

Artificial Intelligence

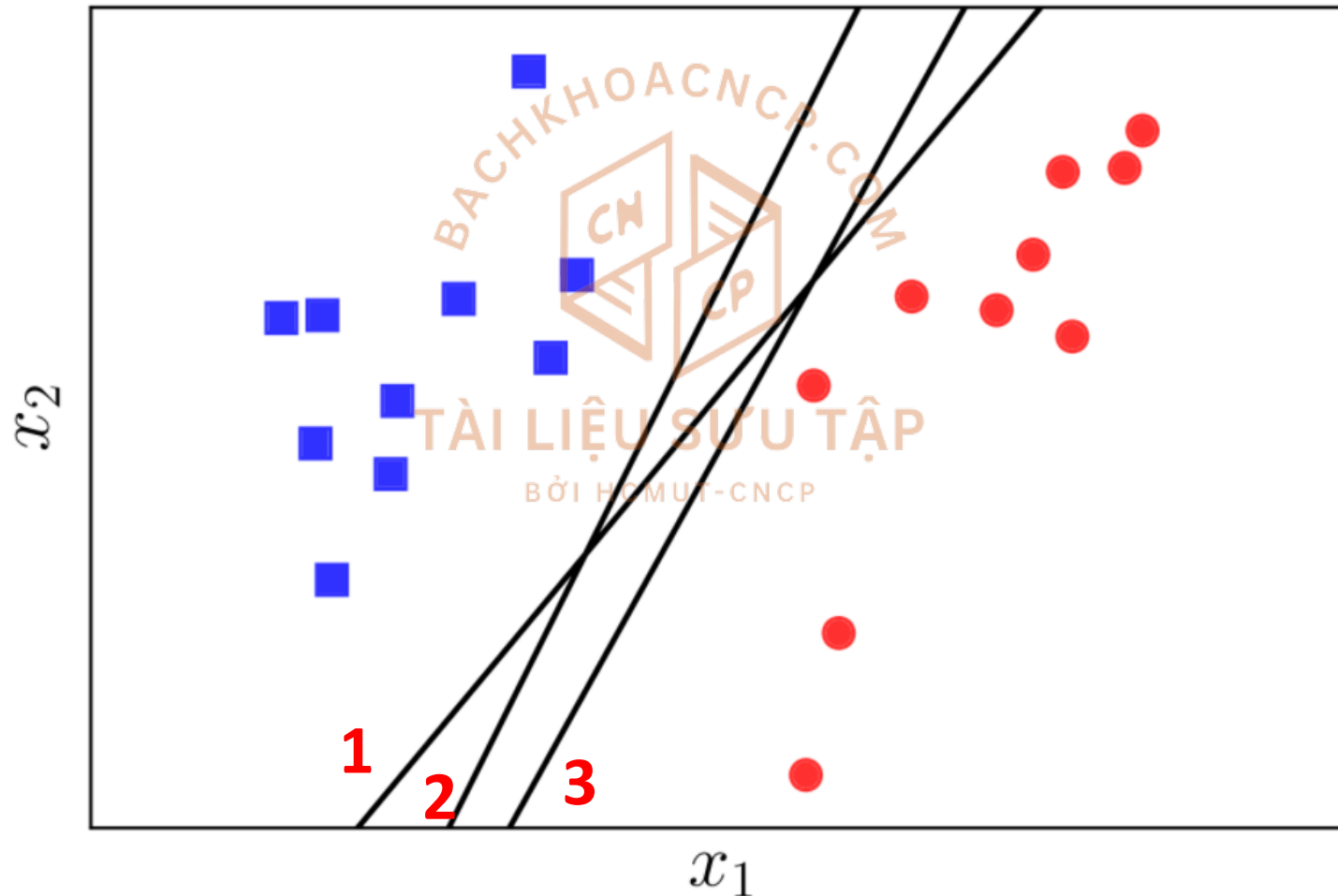
Soft margin

Support Vector Machine

A watermark logo is centered over the text. It consists of a 3D cube with the letters 'CH' on the front face and 'CP' on the right face. Above the cube, the text 'BACH KHOA CNCP.COM' is written in an arc. Below the cube, the text 'TÀI LIỆU SƯU TẬP' is written in a horizontal line, and 'BỞI HCMUT-CNCP' is written in a smaller horizontal line below that.

