# Chapter 3:

# Low Noise Amplifier (LNA)

# Origin of Noise (1)

- **Resistor thermal noise:** Probably the most well known noise source is the **thermal noise** of a resistor (also called Johnson noise). It is generated by thermal energy causing random electron motion. It is **white noise** since the PSD of the noise signal is flat throughout the frequency band.

  The noise is also called **Gaussian** which means the amplitude of the noise signal has random characteristics with a **Gaussian distribution**. We are able to apply statistic measures such as the **mean square values**. The noise power is proportional to absolute temperature.

  The **thermal noise spectral density** in a resistor is given by

$$N_{\mathrm{resistor}} = 4kTR$$

  where $k$ is Boltzmann's constant ($\sim 1.38 \times 10-23$ J/K), $T$ is the absolute temperature in Kelvin temperature of the resistor, and $R$ is the value of the resistor.

# Origin of Noise (2)

Noise power spectral density is expressed using volts squared per hertz (power spectral density). In order to find out how much power a resistor produces in a finite bandwidth of interest $\Delta f$ , we use:

$$v_n^2 = 4kTR\Delta f$$

where $v_n$ is the rms value of the noise voltage in the bandwidth $\Delta f$ . This can also be written equivalently as a noise current rather than a noise voltage:

$$i_n^2 = \frac{4kT\Delta f}{R}$$

Maximum power is transferred to the load when $R_{\text{LOAD}}$ is equal to $R$. Then $v_o$ is equal to $v_n$ /2. The **output power spectral density** $P_o$ is then given by

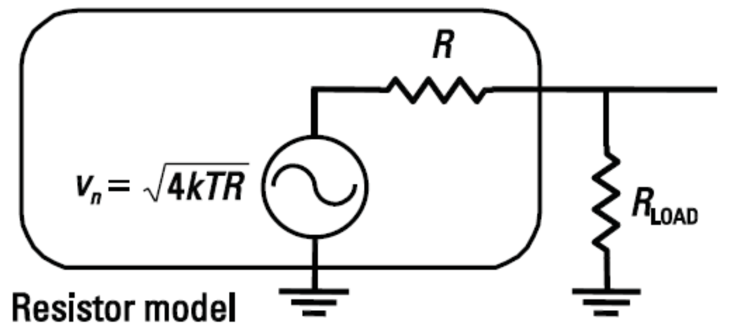
$$P_o = \frac{v_o^2}{R} = \frac{v_n^2}{4R} = kT$$

Thus, **available noise power** is $kT$, independent of resistor size. Note that $kT$ is in watts per hertz, which is a power density.

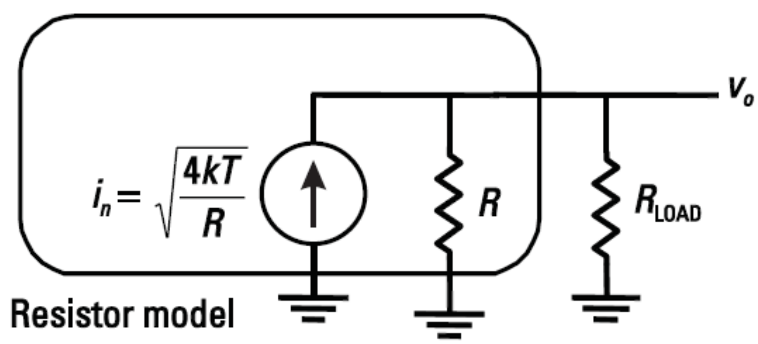
# Origin of Noise (3)

To get **total power out** $P_{out}$ in watts, multiply by the bandwidth, with the result that:

$$P_{\text{out}} = kTB$$




(a)




(b)

# Origin of Noise (4)

**Available power from antenna:** The noise from an antenna can be modeled as a resistor. Thus, the available power from an antenna is given by:

$$P_{\text{available}} = kT = 4 \times 10^{-21} \text{ W/Hz}$$

at $T$ = 290K, or in dBm per hertz:

$$P_{\text{available}} = 10 \log_{10}\left(\frac{4 \times 10^{-21}}{1 \times 10^{-3}}\right) = -174 \text{ dBm/Hz}$$

