# Chapter 7 - Tree

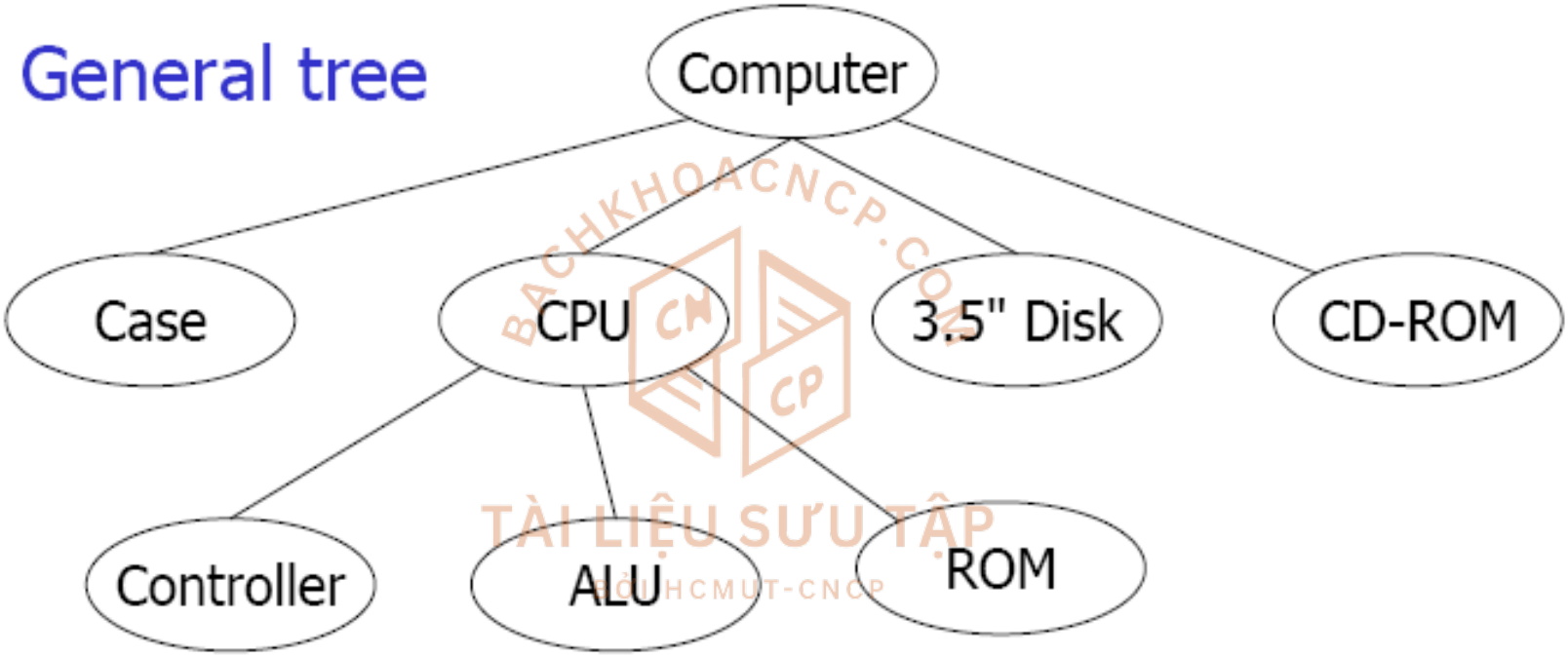
- Basic tree concepts
- Binary trees
- Binary Search Tree (BST)

# Basic Tree Concepts

- A tree consists of:
  - nodes: finite set of elements
  - branches: directed lines connecting the nodes
- For a node:
  - degree: number of branches associated with the node
  - indegree: number of branches towards the node
  - outdegree: number of branches away from the node
- For a tree:
  - root: node with indegree 0
  - nodes different from the root must have indegree 1

# Tree Representation

General tree



# Terminology

- Leaf: node with outdegree 0
- Internal node: not a root or a leaf
- Parent: node with outdegree greater than 0
- Child: node with indegree greater than 0
- Siblings: nodes with the same parent
- Path: sequence of adjacent nodes

# Terminology

- Ancestor: node in the path from the root to the node
- Descendent: node in a path from the node to a leaf
- Level: the node's distance from the root (at level 0)
- Height (Depth): the level of the leaf in the longest path from the root plus 1
- Sub-tree: connected structure below the root

# Tree Representation

## Indented list

Computer
- Case
- CPU
  - Controller
  - ALU
  - ROM
  - ...
- 3.5" Disk
- CD-ROM

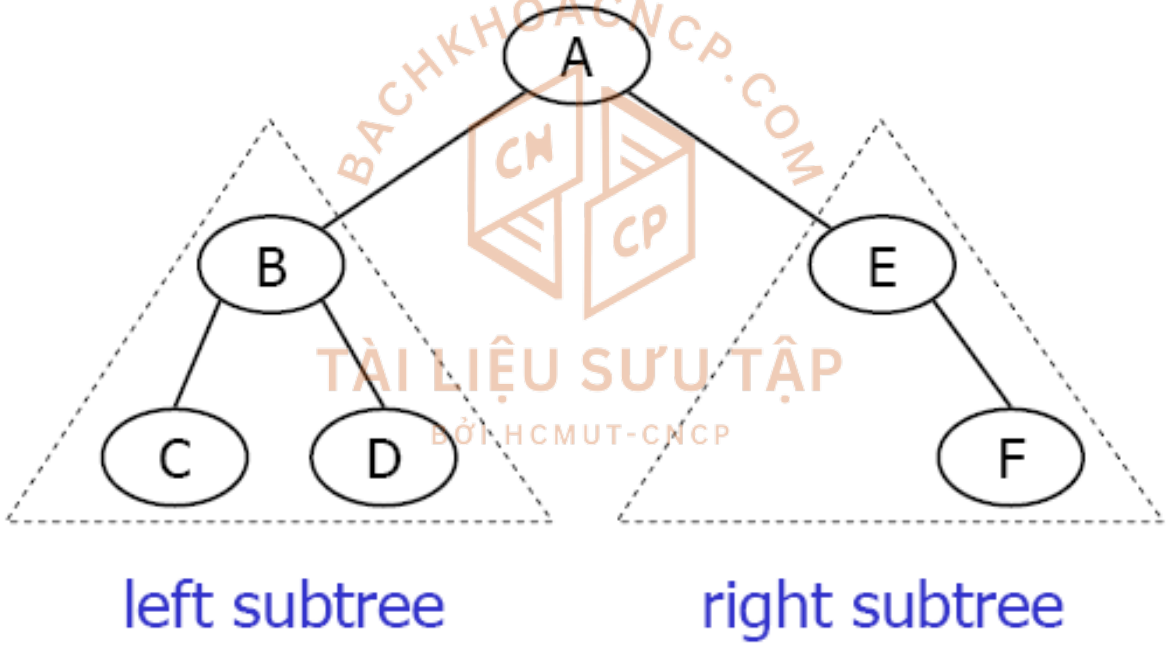


## Parenthetical listing

Computer (Case  CPU (Controller ALU ROM ...)  3.5" Disk  CD-ROM)

# Binary Trees

- A node cannot have more than two sub-trees:



# Binary Tree Properties

- Height of binary trees:

$$H_{max} = N$$

$$H_{min} = \lfloor \log_2 N \rfloor + 1$$

$$N_{min} = H$$

$$N_{max} = 2^H - 1$$

# Binary Tree Properties

- Balance:
  - Balance factor: $B = H_L - H_R$
  - Balanced tree: balance factor is 0, -1, or 1

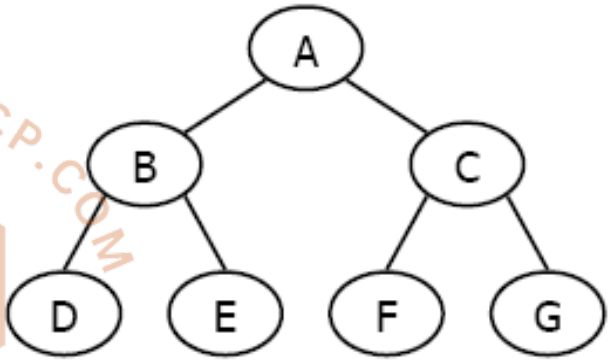
    sub-trees are balanced

# Binary Tree Properties

- Completeness:
  - Complete tree:

    $N = N_{max} = 2^H - 1$

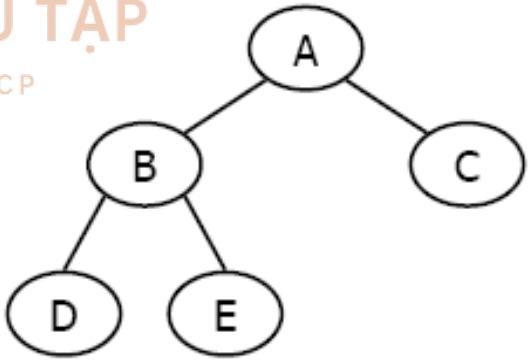
    (last level is full)




  - Nearly complete tree:

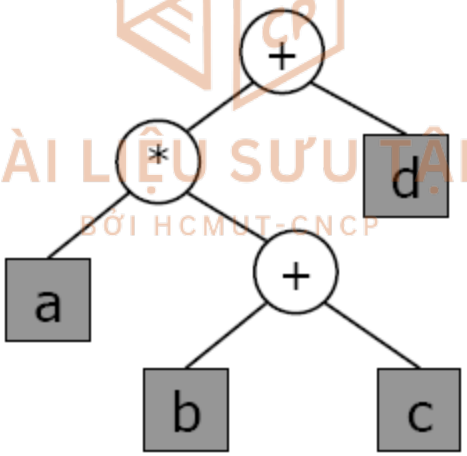
    $H = H_{min} = \lfloor \log_2 N \rfloor + 1$

    nodes in the last level are on the left



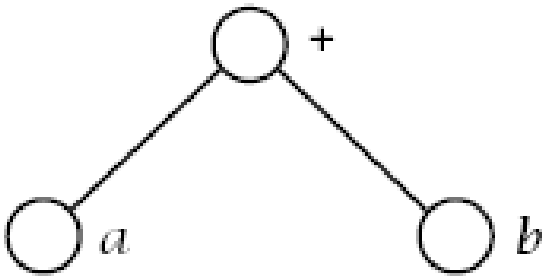
# Expression Trees

- Each leaf is an operand
- The root and internal nodes are operators
- Sub-trees are sub-expressions

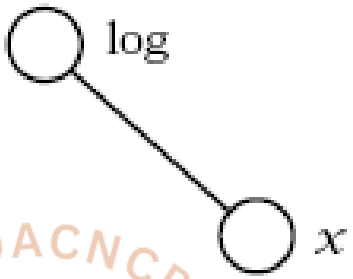



a * (b + c) + d

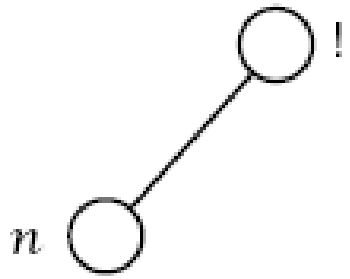
# Expression Trees



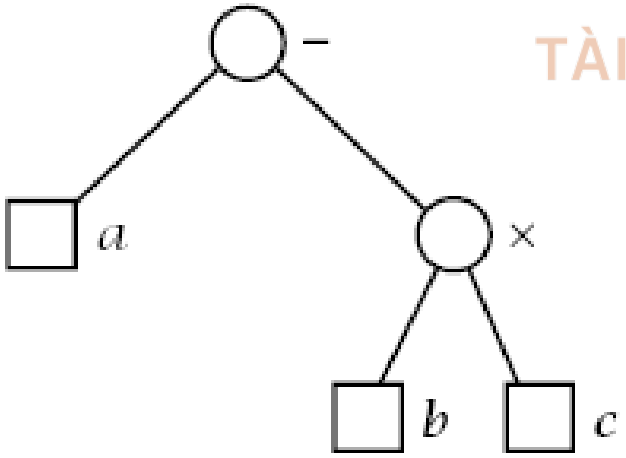

$a + b$



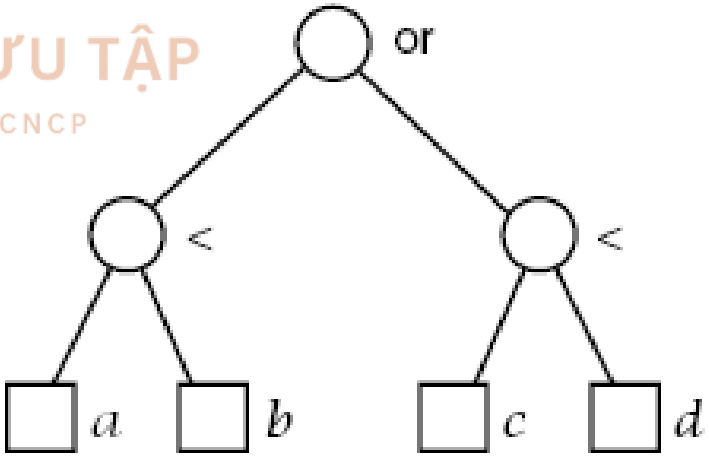

$\log x$




$n!$




$a - (b \times c)$




$(a < b)$ or $(c < d)$

# Binary Tree ADT

**DEFINITION**: A binary tree ADT is either empty, or it consists of a node called root together with two binary trees called the left and the right subtree of the root.

**Basic operations**:

- *Construct* a tree, leaving it empty.
- *Insert* an element.
- *Remove* an element.
- *Search* an element.
- *Retrieve* an element.
- *Traverse* the tree, performing a given operation on each element.

# Binary Tree ADT

**Extended operations**:

- Determine whether the tree is *empty* or not.
- Find the *size* of the tree.
- *Clear* the tree to make it empty.

# Specifications for Binary Tree

<void> Create()

<boolean> isFull()

<boolean> isEmpty()

<integer> Size()

<void> Clear()

<ErrorCode> Search (ref DataOut <DataType>)

<ErrorCode> Insert (val DataIn <DataType>)

<ErrorCode> Remove (val key <KeyType>)

<ErrorCode> Retrieve (ref DataOut <DataType>)

Depend on various types of binary trees (BST, AVL, 2d-tree)

# Specifications for Binary Tree

- ***Binary Tree Traversal***: Each node is processed once and only once in a predetermined sequence.

- ***Depth-First Traverse***:

<void> preOrderTraverse (ref<void>Operation(ref Data <DataType>))

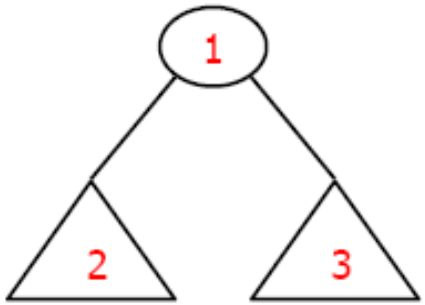
<void> inOrderTraverse (ref<void>Operation(ref Data <DataType>))

<void> postOrderTraverse (ref<void>Operation(ref Data <DataType>))

- ***Breadth-First Traverse***:

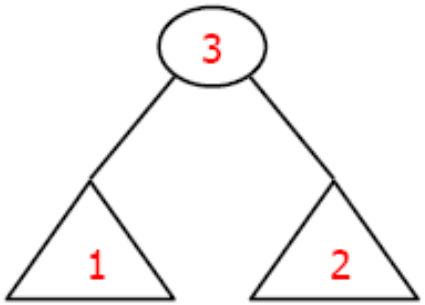
<void> BreadthFirstTraverse (ref<void>Operation(ref Data <DataType>))

