Artificial Intelligence

Uncertainty

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Content

- Probability: Brief review
- Robot localization problem
- Baysian filter
- Particle filter

Probability

- Probability that a random variable *X* has value *x*

$$p(X = x)$$

- Example:

$$p(X = \text{head}) = p(X = \text{tail}) = \frac{1}{2}$$

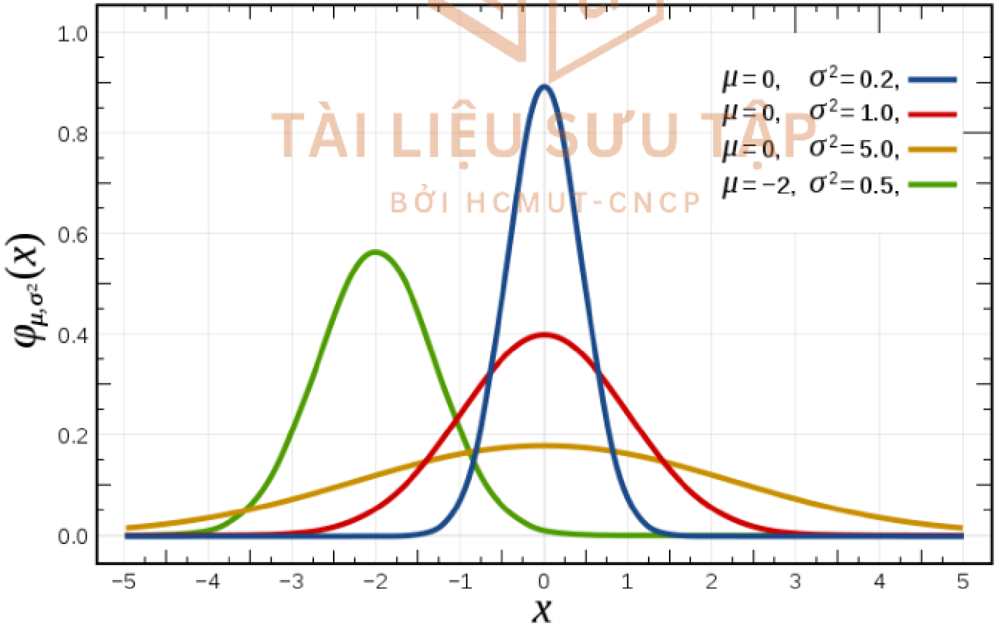
- Abbreviation:

$$p(x)$$

Probability

- Probability density function
  - Univariate normal distribution $\mathcal{N}(x;\mu,\sigma^2)$

$$p(x) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

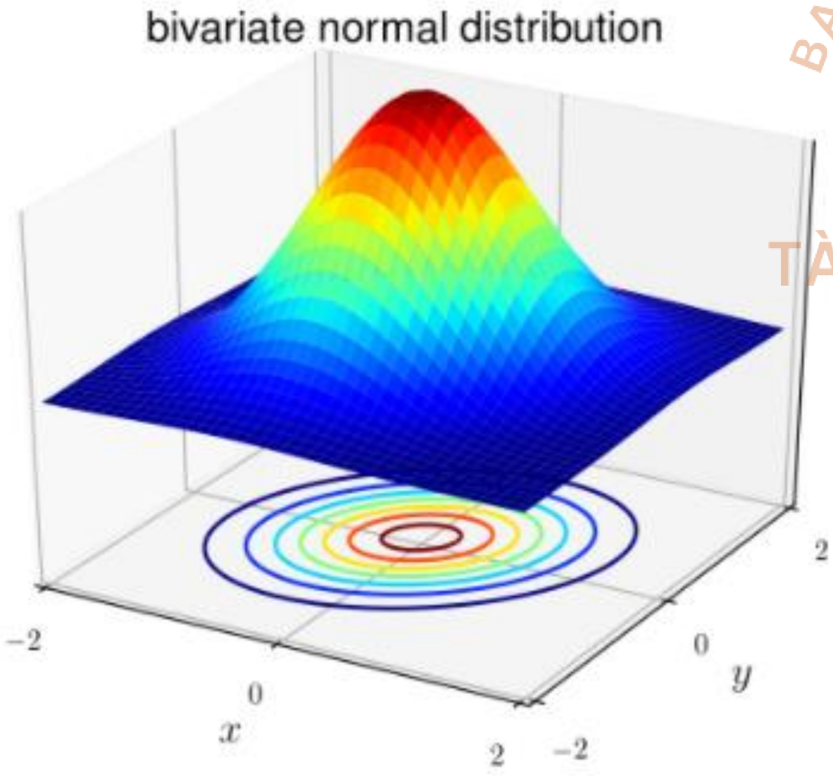


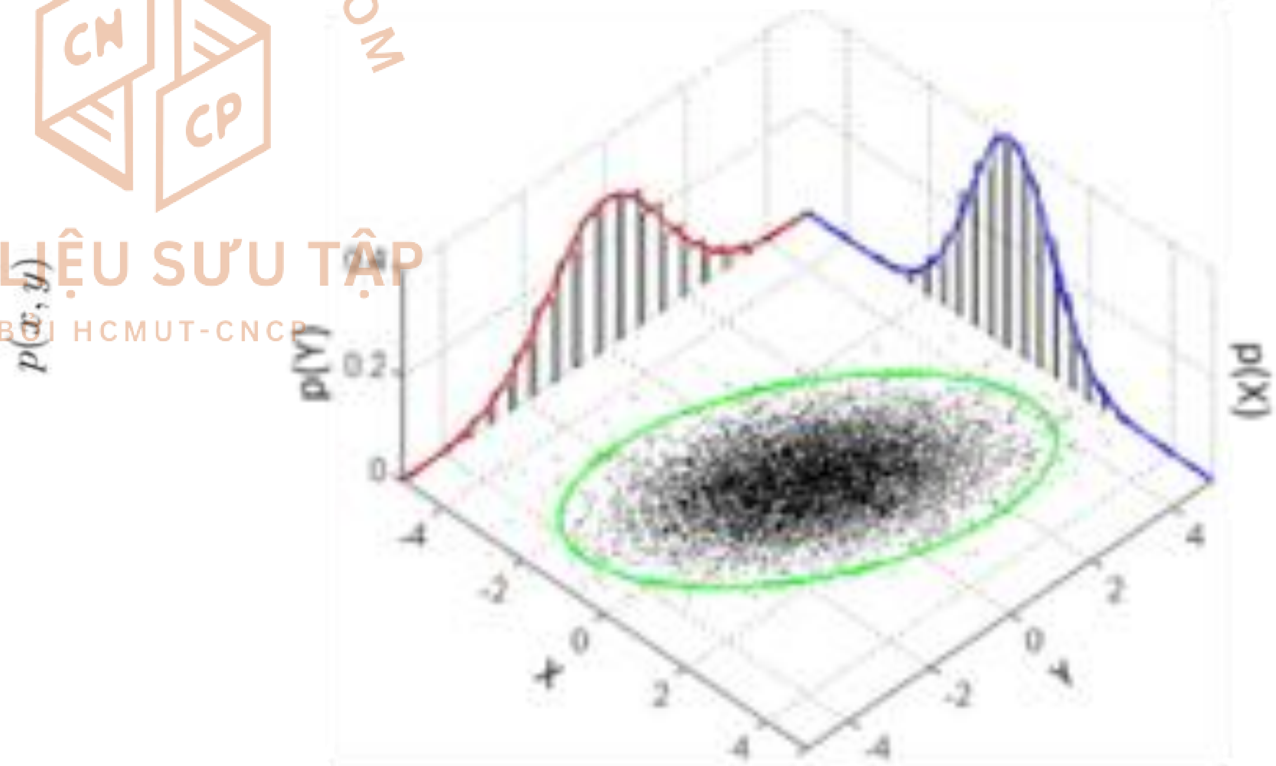
Probability

- Probability density function
  - Multivariate normal distributions

mean vector   covariance matrix

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right\}$$





Probability

- Probability density function
  - Probability of the random variable falling within a particular range of values

$$\Pr[a \le X \le b] = \int_a^b f_X(x)\,dx$$

$$\int p(x)\,dx = 1$$

Probability

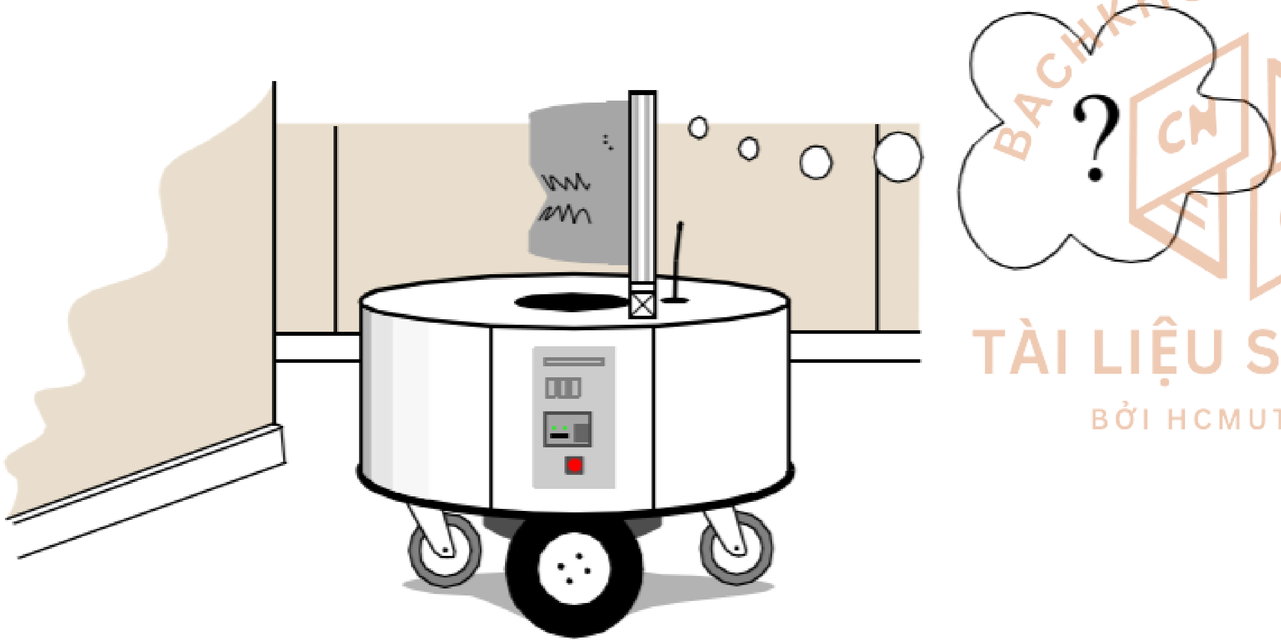
- Joint distribution of 2 random variables $$p(x, y) = p(X = x \text{ and } Y = y)$$
- Conditional probability
$$p(x \mid y) = p(X = x \mid Y = y) \qquad p(x \mid y) = \frac{p(x, y)}{p(y)}$$
- Bayes rule $$p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)}$$