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- ✓ Two-class classification problem with linearly separable data



✓ Optimization problem

$$(\mathbf{w}, b) = \arg \max_{\mathbf{w}, b} \left\{ \min_n \frac{y_n(\mathbf{w}^T \mathbf{x}_n + b)}{\|\mathbf{w}\|_2} \right\}$$

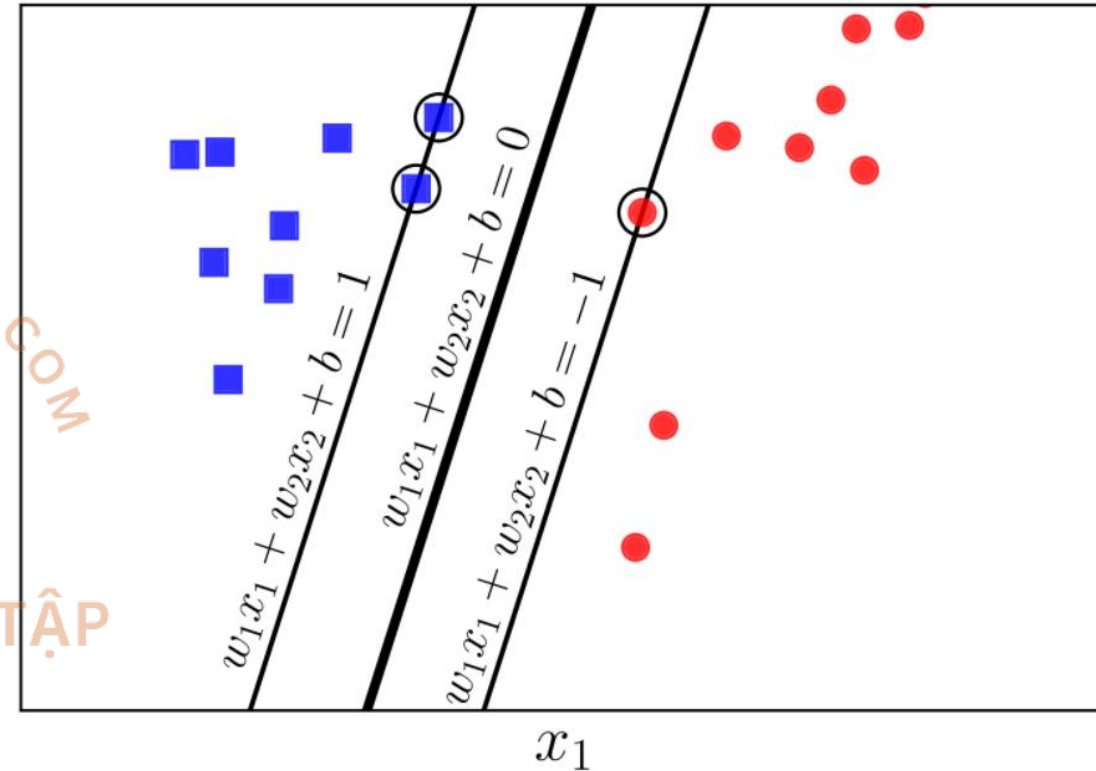
$$\arg \max_{\mathbf{w}, b} \left\{ \frac{1}{\|\mathbf{w}\|_2} \min_n y_n(\mathbf{w}^T \mathbf{x}_n + b) \right\}$$

✓ Assume that

$$\min y_n(\mathbf{w}^T \mathbf{x}_n + b) = 1$$

$$(\mathbf{w}, b) = \arg \max_{\mathbf{w}, b} \frac{1}{\|\mathbf{w}\|_2} \text{ subject to: } y_n(\mathbf{w}^T \mathbf{x}_n + b) \geq 1, \forall n = 1, 2, \dots, N$$

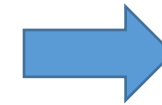
$$(\mathbf{w}, b) = \arg \min_{\mathbf{w}, b} \frac{1}{2} \|\mathbf{w}\|_2^2 \text{ subject to: } 1 - y_n(\mathbf{w}^T \mathbf{x}_n + b) \leq 0, \forall n = 1, 2, \dots, N$$



