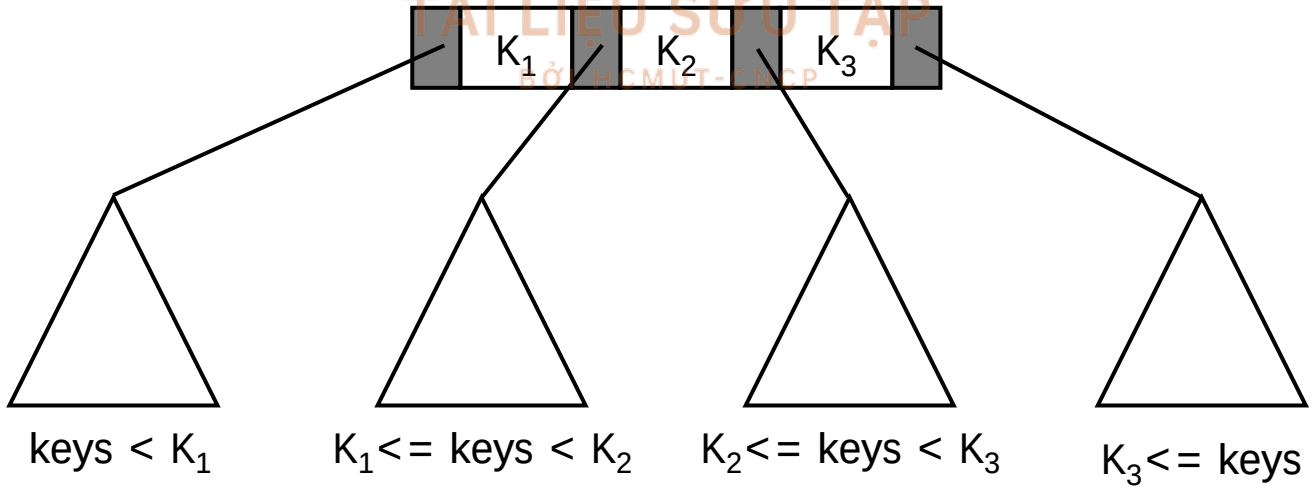
Chapter 6: Multiway Trees

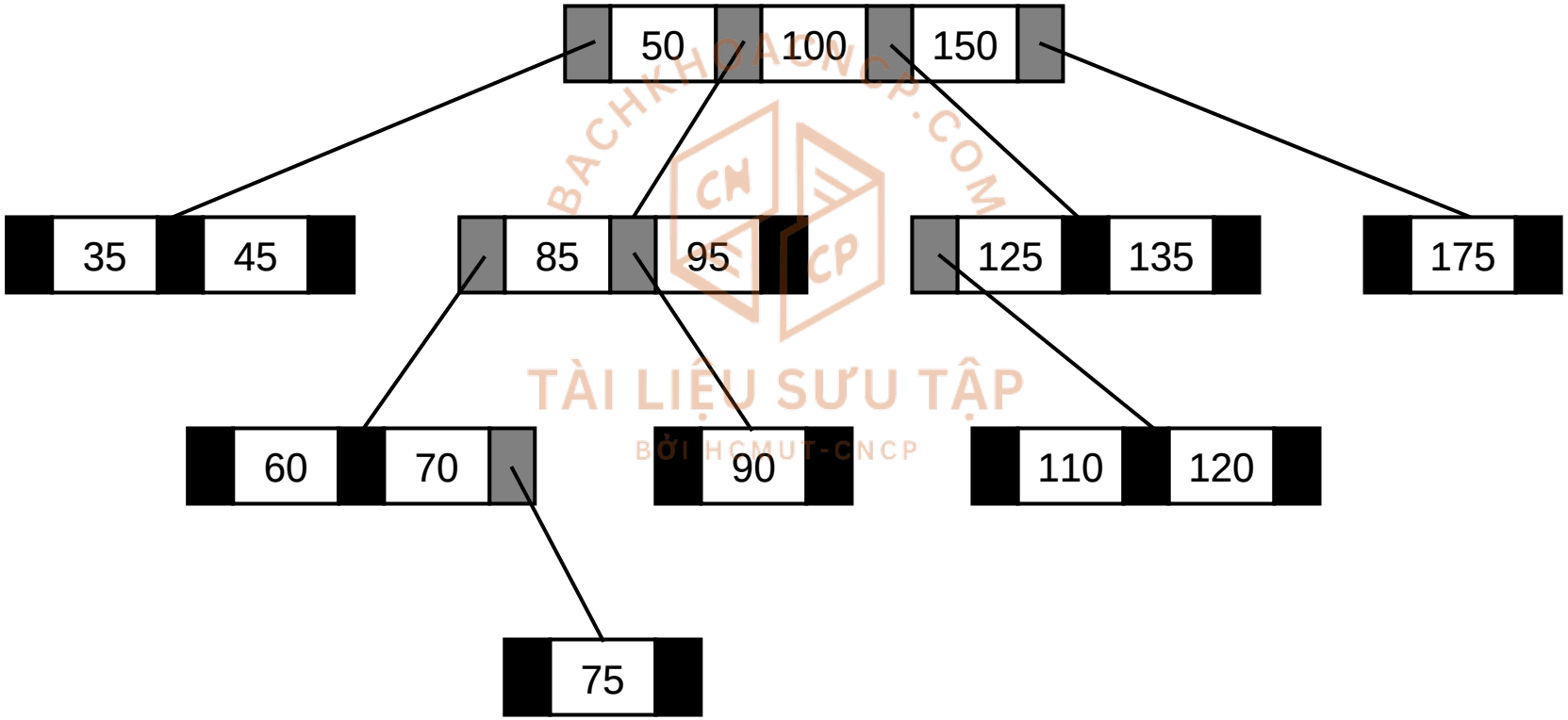
- Tree whose outdegree is not restricted to 2 while retaining the general properties of binary search trees.