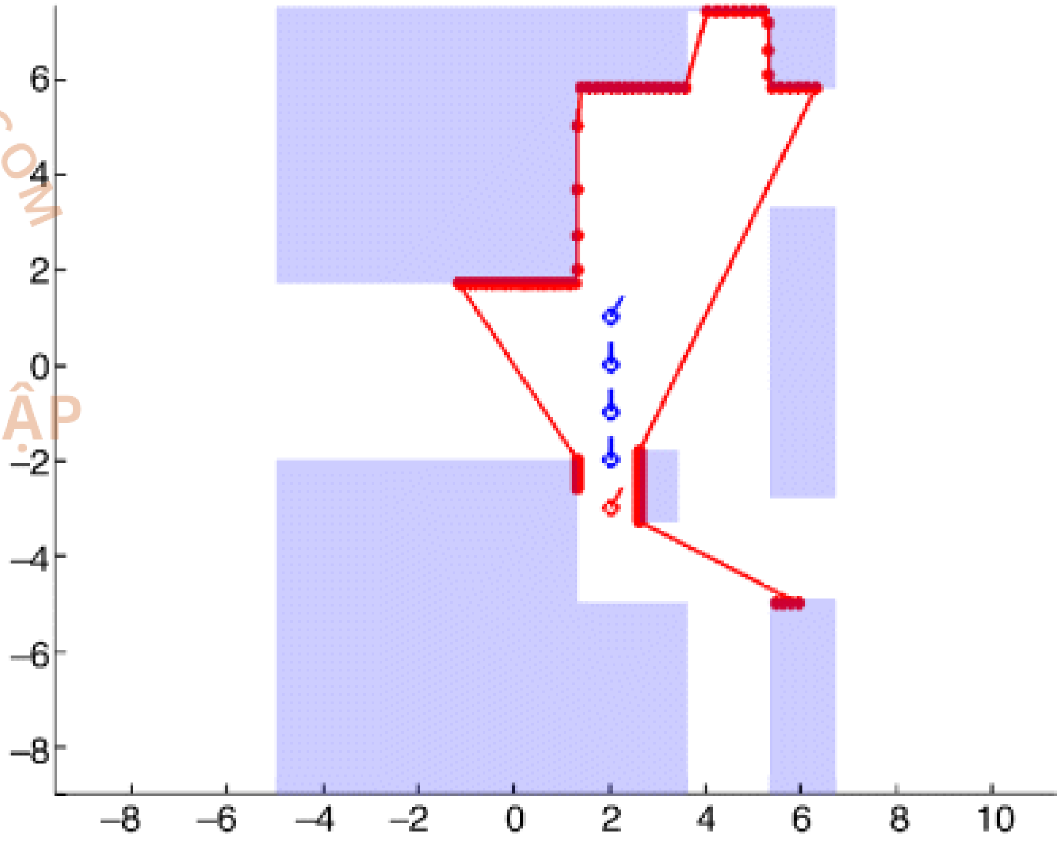
p(*y*) doesn't depent on *x* $$p(x \mid y) = \eta\, p(y \mid x)\, p(x)$$

- Conditional independence
$$p(x \mid y, z) = \frac{p(y \mid x, z)\, p(x \mid z)}{p(y \mid z)}$$
$$p(x, y \mid z) = p(x \mid z)\, p(y \mid z)$$
$$\begin{aligned} p(a, b|c) &= p(a|b, c)p(b|c) \\ &= p(a|c)p(b|c). \end{aligned}$$

