BKU, VNU-HCM

FACULTY OF CS AND ENGINEERING

MIDTERM

Subject: **Maths Modelling** (CO2011)

Class: **CC15KHMT** Group: **CC01**

Time: **60 minutes** ***(Closed book test)***

Test date: **March 26, 2018**

**Student's Name:** ____________________ **Student's ID:** | | | | | | | |

(*There are **20** MCQs, each question is worth **0.5** points. Answers in bold* : ■; *cancel out to deselect:* ☒.)

**Question 1.** Which one of the following statements is true?

(A) All sentences in predicate calculus are either satisfied by all models or by none.
(B) No sentence in predicate calculus is satisfied by all models.
(C) Some sentences in predicate calculus may be satisfied by one model, but not by another one.
(D) Every sentence is satisfied by at least one model.

**Question 2.** If a linear program has an optimal solution, then

(A) the feasible set is non-empty and objective function is bounded.
(B) the objective function might not be bounded.
(C) the feasible set can be empty.
(D) only feasible set is non-empty.

**Question 3.** Consider a linear program finding minimum which has the initial simplex tableau as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | 0 |

Suppose that $x_2, x_5, x_4$ are basic variables. Then, the value of reduced cost $r_i$, for $i = 1, 2, \ldots, 5$, should be

(A) $(-2, 0, 1, 0, 0)$. (B) $(0, 2, 1, 0, 0)$ (C) $(0, 1, 2, 0, 0)$ (D) $(2, 0, -1, 0, 0)$

**Question 4.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & x_1 + x_3 - x_4 \\ \text{s. t.} \quad & x_1 - x_3 = 1, \\ & x_3 + x_4 = 6, \\ & x_2 - 2x_3 = 3, \\ & x_i \geq 0, \text{ for } i = 1, 2, \ldots, 4. \end{aligned}$$

Then, the point $(1, 3, 0, 6)$

(A) is a basic feasible solution. (B) is not a basic solution.
(C) is not a basic feasible solution. (D) is not in the feasible set.

**Question 5.** Given the following predicates

$Q(x)$ : $x$ is a politician,
$T(z)$ : $z$ is a time,
$P(y)$ : $y$ is a person,
$F(x, y, z)$ : person $x$ fools person $y$ at time $z$.

Represent the following sentences in predicate logic:
"***Politicians can't fool all of the people all of the time.***"

(A) $\forall x[Q(x) \to \forall y \forall z((P(y) \land T(z)) \to \neg F(x, y, z))]$.
(B) $\forall x[Q(x) \to \exists y \exists z((P(y) \land T(z)) \to \neg F(x, y, z))]$.
(C) $\forall x \exists y \exists z[Q(x) \to (P(y) \land T(z) \land F(x, y, z))]$.
(D) $\forall x[Q(x) \to \exists y \exists z(P(y) \land T(z) \land \neg F(x, y, z))]$.

**Question 6.** Consider the following program.

```
if (x < 5)
        x = x*x;
else
        x = x+1;
{ x >= 9 }
```

If we know that its (`postcondition`) is $\{x \geq 9\}$, then which of the following is its `precondition`?

(A) $\{(x \geq -3 \land x < 5) \lor (x \geq 8)\}$.
(B) $\{(x \leq -3) \lor (x \geq 3 \land x < 5) \lor (x \geq 8)\}$.
(C) $\{(x \leq -3) \lor (x \geq 3 \land x < 5)\}$.
(D) $\{(x < -3) \lor (x > 8)\}$.

**Question 7.** Consider a general linear program

$$\begin{aligned} \min_{x_1, x_2} \quad & -2x_1 + 3x_2 \\ \text{s. t.} \quad & 3x_1 + 4x_2 \leq 24, \\ & 7x_1 - 4x_2 \leq 16, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Which one of followings can change the problem into standard form?

(A) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \leq 0$.
(B) $3x_1 + 4x_2 - x_3 = 24, 7x_1 - 4x_2 - x_4 = 16$, where $x_3, x_4 \geq 0$.
(C) $x_3 - 3x_1 - 4x_2 = 24, x_4 - 7x_1 + 4x_2 = 16$, where $x_3, x_4 \leq 0$.
(D) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \geq 0$.



**Question 8.** Consider the predicate formula $\phi$ as follow.

$$(\exists x P(y, y) \longrightarrow \exists y P(y, z)).$$

What is the result of the substitution $[y \Rightarrow f(z)]\phi$?

(A) $(\exists x P(f(z), f(z)) \longrightarrow \exists y P(f(z), z))$.
(B) $(\exists x P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.
(C) $(\exists z P(f(z), f(z)) \longrightarrow \exists z P(f(z), z))$.
(D) $(\exists z P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.

**Question 9.** The two propositional operators $|$ (or $NAND$), and $\oplus$ (or $XOR$) are defined as follow, respectively: $p|q := \neg(p \land q)$, and $p \oplus q$ is the proposition with true value T if and only if there is exactly one of $p$, $q$ with true value T. Which of the following is correct?

(A) The set $\{|\}$ is not an adequate set of propositional operators.
(B) The set $\{|, \oplus\}$ is not an adequate set of propositional operators.
(C) The set $\{\oplus\}$ is an adequate set of propositional operators.
(D) The set $\{|\}$ is an adequate set of propositional operators.