# Depth-First Traversal



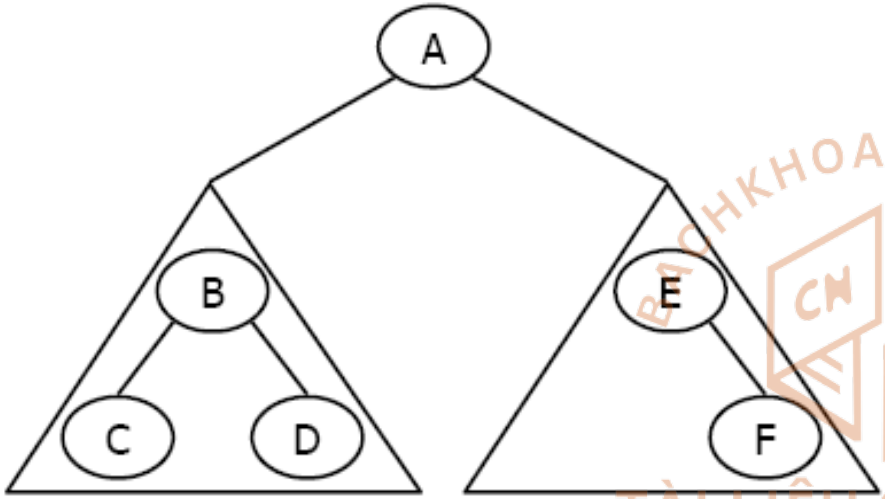

PreOrder
NLR

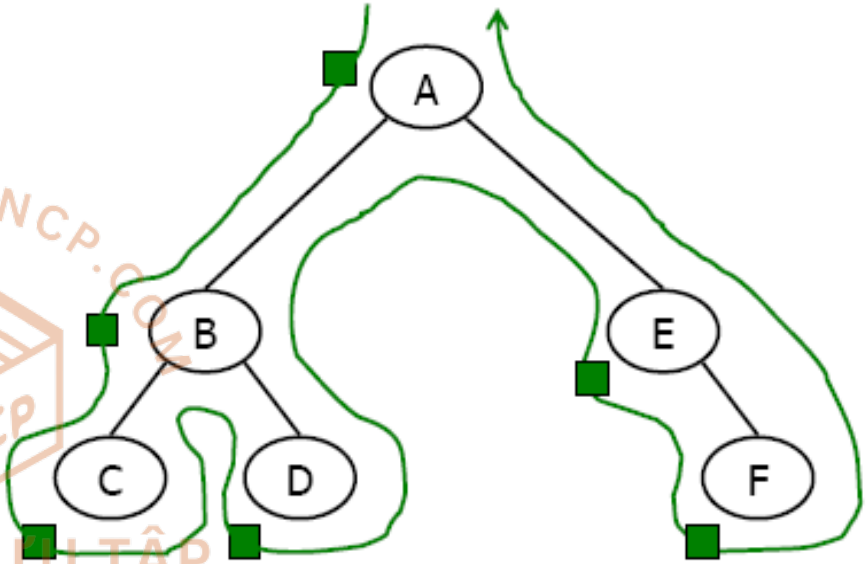

InOrder
LNR



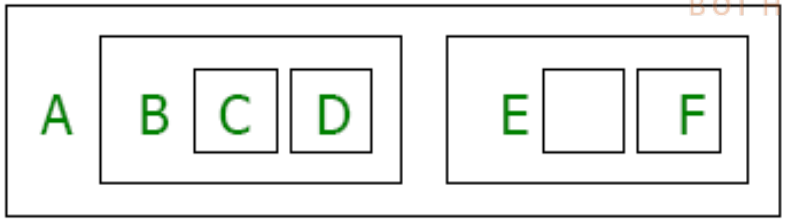

PostOrder
LRN

# PreOrder Traversal



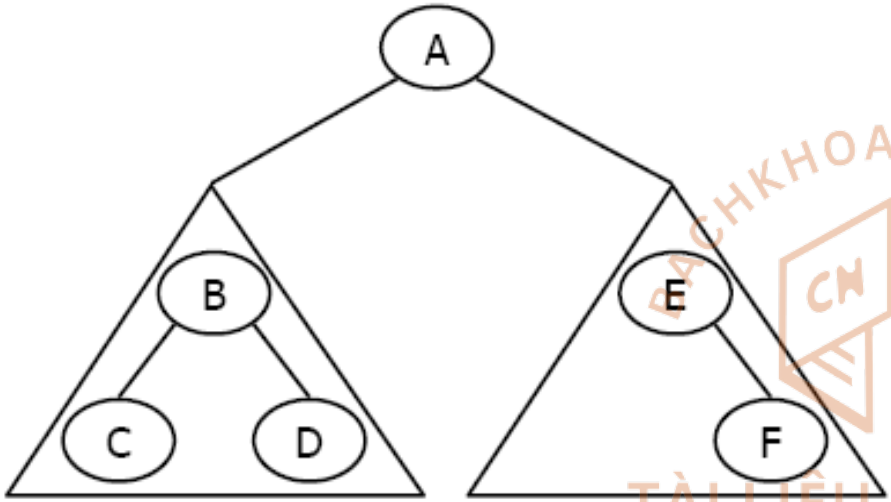



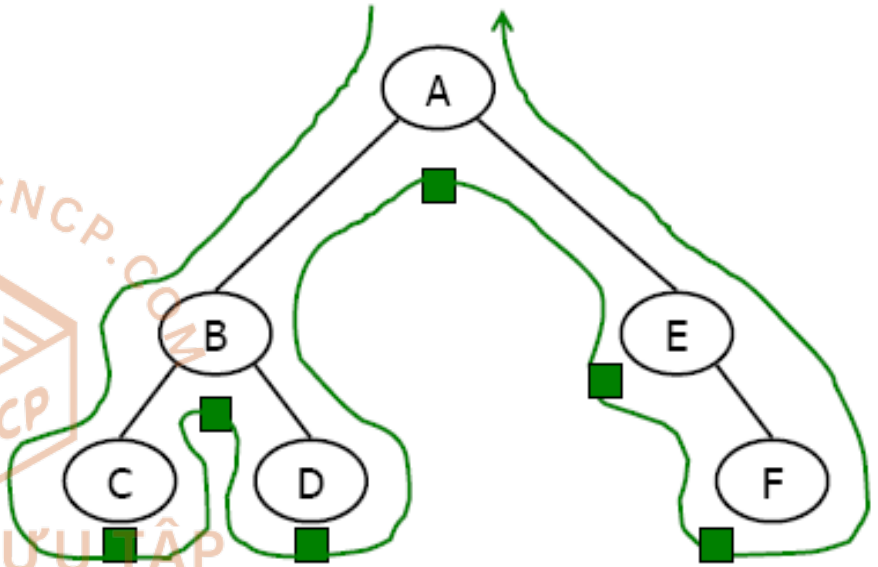

Walking order



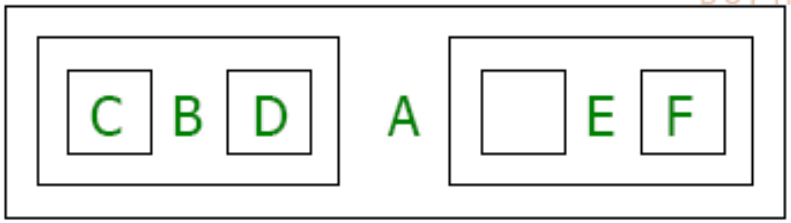

Processing order

# InOrder Traversal



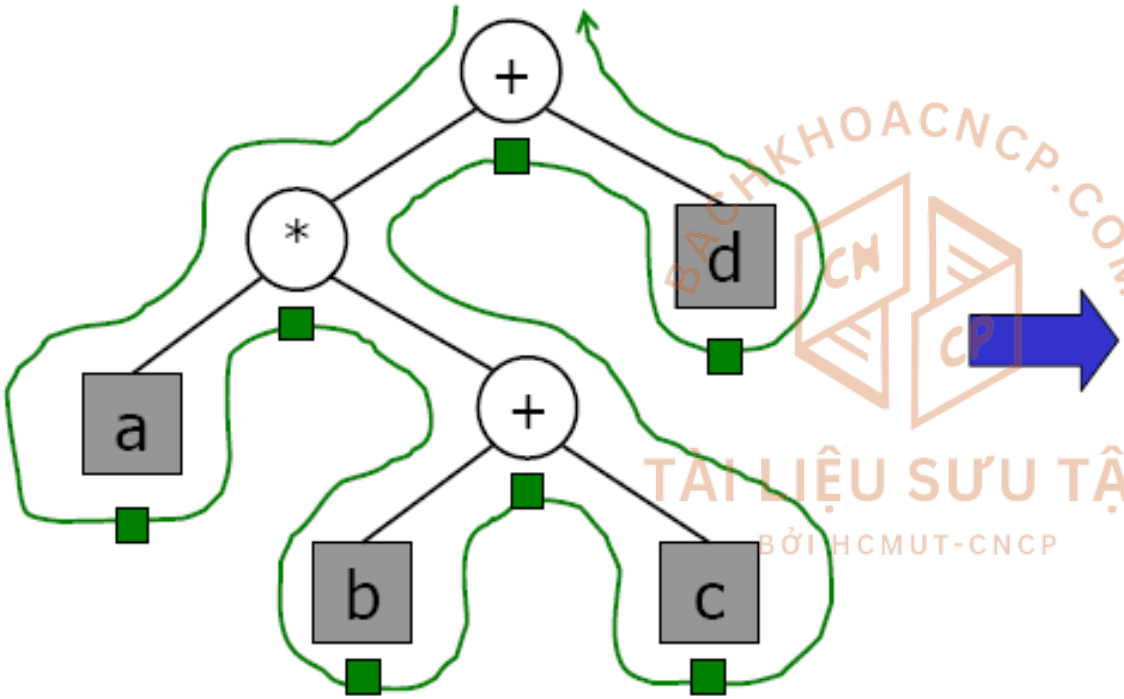




Walking order



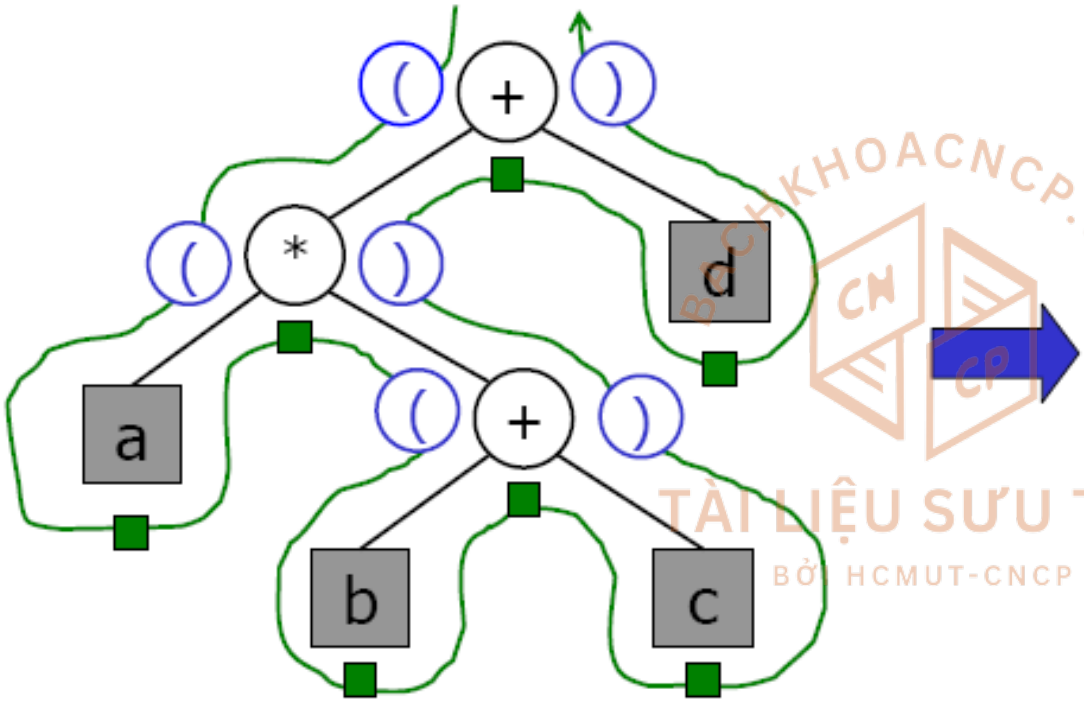

Processing order

# InOrder Traversal



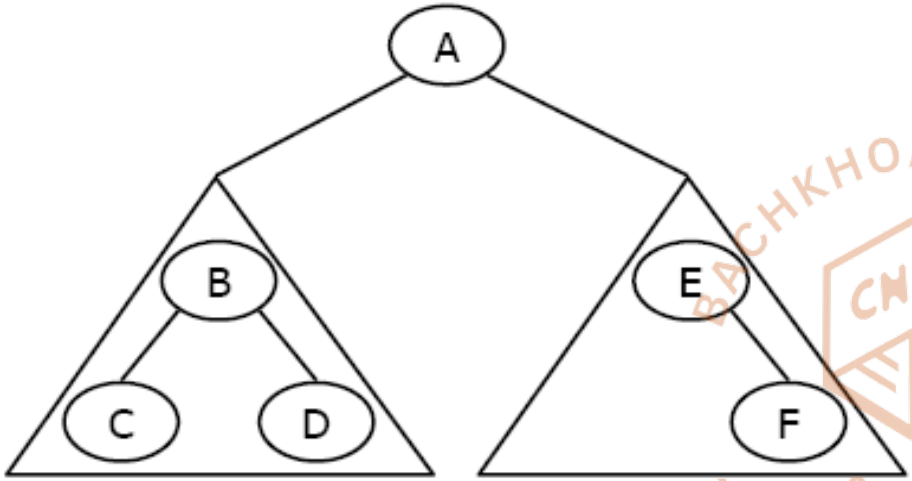

a * b + c + d

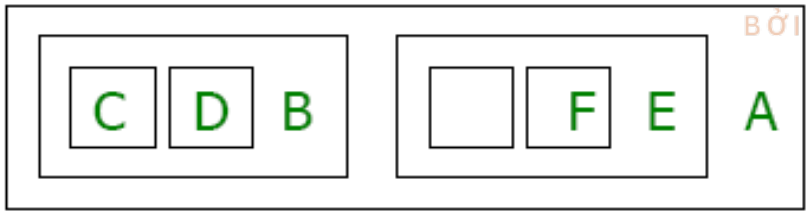
# InOrder Traversal



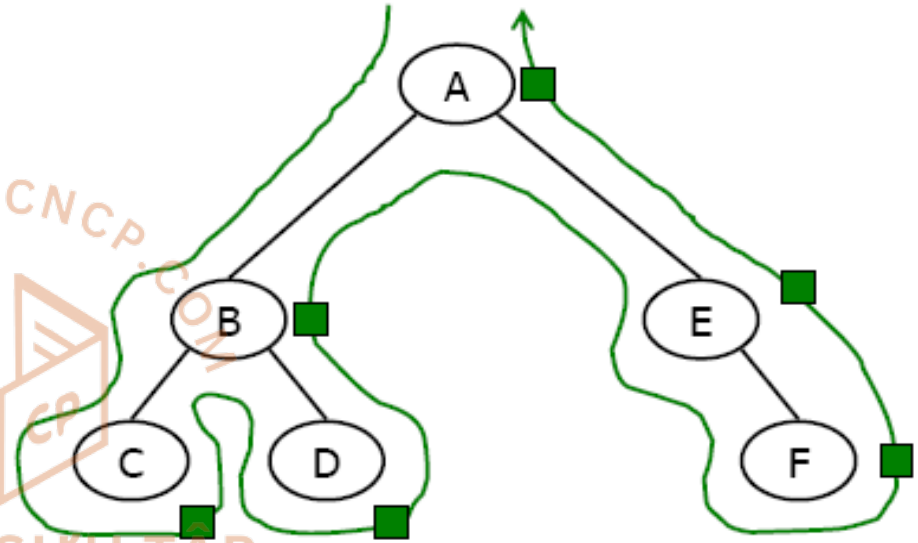

((a * (b + c)) + d)

# PostOrder Traversal




Processing order



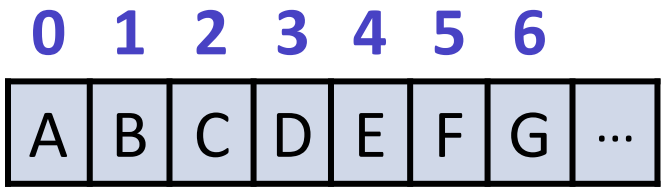

Walking order

# Contiguous Implementation of Binary Tree

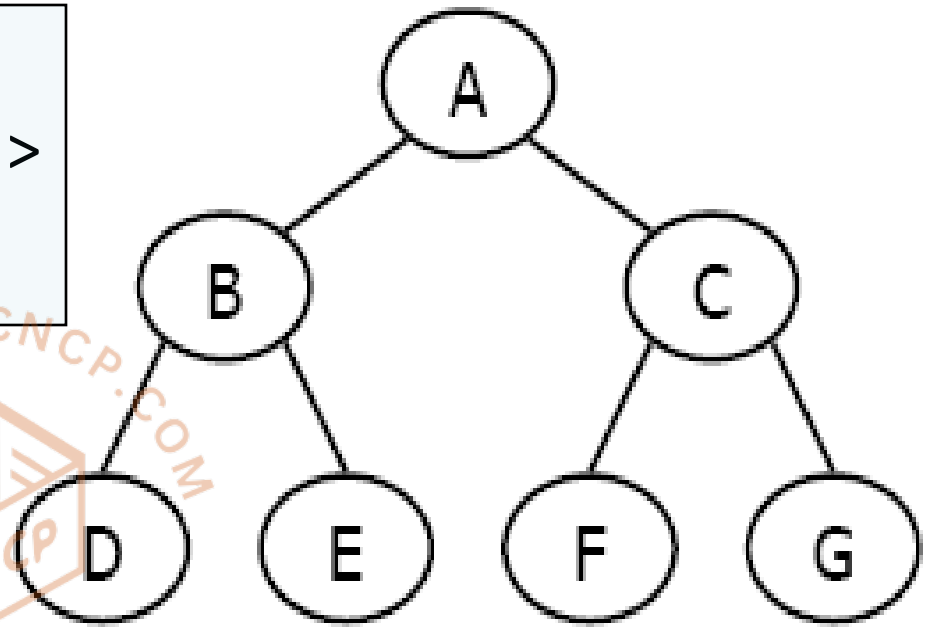
**BinaryTree**

Data <Array of <DataType> >

**End BinaryTree**



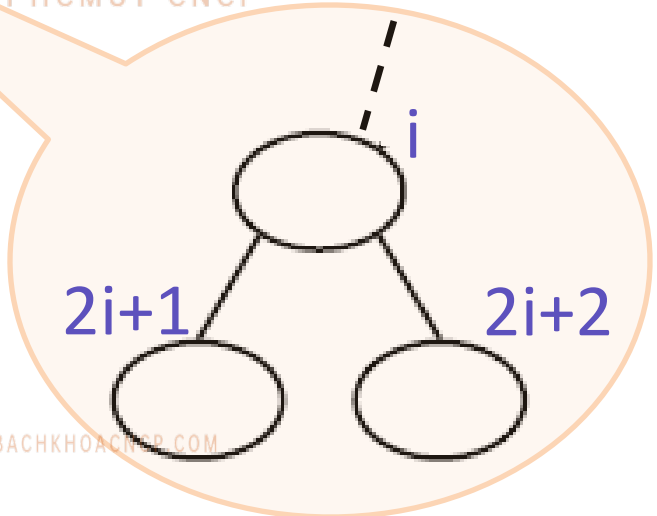

***Physical***




***Conceptual***



*(suitable for complete tree, nearly complete tree, and bushy tree)*



# Contiguous Implementation of Binary Tree

**Record**

Data <DataType>

Parent <DataType>

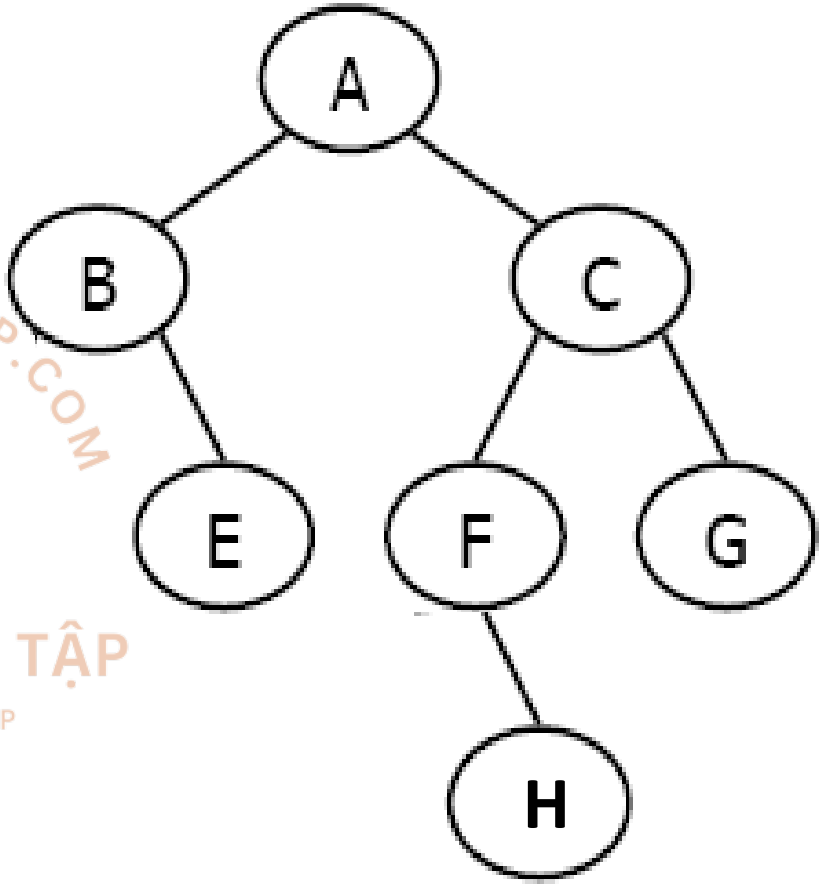
Flag <ChildType>

**End Record**

**BinaryTree**

Data <Array of <Record> >

**End BinaryTree**




***Conceptual***

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|---|
| Data | A | B | E | C | F | G | H | … |
| Parent | - | A | B | A | C | C | F | … |
| Flag | - | L | R | R | L | R | R | … |