M-Way Search Trees

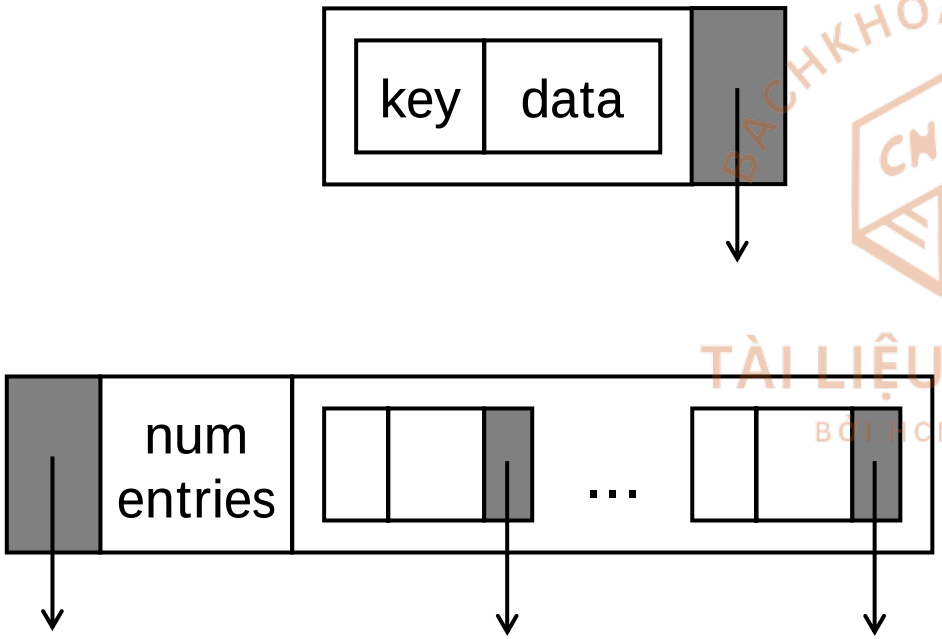
- Each node has m - 1 data entries and m subtree pointers.
- The key values in a subtree such that:
  - >= the key of the left data entry
  - < the key of the right data entry.