<u>Example</u>: For any receiver required to receive a given signal bandwidth, the minimum detectable signal can now be determined. From $P_{out} = kTB$, the noise floor depends on the bandwidth. For example, with a bandwidth of 200 kHz, the noise floor is

$$\text{Noise floor} = kTB = 4 \times 10^{-21} \times 200{,}000 = 8 \times 10^{-16}$$

or in dBm:

$$\text{Noise floor} = -174 \text{ dBm/Hz} + 10 \log_{10}(200{,}000) = -121 \text{ dBm}$$

# Origin of Noise (5)

Thus, we can now also formally define **signal-to-noise ratio (SNR)**. If the signal has a power of *S*, then the SNR is

$$\mathrm{SNR} = \frac{S}{\text{Noise floor}}$$

Thus, if the electronics added no noise and if the detector required a SNR of 0 dB, then a signal at -121 dBm could just be detected. The <u>minimum detectable signal</u> in a receiver is also referred to as the **receiver sensitivity.**

However, the SNR required to detect bits <u>reliably</u> (e.g., bit error rate (BER) = $10^{-3}$) is typically not 0 dB. Typical results for a bit error rate of $10^{-3}$ (for voice transmission) is about 7 dB for quadrature phase shift keying (QPSK), about 12 dB for 16 quadrature amplitude modulation (QAM), and about 17 dB for 64 QAM. For data transmission, lower BER is often required (e.g., $10^{-6}$), resulting in an SNR requirement of 11 dB or more for QPSK.

# Origin of Noise (6)

- **<u>Shot noise</u>:** Shot noise is generated if current flows through a potential barrier such as a *pn* junction. The square root of the shot noise current can be described by

$$i_{sh}^2 = 2qI_{dc}\Delta f$$

with $q$ as the electron charge. As expected, the shot noise increases with DC current $I_{dc}$ since it determines the number of available carriers.

Thus, shot noise can be minimised by reducing the DC current. However, a reduced DC current may decrease the maximum possible gain and large signal properties of transistors. Consequently, a tradeoff has to be found.

Shot noise plays an important role in BJTs since they consist of *pn* junctions (especially for the forward biased base emitter junction).

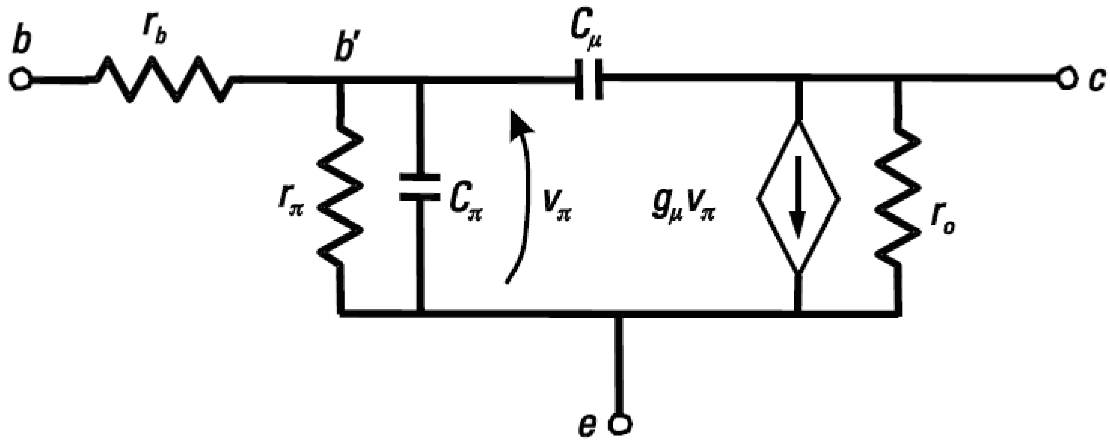
# Origin of Noise (7)

Usually, the shot noise of FETs is very small since there are no relevant *pn*-junctions, and the current flowing through them is weaker than for BJTs. However, the aggressively scaling of MOSFETs can introduce a significant current from the gate to the channel, which may generate shot noise.