Localization Problem

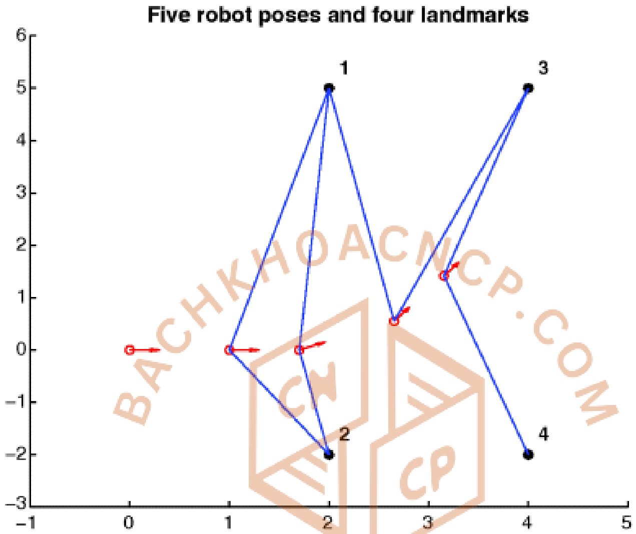
- ✓ Indoor

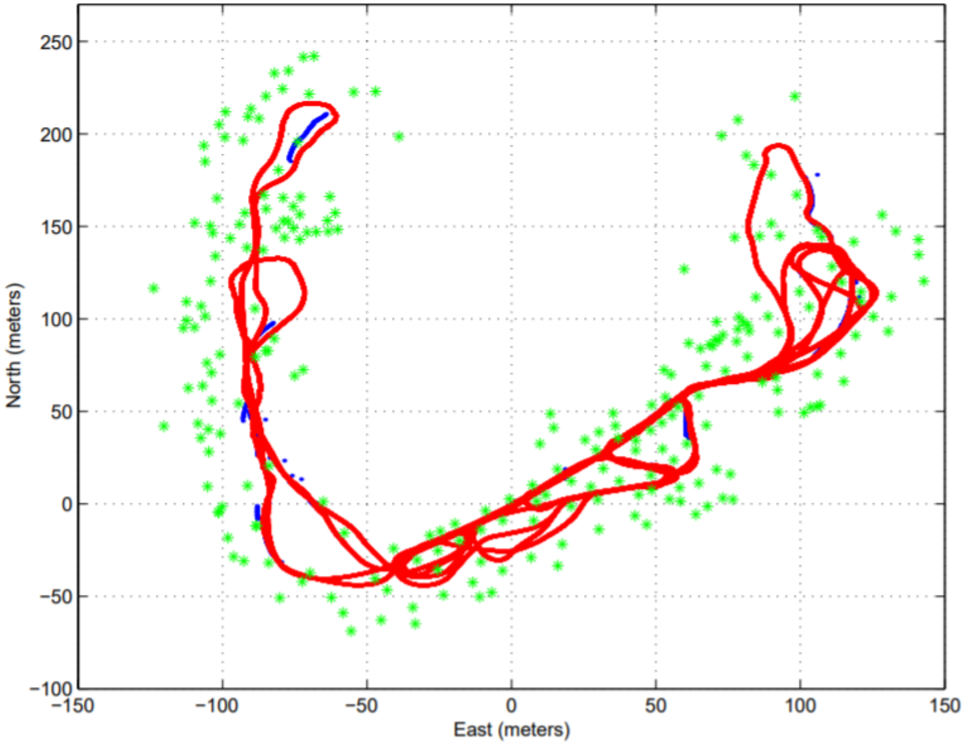




Localization Problem

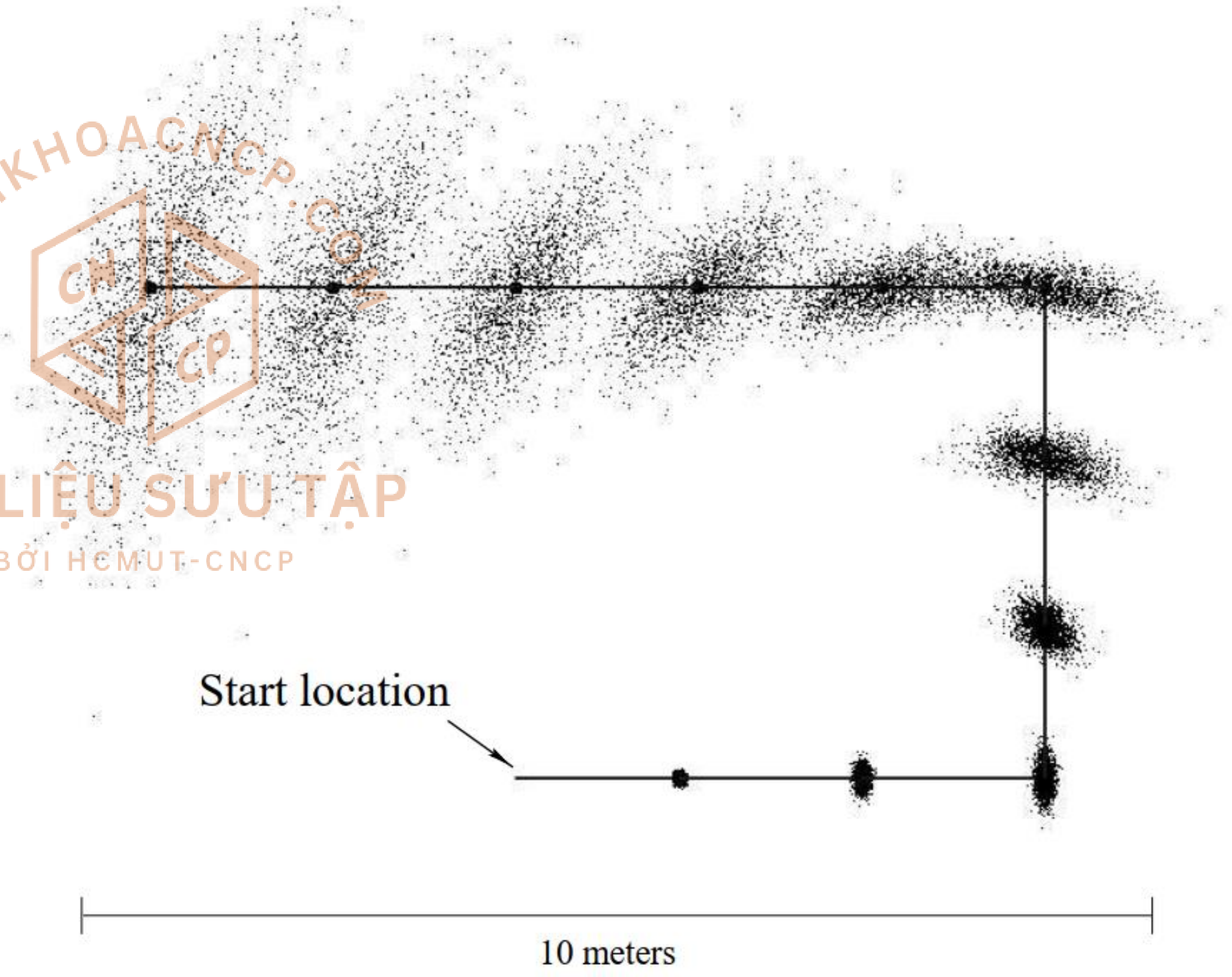
✓ Outdoor





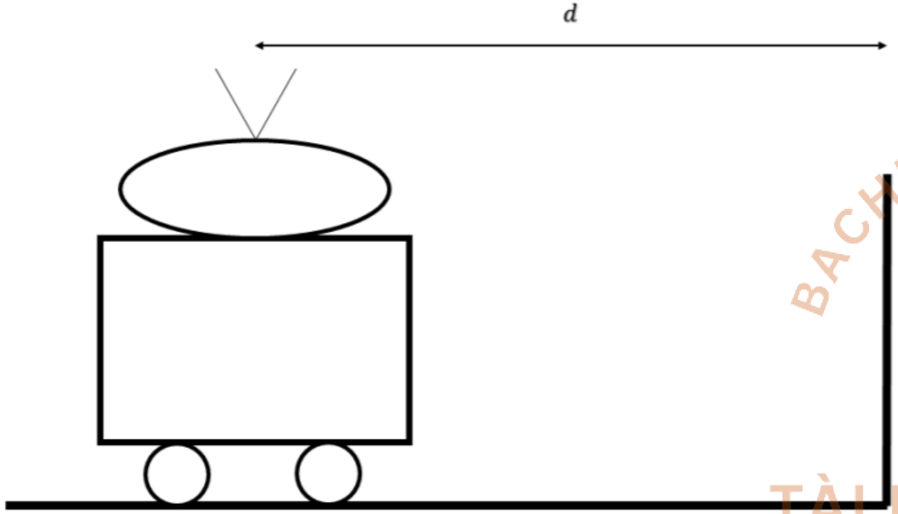
Why localization problem?

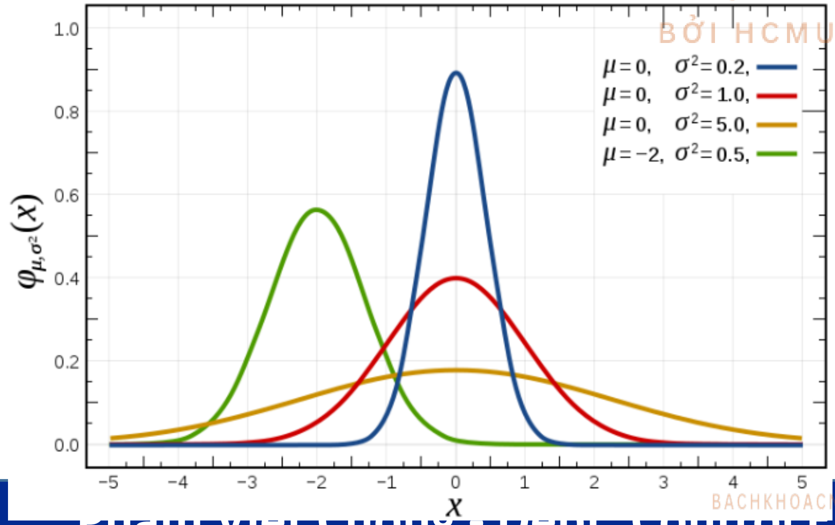
- ✓ Fundamental problem in robotics
- ✓ Control noise