**Question 10.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & 2x_1 - 3x_2 + 2x_3 - 2x_4 \\ \text{s. t.} \quad & 5x_1 + 2x_3 - 6x_4 = 5, \\ & 3x_2 - x_3 + 2x_4 = 5, \\ & x_i \geq \ 0, \ \text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

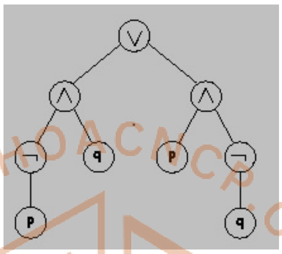
Given non-basic variables $x_2$ and $x_4$, then the corresponding basic solution of the problem is

(A) $(3, 0, -5, 0)$, and also feasible.
(B) $(3, 0, -5, 0)$, and not feasible.
(C) $(0, 3, 0, -5)$, and also feasible.
(D) $(0, 3, 0, -5)$, and not feasible.

**Question 11.** The relaxation in branch-and-bound approach for solving a linear integer program, performs to

(A) assign all variables into zero.
(B) assign all variables into one.
(C) drop all integer variables
(D) drop integer constraint of variables.

**Question 12.** Which of the following formulas has the parse tree:




(A) $(p \wedge \neg q) \vee (\neg p \wedge q)$.
(B) $(\neg p \wedge q) \vee (p \wedge \neg q)$.
(C) $(\neg p \vee q) \wedge (p \vee \neg q)$.
(D) $(p \vee \neg q) \wedge (\neg p \vee q)$.

**Question 13.** Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$. Then the negation of the formula "**For each** $s$ **in** $\mathbb{R}$, **there exists** $r$ **in** $\mathbb{R}$ **such that if** $f(r) > 0$, **then** $g(s) > 0$" is the following formula.

(A) For every $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(B) For every $s$ in $\mathbb{R}$, there does not exist $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(C) There exists $s$ in $\mathbb{R}$ and there exists $r$ in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(D) There exists $s$ in $\mathbb{R}$ such that for every $r$ in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.

**Question 14.** A basic feasible solution of a linear program consists of

(A) all variables of zero
(B) basic variables of zero, non-basic variables of non-zero.
(C) basic variable of non-nagative value, non-basic variables of zero.
(D) basic variables of zero, non-basic variables of positive value.

**Question 15.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| 2 | 0 | $-1$ | 0 | 0 | $-f(x)$ |

Since $r_3 < 0$ and $\bar{a}_{i3}$ is not negative (for $i = 1, 2, 3$), we need to compute the next simplex tableau. Hence, new basic variables should be

(A) $\{x_2, x_5, x_4\}$.
(B) $\{x_3, x_5, x_4\}$.
(C) $\{x_2, x_3, x_4\}$.
(D) $\{x_2, x_5, x_3\}$.

**Question 16.** If a linear program has an optimal solution, then the solution

(A) is a point of the interior of the feasible set.
(B) is an interior point of the boundary of the feasible set.
(C) does not belong to the feasible set.
(D) is an extreme point of the feasible set.

**Question 17.** Let $\phi$ be a propositional formula. Consider the following statements on $\phi$.

I. $\phi$ is satisfiable or $\neg\phi$ is satisfiable.

II. $\phi$ is a tautology or $\neg\phi$ is a tautology.

Then

(A) Both I and II are correct.
(B) Both I and II are incorrect.
(C) I is correct and II is incorrect.
(D) I is incorrect and II is correct.

**Question 18.** Which of the following statements about Natural Deduction is true?

(A) $p \vee \neg p$ cannot be proved in natural deduction.
(B) Boxes are not used to delineate the scope of assumptions.
(C) The rules *Modus Tollens* (MT) cannot be derived from the rules $\wedge i$ and $\wedge e$ alone.
(D) Contradictions don't play an important role in natural deduction.

**Question 19.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| $-2$ | $3$ | $0$ | $0$ | |
|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | rhs |
| 3 | 4 | 1 | 0 | 24 |
| 7 | $-4$ | 0 | 1 | 16 |
| $-2$ | 3 | 0 | 0 | 0 |

Then, the pivot element (phần tử trục/xoay)

(A) can not be determined.
(B) $\bar{a}_{11} = 3$, with in-variable $x_1$ and out-variable $x_3$.
(C) $\bar{a}_{21} = 7$, with in-variable $x_1$ and out-variable $x_4$.
(D) $\bar{a}_{12} = 4$, with in-variable $x_2$ and out-variable $x_3$.

**Question 20.** Suppose that we are proving the validity of the sequent

$$\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \to \phi_2$$

as follows.

| | | |
|---|---|---|
| 1. | $\neg\phi_1 \wedge \neg\phi_2$ | premise |
| 2. | $\phi_1$ | assumption |
| 3. | $\neg\phi_1$ | $\wedge e_1 1$ |
| 4. | $\perp$ | $\neg e 2,3$ |
| 5. | $\phi_2$ | $\perp e 4$ |
| 6. | $\phi_1 \to \phi_2$ | $\to i 2,5$ |

Which of the following is correct?

(A) That is not a correct proof since Line 1 has the premise $\neg\phi_1$, so we are not allowed to introduce the assumption $\phi_1$ in Line 2.
(B) That is not a correct proof since on Line 4 we have seen a contradiction.
(C) That is a correct proof.
(D) That is not a correct proof since we did not use the fact $\neg\phi_2$ in the premise at all.