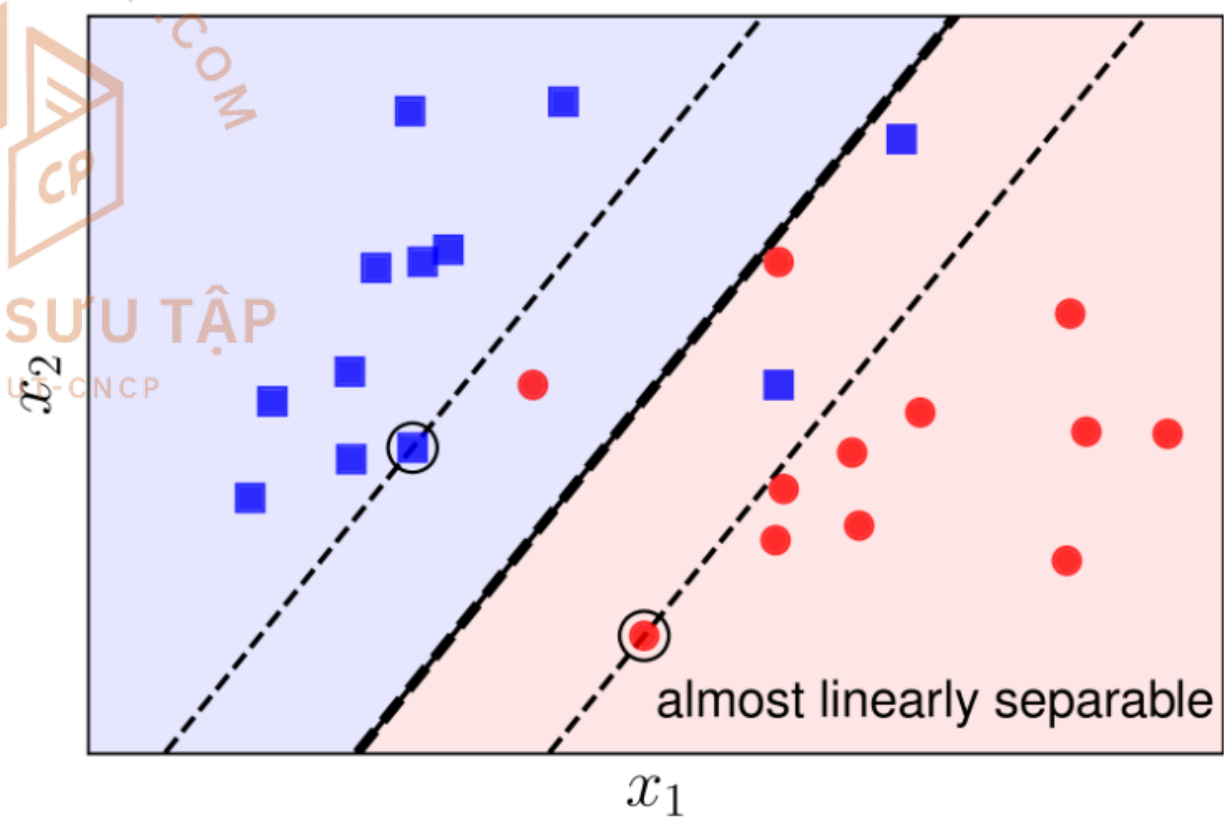
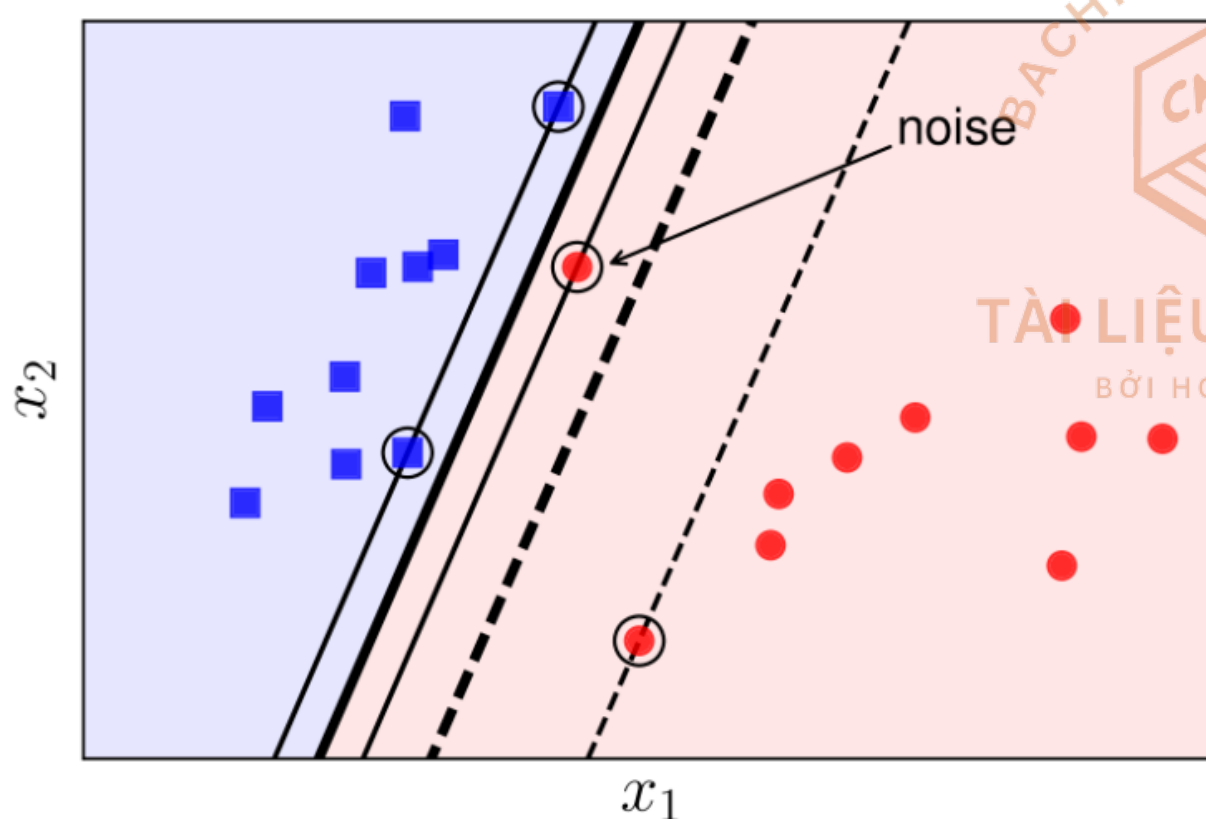
✓ Two-class classification problem