In contradiction to thermal noise, <u>shot noise does not occur in an ideal resistor</u>.

# Noise in Bipolar Transistors (1)

- **Small-signal equivalent circuit of BJT at hight frequencies (without noise):**



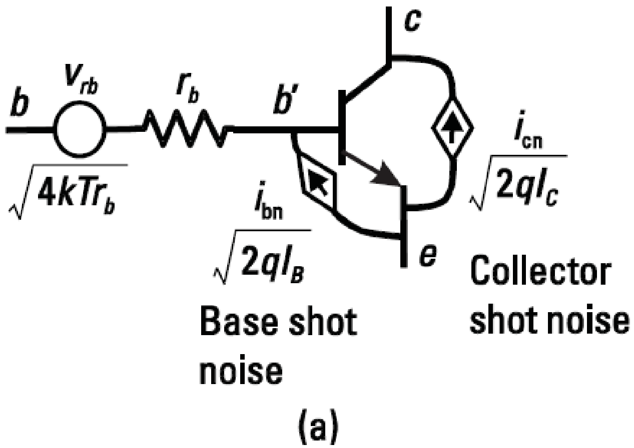

$r_b = r_{bb'}$, $r_\pi = r_{b'e}$, $C_\pi = C_{b'e}$, $C_\mu = C_{b'c}$, $\quad g_m = \dfrac{i_c}{v_\pi} = \dfrac{I_C}{v_T} = \dfrac{I_c q}{kT}$

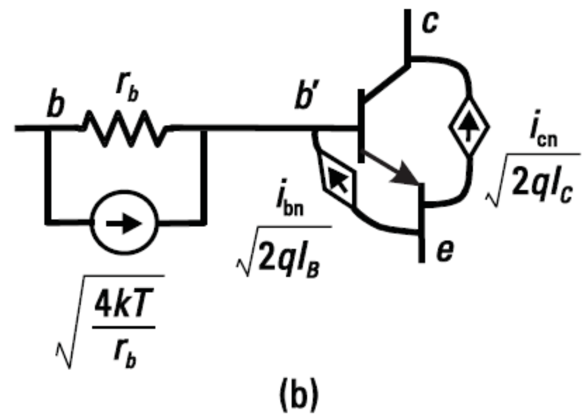
$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} \approx \frac{1}{2\pi r_\pi C_\pi}$$

$$f_T = \beta f_\beta = \frac{g_m}{2\pi (C_\pi + C_\mu)} \approx \frac{g_m}{2\pi C_\pi} = \frac{I_C}{2\pi C_\pi v_T} \qquad \beta = g_m r_\pi$$

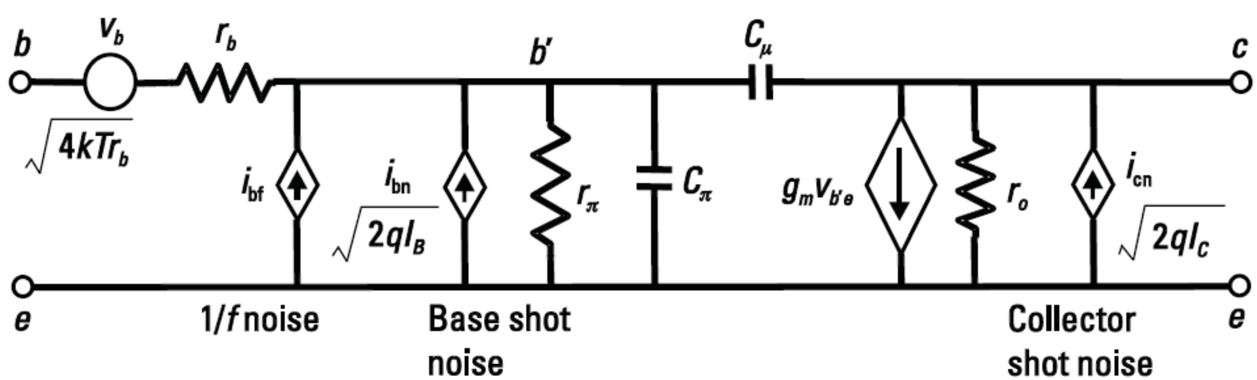
# Noise in Bipolar Transistors (2)