***Physical*** (*suitable for sparse tree*)

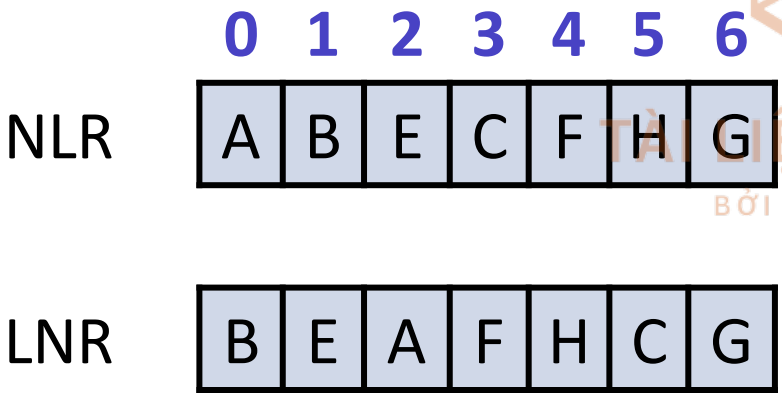
# Contiguous Implementation of Binary Tree

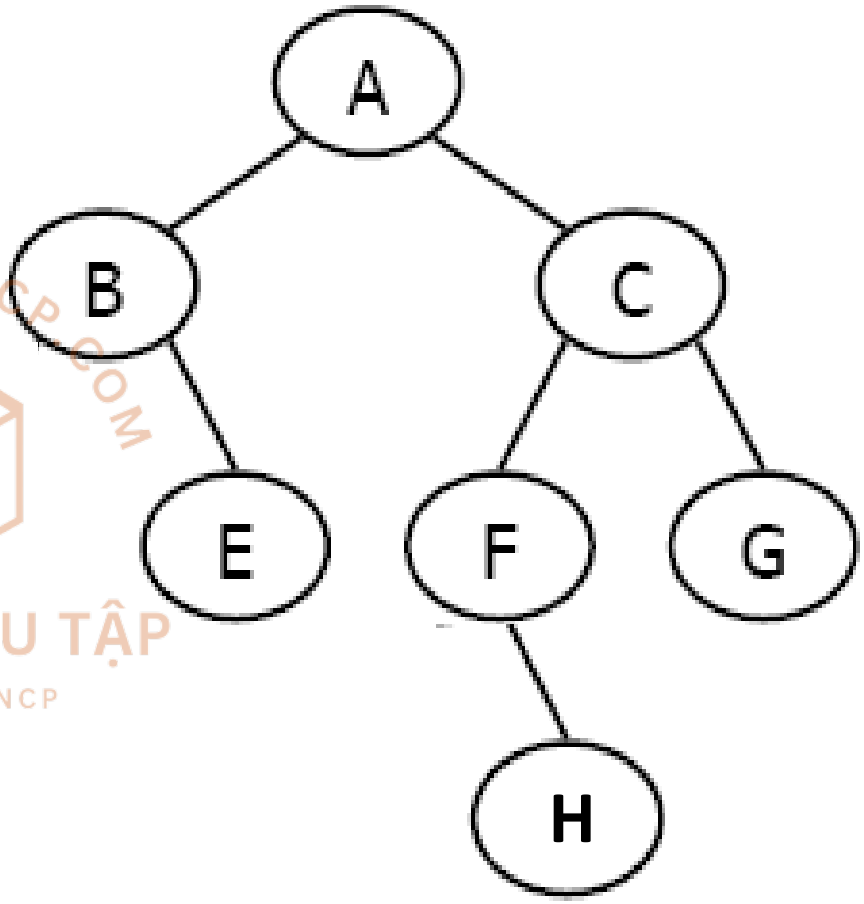
**BinaryTree**

NLR <Array of <DataType> >

LNR <Array of <DataType>>

**End BinaryTree**




***Physical*** ***Conceptual***

(*A binary tree can be restored from two array of LNR and NLR traverse*)

# Linked Implementation of Binary Tree

**BinaryNode**

data <DataType>

left <pointer>

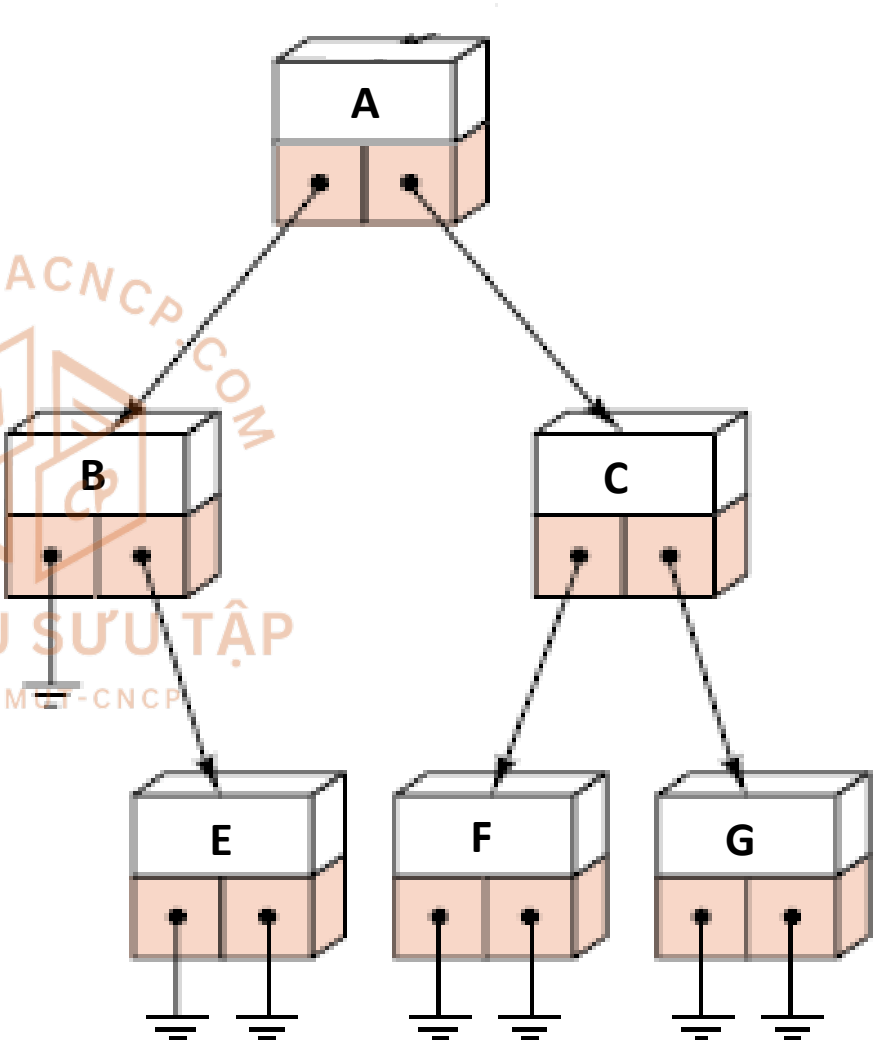
right <pointer>

**End BinaryNode**

**BinaryTree**

root <pointer>

**End BinaryTree**






***Physical***

# Depth-First Traversal

Auxiliary functions for Depth_First Traversal:

recursive_preOrder

recursive_inOrder

recursive_postOrder

# PreOrder Traversal

**Algorithm recursive_preOrder** (val subroot <pointer>,
ref<void>Operation(ref Data <DataType>))

Traverses a binary tree in *node-left-right* sequence.

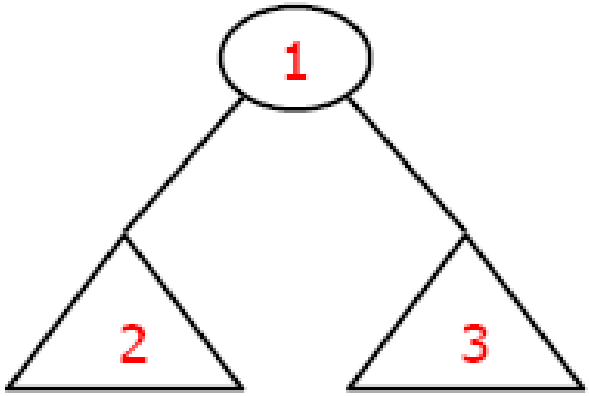
**Pre** subroot points to the root of a tree/ subtree.

**Post** each node has been processed in order.



1. **if** (subroot is not NULL)
    1. Operation(subroot->data)
    2. recursive_preOrder(subroot->left)
    3. recursive_preOrder(subroot->right)

End recursive_preOrder




PreOrder
NLR

# InOrder Traversal

**Algorithm recursive_inOrder** (val subroot <pointer>,
ref<void>Operation(ref Data <DataType>))

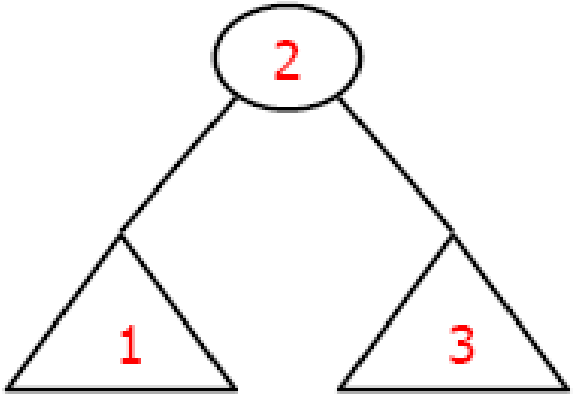
Traverses a binary tree in *left-node-right* sequence

**Pre** subroot points to the root of a tree/ subtree

**Post** each node has been processed in order

1. **if** (subroot is not NULL)
    1. recursive_inOrder(subroot->left)
    2. Operation(subroot->data)
    3. recursive_inOrder(subroot->right)

End recursive_inOrder




InOrder
LNR

# PostOrder Traversal

**Algorithm recursive_postOrder** (val subroot <pointer>,
ref<void>Operation(ref Data <DataType>))

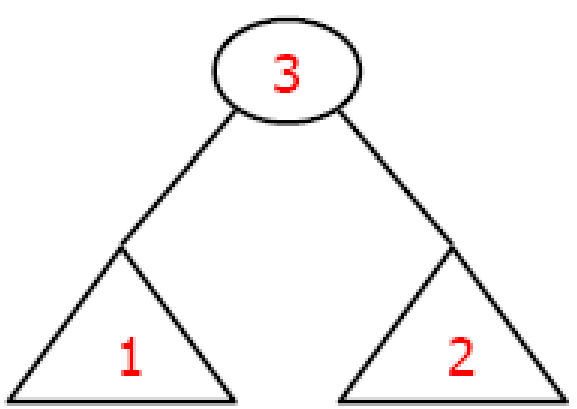
Traverses a binary tree in *left-right-node* sequence

**Pre** subroot points to the root of a tree/ subtree

**Post** each node has been processed in order

1. **if** (subroot is not NULL)
    1. recursive_postOrder(subroot->left)
    2. recursive_postOrder(subroot->right)
    3. Operation(subroot->data)

End recursive_postOrder




PostOrder
LRN

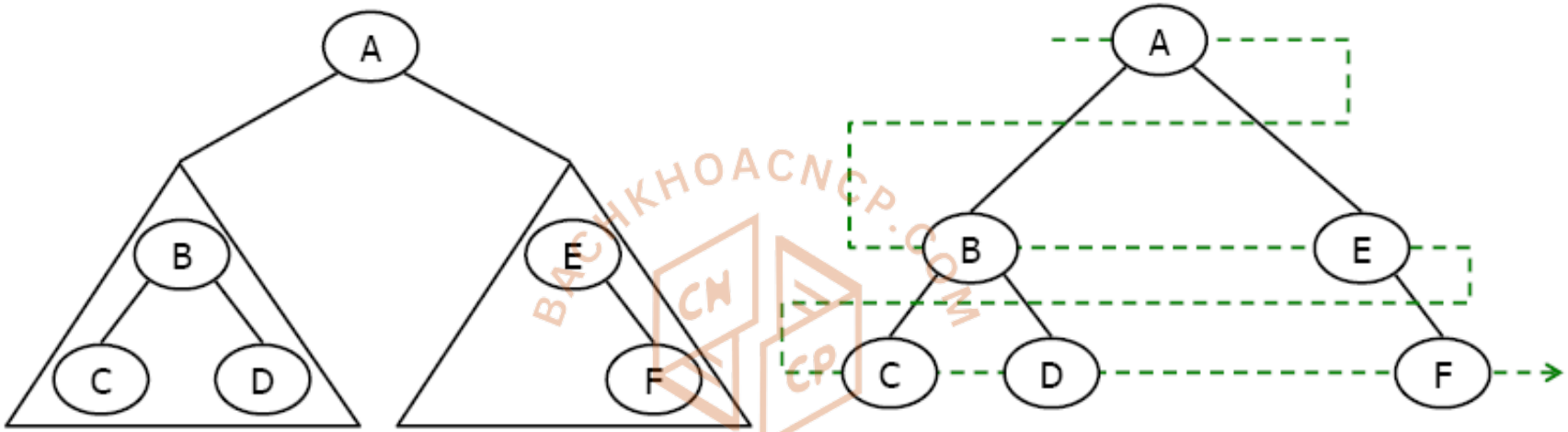
# Depth-First Traversal

```
<void> preOrderTraverse (ref<void>Operation(ref Data <DataType>))
1.      recursive_preOrder(root, Operation)
End preOrderTraverse
```

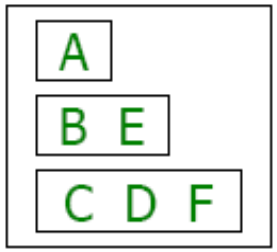
```
<void> inOrderTraverse (ref<void>Operation(ref Data <DataType>))
1.      recursive_inOrder(root, Operation)
End inOrderTraverse
```

```
<void> postOrderTraverse (ref<void>Operation(ref Data <DataType>))
1.      recursive_postOrder(root, Operation)
End postOrderTraverse
```

# Breadth-First Traversal






Processing order

Walking order

# Breadth-First Traversal

**Algorithm BreadthFirstTraverse**

(ref<void>Operation(ref Data <DataType>))

Traverses a binary tree in sequence from lowest level to highest level, in each level traverses from left to right.

**Post** each node has been processed in order

**Uses** Queue ADT

# Breadth-First Traversal

**Algorithm BreadthFirstTraverse**

(ref<void>Operation(ref Data <DataType>))

```
1.  if (root is not NULL)
    1.  queueObj <Queue>
    2.  queueObj.EnQueue(root)
    3.  loop (not queueObj.isEmpty())
        1.  queueObj.QueueFront(pNode)
        2.  queueObj.DeQueue()
        3.  Operation(pNode->data)
        4.  if (pNode->left is not NULL)
            1.  queueObj.EnQueue(pNode->left)
        5.  if (pNode->right is not NULL)
            1.  queueObj.EnQueue(pNode->right)
End BreadthFirstTraverse
```

# Binary Search Tree (BST)

- All items in the left subtree < the root.
- All items in the right subtree >= the root.
- Each subtree is itself a binary search tree.

# Binary Search Tree (BST)

- BST is one of implementations for ordered list.
- In BST we can search quickly (as with binary search on a *contiguous list*).
- In BST we can make insertions and deletions quickly (as with a *linked list*).
- When a BST is traversed in *inorder*, the keys will come out in sorted order.

# Binary Search Tree (BST)

Auxiliary functions for Search:

recursive_Search

iterative_Search

# Search node in BST (recursive version)

<pointer> **recursive_Search** (val subroot <pointer>,
val target <KeyType>)

Searches target in the subtree.

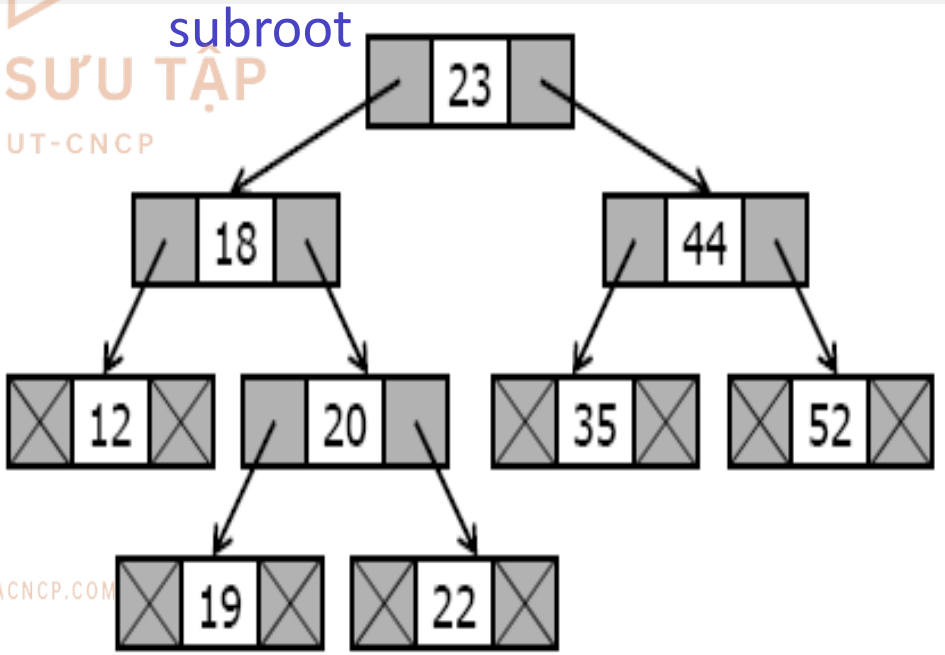
**Pre** subroot points to the root of a tree/ subtree.

**Post** If target is not in the subtree, NULL is returned. Otherwise, a pointer to the node containing the target is returned.