❖ Noise

❖ Non linearly separable data



Soft margin SVM





Objective



Large margin



Small violations



Slack variable

Without violation: $\xi_n = 0$ 

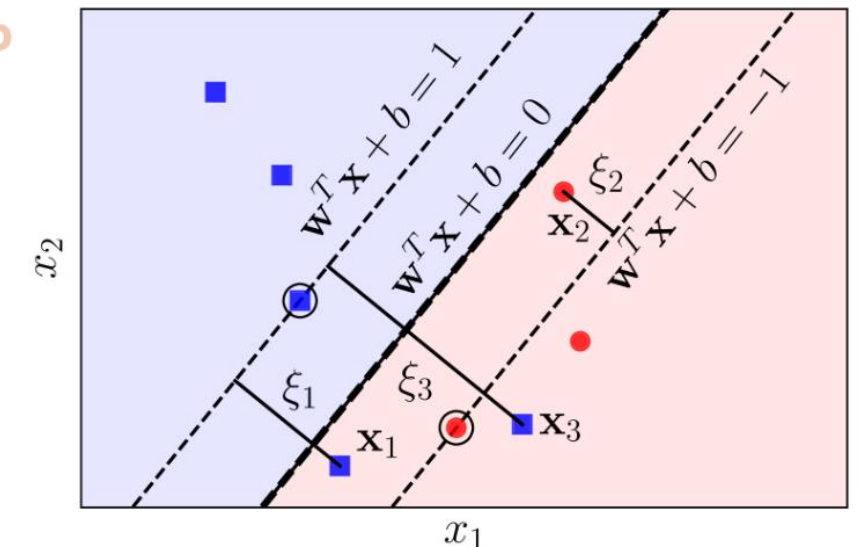
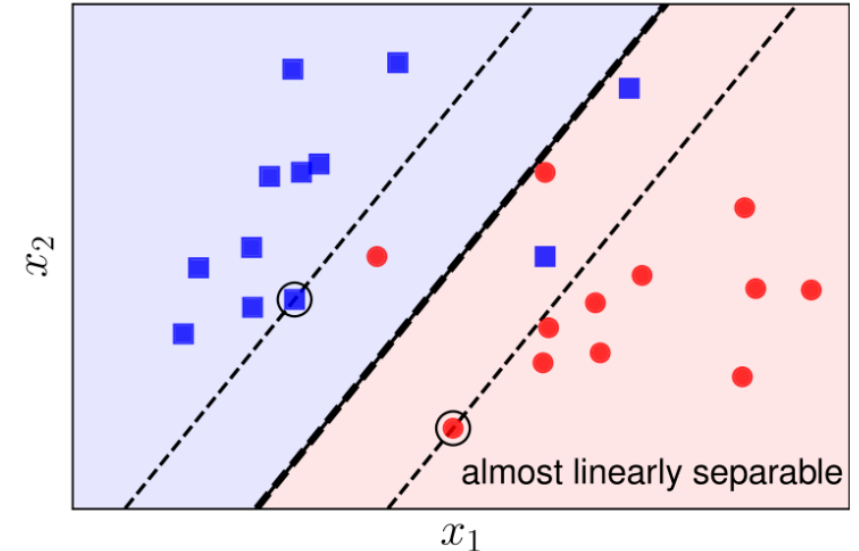
With violation:

$$0 < \xi_n < 1$$

or

$$\xi_n > 1$$

$$\xi_i = |\mathbf{w}^T \mathbf{x}_i + b - y_i|$$

TÀI LIỆU SƯU TẬP
BỞI HCMUT-CNCP

✓ SVM $(\mathbf{w}, b) = \arg \min_{\mathbf{w}, b} \frac{1}{2} \|\mathbf{w}\|_2^2$

subject to: $y_n(\mathbf{w}^T \mathbf{x}_n + b) \geq 1, \forall n = 1, 2, \dots, N$

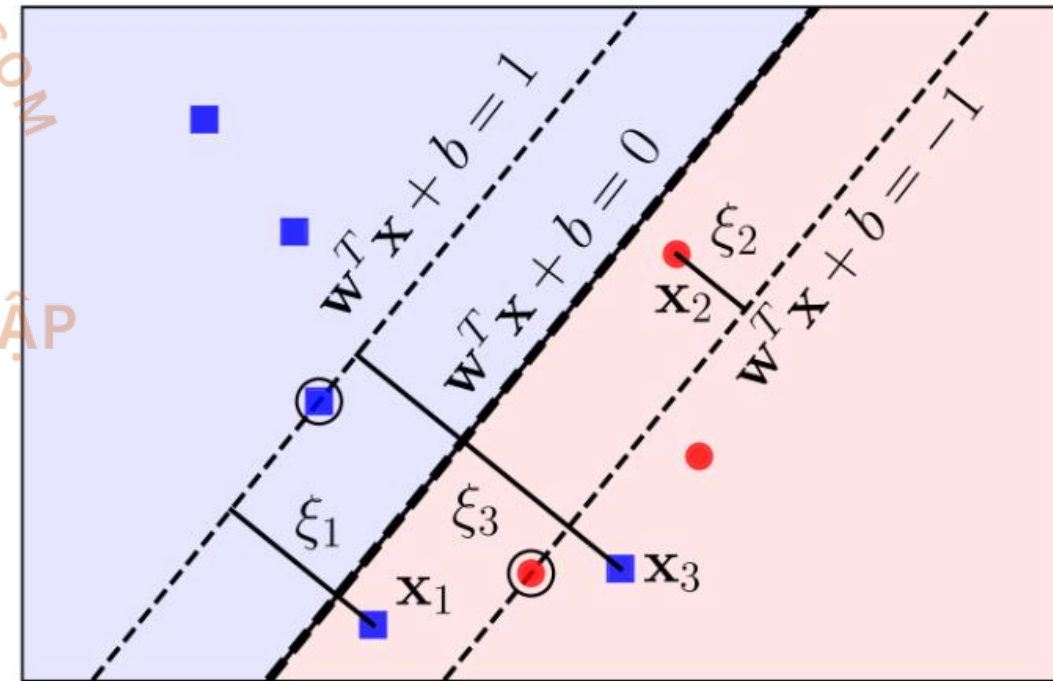
✓ Soft margin SVM

❖ Objective function:

$$\frac{1}{2} \|\mathbf{w}\|_2^2 + C \sum_{n=1}^N \xi_n$$

❖ Constraint:

$$y_n(\mathbf{w}^T \mathbf{x}_n + b) \geq 1 - \xi_n$$



$$1 - \xi_n - y_n(\mathbf{w}^T \mathbf{x}_n + b) \leq 0, \quad \forall n = 1, 2, \dots, n \quad \xi_n \geq 0, \quad \forall n = 1, 2, \dots, N$$

- ✓ Soft margin SVM
 - ❖ Objective function

$$\frac{1}{2} \|\mathbf{w}\|_2^2 + C \sum_{n=1}^N \xi_n$$

- ❖ How to choose C?