M-Way Search Trees



M-Way Node Structure




```
entry
        key <key type>
        data <data type>
        rightPtr <pointer>
end entry

node
        firstPtr <pointer>
        numEntries <integer>
        entries <array[1 .. m-1] of entry>
end node
```

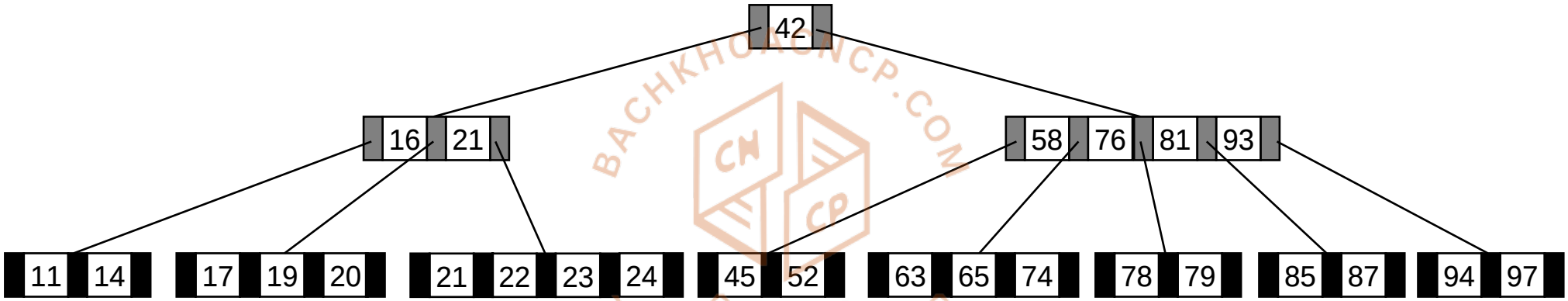
B-Trees

- M-way trees are unbalanced.
- Bayer, R. & McCreight, E. (1970) created B-Trees.

B-Trees

- A B-tree is an m-way tree with the following additional properties (m >= 3):
  - The root is either a leaf or has at least 2 subtrees.
  - All other nodes have at least $\lceil m/2 \rceil - 1$ entries.
  - All leaf nodes are at the same level.

B-Trees



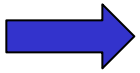
m = 5

B-Tree Insertion

- Insert the new entry into a leaf node.
- If the leaf node is overflow, then split it and insert its median entry into its parent.