**MIDTERM**
Subject: **Maths Modelling** (CO2011)
Class: **CC15KHMT** Group: **CC01**
Time: **60 minutes** ***(Closed book test)***
Test date: **March 26, 2018**

**Student's Name:** ____________________ **Student's ID:** | | | | | | | |

(*There are **20** MCQs, each question is worth **0.5** points. Answers in bold* : ■; *cancel out to deselect:* ☒.)

**Question 1.** Consider the predicate formula $\phi$ as follow.

$$(\exists x P(y, y) \longrightarrow \exists y P(y, z)).$$

What is the result of the substitution $[y \Rightarrow f(z)]\phi$?

(A) $(\exists z P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.
(B) $(\exists x P(f(z), f(z)) \longrightarrow \exists y P(f(z), z))$.
(C) $(\exists x P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.
(D) $(\exists z P(f(z), f(z)) \longrightarrow \exists z P(f(z), z))$.

**Question 2.** If a linear program has an optimal solution, then the solution

(A) is an extreme point of the feasible set.
(B) is a point of the interior of the feasible set.
(C) is an interior point of the boundary of the feasible set.
(D) does not belong to the feasible set.

**Question 3.** Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$. Then the negation of the formula "**For each** $s$ **in** $\mathbb{R}$**, there exists** $r$ **in** $\mathbb{R}$ **such that if** $f(r) > 0$**, then** $g(s) > 0$" is the following formula.

(A) There exists $s$ in $\mathbb{R}$ such that for every $r$ in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(B) For every $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(C) For every $s$ in $\mathbb{R}$, there does not exist $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(D) There exists $s$ in $\mathbb{R}$ and there exists $r$ in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

**Question 4.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & 2x_1 - 3x_2 + 2x_3 - 2x_4 \\ \text{s. t.} \quad & 5x_1 + 2x_3 - 6x_4 = 5, \\ & 3x_2 - x_3 + 2x_4 = 5, \\ & x_i \geq \ 0, \ \text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

Given non-basic variables $x_2$ and $x_4$, then the corresponding basic solution of the problem is

(A) $(0, 3, 0, -5)$, and not feasible.
(B) $(3, 0, -5, 0)$, and also feasible.
(C) $(3, 0, -5, 0)$, and not feasible.
(D) $(0, 3, 0, -5)$, and also feasible.

**Question 5.** Which one of the following statements is true?

(A) Every sentence is satisfied by at least one model.
(B) All sentences in predicate calculus are either satisfied by all models or by none.
(C) No sentence in predicate calculus is satisfied by all models.
(D) Some sentences in predicate calculus may be satisfied by one model, but not by another one.

**Question 6.** Consider the following program.

```
if (x < 5)
        x = x*x;
else
        x = x+1;
{ x >= 9 }
```

If we know that its (`postcondition`) is $\{x \geq 9\}$, then which of the following is its `precondition`?

Ⓐ $\{(x < -3) \vee (x > 8)\}$. Ⓑ $\{(x \geq -3 \wedge x < 5) \vee (x \geq 8)\}$.
Ⓒ $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5) \vee (x \geq 8)\}$. Ⓓ $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5)\}$.

**Question 7.** The two propositional operators $|$ (or $NAND$), and $\oplus$ (or $XOR$) are defined as follow, respectively: $p|q := \neg(p \wedge q)$, and $p \oplus q$ is the proposition with true value T if and only if there is exactly one of $p$, $q$ with true value T. Which of the following is correct?

Ⓐ The set $\{|\}$ is an adequate set of propositional operators.
Ⓑ The set $\{|\}$ is not an adequate set of propositional operators.
Ⓒ The set $\{|, \oplus\}$ is not an adequate set of propositional operators.
Ⓓ The set $\{\oplus\}$ is an adequate set of propositional operators.

**Question 8.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| $-2$ | $3$ | $0$ | $0$ | |
|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | rhs |
| 3 | 4 | 1 | 0 | 24 |
| 7 | $-4$ | 0 | 1 | 16 |
| $-2$ | 3 | 0 | 0 | 0 |

Then, the pivot element (phần tử trục/xoay)

Ⓐ $\bar{a}_{12} = 4$, with in-variable $x_2$ and out-variable $x_3$.
Ⓑ can not be determined.
Ⓒ $\bar{a}_{11} = 3$, with in-variable $x_1$ and out-variable $x_3$.
Ⓓ $\bar{a}_{21} = 7$, with in-variable $x_1$ and out-variable $x_4$.



**Question 9.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & x_1 + x_3 - x_4 \\ \text{s. t.} \quad & x_1 - x_3 = 1, \\ & x_3 + x_4 = 6, \\ & x_2 - 2x_3 = 3, \\ & x_i \geq \ 0, \ \text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

Then, the point $(1, 3, 0, 6)$

Ⓐ is not in the feasible set. Ⓑ is a basic feasible solution.
Ⓒ is not a basic solution. Ⓓ is not a basic feasible solution.

**Question 10.** Suppose that we are proving the validity of the sequent

$$\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \rightarrow \phi_2$$

as follows.

| | | |
|---|---|---|
| 1. | $\neg\phi_1 \wedge \neg\phi_2$ | premise |
| 2. | $\phi_1$ | assumption |
| 3. | $\neg\phi_1$ | $\wedge e_1 1$ |
| 4. | $\perp$ | $\neg e 2,3$ |
| 5. | $\phi_2$ | $\perp e 4$ |
| 6. | $\phi_1 \rightarrow \phi_2$ | $\rightarrow i 2,5$ |

Which of the following is correct?