- BJT with base shot noise, collector shot noise, and thermal noise at $r_b$:




- Small-signal equivalent circuit of BJT with noise:



# Noise Figure (1)

- **Noise from the electronics** (e.g. thermal noise, shot noise…) is described by **<u>noise factor</u>** $F$, which is a measure of **how much the signal-to-noise ratio is degraded through the system**. We note that:

$$S_o = G \cdot S_i$$

where $S_i$ is the input signal power, $S_o$ is the output signal power, and $G$ is the power gain $S_o/S_i$. Then, the noise factor is:

$$F = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o} = \frac{S_i / N_{i\,(\text{source})}}{S_o / N_{o\,(\text{total})}} = \frac{S_i / N_{i\,(\text{source})}}{(S_i \cdot G) / N_{o\,(\text{total})}} = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}}$$

where $N_{o(\text{total})}$ is the total noise at the output. If $N_{o(\text{source})}$ is the noise at the output originating at the source, and $N_{o(\text{added})}$ is the noise at the output added by the electronic circuitry, then we can write:

$$N_{o\,(\text{total})} = N_{o\,(\text{source})} + N_{o\,(\text{added})}$$

# Noise Figure (2)

Noise factor can be written in useful alternative form:

$$F = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}} = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = \frac{N_{o\,(\text{source})} + N_{o\,(\text{added})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o\,(\text{added})}}{N_{o\,(\text{source})}}$$

This shows that the minimum possible noise factor, which occurs if the electronics adds no noise, is equal to 1.

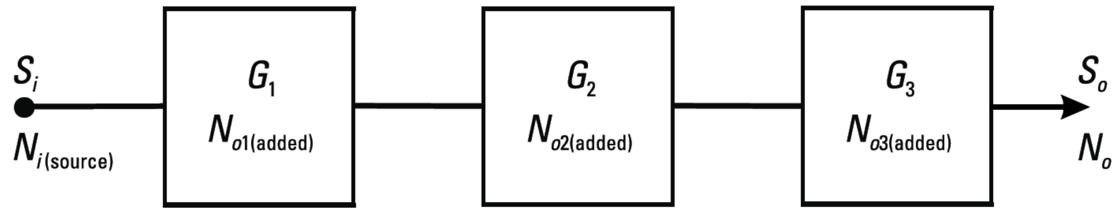
**<u>Noise figure</u>** NF is related to noise factor *F* by:

$$\text{NF} = 10 \log_{10} F$$

Thus, an electronic system that adds no noise has a noise figure of 0 dB.

# Noise Figure (3)

- **<u>Noise Figure of stages in series</u>:** Consider three stages in series as




The output signal $S_o$ is given by: $S_o = S_i \cdot G_i \cdot G_2 \cdot G_3$

The input noise is: $N_{i\,(\text{source})} = kT$

The total output noise is:

$$N_{o\,(\text{total})} = N_{i\,(\text{source})}\, G_1\, G_2\, G_3 + N_{o1(\text{added})}\, G_2\, G_3$$
$$+ N_{o2(\text{added})}\, G_3 + N_{o3(\text{added})}$$

The output noise due to the source is:

$$N_{o\,(\text{source})} = N_{i\,(\text{source})}\, G_1\, G_2\, G_3$$

# Noise Figure (4)

Finally, the noise factor can be determined as:

$$F = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o1(\text{added})}}{N_{i\,(\text{source})}\,G_1} + \frac{N_{o2(\text{added})}}{N_{i\,(\text{source})}\,G_1\,G_2} + \frac{N_{o3(\text{added})}}{N_{i\,(\text{source})}\,G_1\,G_2\,G_3}$$

$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1\,G_2}$$

The above formula shows how the presence of gain preceding a stage causes the effective noise figure to be **reduced** compared to the measured noise figure of a stage by itself. For this reason, **we typically design systems with a low-noise amplifier at the front of the system.**

Question: Derive the formula for *N* stages in series?