# Search node in BST (recursive version)

1. **if** (subroot is NULL) OR (subroot->data = target)
    1. return subroot
2. **else if** (target < subroot->data)
    1. return recursive_Search(subroot->left, target)
3. **else**
    1. return recursive_Search(subroot->right, target)

End recursive_Search



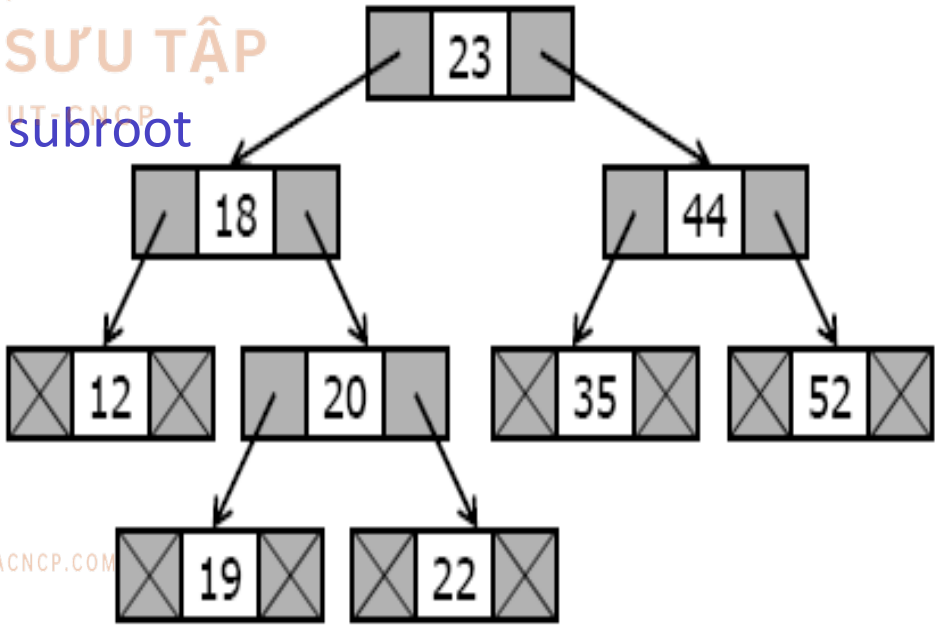

Target = 22

# Search node in BST (recursive version)

1. **if** (subroot is NULL) OR (subroot->data = target)
    1. return subroot
2. **else if** (target < subroot->data)
    1. return recursive_Search(subroot->left, target)
3. **else**
    1. return recursive_Search(subroot->right, target)

End recursive_Search





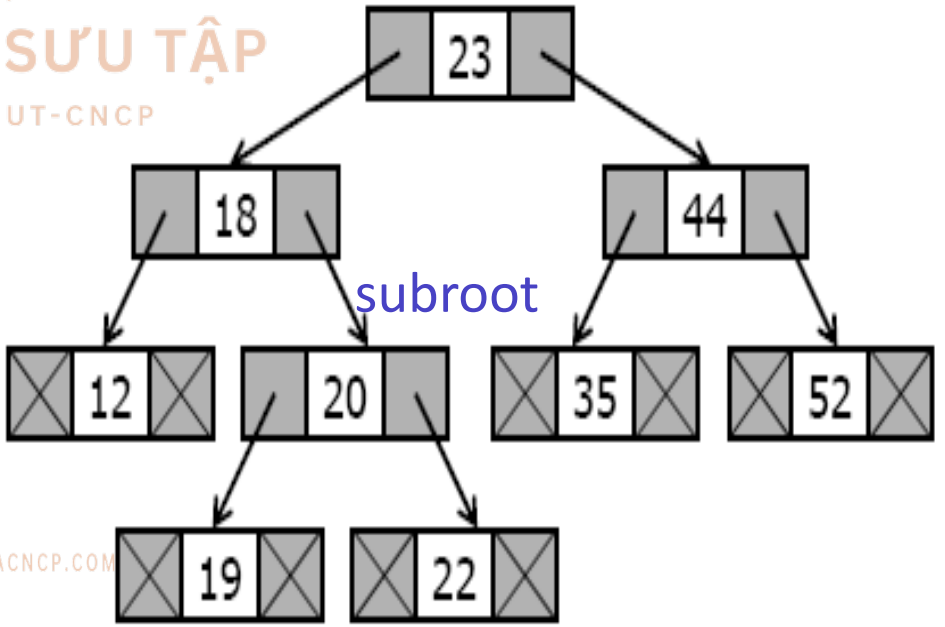

Target = 22

# Search node in BST (recursive version)

1. **if** (subroot is NULL) OR (subroot->data = target)
    1. return subroot
2. **else if** (target < subroot->data)
    1. return recursive_Search(subroot->left, target)
3. **else**
    1. return recursive_Search(subroot->right, target)

End recursive_Search





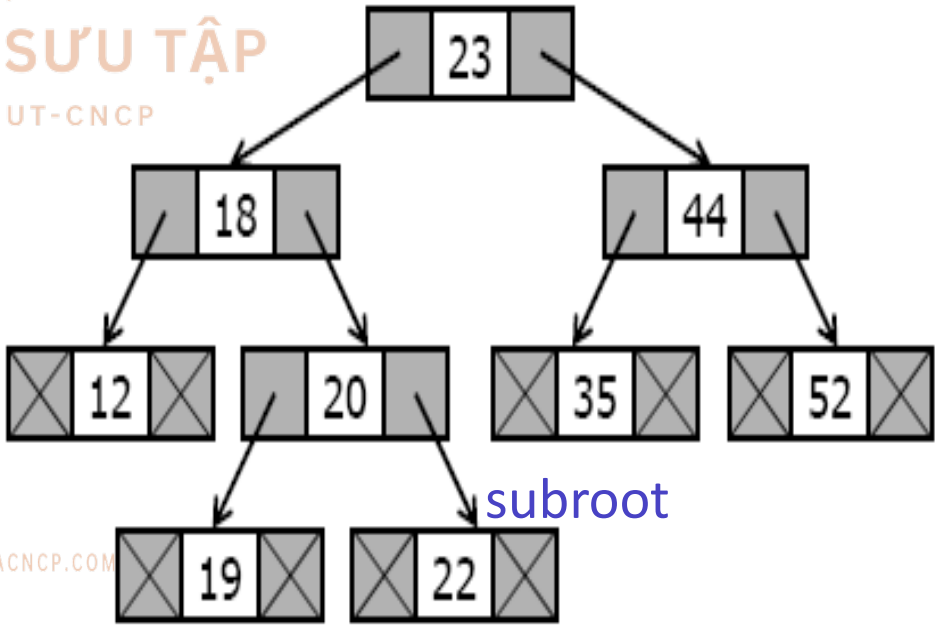

Target = 22

# Search node in BST (recursive version)

1. **if** (subroot is NULL) OR (subroot->data = target)
    1. return subroot
2. **else if** (target < subroot->data)
    1. return recursive_Search(subroot->left, target)
3. **else**
    1. return recursive_Search(subroot->right, target)

End recursive_Search






Target = 22

# Search Node in BST (nonrecursive version)

<pointer> **iterative_Search** (val subroot <pointer>,
val target <KeyType>)

Searches target in the subtree.

**Pre** subroot points to the root of a tree/ subtree.

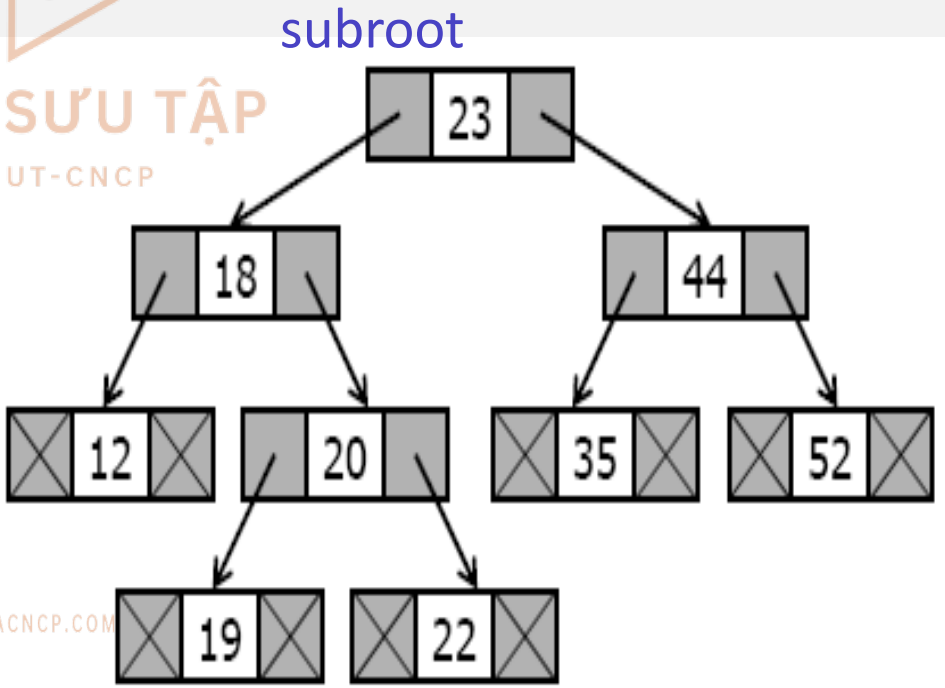
**Post** If target is not in the subtree, NULL is returned. Otherwise, a pointer to the node containing the target is returned.

# Search Node in BST (nonrecursive version)

1. **while** (subroot is not NULL) AND (subroot->data.key <> target)
    1. **if** (target < subroot->data.key)
        1. subroot = subroot->left
    2. **else**
        1. subroot = subroot->right
2. return subroot

End iterative_Search





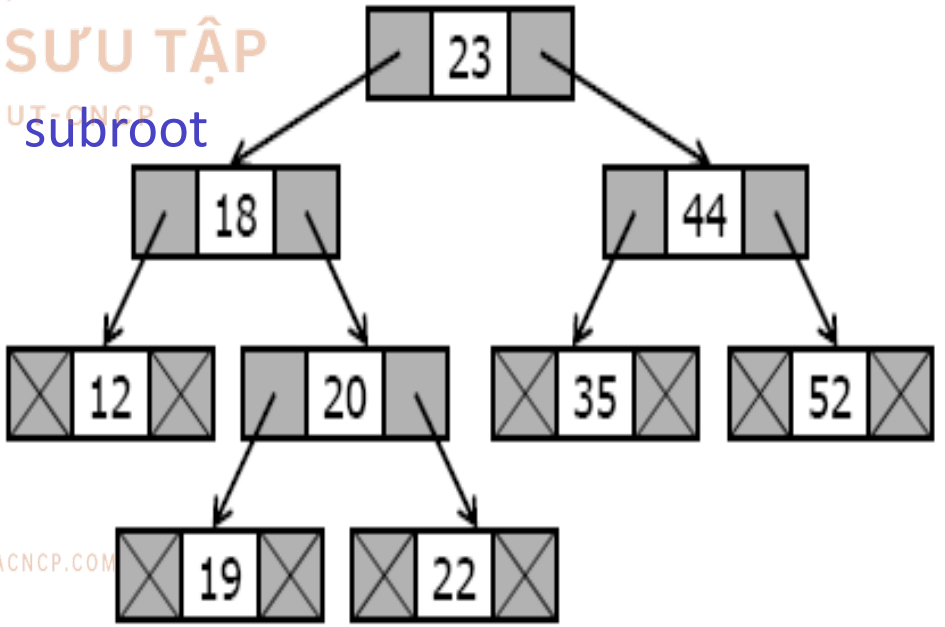

Target = 22

# Search Node in BST (nonrecursive version)

1. **while** (subroot is not NULL) AND (subroot->data.key <> target)
    1. **if** (target < subroot->data.key)
        1. subroot = subroot->left
    2. **else**
        1. subroot = subroot->right
2. return subroot

End iterative_Search





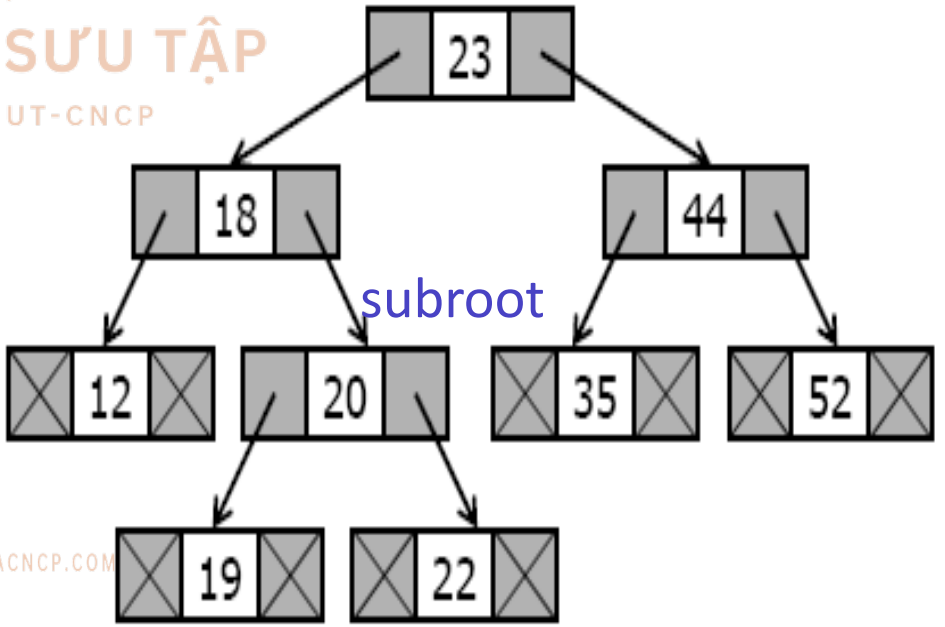

Target = 22

# Search Node in BST (nonrecursive version)

1. **while** (subroot is not NULL) AND (subroot->data.key <> target)
    1. **if** (target < subroot->data.key)
        1. subroot = subroot->left
    2. **else**
        1. subroot = subroot->right
2. return subroot

End iterative_Search





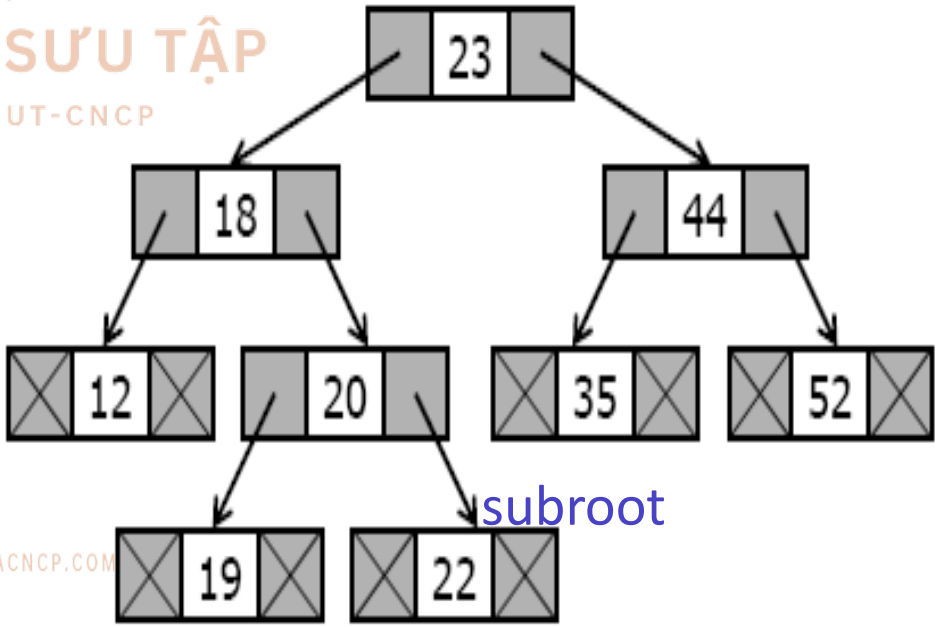

Target = 22

# Search Node in BST (nonrecursive version)

1. **while** (subroot is not NULL) AND (subroot->data.key <> target)
    1. **if** (target < subroot->data.key)
        1. subroot = subroot->left
    2. **else**
        1. subroot = subroot->right
2. return subroot

End iterative_Search






Target = 22

# Search node in BST

<ErrorCode> **Search** (ref DataOut <DataType>)

Searches target in the subtree.

**Pre** DataOut contains value needs to be found in key field.

**Post** DataOut will reveive all other values in other fields if that key is found.

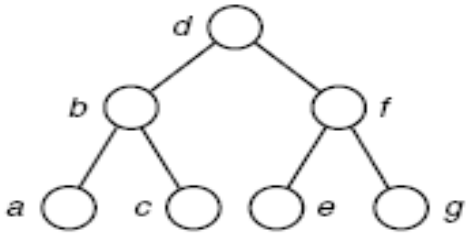
**Return** *success* or *notPresent*

**Uses** Auxiliary function recursive_Search or iterative_Search

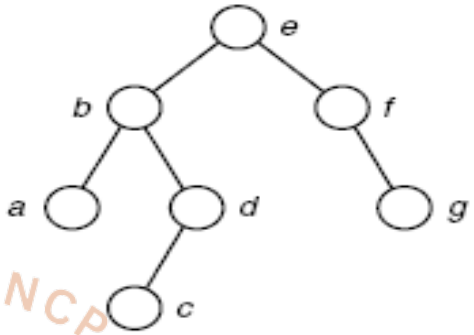
1. pNode = recursive_Search(root, DataOut.key)
2. **if** (pNode is NULL)
    1. return *notPresent*
3. dataOut = pNode->data
4. return *success*

End Search

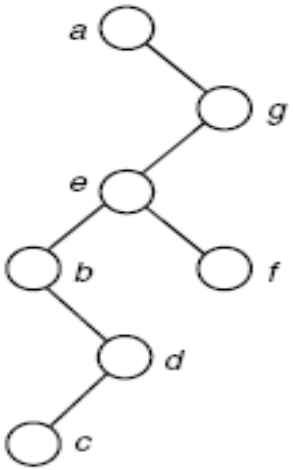
# Binary Search Trees with the Same Keys



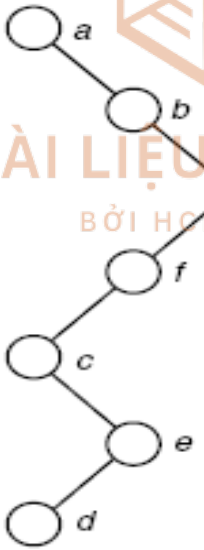

(a)



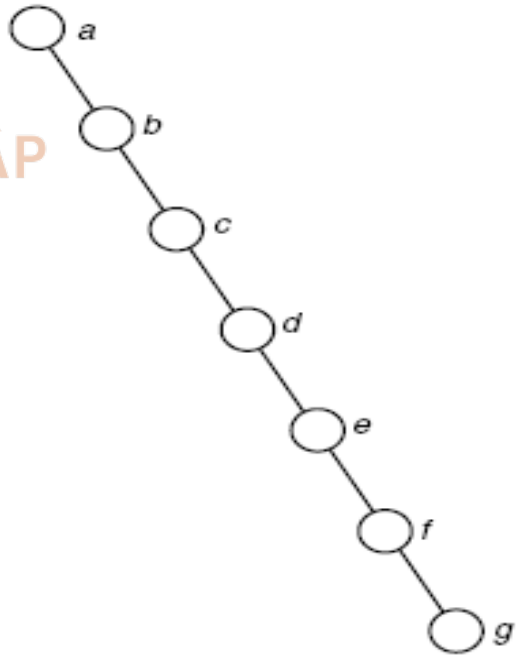

(b)