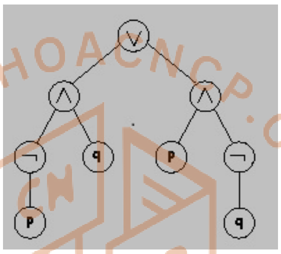
(A) That is not a correct proof since we did not use the fact $\neg\phi_2$ in the premise at all.

(B) That is not a correct proof since Line 1 has the premise $\neg\phi_1$, so we are not allowed to introduce the assumption $\phi_1$ in Line 2.

(C) That is not a correct proof since on Line 4 we have seen a contradiction.

(D) That is a correct proof.

**Question 11.** Which of the following formulas has the parse tree:




(A) $(p \vee \neg q) \wedge (\neg p \vee q)$.

(B) $(p \wedge \neg q) \vee (\neg p \wedge q)$.

(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$.

(D) $(\neg p \vee q) \wedge (p \vee \neg q)$.

**Question 12.** A basic feasible solution of a linear program consists of

(A) basic variables of zero, non-basic variables of positive value.

(B) all variables of zero

(C) basic variables of zero, non-basic variables of non-zero.

(D) basic variable of non-nagative value, non-basic variables of zero.

**Question 13.** Consider a linear program finding minimum which has the initial simplex tableau as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | 0 |

Suppose that $x_2, x_5, x_4$ are basic variables. Then, the value of reduced cost $r_i$, for $i = 1, 2, \ldots, 5$, should be

(A) $(2, 0, -1, 0, 0)$ (B) $(-2, 0, 1, 0, 0)$. (C) $(0, 2, 1, 0, 0)$ (D) $(0, 1, 2, 0, 0)$

**Question 14.** The relaxation in branch-and-bound approach for solving a linear integer program, performs to

(A) drop integer constraint of variables.

(B) assign all variables into zero.

(C) assign all variables into one.

(D) drop all integer variables

**Question 15.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| −1 | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | −1 | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| 2 | 0 | −1 | 0 | 0 | $-f(x)$ |

Since $r_3 < 0$ and $\bar{a}_{i3}$ is not negative (for $i = 1, 2, 3$), we need to compute the next simplex tableau. Hence, new basic variables should be

(A) $\{x_2, x_5, x_3\}$. (B) $\{x_2, x_5, x_4\}$. (C) $\{x_3, x_5, x_4\}$. (D) $\{x_2, x_3, x_4\}$.

**Question 16.** If a linear program has an optimal solution, then

(A) only feasible set is non-empty.
(B) the feasible set is non-empty and objective function is bounded.
(C) the objective function might not be bounded.
(D) the feasible set can be empty.

**Question 17.** Which of the following statements about Natural Deduction is true?

(A) Contradictions don't play an important role in natural deduction.
(B) $p \vee \neg p$ cannot be proved in natural deduction.
(C) Boxes are not used to delineate the scope of assumptions.
(D) The rules *Modus Tollens* (MT) cannot be derived from the rules $\wedge i$ and $\wedge e$ alone.

**Question 18.** Consider a general linear program

$$\begin{aligned} \min_{x_1, x_2} \quad & -2x_1 + 3x_2 \\ \text{s. t.} \quad & 3x_1 + 4x_2 \leq 24, \\ & 7x_1 - 4x_2 \leq 16, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Which one of followings can change the problem into standard form?

(A) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \geq 0$.
(B) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \leq 0$.
(C) $3x_1 + 4x_2 - x_3 = 24, 7x_1 - 4x_2 - x_4 = 16$, where $x_3, x_4 \geq 0$.
(D) $x_3 - 3x_1 - 4x_2 = 24, x_4 - 7x_1 + 4x_2 = 16$, where $x_3, x_4 \leq 0$.

**Question 19.** Given the following predicates

$Q(x)$ : $x$ is a politician,
$P(y)$ : $y$ is a person,
$T(z)$ : $z$ is a time,
$F(x, y, z)$ : person $x$ fools person $y$ at time $z$.

Represent the following sentences in predicate logic:

"***Politicians can't fool all of the people all of the time.***"

(A) $\forall x[Q(x) \rightarrow \exists y \exists z(P(y) \wedge T(z) \wedge \neg F(x, y, z))]$.
(B) $\forall x[Q(x) \rightarrow \forall y \forall z((P(y) \wedge T(z)) \rightarrow \neg F(x, y, z))]$.
(C) $\forall x[Q(x) \rightarrow \exists y \exists z((P(y) \wedge T(z)) \rightarrow \neg F(x, y, z))]$.
(D) $\forall x \exists y \exists z[Q(x) \rightarrow (P(y) \wedge T(z) \wedge F(x, y, z))]$.

**Question 20.** Let $\phi$ be a propositional formula. Consider the following statements on $\phi$.

I. $\phi$ is satisfiable or $\neg\phi$ is satisfiable.

II. $\phi$ is a tautology or $\neg\phi$ is a tautology.

Then

(A) I is incorrect and II is correct.
(B) Both I and II are correct.
(C) Both I and II are incorrect.
(D) I is correct and II is incorrect.

**BKU, VNU-HCM**
FACULTY OF CS AND ENGINEERING

**MIDTERM**
Subject: **Maths Modelling** (CO2011)
Class: **CC15KHMT** Group: **CC01**
Time: **60 minutes** ***(Closed book test)***
Test date: **March 26, 2018**

**Student's Name:** ____________________ **Student's ID:** | | | | | | | |

(*There are **20** MCQs, each question is worth **0.5** points. Answers in bold :* ■; *cancel out to deselect:* ☒.)