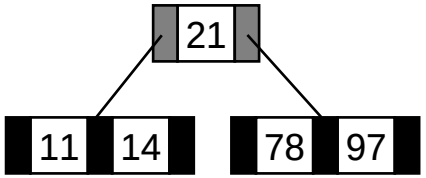
B-Tree Insertion

Insert 78, 21, 14, 11

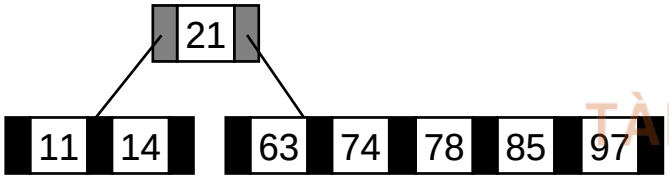


Insert 97

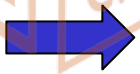
overflow

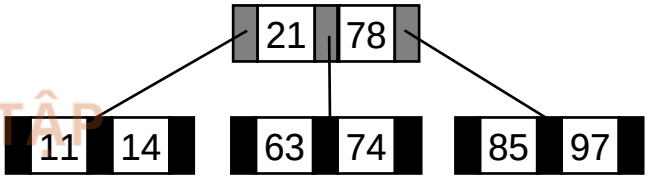


Insert 85, 74, 63

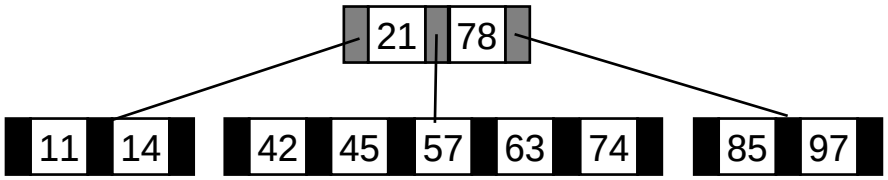


overflow

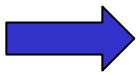


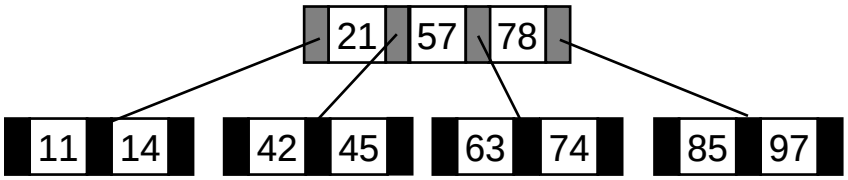


Insert 45, 42, 57



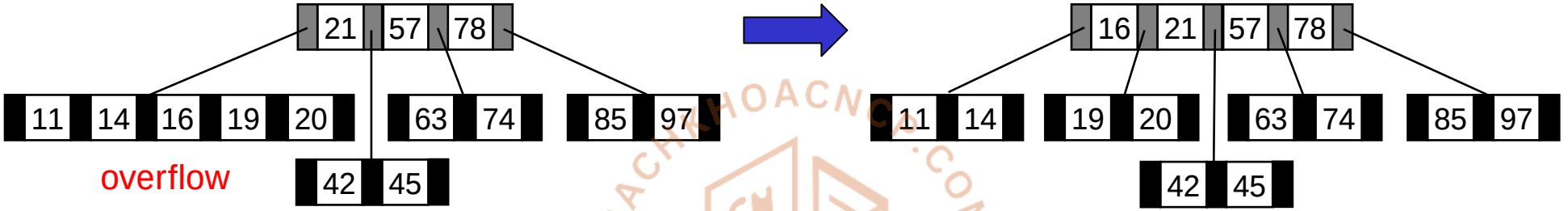
overflow



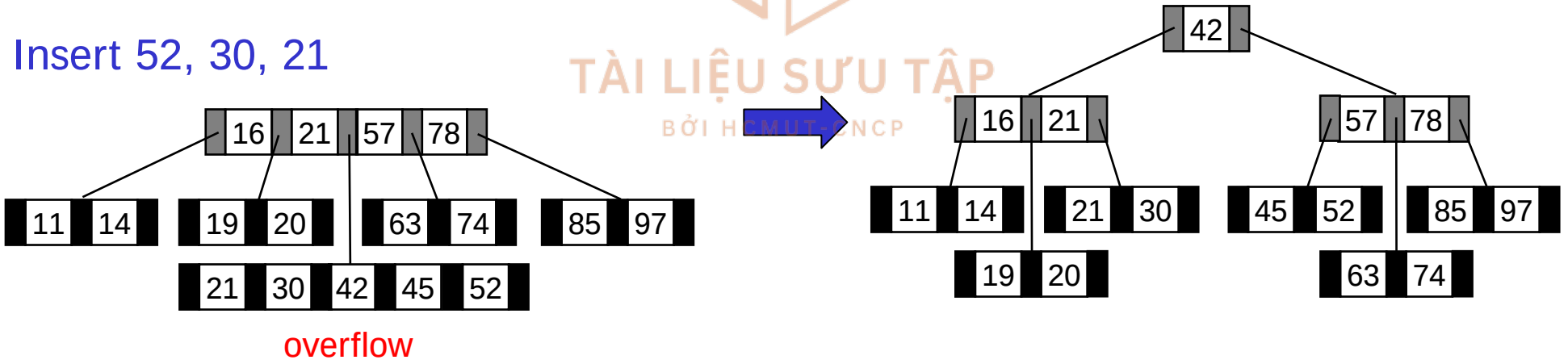


B-Tree Insertion

Insert 20, 16, 19



Insert 52, 30, 21



B-Tree Insertion

Algorithm BTreeInsert (val root <pointer>, val data <record>)

Inserts data into B-tree. Equal keys placed on right branch.

Pre root is a pointer to the B-tree. May be null.
Post data inserted.
Return pointer to B-tree root.

```
1   taller = insertNode(root, data, upEntry)
2   if (taller true)
        Tree has grown. Create new root.
    1   allocate (newPtr)
    2   newPtr –> entries[1] = upEntry
    3   newPtr –> firstPtr = root
    4   newPtr –> numEntries = 1
    5   root = newPtr
3   return root
```

End BTreeInsert

B-Tree Insertion

Algorithm insertNode (val root <pointer>, val data <record>,
ref upEntry <entry>)

Recursively searches tree to locate leaf for data. If node overflow, inserts median key's data into parent.