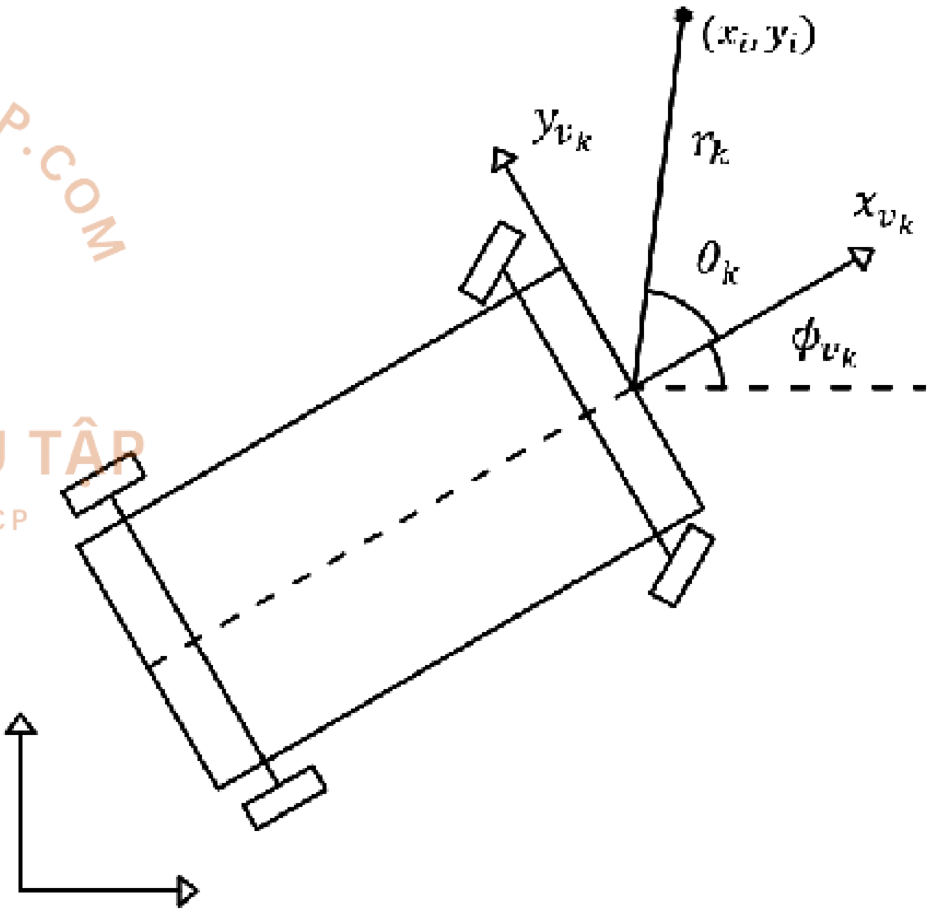
Localization Problem

- ✓ 1D localization





- ✓ 2D localization



Robot & Environment

- ✓ State

  x_t

- ✓ Control actions

  u_t

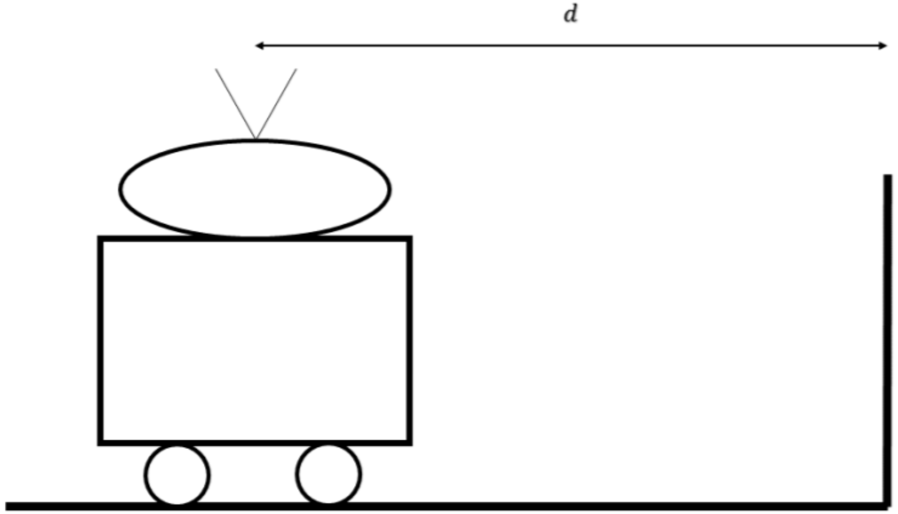
- ✓ Sensor measurements

  z_t

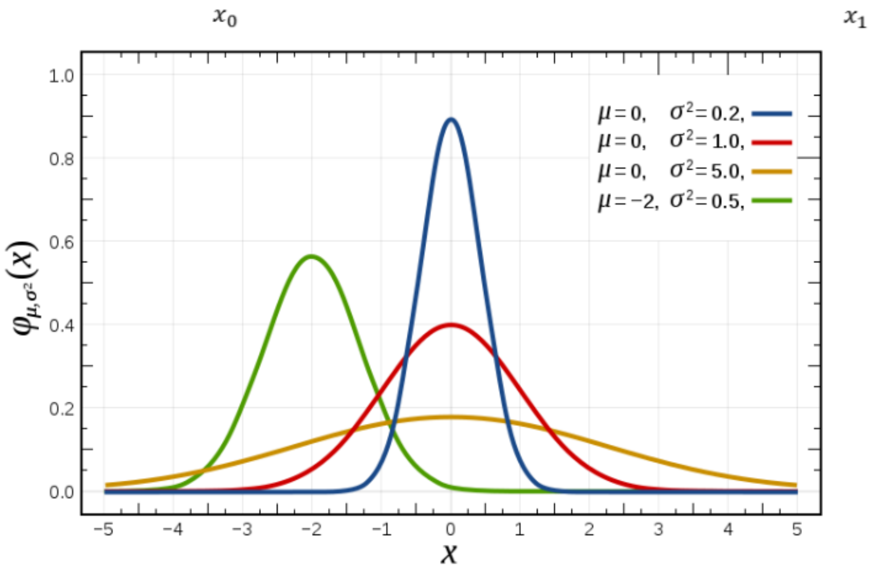
Robot & Environment

✓ State evolution: state transition probability

$$p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) \;=\; p(x_t \mid x_{t-1}, u_t)$$



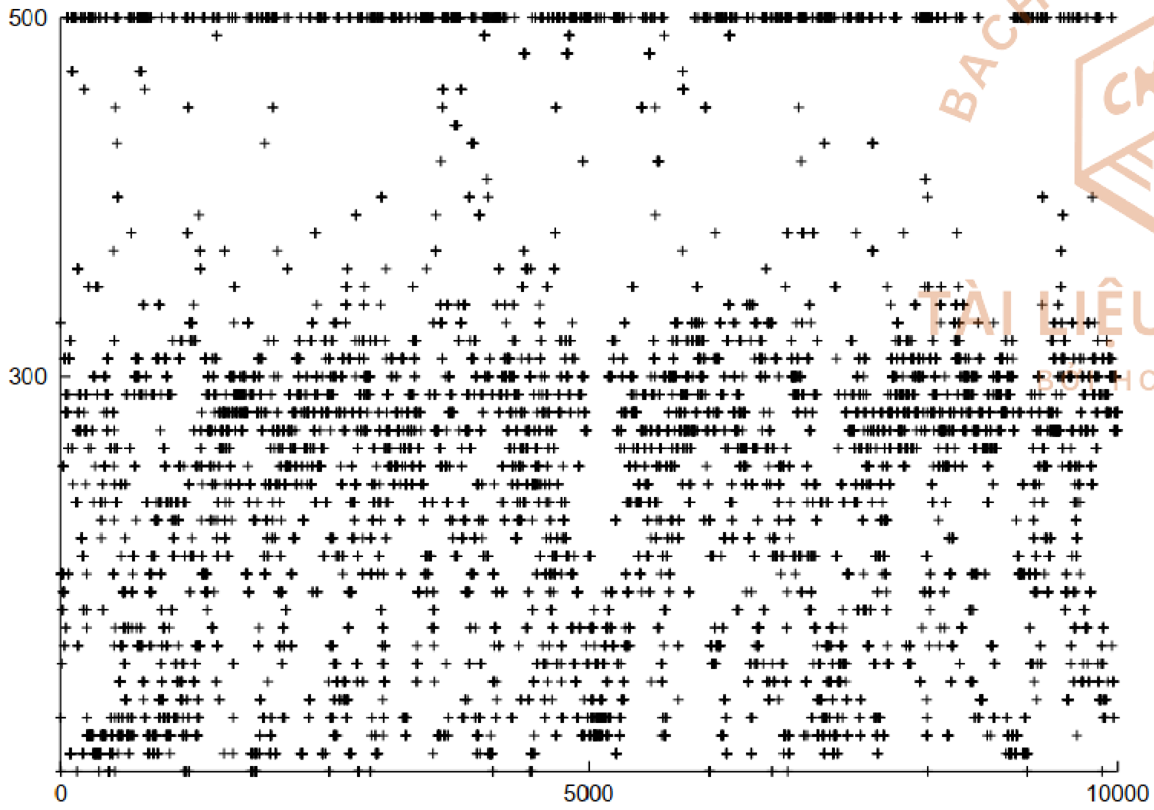
Markov process: a process for which predictions can be made regarding future outcomes based solely on its present state (as good as the ones that could be made knowing the process's full history)



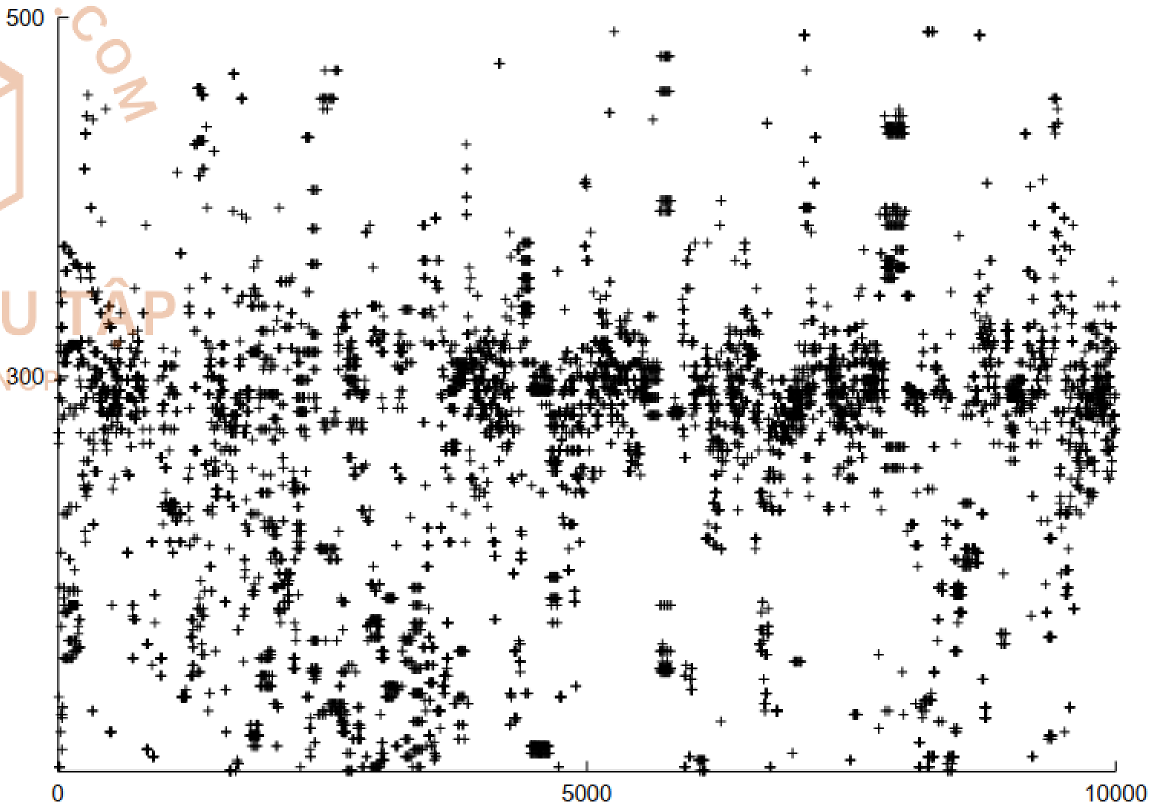
Robot & Environment

✓ Measurement model: $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) \;=\; p(z_t \mid x_t)$