**Question 1.** Given the following predicates

$Q(x)$ : $x$ is a politician,
$P(y)$ : $y$ is a person,
$T(z)$ : $z$ is a time,
$F(x,y,z)$ : person $x$ fools person $y$ at time $z$.

Represent the following sentences in predicate logic:
"***Politicians can't fool all of the people all of the time.***"

(A) $\forall x[Q(x) \to \forall y \forall z((P(y) \wedge T(z)) \to \neg F(x,y,z))]$.
(B) $\forall x[Q(x) \to \exists y \exists z(P(y) \wedge T(z) \wedge \neg F(x,y,z))]$.
(C) $\forall x[Q(x) \to \exists y \exists z((P(y) \wedge T(z)) \to \neg F(x,y,z))]$.
(D) $\forall x \exists y \exists z[Q(x) \to (P(y) \wedge T(z) \wedge F(x,y,z))]$.

**Question 2.** Which of the following statements about Natural Deduction is true?

(A) $p \vee \neg p$ cannot be proved in natural deduction.
(B) Contradictions don't play an important role in natural deduction.
(C) Boxes are not used to delineate the scope of assumptions.
(D) The rules *Modus Tollens* (MT) cannot be derived from the rules $\wedge i$ and $\wedge e$ alone.

**Question 3.** Consider the following program.

```
if (x < 5)
        x = x*x;
else
        x = x+1;
{ x >= 9 }
```

If we know that its `(postcondition)` is $\{x \geq 9\}$, then which of the following is its `precondition`?

(A) $\{(x \geq -3 \wedge x < 5) \vee (x \geq 8)\}$.
(B) $\{(x < -3) \vee (x > 8)\}$.
(C) $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5) \vee (x \geq 8)\}$.
(D) $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5)\}$.

**Question 4.** Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$. Then the negation of the formula "**For each** $s$ **in** $\mathbb{R}$**, there exists** $r$ **in** $\mathbb{R}$ **such that if** $f(r) > 0$**, then** $g(s) > 0$" is the following formula.

(A) For every $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(B) There exists $s$ in $\mathbb{R}$ such that for every $r$ in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(C) For every $s$ in $\mathbb{R}$, there does not exist $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(D) There exists $s$ in $\mathbb{R}$ and there exists $r$ in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

**Question 5.** A basic feasible solution of a linear program consists of

(A) all variables of zero
(B) basic variables of zero, non-basic variables of positive value.
(C) basic variables of zero, non-basic variables of non-zero.
(D) basic variable of non-nagative value, non-basic variables of zero.

**Question 6.** Let $\phi$ be a propositional formula. Consider the following statements on $\phi$.

I. $\phi$ is satisfiable or $\neg\phi$ is satisfiable.

II. $\phi$ is a tautology or $\neg\phi$ is a tautology.

Then

(A) Both I and II are correct.
(B) I is incorrect and II is correct.
(C) Both I and II are incorrect.
(D) I is correct and II is incorrect.

**Question 7.** If a linear program has an optimal solution, then the solution

(A) is a point of the interior of the feasible set.
(B) is an extreme point of the feasible set.
(C) is an interior point of the boundary of the feasible set.
(D) does not belong to the feasible set.

**Question 8.** Consider the following linear program

$$\min_{x_i} \quad 2x_1 - 3x_2 + 2x_3 - 2x_4$$
$$\text{s. t.} \quad 5x_1 + 2x_3 - 6x_4 = 5,$$
$$3x_2 - x_3 + 2x_4 = 5,$$
$$x_i \geq 0, \text{ for } i = 1, 2, \ldots, 4.$$

Given non-basic variables $x_2$ and $x_4$, then the corresponding basic solution of the problem is

(A) $(3, 0, -5, 0)$, and also feasible.
(B) $(0, 3, 0, -5)$, and not feasible.
(C) $(3, 0, -5, 0)$, and not feasible.
(D) $(0, 3, 0, -5)$, and also feasible.

**Question 9.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| 2 | 0 | $-1$ | 0 | 0 | $-f(x)$ |

Since $r_3 < 0$ and $\bar{a}_{i3}$ is not negative (for $i = 1, 2, 3$), we need to compute the next simplex tableau. Hence, new basic variables should be

(A) $\{x_2, x_5, x_4\}$.
(B) $\{x_2, x_5, x_3\}$.
(C) $\{x_3, x_5, x_4\}$.
(D) $\{x_2, x_3, x_4\}$.



**Question 10.** Consider a general linear program

$$\min_{x_1, x_2} \quad -2x_1 + 3x_2$$
$$\text{s. t.} \quad 3x_1 + 4x_2 \leq 24,$$
$$7x_1 - 4x_2 \leq 16,$$
$$x_1, x_2 \geq 0.$$

Which one of followings can change the problem into standard form?

(A) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \leq 0$.
(B) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \geq 0$.
(C) $3x_1 + 4x_2 - x_3 = 24, 7x_1 - 4x_2 - x_4 = 16$, where $x_3, x_4 \geq 0$.
(D) $x_3 - 3x_1 - 4x_2 = 24, x_4 - 7x_1 + 4x_2 = 16$, where $x_3, x_4 \leq 0$.

**Question 11.** If a linear program has an optimal solution, then

(A) the feasible set is non-empty and objective function is bounded.
(B) only feasible set is non-empty.
(C) the objective function might not be bounded.
(D) the feasible set can be empty.