(c)




(d)



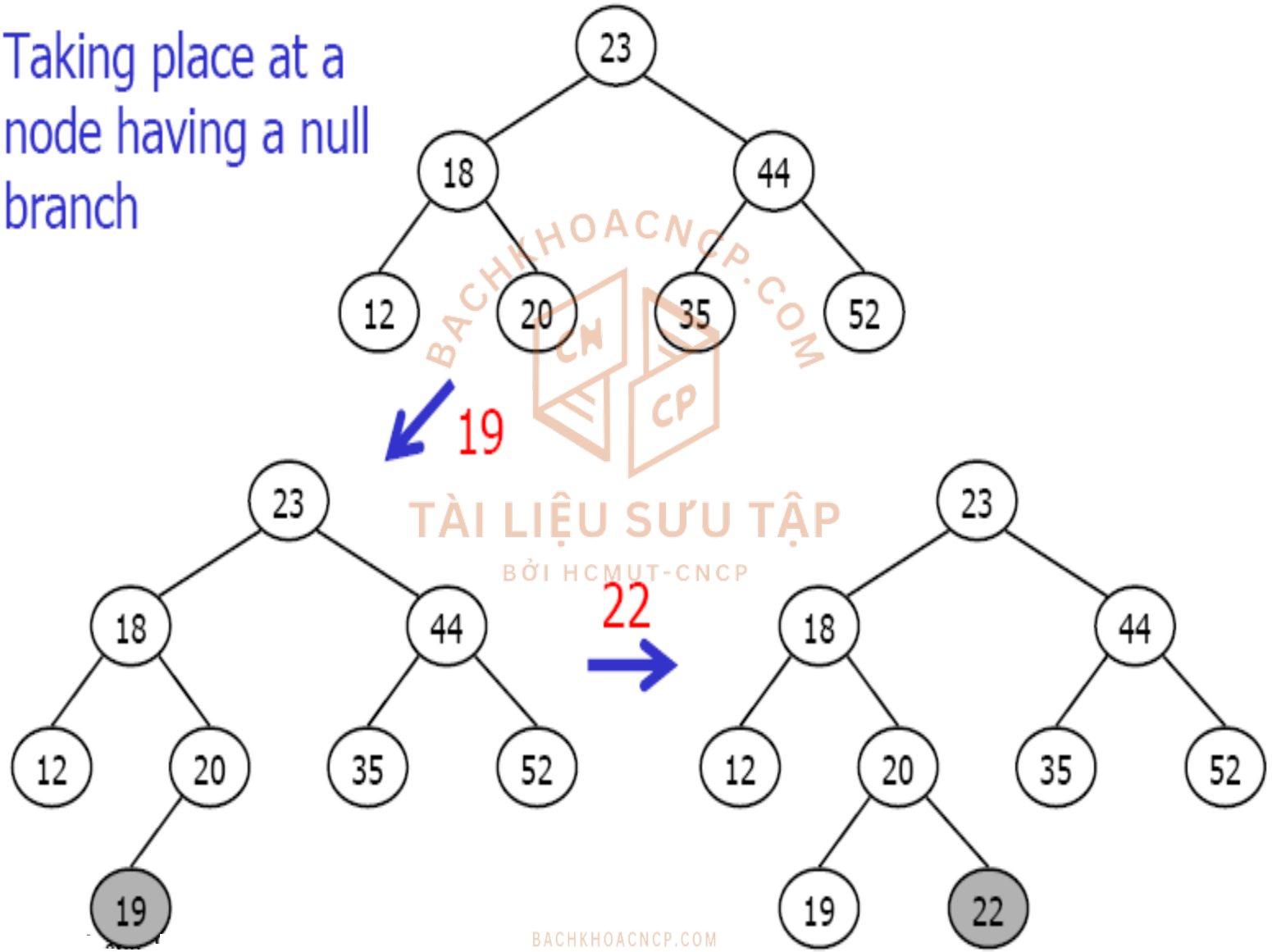

(e)

# Search node in BST

- The same keys may be built into BST of many different shapes.
- Search in bushy BST with n nodes will do O(log n) comparisons of keys
- If the tree degenerates into a long chain, search will do $\Theta(n)$ comparisons on n vertices.
- The bushier the tree, the smaller the number of comparisons of keys need to be done.

# Insert Node into BST

Taking place at a node having a null branch



# Insert Node into BST


*Question*:

Can Insert method use recursive_Search or iterative_Search instead of recursive_Insert like that:

```
ErrorCode Insert (val DataIn <DataType>)

1. pNode = recursive_Search (root, DataIn.key)
2. if (pNode is NULL)
    1. Allocate pNode
    2. pNode->data = DataIn
    3. return success
3. else
    1. return duplicate_error
End Insert
```

# Insert Node into BST

Auxiliary functions for Insert:

recursive_Insert

iterative_Insert

# Recursive Insert

<ErrorCode> **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

Inserts a new node into a BST.

**Pre** subroot points to the root of a tree/ subtree.
DataIn contains data to be inserted into the subtree.

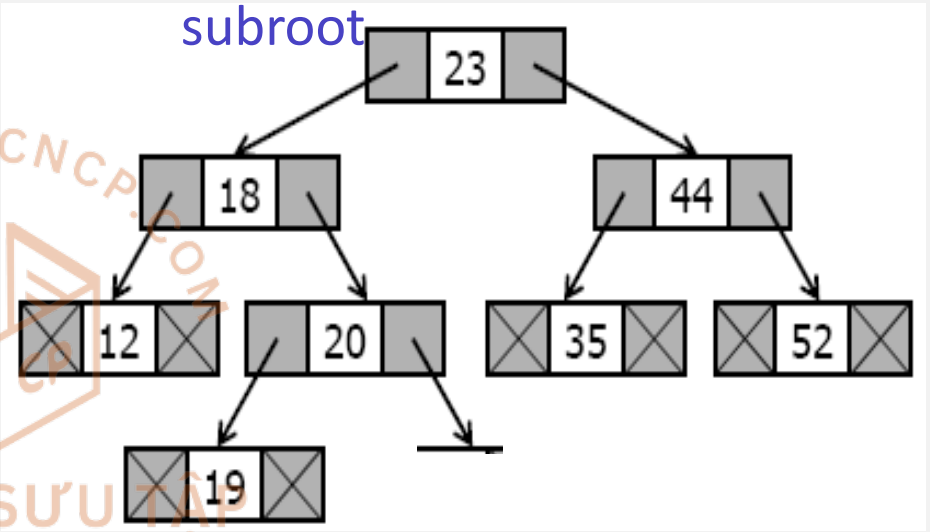
**Post** If the key of DataIn already belongs to the subtree, *duplicate_error* is returned. Otherwise, DataIn is inserted into the subtree in such a way that the properties of a BST are preserved.

**Return** *duplicate_error* or *success*.

**Uses** recursive_Insert function.

# Recursive Insert (cont.)

ErrorCode **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

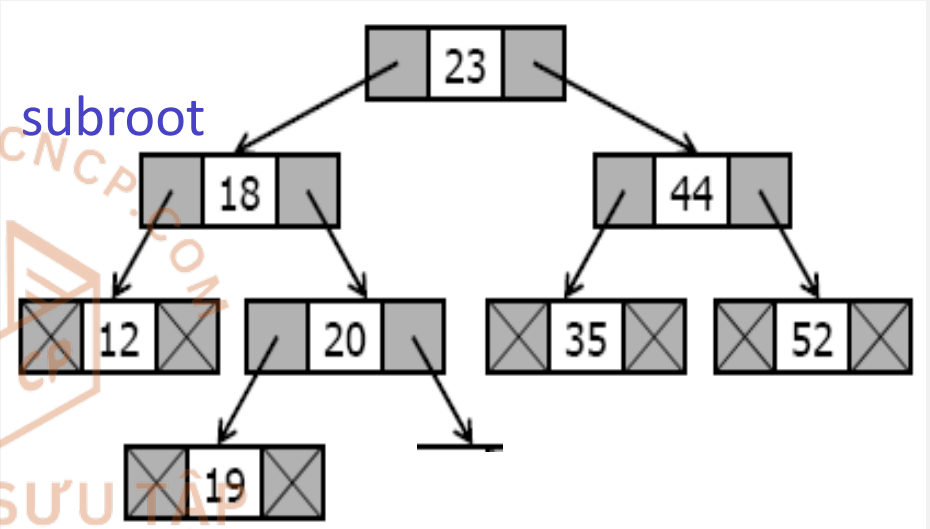



1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else if** (DataIn.key < subroot->data.key)
    1. return recursive_Insert(subroot->left, DataIn)
3. **else if** (DataIn.key > subroot->data.key)
    1. return recursive_Insert(subroot->right, DataIn)
4. **else**
    1. return *duplicate_error*
5. End recursive_Insert

DataIn.key = 22

# Recursive Insert (cont.)

ErrorCode **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

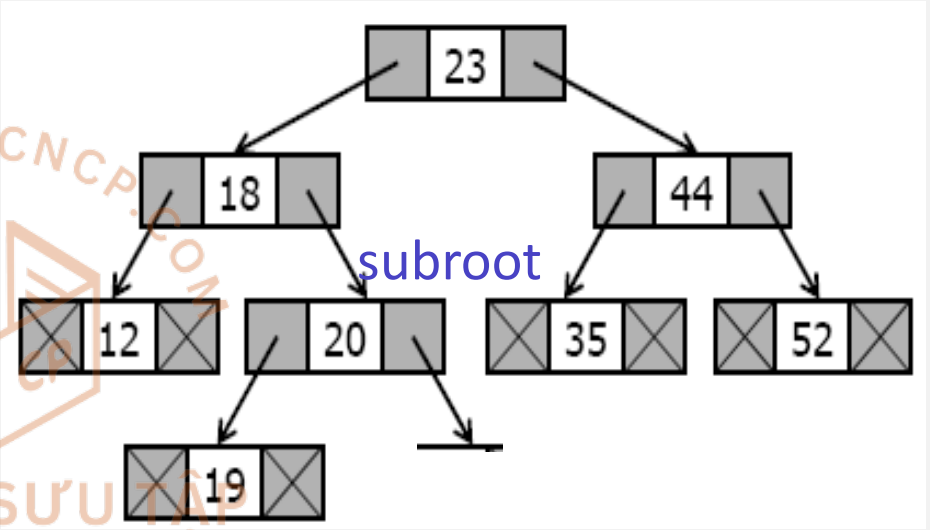



1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else if** (DataIn.key < subroot->data.key)
    1. return recursive_Insert(subroot->left, DataIn)
3. **else if** (DataIn.key > subroot->data.key)
    1. return recursive_Insert(subroot->right, DataIn)
4. **else**
    1. return *duplicate_error*
5. End recursive_Insert

DataIn.key = 22

# Recursive Insert (cont.)

ErrorCode **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

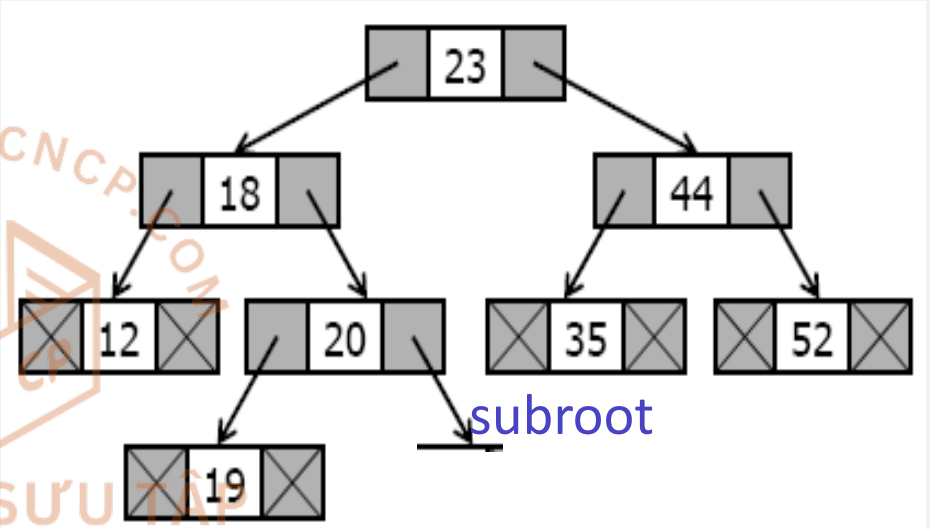



1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else if** (DataIn.key < subroot->data.key)
    1. return recursive_Insert(subroot->left, DataIn)
3. **else if** (DataIn.key > subroot->data.key)
    1. return recursive_Insert(subroot->right, DataIn)
4. **else**
    1. return *duplicate_error*
5. End recursive_Insert

DataIn.key = 22

# Recursive Insert (cont.)

ErrorCode **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

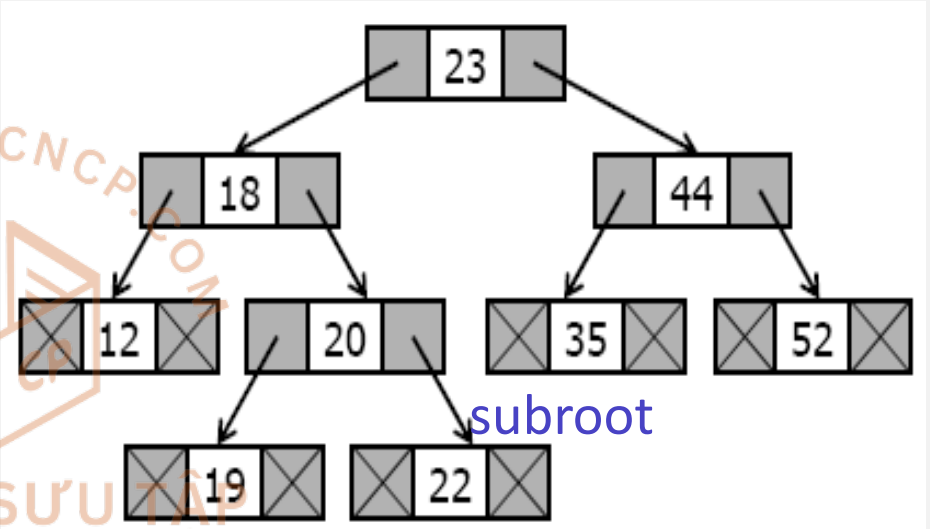



1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else if** (DataIn.key < subroot->data.key)
    1. return recursive_Insert(subroot->left, DataIn)
3. **else if** (DataIn.key > subroot->data.key)
    1. return recursive_Insert(subroot->right, DataIn)
4. **else**
    1. return *duplicate_error*
5. End recursive_Insert

DataIn.key = 22

# Recursive Insert (cont.)

ErrorCode **recursive_Insert** (ref subroot <pointer>,
val DataIn <DataType>)




1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else if** (DataIn.key < subroot->data.key)
    1. return recursive_Insert(subroot->left, DataIn)
3. **else if** (DataIn.key > subroot->data.key)
    1. return recursive_Insert(subroot->right, DataIn)
4. **else**
    1. return *duplicate_error*
5. End recursive_Insert

DataIn.key = 22

# Iterative Insert

ErrorCode **iterative_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

Inserts a new node into a BST.

**Pre** subroot is NULL or points to the root of a subtree. DataIn contains data to be inserted into the subtree.

**Post** If the key of DataIn already belongs to the subtree, *duplicate_error* is returned. Otherwise, DataIn is inserted into the subtree in such a way that the properties of a BST are preserved.

**Return** *duplicate_error* or *success*.

# Iterative Insert (cont.)

ErrorCode **iterative_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

1. **if** (subroot is NULL)
   1. Allocate subroot
   2. subroot->data = DataIn
   3. return *success*
2. **else**

subroot

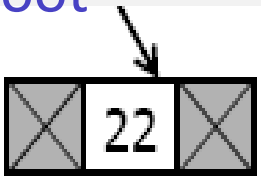
DataIn.key = 22

# Iterative Insert (cont.)

ErrorCode **iterative_Insert** (ref subroot <pointer>,
val DataIn <DataType>)

1. **if** (subroot is NULL)
    1. Allocate subroot
    2. subroot->data = DataIn
    3. return *success*
2. **else**

subroot

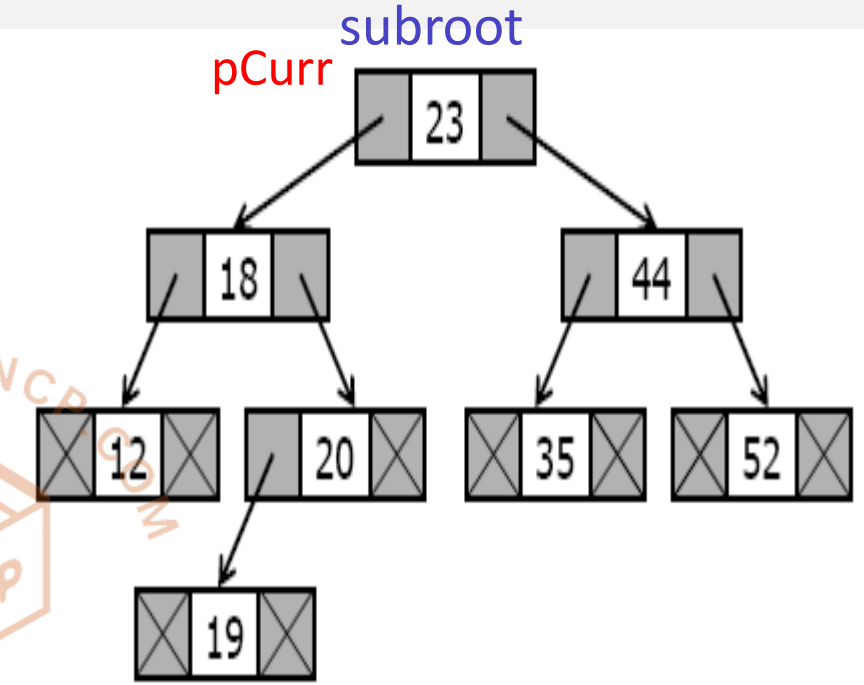



DataIn.key = 22

# Iterative Insert (cont.)

2. else
    1. pCurr = subroot
    2. **loop** (pCurr is not NULL)
        1. **if** (pCurr->data.key = DataIn.key)
            1. return *duplicate_error*
        2. parent = pCurr
        3. **if** (DataIn.key < parent->data.key)
            1. pCurr = parent -> left
        4. **else**
            1. pCurr = parent -> right
    3. **if** (DataIn.key < parent->data.key)
        1. Allocate parent->left
        2. parent->left.data = DataIn
    4. **else**
        1. Allocate parent->right
        2. parent->right.data = DataIn
    5. return *success*