(a) Sonar data

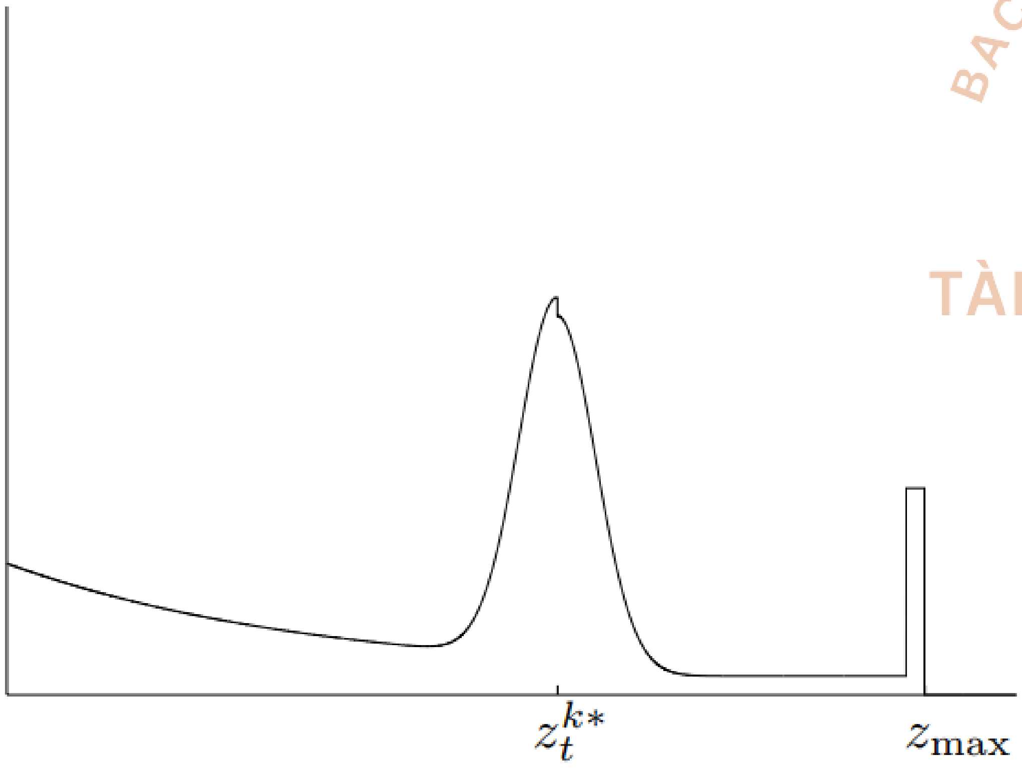


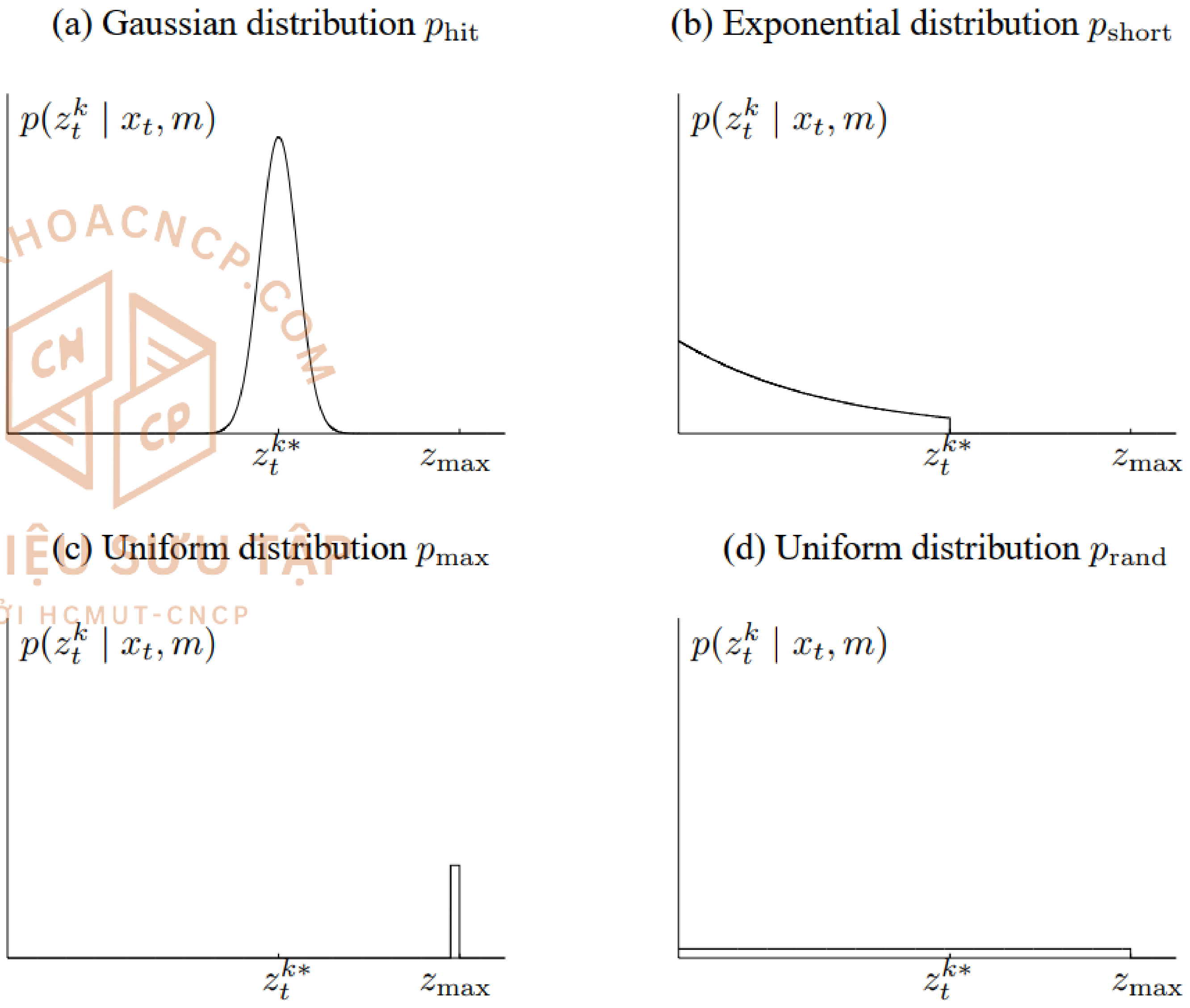
(b) Laser data



✓ Measurement model:

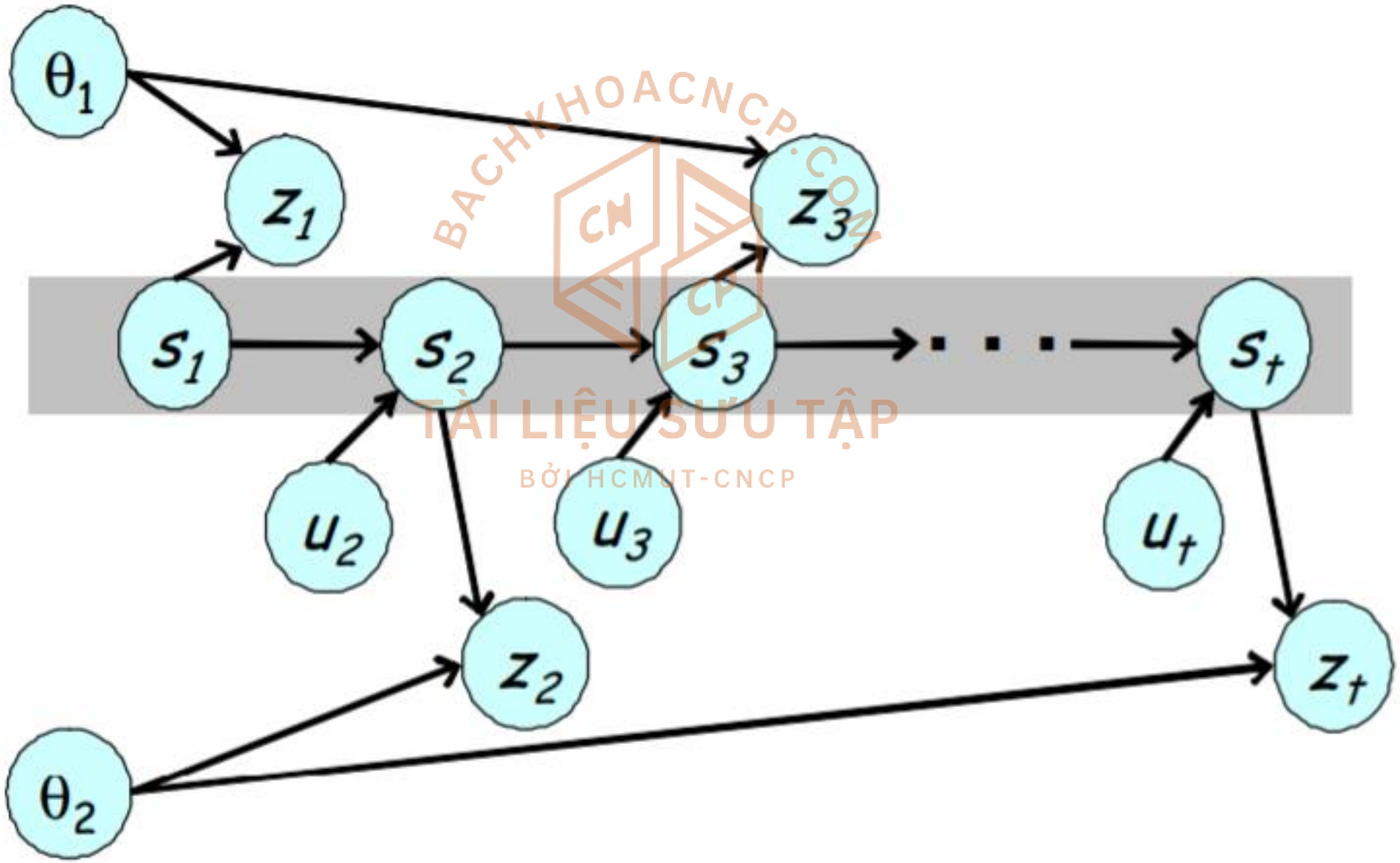
$$p(z_t \mid x_t)$$





Robot & Environment

✓ Hidden Markov chain:

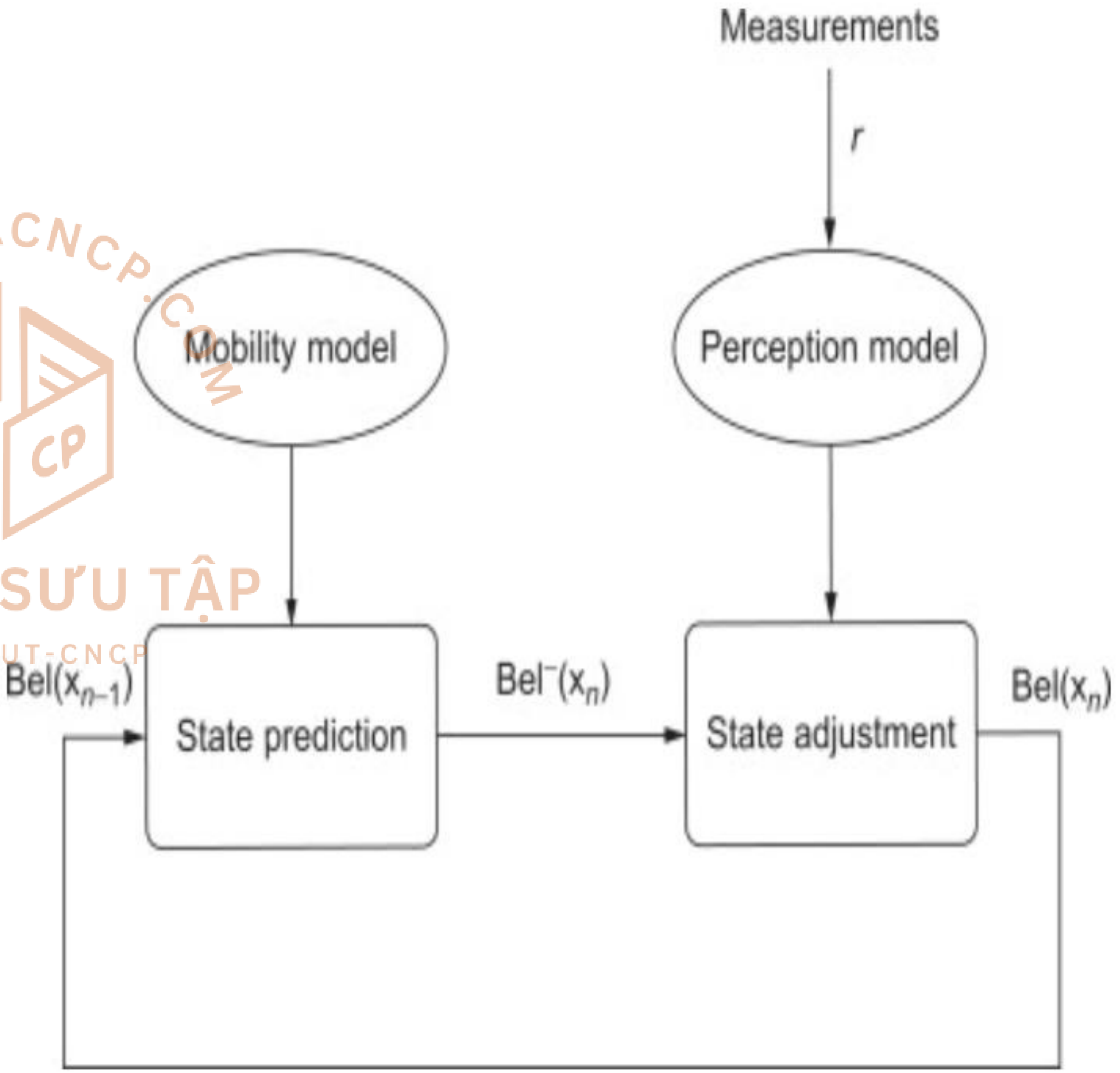


Belief

✓ Belief

$$\overline{bel}(x_t) = p(x_t \mid z_{1:t-1}, u_{1:t})$$

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$



Bayesian filter

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$)**:**

for all x_t do

$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx$

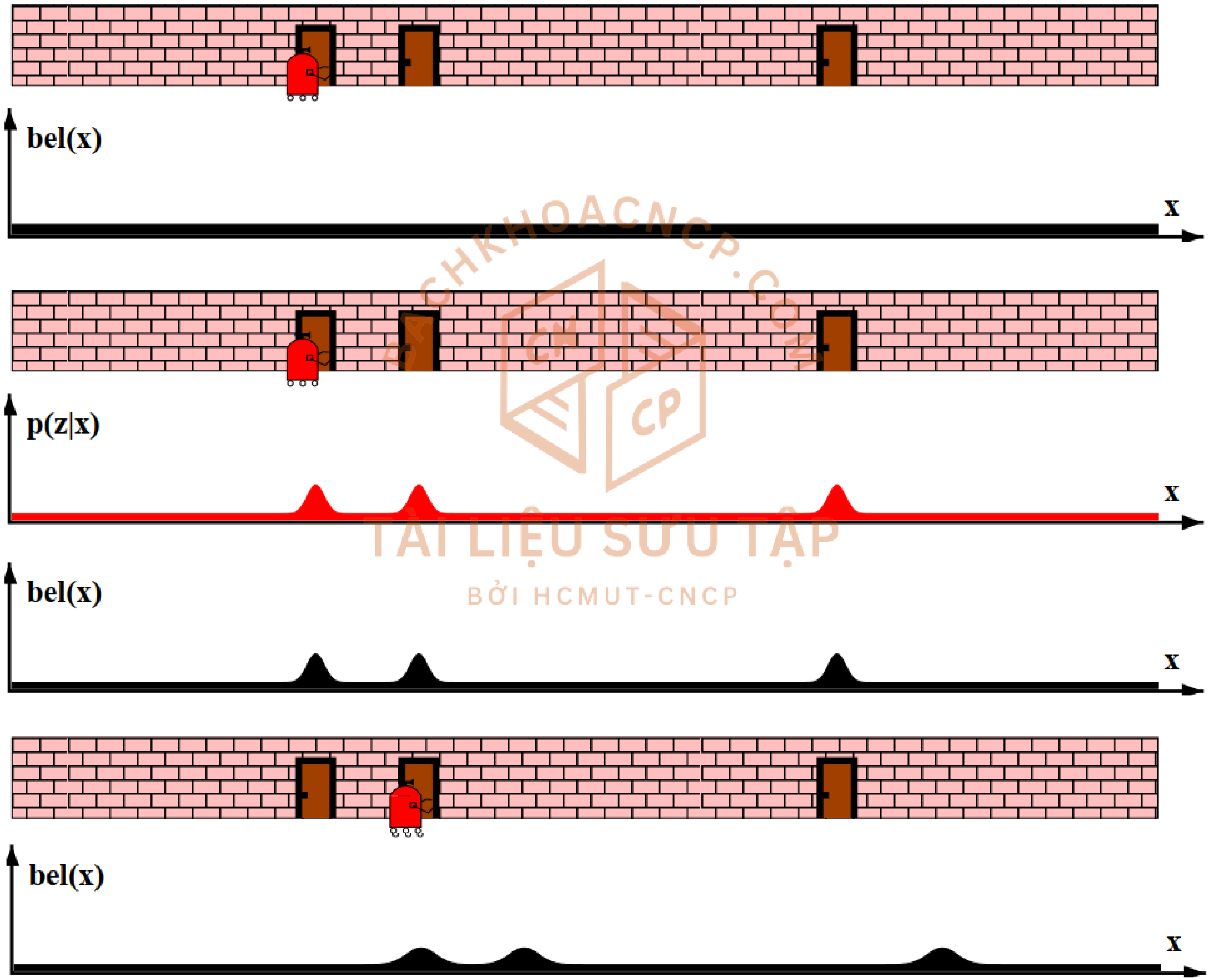
$bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$

