x) \vee Q(x))$.

(A) III only.
(B) I only.
(C) I and III.
(D) I and II.