End Iterative_Insert

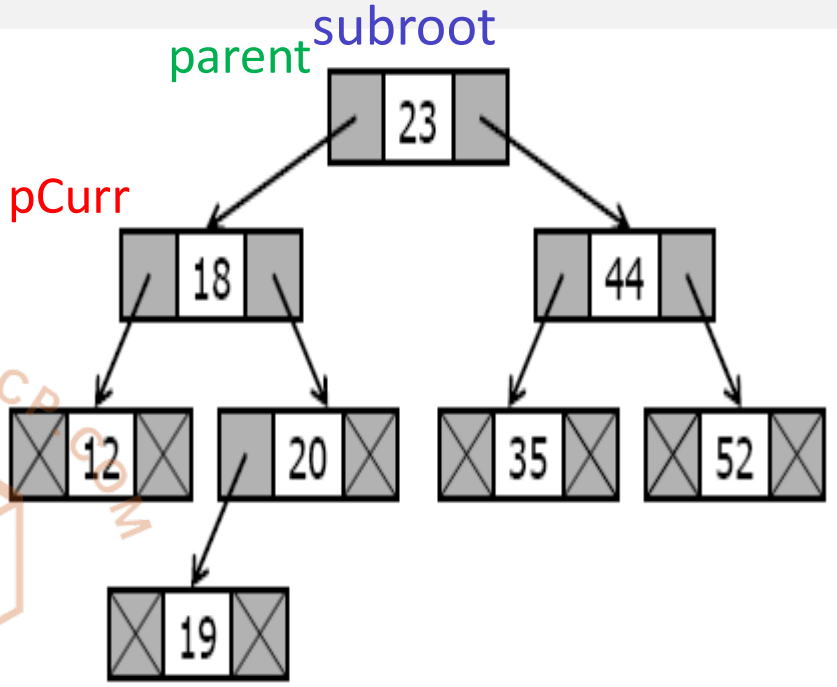



DataIn.key = 22

# Iterative Insert (cont.)

2. else
    1. pCurr = subroot
    2. **loop** (pCurr is not NULL)
        1. **if** (pCurr->data.key = DataIn.key)
            1. return *duplicate_error*
        2. parent = pCurr
        3. **if** (DataIn.key < parent->data.key)
            1. pCurr = parent -> left
        4. **else**
            1. pCurr = parent -> right
    3. **if** (DataIn.key < parent->data.key)
        1. Allocate parent->left
        2. parent->left.data = DataIn
    4. **else**
        1. Allocate parent->right
        2. parent->right.data = DataIn
    5. return *success*

End Iterative_Insert

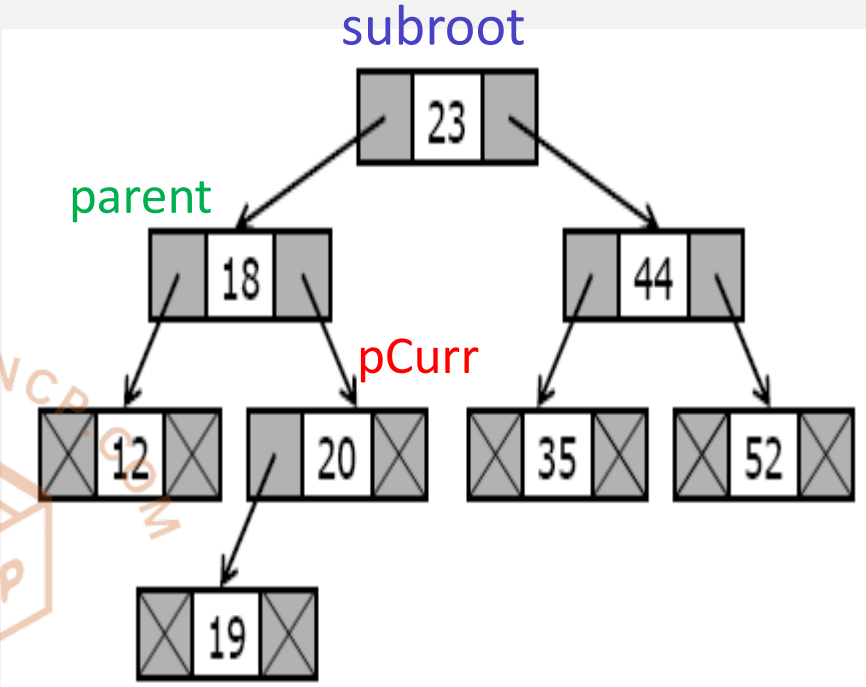



DataIn.key = 22

# Iterative Insert (cont.)

2. else
    1. pCurr = subroot
    2. **loop** (pCurr is not NULL)
        1. **if** (pCurr->data.key = DataIn.key)
            1. return *duplicate_error*
        2. parent = pCurr
        3. **if** (DataIn.key < parent->data.key)
            1. pCurr = parent -> left
        4. **else**
            1. pCurr = parent -> right
    3. **if** (DataIn.key < parent->data.key)
        1. Allocate parent->left
        2. parent->left.data = DataIn
    4. **else**
        1. Allocate parent->right
        2. parent->right.data = DataIn
    5. return *success*

End Iterative_Insert

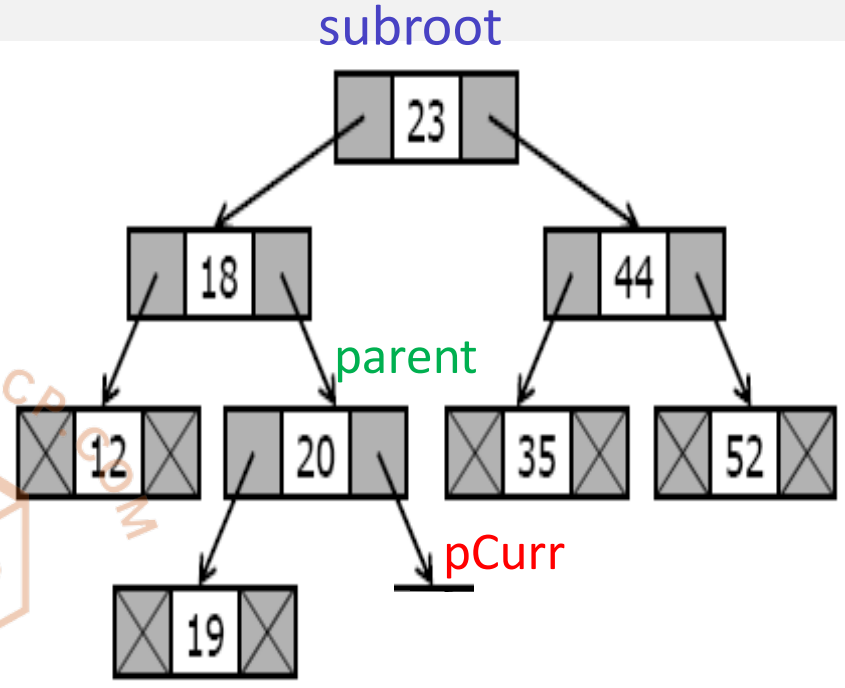



DataIn.key = 22

# Iterative Insert (cont.)

2. else
    1. pCurr = subroot
    2. **loop** (pCurr is not NULL)
        1. **if** (pCurr->data.key = DataIn.key)
            1. return *duplicate_error*
        2. parent = pCurr
        3. **if** (DataIn.key < parent->data.key)
            1. pCurr = parent -> left
        4. **else**
            1. pCurr = parent -> right
    3. **if** (DataIn.key < parent->data.key)
        1. Allocate parent->left
        2. parent->left.data = DataIn
    4. **else**
        1. Allocate parent->right
        2. parent->right.data = DataIn
    5. return *success*

End Iterative_Insert

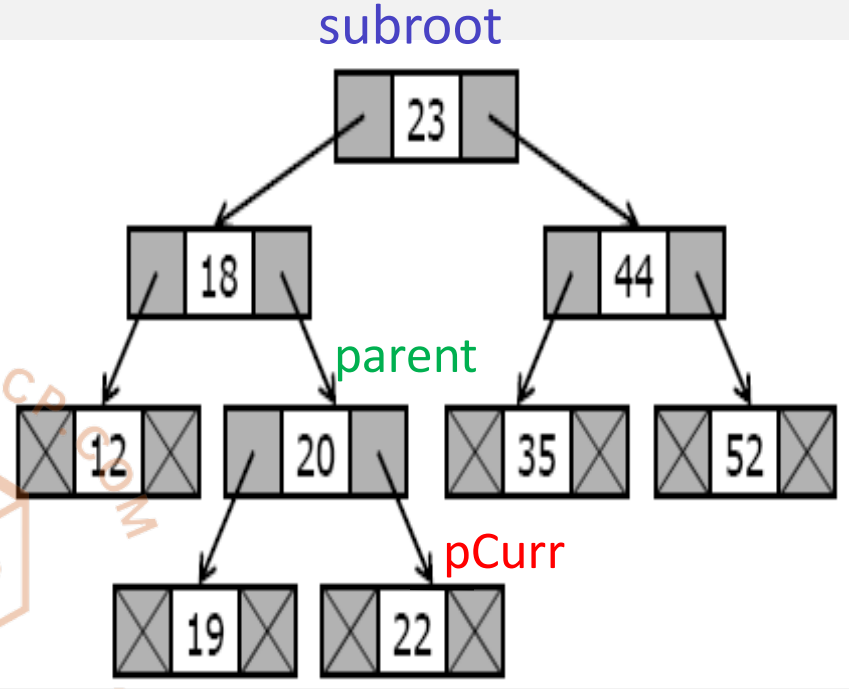



DataIn.key = 22

# Iterative Insert (cont.)

2. else
    1. pCurr = subroot
    2. **loop** (pCurr is not NULL)
        1. **if** (pCurr->data.key = DataIn.key)
            1. return *duplicate_error*
        2. parent = pCurr
        3. **if** (DataIn.key < parent->data.key)
            1. pCurr = parent -> left
        4. **else**
            1. pCurr = parent -> right
    3. **if** (DataIn.key < parent->data.key)
        1. Allocate parent->left
        2. parent->left.data = DataIn
    4. **else**
        1. Allocate parent->right
        2. parent->right.data = DataIn
    5. return *success*

End Iterative_Insert




DataIn.key = 22

# Insert Node into BST

ErrorCode **Insert** (val DataIn <DataType>)

Inserts a new node into a BST.

**Post** If the key of DataIn already belongs to the BST, *duplicate_error* is returned. Otherwise, DataIn is inserted into the tree in such a way that the properties of a BST are preserved.

**Return** *duplicate_error* or *success*.

**Uses** recursive_Insert or iterative_Insert function.



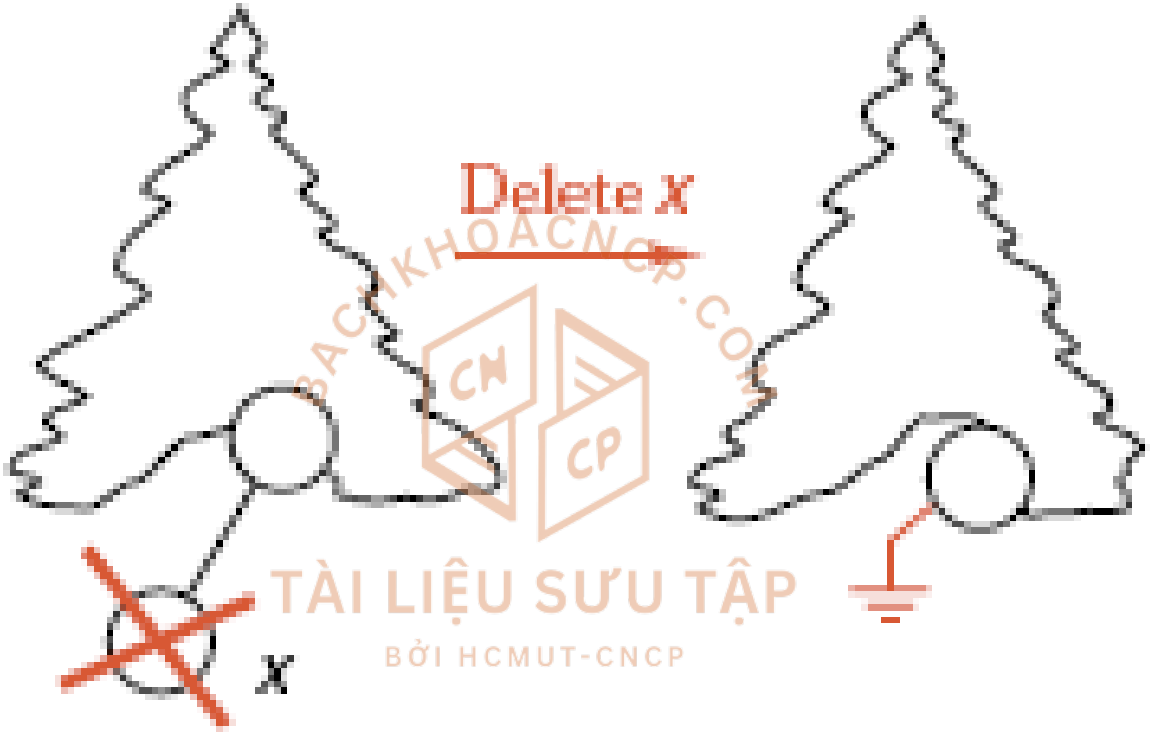
1. return recursive_Insert (root, DataIn)

End Insert

# Insert Node into BST

- Insertion a new node into a random BST with n nodes takes O(log n) steps.
- Insertion may take n steps when BST degenerates to a chain.
- If the keys are inserted in sorted order into an empty tree, BST becomes a chain.

# Delete node from BST






- Deletion of a leaf:

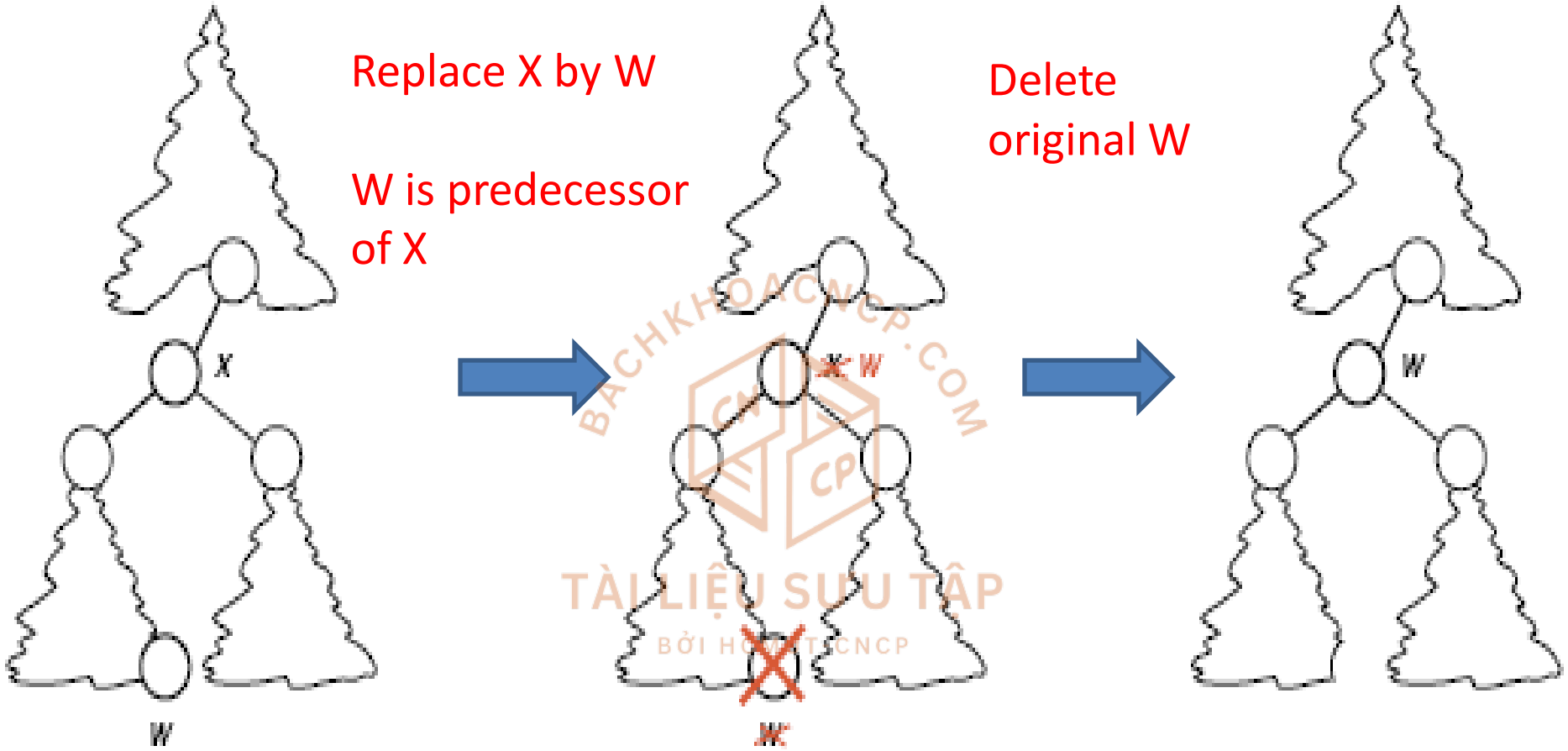
  Set the deleted node's parent link to NULL.

# Delete node from BST


- Deletion of a node having only right subtree or left subtree:

  Attach the subtree to the deleted node's parent.