endfor

return $bel(x_t)$

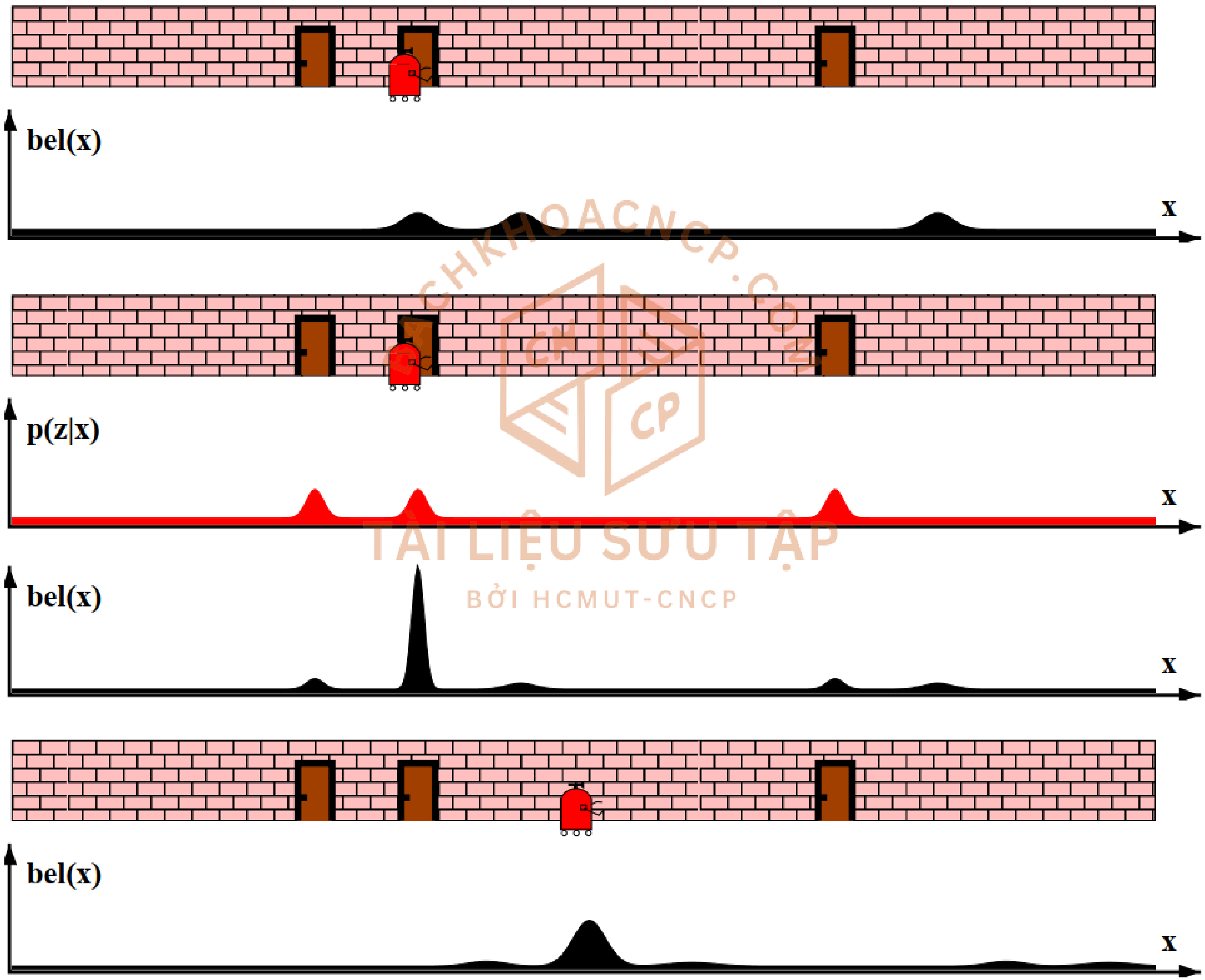
Bayes filter

- ✓ Example



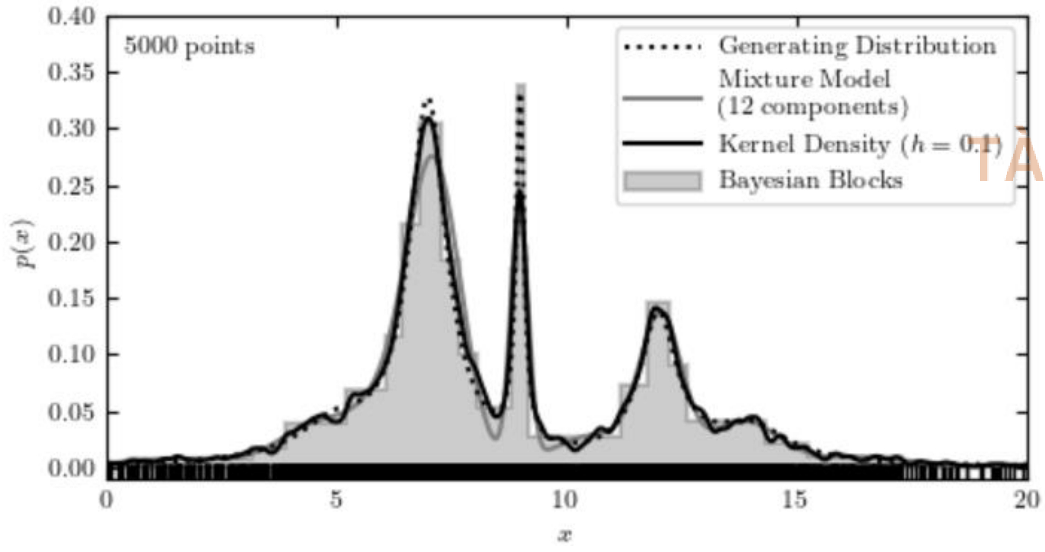
Bayes filter

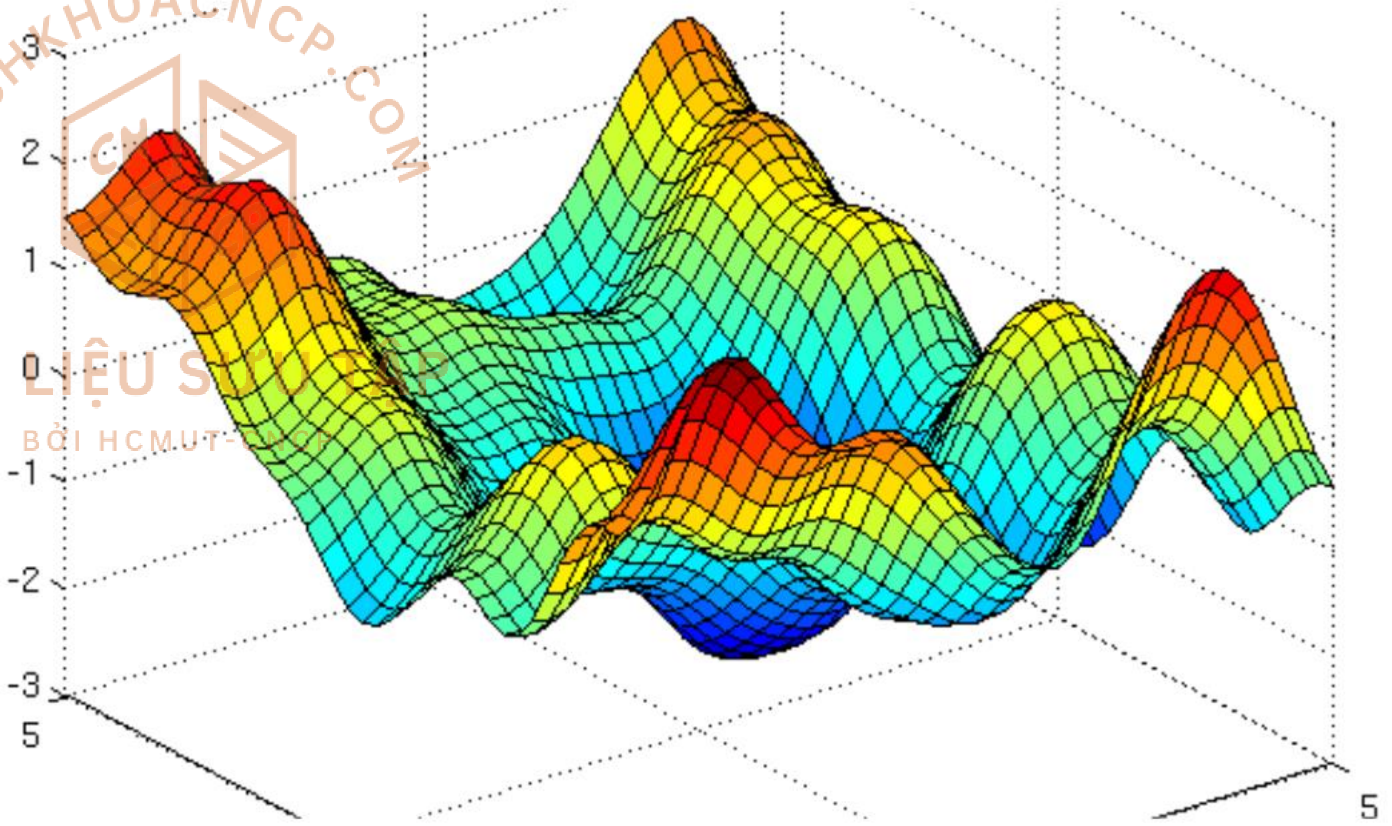
- Example



Bayes filter

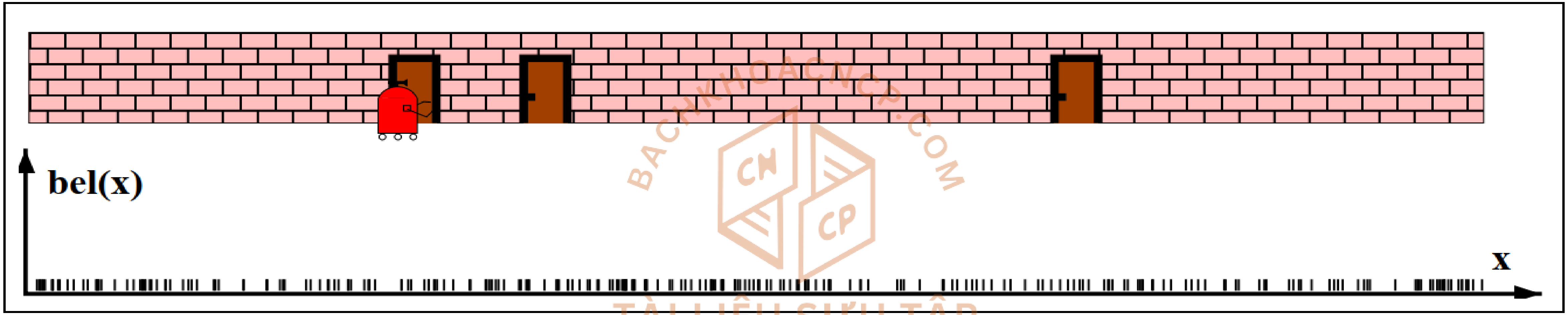
- No closed form solution in general
- Two approaches:
  - Gaussian filters
  - Nonparametric filters

