Artificial Intelligence

Perceptron Learning Algorithm

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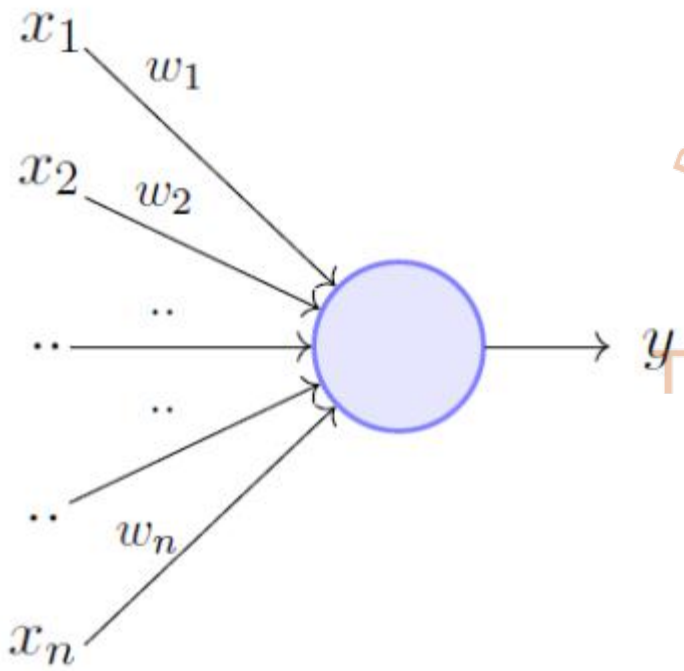
Perceptron Learning Algorithm

- ✓ Proposed by Minsky and Papert in 1969



$$y = 1 \quad if \sum_{i=1}^{n} w_i * x_i \geq \theta$$

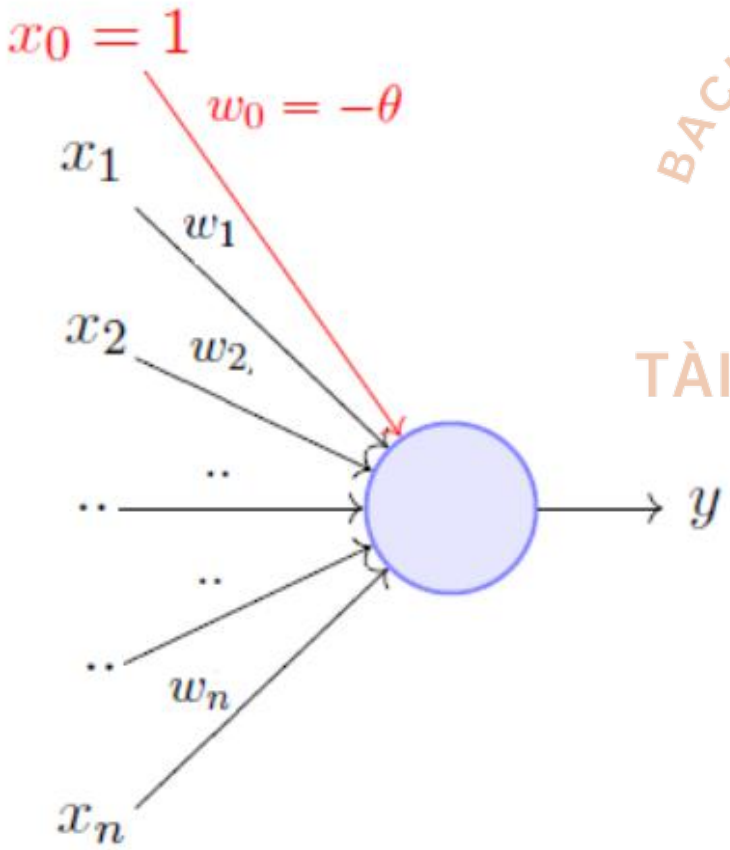
$$= 0 \quad if \sum_{i=1}^{n} w_i * x_i < \theta$$