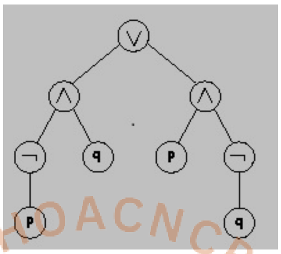
**Question 12.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| | | | | |
|---|---|---|---|---|
| $-2$ | $3$ | $0$ | $0$ | |
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | rhs |
| $3$ | $4$ | $1$ | $0$ | $24$ |
| $7$ | $-4$ | $0$ | $1$ | $16$ |
| $-2$ | $3$ | $0$ | $0$ | $0$ |

Then, the pivot element (phần tử trục/xoay)

(A) can not be determined.
(B) $\bar{a}_{12} = 4$, with in-variable $x_2$ and out-variable $x_3$.
(C) $\bar{a}_{11} = 3$, with in-variable $x_1$ and out-variable $x_3$.
(D) $\bar{a}_{21} = 7$, with in-variable $x_1$ and out-variable $x_4$.

**Question 13.** Which of the following formulas has the parse tree:



(A) $(p \wedge \neg q) \vee (\neg p \wedge q)$.
(B) $(p \vee \neg q) \wedge (\neg p \vee q)$.
(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$.
(D) $(\neg p \vee q) \wedge (p \vee \neg q)$.



**Question 14.** The relaxation in branch-and-bound approach for solving a linear integer program, performs to

(A) assign all variables into zero.
(B) drop integer constraint of variables.
(C) assign all variables into one.
(D) drop all integer variables

**Question 15.** Which one of the following statements is true?

(A) All sentences in predicate calculus are either satisfied by all models or by none.
(B) Every sentence is satisfied by at least one model.
(C) No sentence in predicate calculus is satisfied by all models.
(D) Some sentences in predicate calculus may be satisfied by one model, but not by another one.

**Question 16.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & x_1 + x_3 - x_4 \\ \text{s. t.} \quad & x_1 - x_3 = 1, \\ & x_3 + x_4 = 6, \\ & x_2 - 2x_3 = 3, \\ & x_i \geq \; 0, \;\text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

Then, the point $(1, 3, 0, 6)$

(A) is a basic feasible solution.
(B) is not in the feasible set.
(C) is not a basic solution.
(D) is not a basic feasible solution.

**Question 17.** Consider a linear program finding minimum which has the initial simplex tableau as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | 0 |

Suppose that $x_2, x_5, x_4$ are basic variables. Then, the value of reduced cost $r_i$, for $i = 1, 2, \ldots, 5$, should be

(A) $(-2, 0, 1, 0, 0)$. (B) $(2, 0, -1, 0, 0)$ (C) $(0, 2, 1, 0, 0)$ (D) $(0, 1, 2, 0, 0)$

**Question 18.** Suppose that we are proving the validity of the sequent

$$\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \to \phi_2$$

as follows.

| | | |
|---|---|---|
| 1. | $\neg\phi_1 \wedge \neg\phi_2$ | premise |
| 2. | $\phi_1$ | assumption |
| 3. | $\neg\phi_1$ | $\wedge e_1 1$ |
| 4. | $\bot$ | $\neg$e2,3 |
| 5. | $\phi_2$ | $\bot$e4 |
| 6. | $\phi_1 \to \phi_2$ | $\to$ i2,5 |

Which of the following is correct?

(A) That is not a correct proof since Line 1 has the premise $\neg\phi_1$, so we are not allowed to introduce the assumption $\phi_1$ in Line 2.

(B) That is not a correct proof since we did not use the fact $\neg\phi_2$ in the premise at all.

(C) That is not a correct proof since on Line 4 we have seen a contradiction.

(D) That is a correct proof.



**Question 19.** Consider the predicate formula $\phi$ as follow.

$$(\exists x P(y, y) \longrightarrow \exists y P(y, z)).$$

What is the result of the substitution $[y \Rightarrow f(z))]\phi$?

(A) $(\exists x P(f(z), f(z)) \longrightarrow \exists y P(f(z), z))$.

(B) $(\exists z P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.

(C) $(\exists x P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.

(D) $(\exists z P(f(z), f(z)) \longrightarrow \exists z P(f(z), z))$.

**Question 20.** The two propositional operators $|$ (or $NAND$), and $\oplus$ (or $XOR$) are defined as follow, respectively: $p|q := \neg(p \wedge q)$, and $p \oplus q$ is the proposition with true value T if and only if there is exactly one of $p$, $q$ with true value T. Which of the following is correct?

(A) The set $\{|\}$ is not an adequate set of propositional operators.

(B) The set $\{|\}$ is an adequate set of propositional operators.

(C) The set $\{|, \oplus\}$ is not an adequate set of propositional operators.

(D) The set $\{\oplus\}$ is an adequate set of propositional operators.

**BKU, VNU-HCM**
FACULTY OF CS AND ENGINEERING

**MIDTERM**
Subject: **Maths Modelling** (CO2011)
Class: **CC15KHMT** Group: **CC01**
Time: **60 minutes** ***(Closed book test)***
Test date: **March 26, 2018**

**Student's Name:** ____________________ **Student's ID:** | | | | | | | |

(*There are* ***20*** *MCQs, each question is worth* ***0.5*** *points. Answers in bold :* ■; *cancel out to deselect:* ☒.)