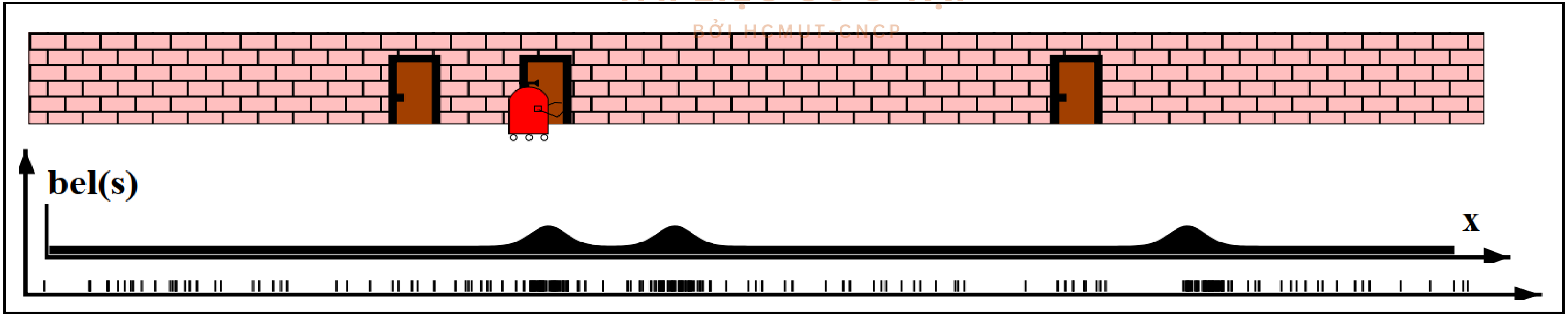




Particle filter

- ✓ Represent the posterior bel(x_t) by a set of random state samples





Particle filter

- Approximate the belief bel(X_t) by the set of particles X_t

$$\mathcal{X}_t \quad := \quad x_t^{[1]}, x_t^{[2]}, \ldots, x_t^{[M]}$$

- Particles: samples of a posterior distribution

$$x_t^{[m]} \quad \sim \quad p(x_t \mid z_{1:t}, u_{1:t})$$

Particle filter

Algorithm Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$)**:**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ to M do

  sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

  $w_t^{[m]} = p(z_t \mid x_t^{[m]})$

  $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ to M do

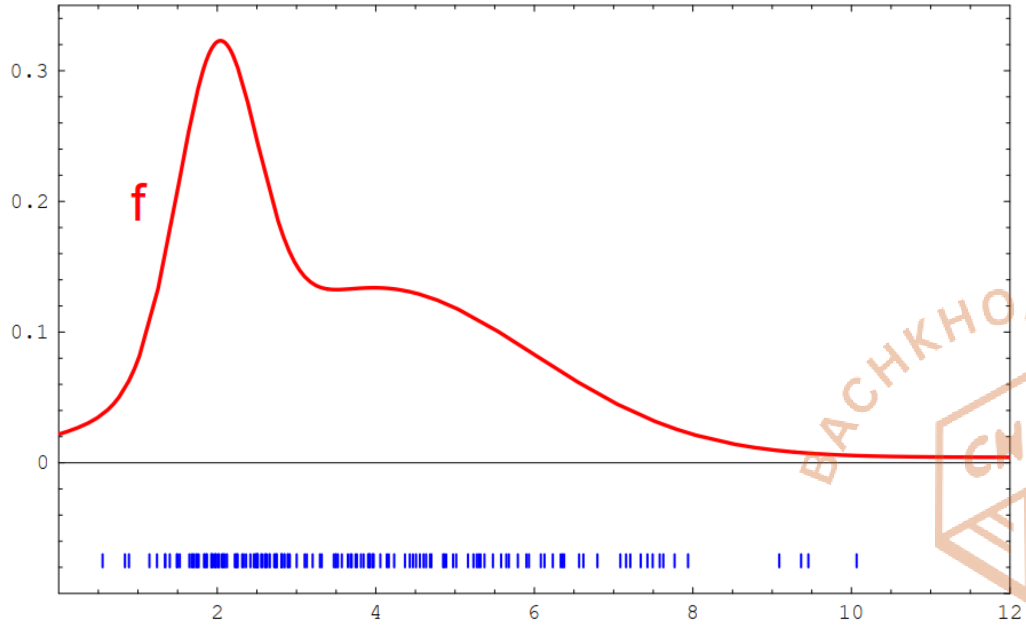
  draw i with probability $\propto w_t^{[i]}$

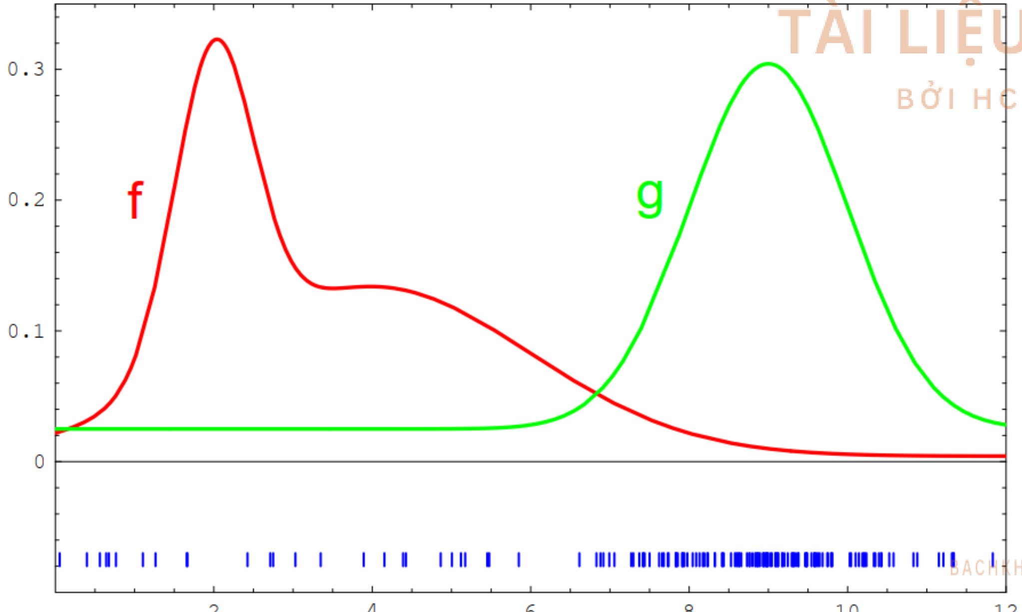
  add $x_t^{[i]}$ to $\mathcal{X}_t$

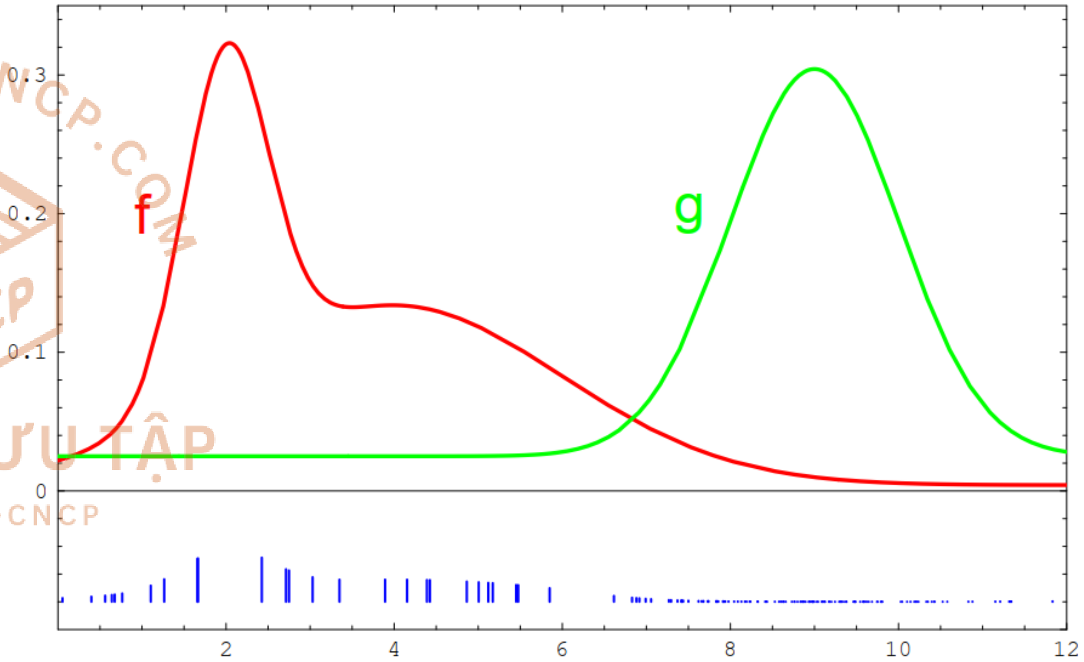
endfor

return $\mathcal{X}_t$

Particle filter







Particle filter

$$
\begin{aligned}
p(x_{0:t} \mid z_{1:t}, u_{1:t}) & \\
\overset{\text{Bayes}}{=} \quad & \eta\, p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t})\, p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
\overset{\text{Markov}}{=} \quad & \eta\, p(z_t \mid x_t)\, p(x_{0:t} \mid z_{1:t-1}, u_{1:t}) \\
= \quad & \eta\, p(z_t \mid x_t)\, p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t})\, p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t}) \\
\overset{\text{Markov}}{=} \quad & \eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_t)\, p(x_{0:t-1} \mid z_{1:t-1}, u_{1:t-1})
\end{aligned}
$$

$$
p(x \mid y, z) = \frac{p(y \mid x, z)\, p(x \mid z)}{p(y \mid z)}
$$

$$
\begin{aligned}
p(a, b|c) &= p(a|b, c)p(b|c) \\
&= p(a|c)p(b|c).
\end{aligned}
$$