$$y = 1 \quad if \sum_{i=1}^{n} w_i * x_i - \theta \geq 0$$

$$= 0 \quad if \sum_{i=1}^{n} w_i * x_i - \theta < 0$$

Perceptron Learning Algorithm

- A single perceptron can only be used to implement linearly separable functions



$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

$$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$$

Perceptron Learning Algorithm

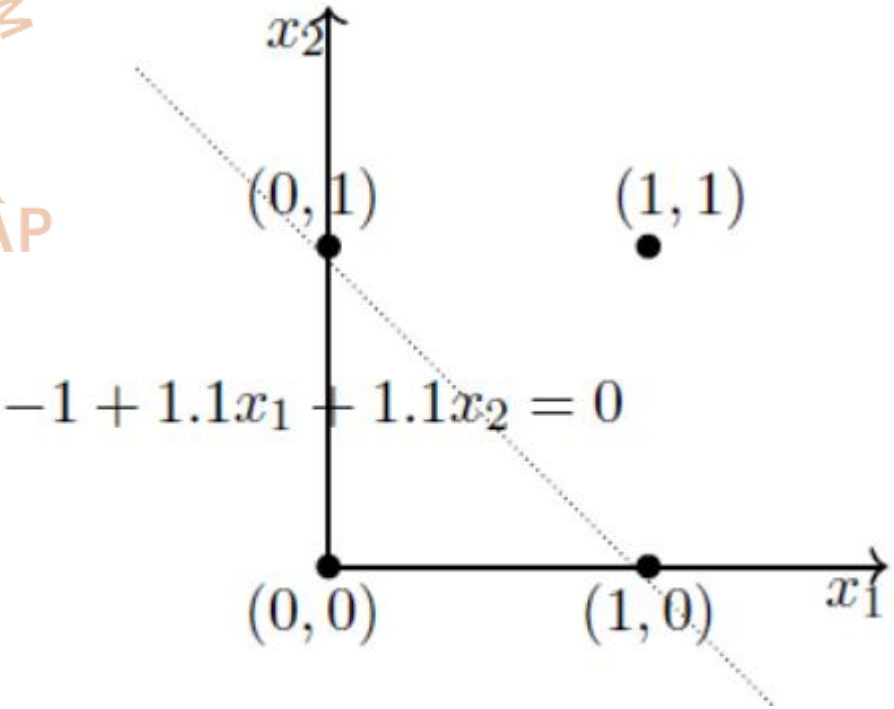
- ✓ OR function using a Perceptron

x_1	x_2	OR	
0	0	0	$w_0 + \sum_{i=1}^{2} w_i x_i < 0$
1	0	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$
0	1	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$
1	1	1	$w_0 + \sum_{i=1}^{2} w_i x_i \geq 0$

$$w_0 + w_1 \cdot 0 + w_2 \cdot 0 < 0 \implies w_0 < 0$$
$$w_0 + w_1 \cdot 0 + w_2 \cdot 1 \geq 0 \implies w_2 > -w_0$$
$$w_0 + w_1 \cdot 1 + w_2 \cdot 0 \geq 0 \implies w_1 > -w_0$$
$$w_0 + w_1 \cdot 1 + w_2 \cdot 1 \geq 0 \implies w_1 + w_2 > -w_0$$