**Question 1.** Consider the following program.

```
if (x < 5)
        x = x*x;
else
        x = x+1;
{ x >= 9 }
```

If we know that its (`postcondition`) is $\{x \geq 9\}$, then which of the following is its `precondition`?

(A) $\{(x \geq -3 \wedge x < 5) \vee (x \geq 8)\}$.
(B) $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5)\}$.
(C) $\{(x \leq -3) \vee (x \geq 3 \wedge x < 5) \vee (x \geq 8)\}$.
(D) $\{(x < -3) \vee (x > 8)\}$.

**Question 2.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & 2x_1 - 3x_2 + 2x_3 - 2x_4 \\ \text{s. t.} \quad & 5x_1 + 2x_3 - 6x_4 = 5, \\ & 3x_2 - x_3 + 2x_4 = 5, \\ & x_i \geq 0, \ \text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

Given non-basic variables $x_2$ and $x_4$, then the corresponding basic solution of the problem is

(A) $(3, 0, -5, 0)$, and also feasible.
(B) $(0, 3, 0, -5)$, and also feasible.
(C) $(3, 0, -5, 0)$, and not feasible.
(D) $(0, 3, 0, -5)$, and not feasible.

**Question 3.** If a linear program has an optimal solution, then the solution

(A) is a point of the interior of the feasible set.
(B) does not belong to the feasible set.
(C) is an interior point of the boundary of the feasible set.
(D) is an extreme point of the feasible set.

**Question 4.** Let $\phi$ be a propositional formula. Consider the following statements on $\phi$.

I. $\phi$ is satisfiable or $\neg\phi$ is satisfiable.

II. $\phi$ is a tautology or $\neg\phi$ is a tautology.

Then

(A) Both I and II are correct.
(B) I is correct and II is incorrect.
(C) Both I and II are incorrect.
(D) I is incorrect and II is correct.

**Question 5.** Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$. Then the negation of the formula "**For each** $s$ **in** $\mathbb{R}$, **there exists** $r$ **in** $\mathbb{R}$ **such that if** $f(r) > 0$, **then** $g(s) > 0$" is the following formula.

(A) For every $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(B) There exists $s$ in $\mathbb{R}$ and there exists $r$ in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(C) For every $s$ in $\mathbb{R}$, there does not exist $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(D) There exists $s$ in $\mathbb{R}$ such that for every $r$ in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.

**Question 6.** Which of the following statements about Natural Deduction is true?

(A) $p \vee \neg p$ cannot be proved in natural deduction.
(B) The rules *Modus Tollens* (MT) cannot be derived from the rules $\wedge i$ and $\wedge e$ alone.
(C) Boxes are not used to delineate the scope of assumptions.
(D) Contradictions don't play an important role in natural deduction.

**Question 7.** The relaxation in branch-and-bound approach for solving a linear integer program, performs to

(A) assign all variables into zero.
(B) drop all integer variables
(C) assign all variables into one.
(D) drop integer constraint of variables.

**Question 8.** Consider a general linear program

$$\begin{aligned} \min_{x_1,x_2} \quad & -2x_1 + 3x_2 \\ \text{s. t.} \quad & 3x_1 + 4x_2 \le 24, \\ & 7x_1 - 4x_2 \le 16, \\ & x_1, x_2 \ge \ 0. \end{aligned}$$

Which one of followings can change the problem into standard form?

(A) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \le 0$.
(B) $x_3 - 3x_1 - 4x_2 = 24, x_4 - 7x_1 + 4x_2 = 16$, where $x_3, x_4 \le 0$.
(C) $3x_1 + 4x_2 - x_3 = 24, 7x_1 - 4x_2 - x_4 = 16$, where $x_3, x_4 \ge 0$.
(D) $3x_1 + 4x_2 + x_3 = 24, 7x_1 - 4x_2 + x_4 = 16$, where $x_3, x_4 \ge 0$.

**Question 9.** Given the following predicates

$Q(x)$ : $x$ is a politician,
$P(y)$ : $y$ is a person,
$T(z)$ : $z$ is a time,
$F(x, y, z)$ : person $x$ fools person $y$ at time $z$.

Represent the following sentences in predicate logic:

"***Politicians can't fool all of the people all of the time.***"

(A) $\forall x[Q(x) \to \forall y \forall z((P(y) \wedge T(z)) \to \neg F(x, y, z))]$.
(B) $\forall x \exists y \exists z[Q(x) \to (P(y) \wedge T(z) \wedge F(x, y, z))]$.
(C) $\forall x[Q(x) \to \exists y \exists z((P(y) \wedge T(z)) \to \neg F(x, y, z))]$.
(D) $\forall x[Q(x) \to \exists y \exists z(P(y) \wedge T(z) \wedge \neg F(x, y, z))]$.



**Question 10.** Consider the following linear program

$$\begin{aligned} \min_{x_i} \quad & x_1 + x_3 - x_4 \\ \text{s. t.} \quad & x_1 - x_3 = 1, \\ & x_3 + x_4 = 6, \\ & x_2 - 2x_3 = 3, \\ & x_i \ge \ 0, \ \text{for } i = 1, 2, \ldots, 4. \end{aligned}$$

Then, the point $(1, 3, 0, 6)$

(A) is a basic feasible solution.
(B) is not a basic feasible solution.
(C) is not a basic solution.
(D) is not in the feasible set.