Pre root is a pointer to tree or subtree. May be null.
Post data inserted.
upEntry is overflow entry to be inserted into parent.
Return tree taller <boolean>.

```
1  if (root null)
   1  upEntry.data = data
   2  upEntry.rightPtr = null
   3  taller = true
2  else
```

B-Tree Insertion

```
2   else
    1   entryNdx = searchNode (root, data.key)
    2   if (entryNdx > 0)
        1   subTree = root –> entries[entryNdx].rightPtr
    3   else
        1   subTree = root –> firstPtr
    4   taller = insertNode(subTree, data, upEntry)
    5   if (taller)
        1   if (node full)
            1   splitNode (root, entryNdx, upEntry)
            2   taller = true
        2   else
            1   insertEntry (root, entryNdx, upEntry)
            2   taller = false
            3   root –> numEntries = root –> numEntries + 1
3   return taller

End     insertNode
```

B-Tree Insertion

Algorithm searchNode (val nodePtr <pointer>, val target <key>)

Search B-tree node for data entry containing key <= target.

Pre nodePtr is pointer to non-null node.
target is key to be located.
Return index to entry with key <= target.
0 if key < first entry in node

```
1  if (target < nodePtr –> entry[1].data.key)
   1  walker = 0
2  else
   1  walker = nodePtr –> numEntries
   2  loop (target < nodePtr –> entries[walker].data.key)
      1  walker = walker - 1
3  return walker
```

End searchNode

B-Tree Insertion

Algorithm splitNode (val node <pointer>, val entryNdx <index>,
ref upEntry <entry>)

Node has overflowed. Split node. No duplicate keys allowed.

Pre node is pointer to node that overflowed.
entryNdx contains index location of parent.
upEntry contains entry being inserted into split node.

Post upEntry now contains entry to be inserted into parent.

```
1  minEntries = minimum number of entries
2  allocate (rightPtr)
   Build right subtree node
3  if (entryNdx <= minEntries)
   1   fromNdx = minEntries + 1
4  else
```

B-Tree Insertion

```
4   else
    1   fromNdx = minEntries + 2
5   toNdx = 1
6   rightPtr -> numEntries = node -> numEntries – fromNdx + 1
7   loop (fromNdx <= node -> numEntries)
    1   rightPtr -> entries[toNdx] = node -> entries[fromNdx]
    2   fromNdx = fromNdx + 1
    3   toNdx = toNdx + 1
8   node -> numEntries = node -> numEntries  – rightPtr -> numEntries
9   if (entryNdx <= minEntries)
    1   insertEntry (node, entryNdx, upEntry)
10  else
```

B-Tree Insertion

```
11  else
        1   insertEntry (rightPtr, entryNdx – minEntries, upEntry)
        2   node -> numEntries = node -> numEntries – 1
        3   rightPtr -> numEntries = rightPtr -> numEntries + 1
        Build entry for parent
12  medianNdx = minEntries + 1
13  upEntry.data = node -> entries[medianNdx].data
14  upEntry.rightPtr = rightPtr
15  rightPtr -> firstPtr = node -> entries[medianNdx]. rightPtr
16  return
End     splitNode
```

B-Tree Insertion

Algorithm insertEntry (val node <pointer>, val entryNdx <index>,
val newEntry <entry>)

Inserts one entry into a node by shifting nodes to make room.

Pre node is pointer to node to contain data.
newEntry contains data to be inserted.
entryNdx is index to location for new data.

Post data have been inserted in sequence.

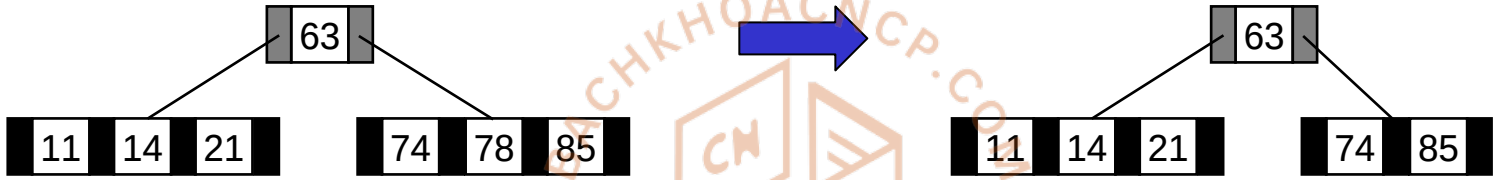
```
1  shifter = node -> numEntries + 1
2  loop (shifter > entryNdx + 1)
   1  node -> entries[shifter] = node -> entries[shifter - 1]
   2  shifter = shifter - 1
3  node -> entries[shifter] = newEntry
4  node -> numEntries = node -> numEntries + 1
5  return
End  insertEntry
```

B-Tree Deletion

- It must take place at a leaf node.
- If the data to be deleted are not in a leaf node, then replace that entry by the largest entry on its left subtree.