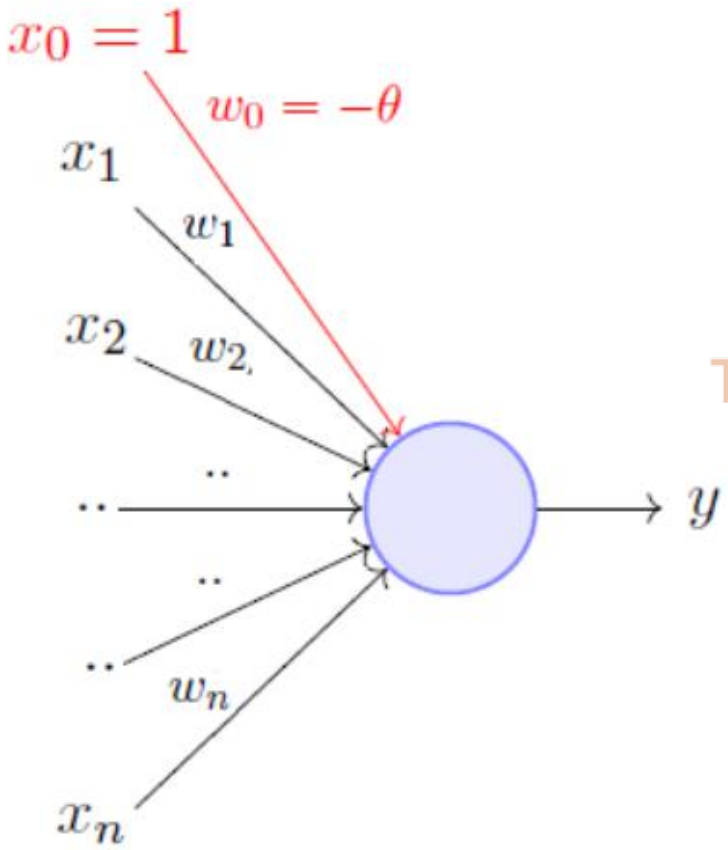
One possible solution is

$$w_0 = -1,\ w_1 = 1.1,\ w_2 = 1.1$$



Perceptron Learning Algorithm

- ✓ Dot product of 2 vectors



$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

$$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$$

$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$

$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$

$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w}^T \mathbf{x} = \sum_{i=0}^{n} w_i * x_i$$

Perceptron Learning Algorithm

- Algorithm
  - Goal: Find vector w that can perfectly classify positive inputs and negative inputs

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

$$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$$

Algorithm: Perceptron Learning Algorithm

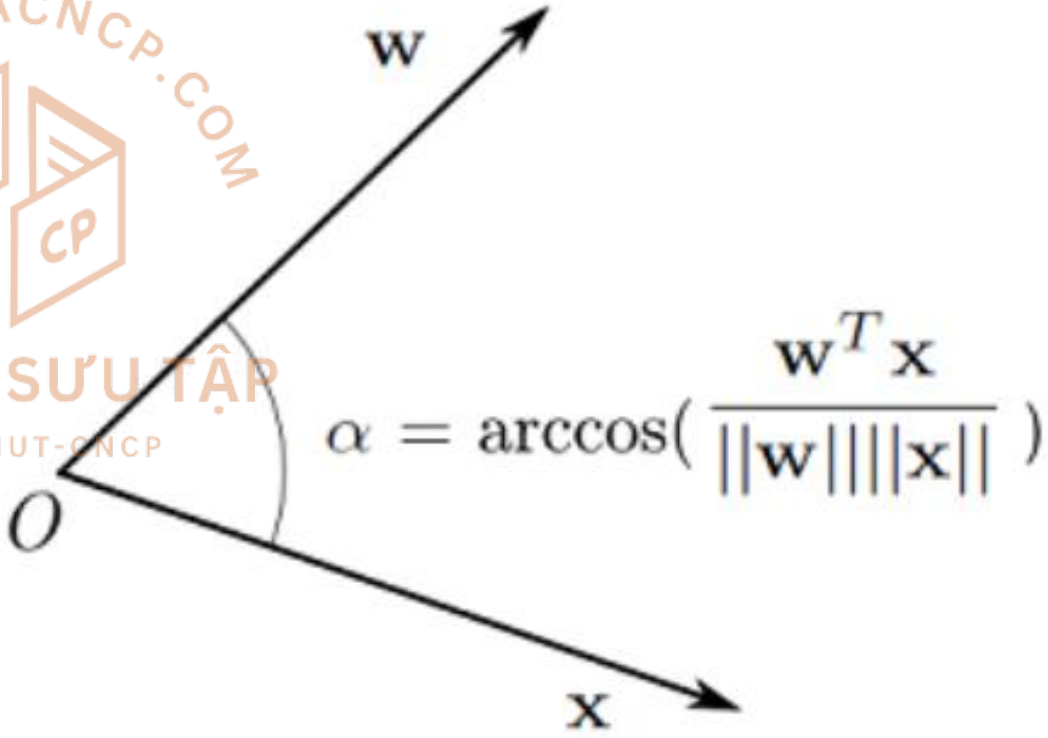
```
P ← inputs with label 1;
N ← inputs with label 0;
Initialize w randomly;
while !convergence do
    Pick random x ∈ P ∪ N ;
    if x ∈ P and w.x < 0 then
        w = w + x ;
    end
    if x ∈ N and w.x ≥ 0 then
        w = w − x ;
    end
end
//the algorithm converges when all the
 inputs are classified correctly
```