**Question 11.** Consider the predicate formula $\phi$ as follow.

$$(\exists x P(y, y) \longrightarrow \exists y P(y, z)).$$

What is the result of the substitution $[y \ \Rightarrow \ f(z)]\phi$?

(A) $(\exists x P(f(z), f(z)) \longrightarrow \exists y P(f(z), z))$.
(B) $(\exists z P(f(z), f(z)) \longrightarrow \exists z P(f(z), z))$.
(C) $(\exists x P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.
(D) $(\exists z P(f(z), f(z)) \longrightarrow \exists y' P(y', z))$.

**Question 12.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| 2 | 0 | $-1$ | 0 | 0 | $-f(x)$ |

Since $r_3 < 0$ and $\bar{a}_{i3}$ is not negative (for $i = 1, 2, 3$), we need to compute the next simplex tableau. Hence, new basic variables should be

(A) $\{x_2, x_5, x_4\}$. (B) $\{x_2, x_3, x_4\}$. (C) $\{x_3, x_5, x_4\}$. (D) $\{x_2, x_5, x_3\}$.

**Question 13.** Consider a linear program finding minimum which has the initial simplex tableau as below.

| 1 | 1 | 1 | 0 | 0 | |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | rhs |
| $-1$ | 1 | 2 | 0 | 0 | 2 |
| 1 | 0 | $-1$ | 0 | 1 | 3 |
| 2 | 0 | 1 | 1 | 0 | 4 |
| $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | 0 |

Suppose that $x_2, x_5, x_4$ are basic variables. Then, the value of reduced cost $r_i$, for $i = 1, 2, \ldots, 5$, should be

(A) $(-2, 0, 1, 0, 0)$. (B) $(0, 1, 2, 0, 0)$ (C) $(0, 2, 1, 0, 0)$ (D) $(2, 0, -1, 0, 0)$



**Question 14.** Which one of the following statements is true?

(A) All sentences in predicate calculus are either satisfied by all models or by none.
(B) Some sentences in predicate calculus may be satisfied by one model, but not by another one.
(C) No sentence in predicate calculus is satisfied by all models.
(D) Every sentence is satisfied by at least one model.

**Question 15.** A basic feasible solution of a linear program consists of

(A) all variables of zero
(B) basic variable of non-nagative value, non-basic variables of zero.
(C) basic variables of zero, non-basic variables of non-zero.
(D) basic variables of zero, non-basic variables of positive value.

**Question 16.** Consider a linear program finding minimum which has the simplex tableau for basic variables $\{x_2, x_5, x_4\}$ as below.

| $-2$ | 3 | 0 | 0 | |
|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | rhs |
| 3 | 4 | 1 | 0 | 24 |
| 7 | $-4$ | 0 | 1 | 16 |
| $-2$ | 3 | 0 | 0 | 0 |

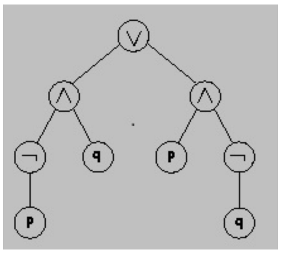
Then, the pivot element (phần tử trục/xoay)

(A) can not be determined.
(B) $\bar{a}_{21} = 7$, with in-variable $x_1$ and out-variable $x_4$.
(C) $\bar{a}_{11} = 3$, with in-variable $x_1$ and out-variable $x_3$.
(D) $\bar{a}_{12} = 4$, with in-variable $x_2$ and out-variable $x_3$.

**Question 17.** The two propositional operators $|$ (or $NAND$), and $\oplus$ (or $XOR$) are defined as follow, respectively: $p|q := \neg(p \wedge q)$, and $p \oplus q$ is the proposition with true value T if and only if there is exactly one of $p$, $q$ with true value T. Which of the following is correct?

(A) The set $\{|\}$ is not an adequate set of propositional operators.
(B) The set $\{\oplus\}$ is an adequate set of propositional operators.
(C) The set $\{|, \oplus\}$ is not an adequate set of propositional operators.
(D) The set $\{|\}$ is an adequate set of propositional operators.

**Question 18.** Which of the following formulas has the parse tree:



(A) $(p \wedge \neg q) \vee (\neg p \wedge q)$.
(B) $(\neg p \vee q) \wedge (p \vee \neg q)$.
(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$.
(D) $(p \vee \neg q) \wedge (\neg p \vee q)$.

**Question 19.** Suppose that we are proving the validity of the sequent

$$\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \to \phi_2$$

as follows.

| | | |
|---|---|---|
| 1. | $\neg\phi_1 \wedge \neg\phi_2$ | premise |
| 2. | $\phi_1$ | assumption |
| 3. | $\neg\phi_1$ | $\wedge e_1 1$ |
| 4. | $\bot$ | $\neg$e2,3 |
| 5. | $\phi_2$ | $\bot$e4 |
| 6. | $\phi_1 \to \phi_2$ | $\to$ i2,5 |

Which of the following is correct?

(A) That is not a correct proof since Line 1 has the premise $\neg\phi_1$, so we are not allowed to introduce the assumption $\phi_1$ in Line 2.
(B) That is a correct proof.
(C) That is not a correct proof since on Line 4 we have seen a contradiction.
(D) That is not a correct proof since we did not use the fact $\neg\phi_2$ in the premise at all.

**Question 20.** If a linear program has an optimal solution, then

(A) the feasible set is non-empty and objective function is bounded.
(B) the feasible set can be empty.
(C) the objective function might not be bounded.
(D) only feasible set is non-empty.