# Delete node from BST

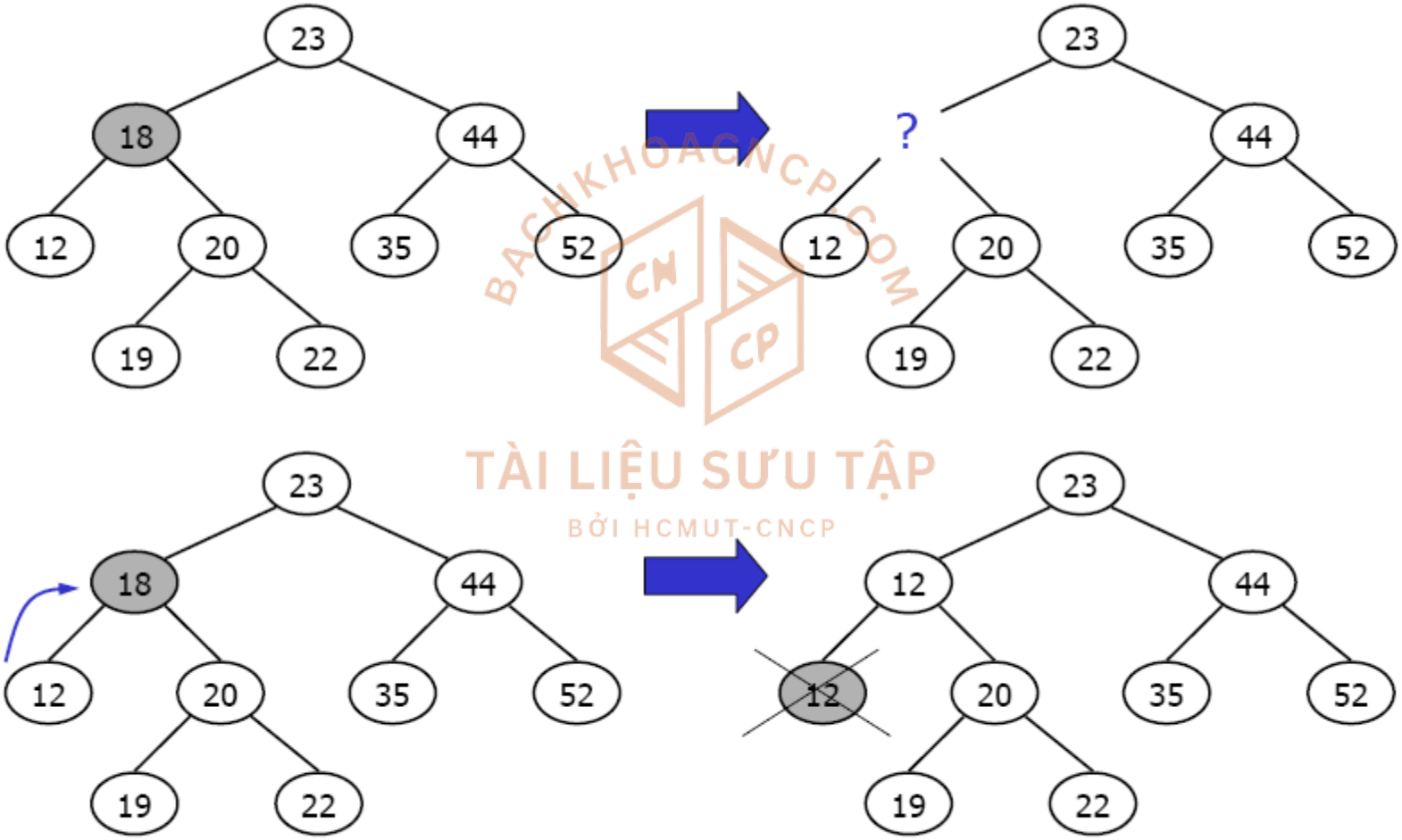



- Deletion of a node having both subtrees:

  Replace the deleted node by its predecessor or by its successor, recycle this node instead.

# Delete node from BST

- Node having both subtrees

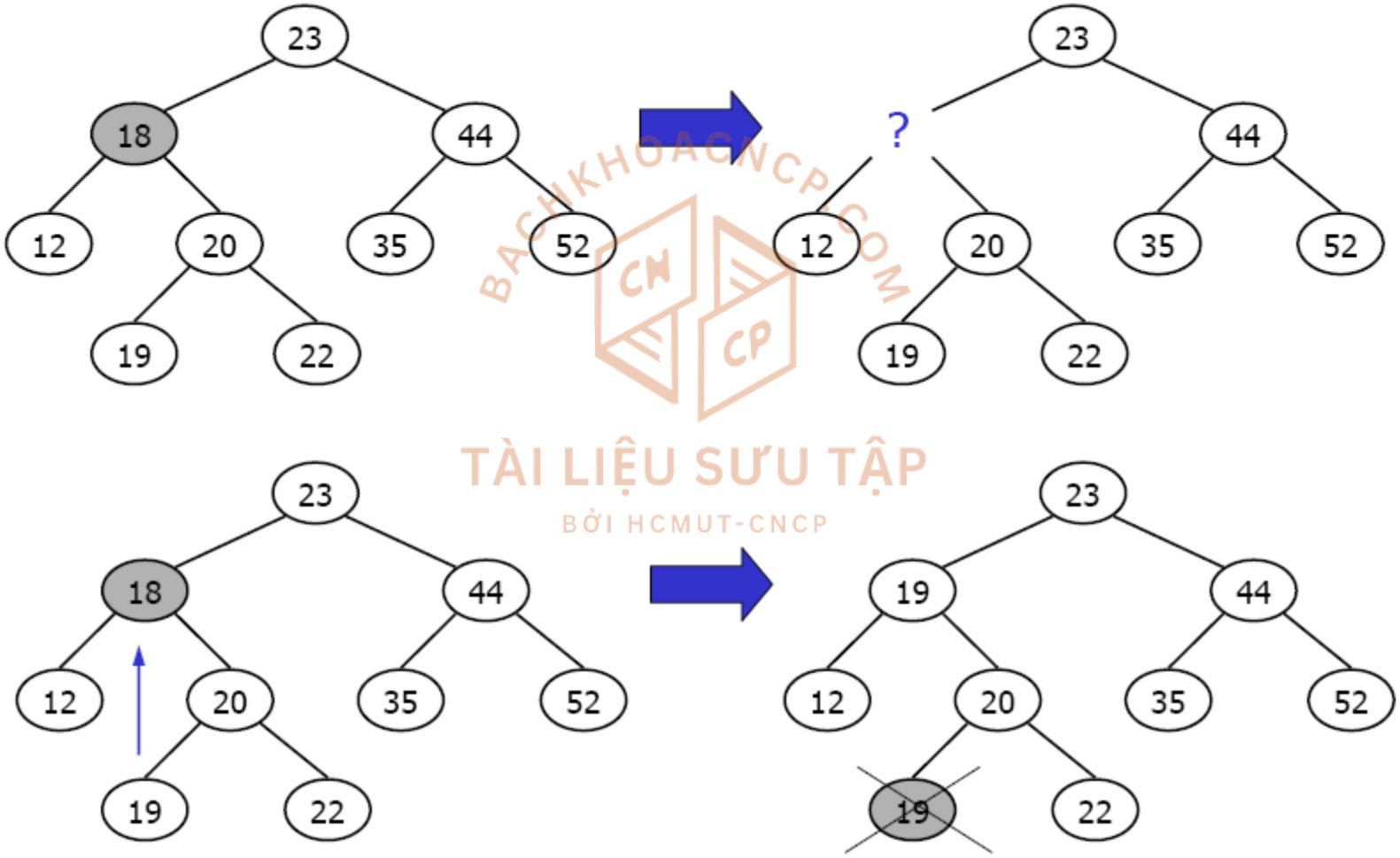





Using largest node in the left subtree

# Delete node from BST

- Node having both subtrees






Using smallest node in the right subtree

# Delete node from BST

Auxiliary functions for Insert:

recursive_Delete

iterative_Delete

# Recursive Delete

<ErrorCode> **recursive_Delete** (ref subroot <pointer>,
val key <KeyType>)

Deletes a node from a BST.

**Pre** subroot is NULL or points to the root of a subtree. Key contains value needs to be removed from BST.

**Post** If key is found, it will be removed from BST.

**Return** *notFound* or *success*.

**Uses** recursive_Delete and RemoveNode functions.

# Recursive Delete (cont.)

<ErrorCode> **recursive_Delete** (ref subroot <pointer>,
val key <KeyType>)

1. **if** (subroot is NULL)
    1. return *notFound*
2. **else if** (key < subroot->data.key)
    1. return recursive_Delete(subroot->left, key)
3. **else if** (key > subroot->data.key)
    1. return recursive_Delete(subroot->right, key)
4. **else**
    1. RemoveNode(subroot)
    2. return *success*

End recursive_Delete

# Delete Node from BST

<ErrorCode> **Delete** (val key <KeyType>)

Deletes a node from a BST.

**Pre** subroot is NULL or points to the root of a subtree. Key contains value needs to be removed from BST.

**Post** If key is found, it will be removed from BST.

**Return** *notFound* or *success*.

**Uses** recursive_Delete and RemoveNode functions.



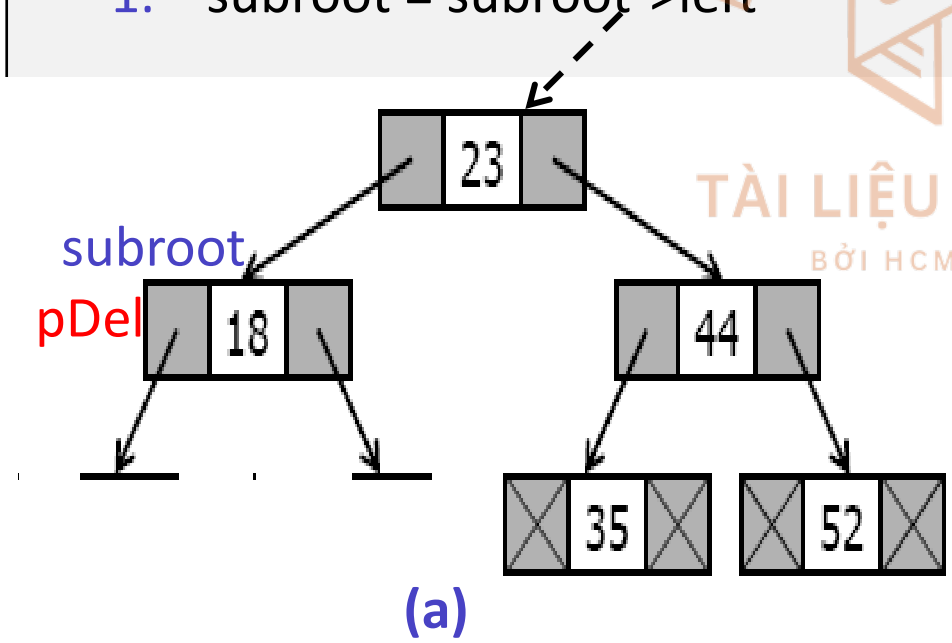
1. return recursive_Delete (root, key)

End Delete

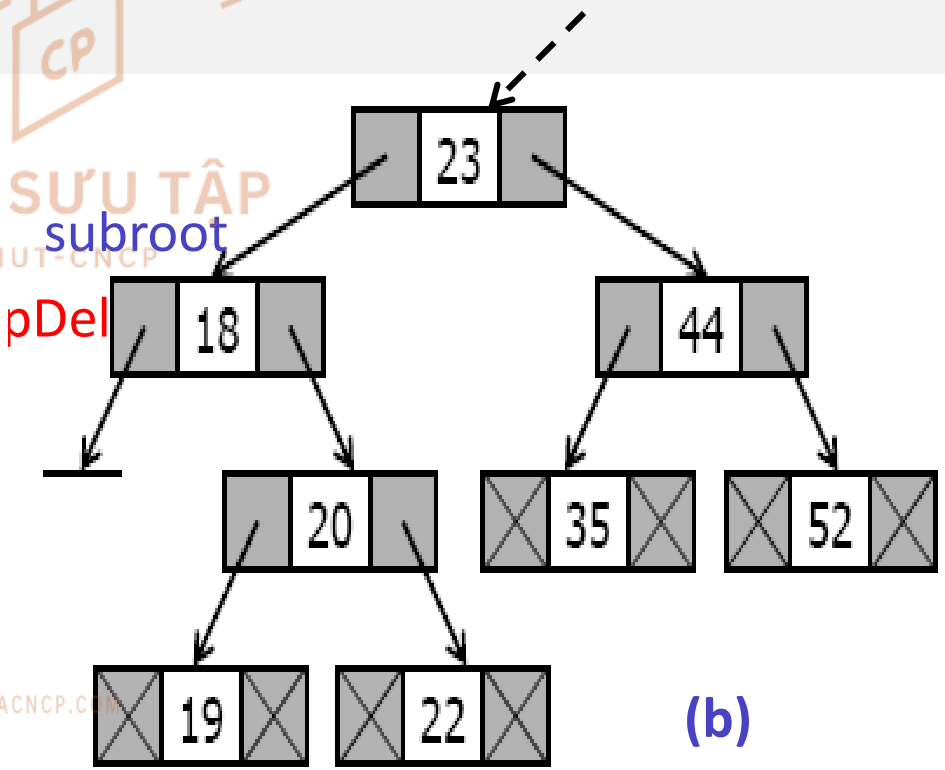
# Auxiliary Function RemoveNode

<void> **RemoveNode** (ref subroot <pointer>, val key <KeyType>)

1. pDel = subroot // remember node to delete at end.
2. **if** (subroot ->left is NULL) // leaf node or node having only right subtree.
    1. subroot = subroot->right // **(a) and (b)**
3. **else if** (subroot->right is NULL) // node having only left subtree.
    1. subroot = subroot->left




(a)



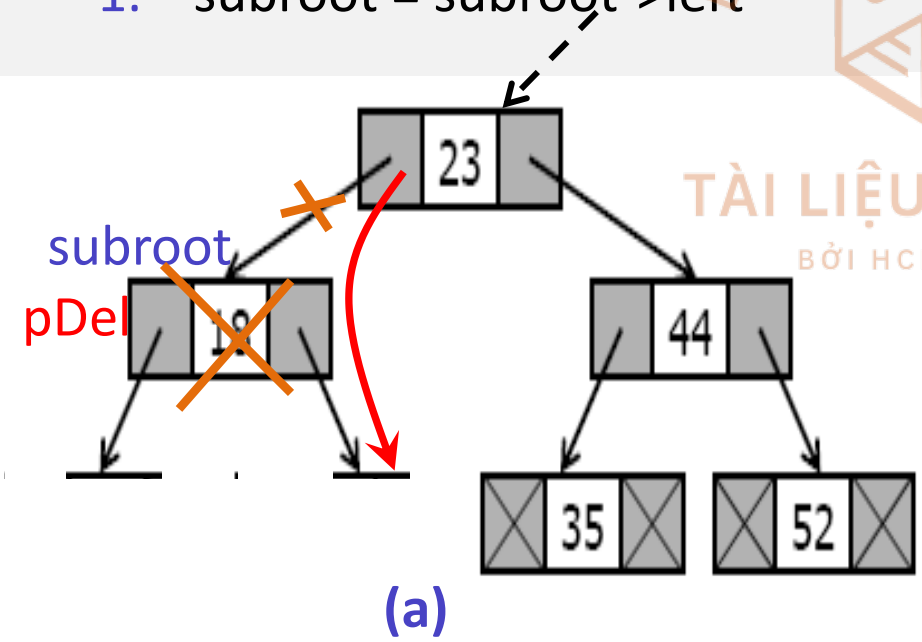

(b)

key needs to be deleted = 18

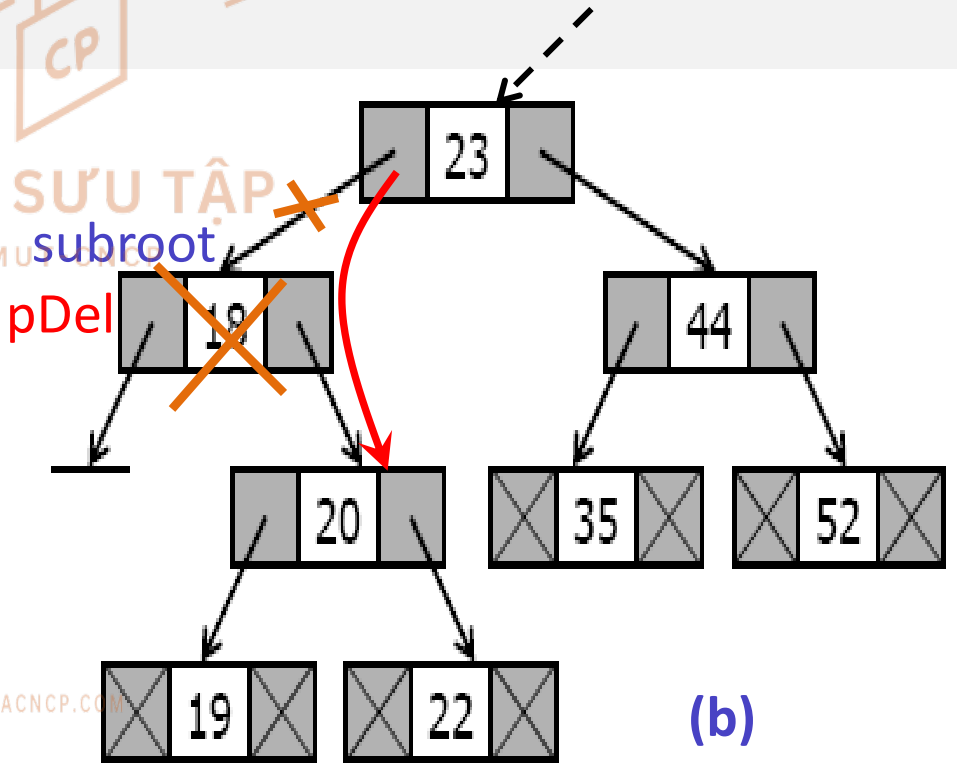
# Auxiliary Function RemoveNode

<void> **RemoveNode** (ref subroot <pointer>, val key <KeyType>)

1. pDel = subroot // remember node to delete at end.
2. **if** (subroot ->left is NULL) // leaf node or node having only right subtree.
    1. subroot = subroot->right // **(a) and (b)**
3. **else if** (subroot->right is NULL) // node having only left subtree.
    1. subroot = subroot->left




(a)




(b)

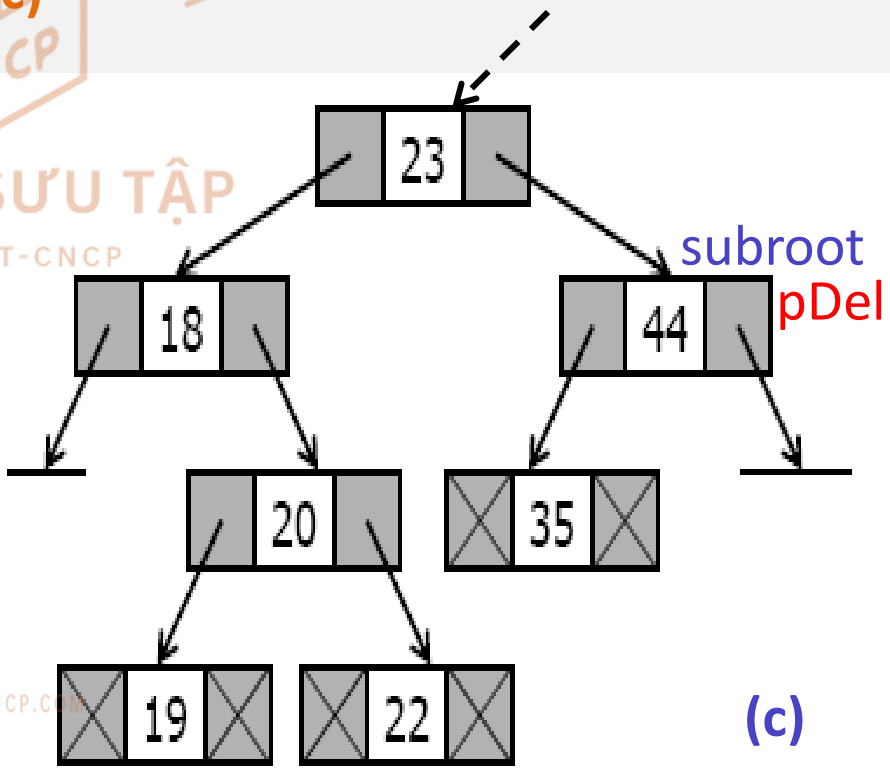
key needs to be deleted = 18

# Auxiliary Function RemoveNode

<void> **RemoveNode** (ref subroot <pointer>, val key <KeyType>)