Perceptron Learning Algorithm

- ✓ Angle between 2 vectors

$$\mathbf{w}^T\mathbf{x} = \|\mathbf{w}\|\|\mathbf{x}\| cos\alpha$$

$$cos\alpha = \frac{\mathbf{w}^T\mathbf{x}}{\|\mathbf{w}\|\|\mathbf{x}\|}$$



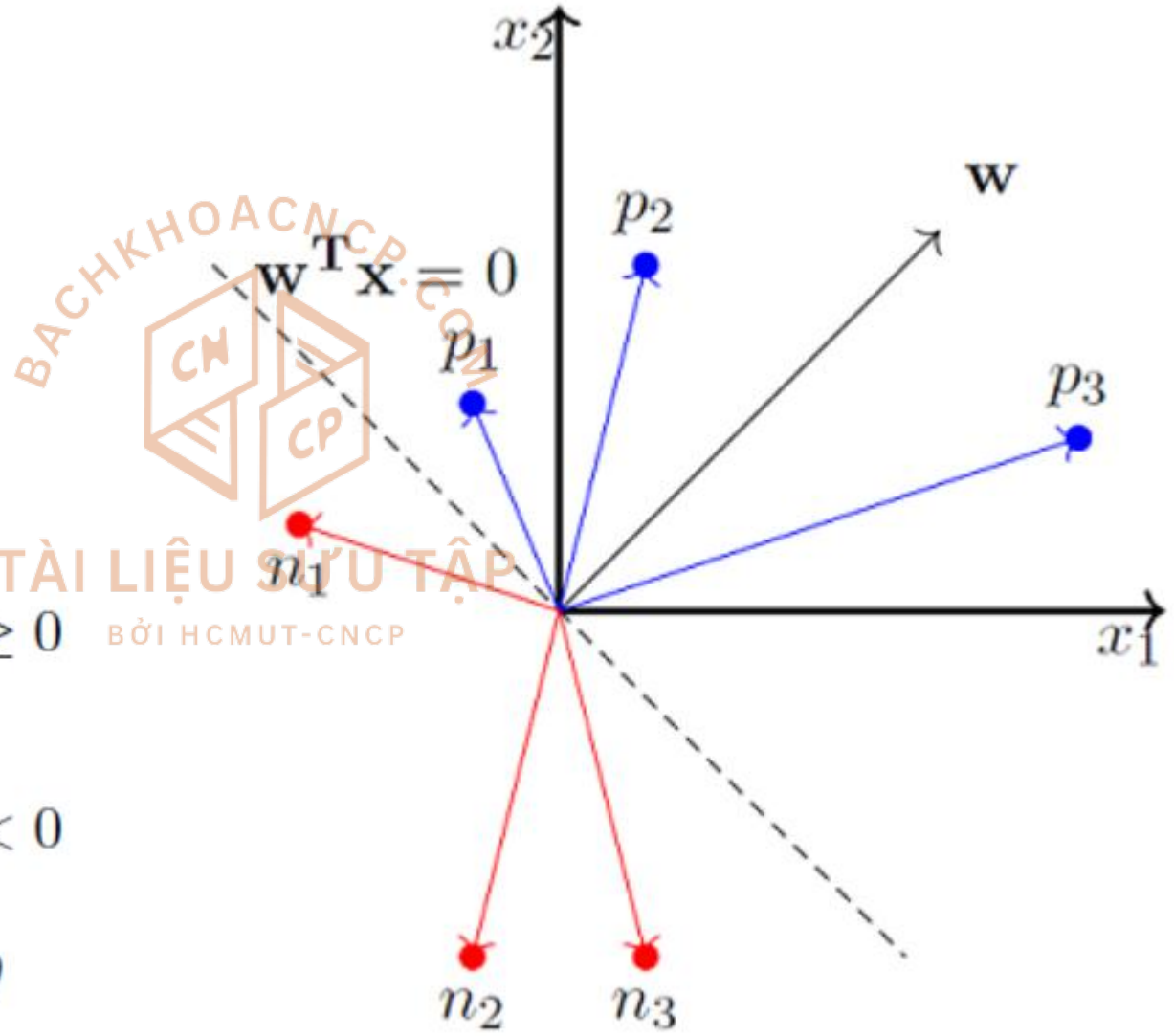
Perceptron Learning Algorithm

- When x belongs to P
  - We want **w**.**x** ≥ 0
  - $\alpha \leq 90^0$
- When x belongs to N
  - We want **w**.**x** < 0
  - $\alpha > 90^0$