B-Tree Deletion

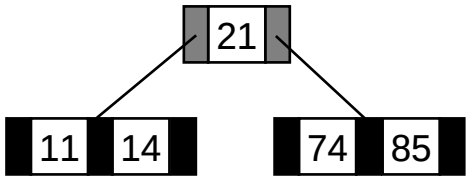
Delete 78

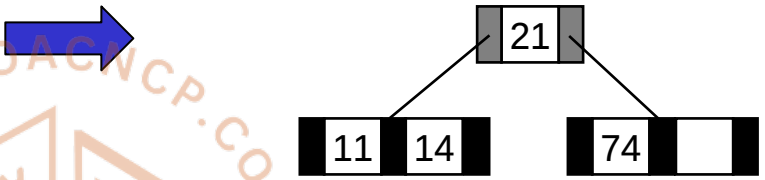


Delete 63

B-Tree Deletion

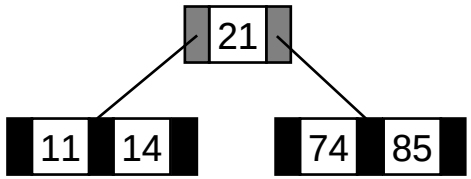
Delete 85

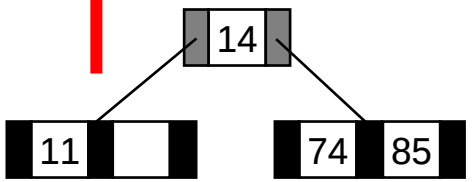




underflow
(node has fewer than the min num of entries)

Delete 21





Reflow

- For each node to have sufficient number of entries:
  - Balance: shift data among nodes.
  - Combine: join data from nodes.