1. pDel = subroot // remember node to delete at end.
2. **if** (subroot ->left is NULL) // leaf node or node having only right subtree.
    1. subroot = subroot->right
3. **else if** (subroot->right is NULL) // node having only left subtree.
    1. subroot = subroot->left // (c)





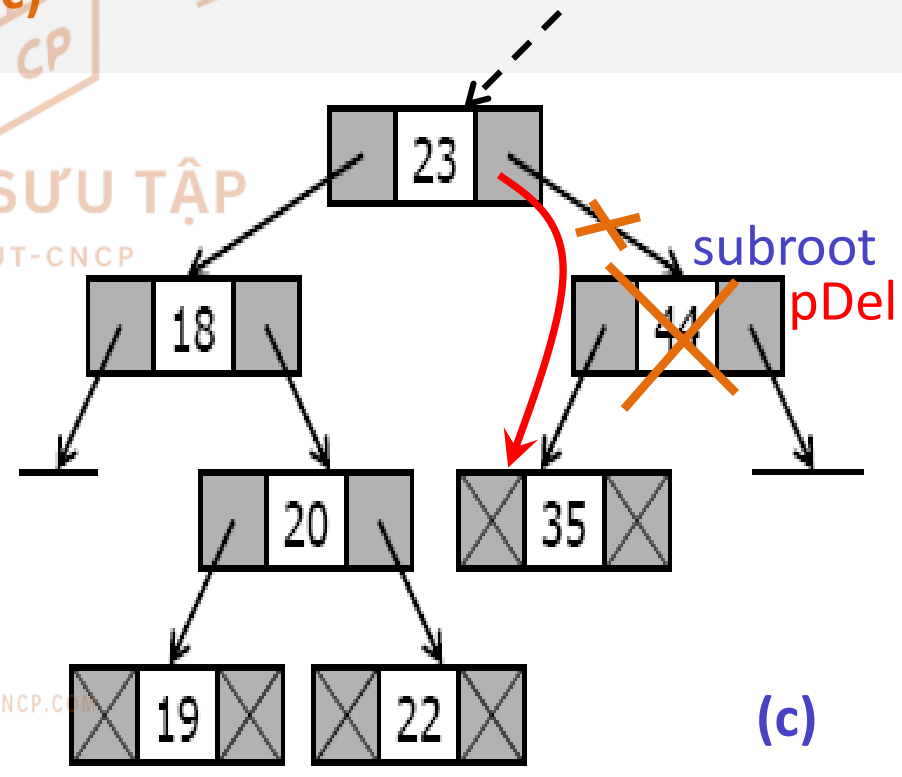

key needs to be deleted = 44

# Auxiliary Function RemoveNode

<void> **RemoveNode** (ref subroot <pointer>, val key <KeyType>)

1. pDel = subroot // remember node to delete at end.
2. **if** (subroot ->left is NULL) // leaf node or node having only right subtree.
    1. subroot = subroot->right
3. **else if** (subroot->right is NULL) // node having only left subtree.
    1. subroot = subroot->left // (c)



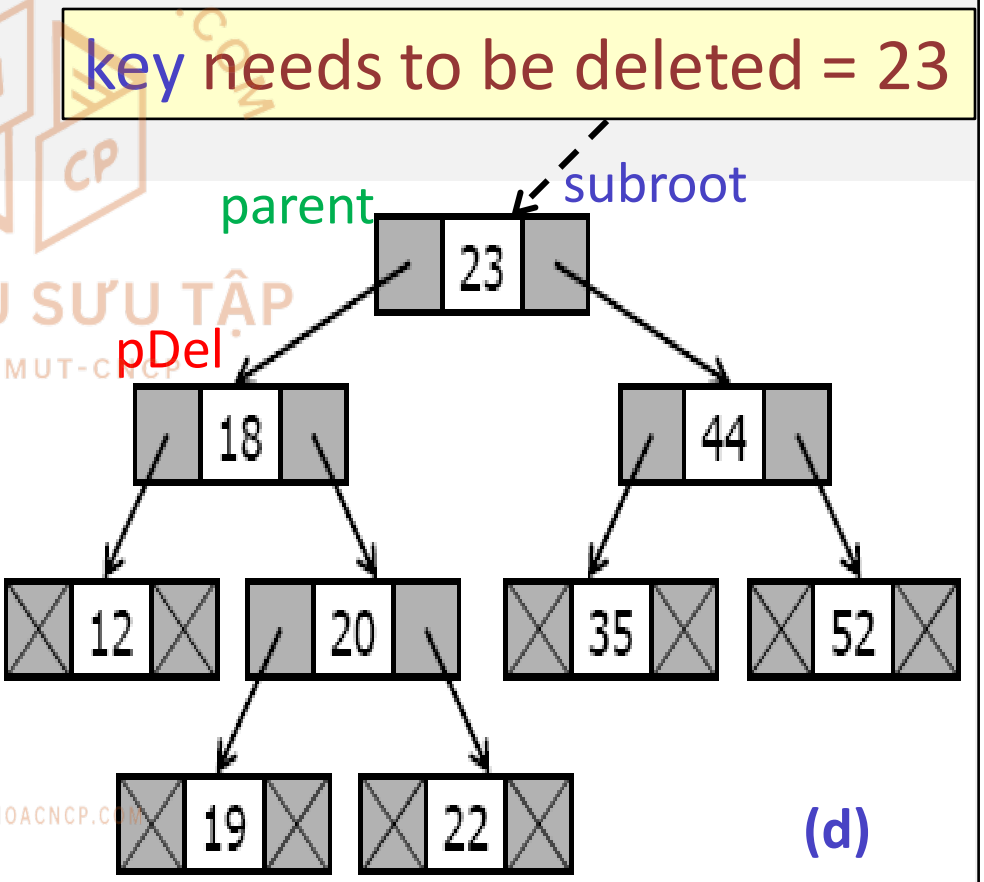



key needs to be deleted = 44

# Auxiliary Function RemoveNode (cont.)

4. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel

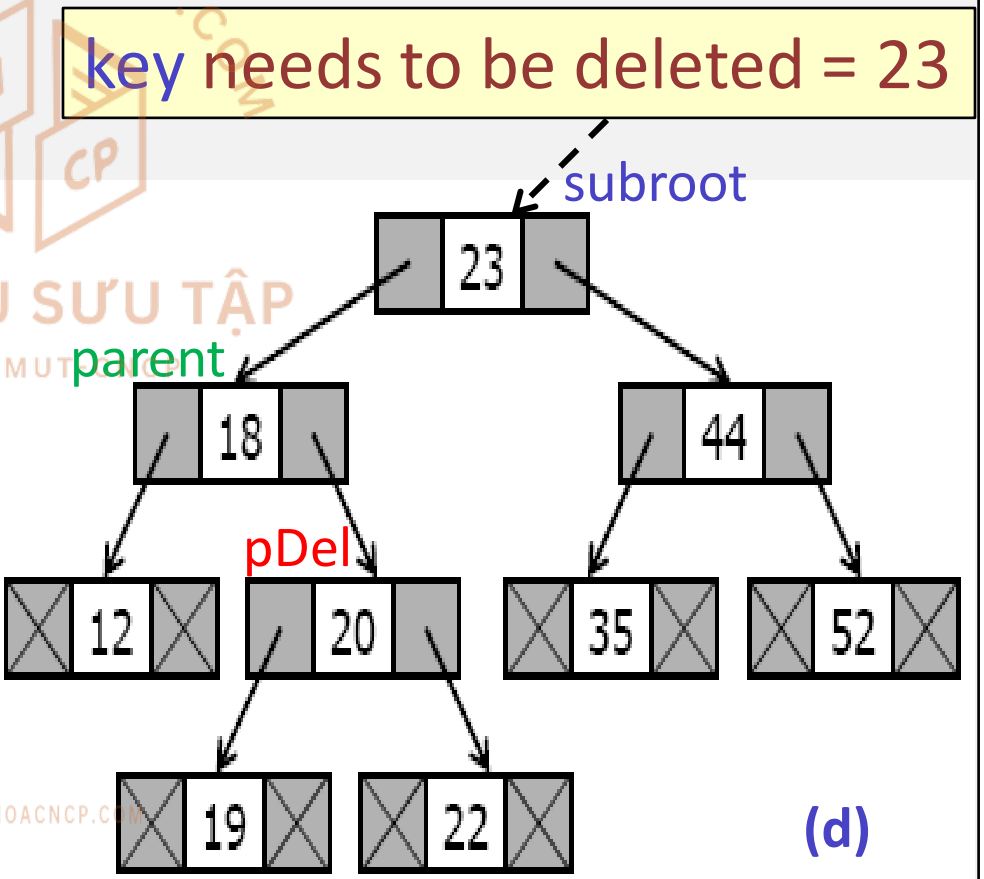
End RemoveNode



# Auxiliary Function RemoveNode (cont.)

1. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel
    8. return *success*

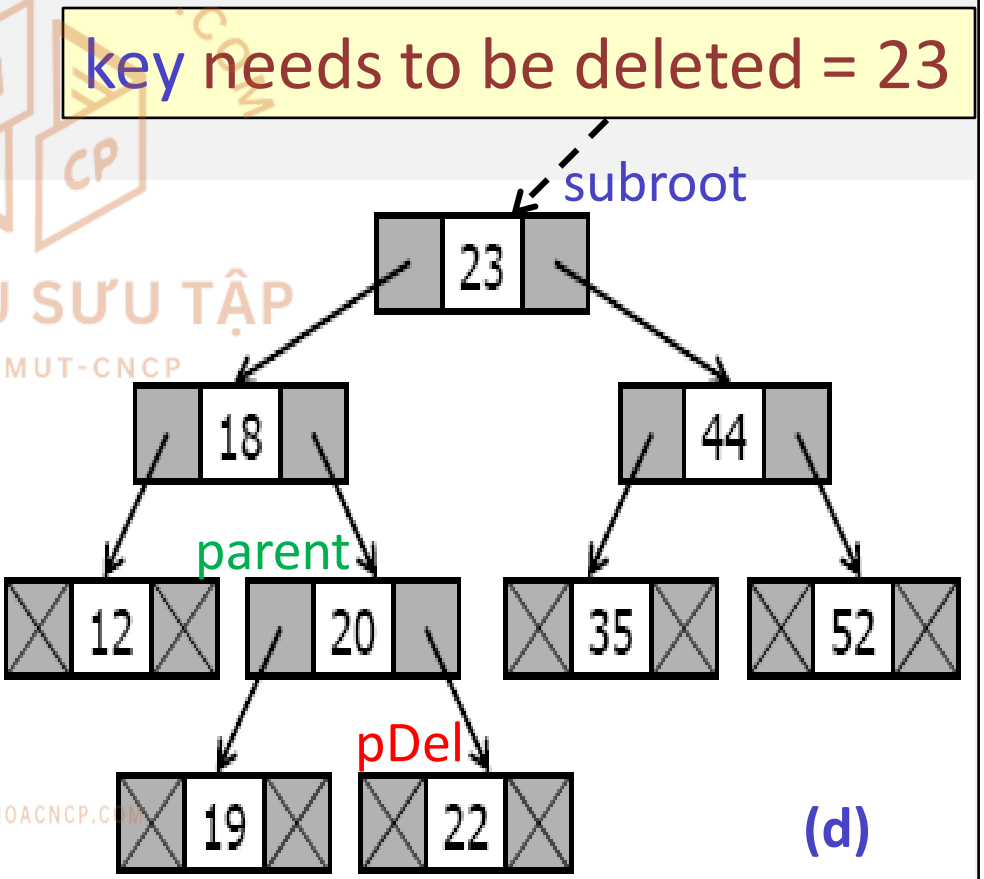
End RemoveNode



# Auxiliary Function RemoveNode (cont.)

1. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel
    8. return *success*

End RemoveNode

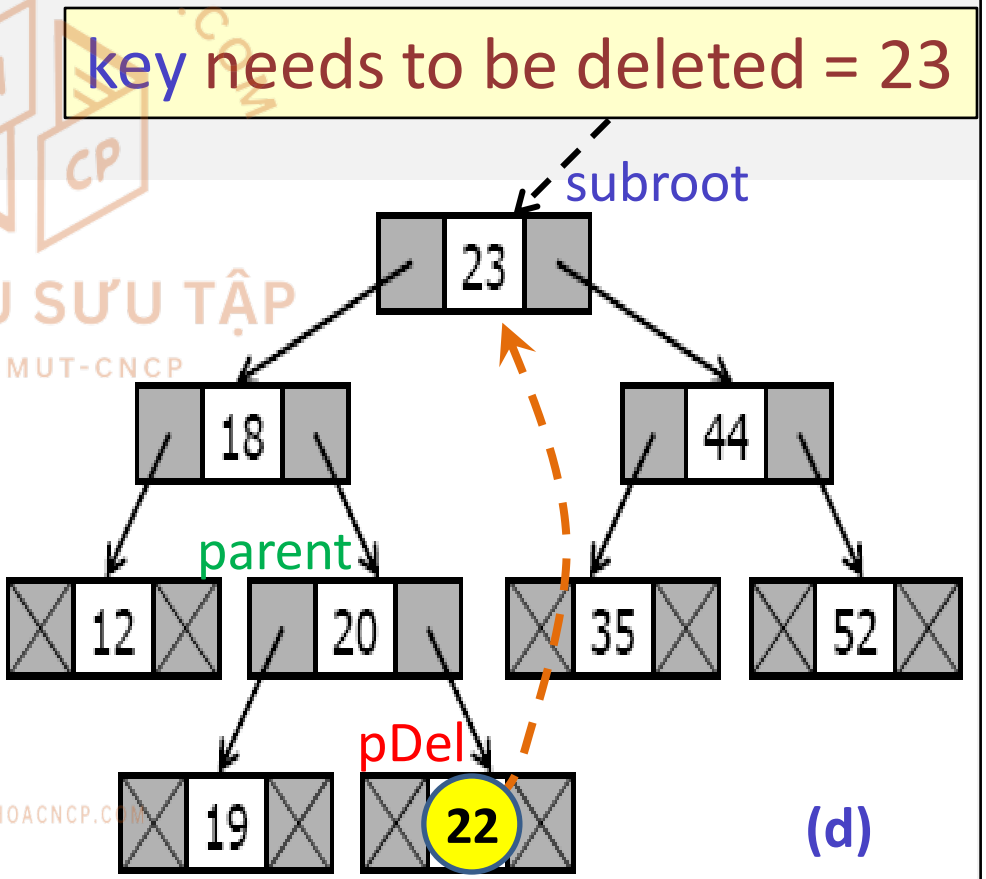



**(d)**

# Auxiliary Function RemoveNode (cont.)

1. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel
    8. return *success*

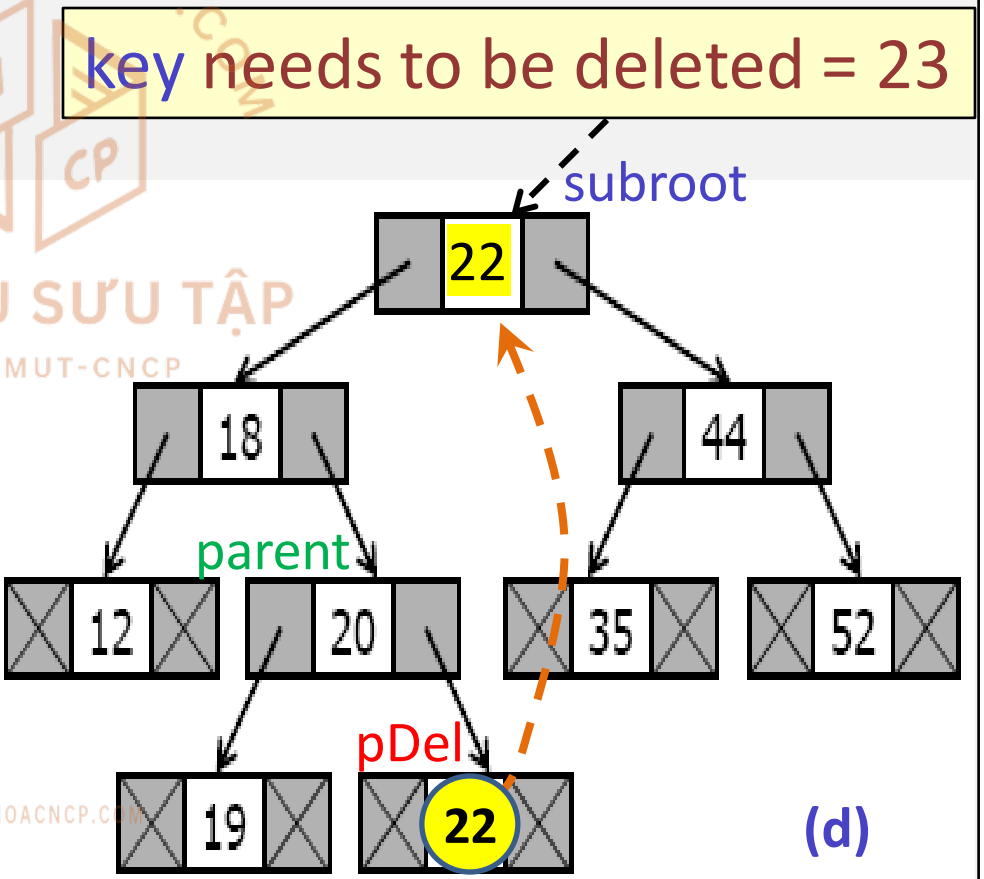
End RemoveNode



# Auxiliary Function RemoveNode (cont.)

1. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel
    8. return *success*

End RemoveNode



# Auxiliary Function RemoveNode (cont.)

1. else // node having both subtrees. **(d)**
    1. parent = subroot
    2. pDel = parent ->left // move left to find the predecessor.
    3. **loop** (pDel->right is not NULL) // pDel is not the predecessor
        1. parent = pDel
        2. pDel = pDel->right
    4. subroot->data = pDel->data
    5. if (parent = subroot)
        1. parent->left = pDel->left
    6. else
        1. parent->right = pDel->left
    7. recycle pDel
    8. return *success*

End RemoveNode