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

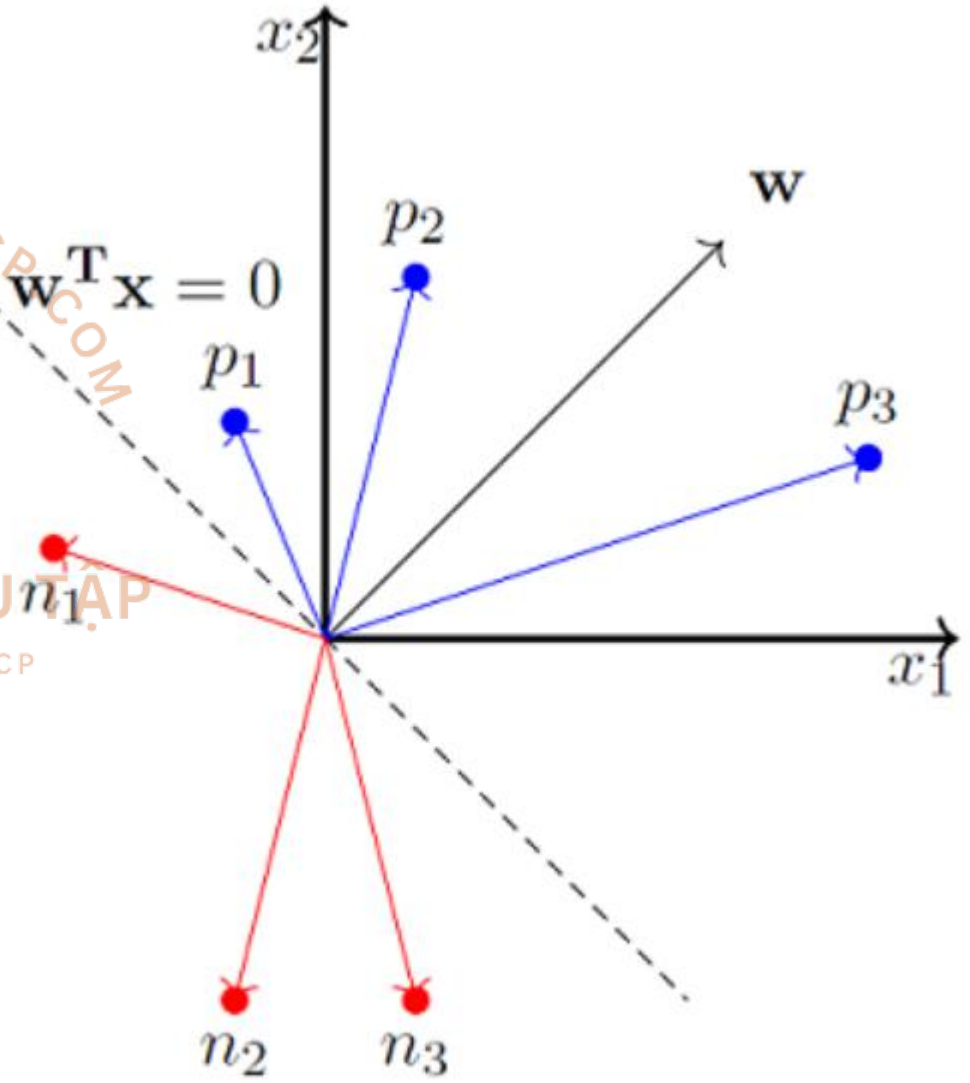
$$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$$



Perceptron Learning Algorithm

Algorithm: Perceptron Learning Algorithm

$P \leftarrow$ *inputs with label* 1;
$N \leftarrow$ *inputs with label* 0;
Initialize $\mathbf{w}$ randomly;
while !*convergence* **do**
  Pick random $\mathbf{x} \in P \cup N$;
  if $\mathbf{x} \in P$ *and* $\mathbf{w}.\mathbf{x} < 0$ **then**
    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
  end
  if $\mathbf{x} \in N$ *and* $\mathbf{w}.\mathbf{x} \geq 0$ **then**
    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
  end
end
//the algorithm converges when all the inputs are classified correctly



Perceptron Learning Algorithm

- ✓ Animation

Perceptron Learning Algorithm

- References
  - https://towardsdatascience.com/perceptron-learning-algorithm-d5db0deab975