Balance

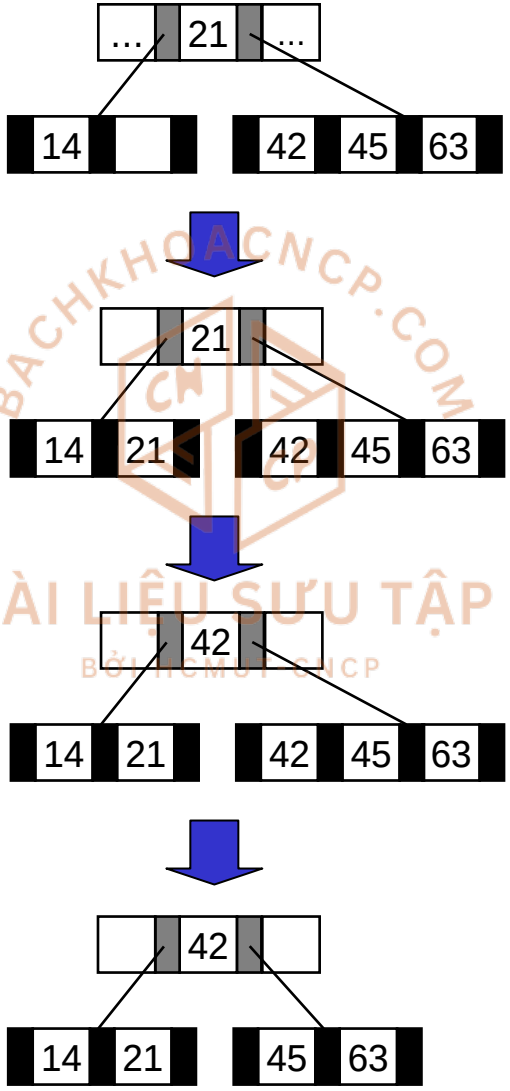
Borrow from right

Original node

Rotate parent data down

Rotate data to parent

Shift entries left



Balance

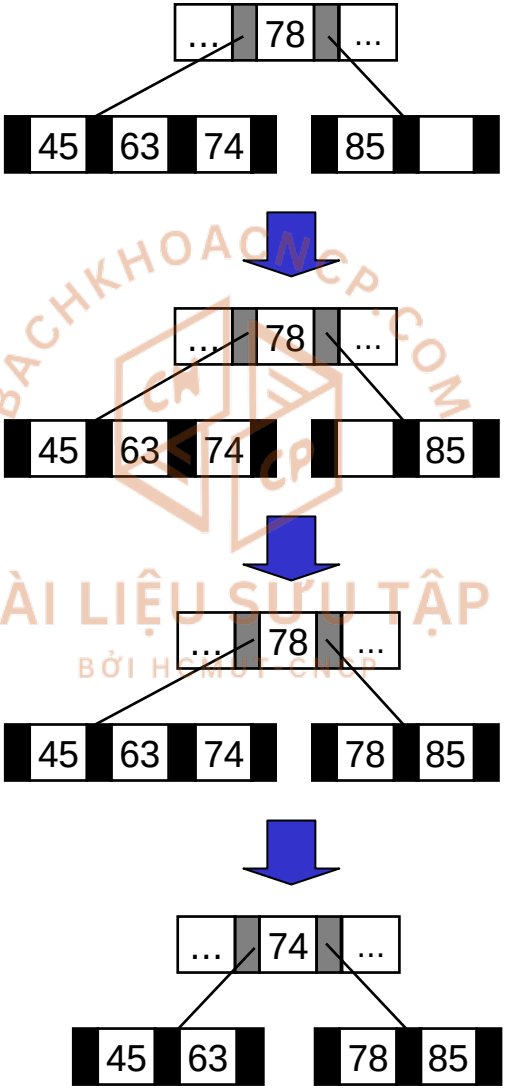
Borrow from left

Original node

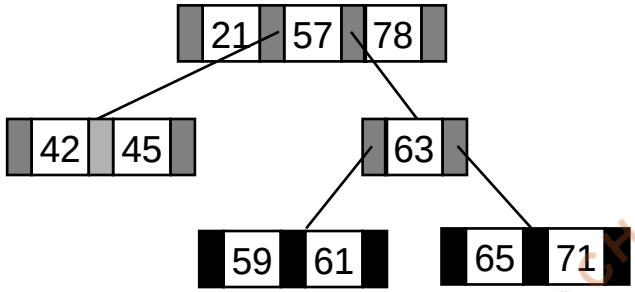
Shift entries right

Rotate parent data down

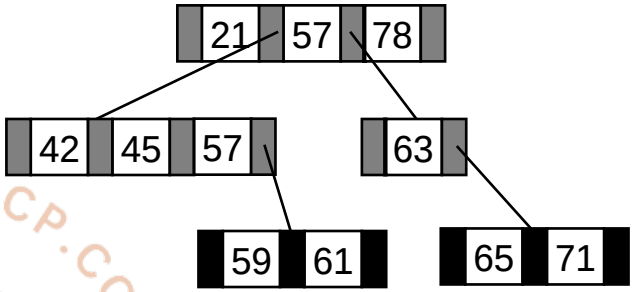
Rotate data up



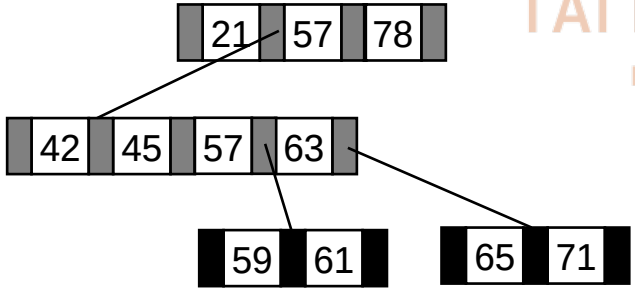
Combine



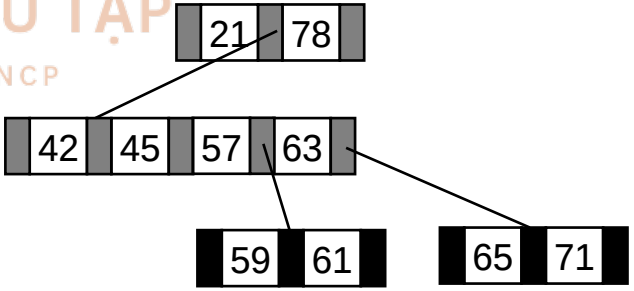
1. After underflow



2. After moving root to subtree

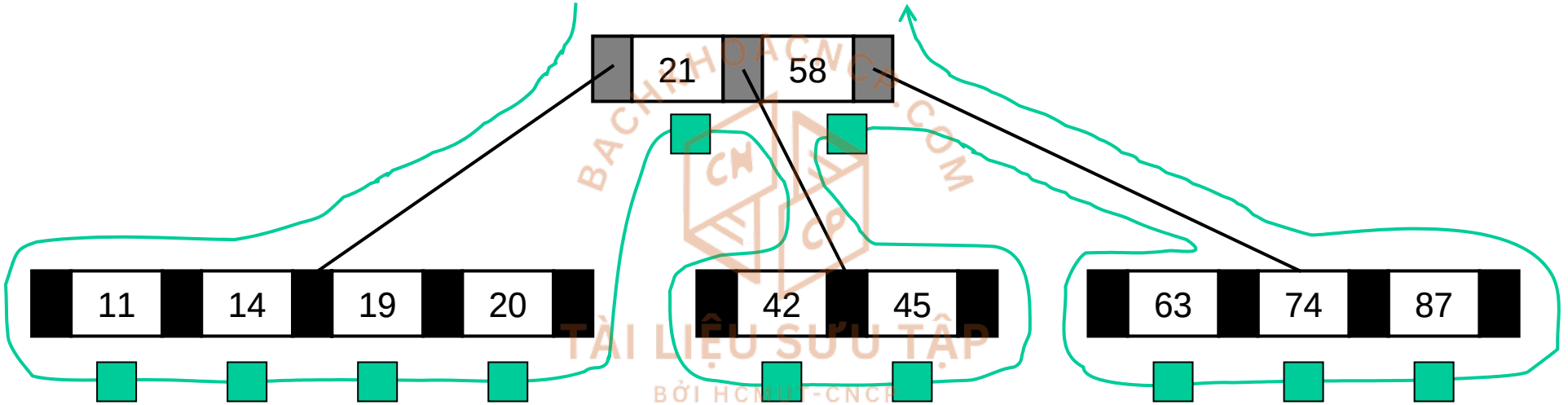


3. After moving right entries



4. After shifting root

B-Tree Traversal



B-Tree Traversal

Algorithm BTreeTraversal (val root <pointer>)

Processes tree using inorder traversal

Pre root is a pointer to B-tree
Post Every entry has been processed in order

```
1  scanCount = 0
2  ptr = root –> firstPtr
3  loop (scanCount <= root –> numEntries)
   1  if (ptr not null)
      1  BTreeTraversal (ptr)
   2  scanCount = scanCount + 1
   3  if (scanCount <= root –> numEntries)
      1  process (root –> entries[scanCount].data)
      2  ptr = root –> entries[scanCount].rightPtr
4  return
End BTreeTraversal
```

B-Tree Search

Algorithm BTreeSearch (val root <pointer>, val target <key>,
ref node <pointer>, ref entryNo <index>)

Recursively searches a B-tree for the target key

Pre root is a pointer to a tree or subtree
target is the data to be located

Post if found --
node is pointer to located node
entryNo is entry within node
if not found --
node is null and entryNo is zero

Return found <boolean>

B-Tree Search

```
1   if (empty tree)
    1   node = null
    2   entryNo = 0
    3   found = false
2   else
    1   if (target < first entry)
        1   return BTreeSearch (root –> firstPtr, target, node, entryNo)
    2   else
        1   entryNo = root –> numEntries
        2   loop (target < root –> entries[entryNo].data.key)
            1   entryNo = entryNo - 1
        3   if (target = root –> entries[entryNo].data.key)
            1   found = true
            2   node = root
        4   else
            1   return BTreeSearch (root –> entries[entryNo].rightPtr, target, node, entryNo)
4   return found

End     BTreeTraversal
```

B-Tree Variations

- B*Tree: the minimum number of (used) entries is two thirds.
- B+Tree:
  - Each data entry must be represented at the leaf level.
  - Each leaf node has one additional pointer to move to the next leaf node.

Reading

- Pseudo code of algorithms for B-Tree Insertion