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# Chapter 4

Automata

Mathematical Modeling

(Materials drawn from this chapter in:

- Peter Linz. An Introduction to Formal Languages and Automata, (5th Ed.), Jones & Bartlett Learning, 2011.

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullamn. *Introduction to Automata Theory, Languages, and Computation* (3rd Ed.), Prentice Hall, 2006.
- Antal Iványi *Algorithms of Informatics*, Kempelen Farkas Hallgatói Információs Központ, 2011. )

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#### **Course outcomes**

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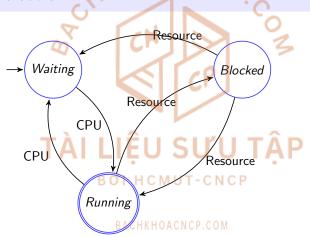
|       |   | _                              |
|-------|---|--------------------------------|
|       | Course learning outcomes \( \( \) \( \)                           |                                |
|       | THOUSE  | BK                             |
| L.O.1 | Understanding of predicate logic                                  | IP.HCM                         |
|       | L.O.1.1 – Give an example of predicate logic                      |                                |
|       | L.O.1.2 – Explain logic expression for some real problems         | C                              |
|       | L.O.1.3 – Describe logic expression for some real problems        | Contents                       |
|       | m (r) 2   | - Motivation                   |
| L.O.2 | Understanding of deterministic modeling using some discrete       | Alphabets, words and languages |
|       | structures  | Regular expression or          |
|       | L.O.2.1 – Explain a linear programming (mathematical statement)   | rationnal expression           |
|       | L.O.2.2 – State some well-known discrete structures               | Non-deterministic              |
|       | L.O.2.3 – Give a counter-example for a given model                | finite automata                |
|       | L.O.2.4 – Construct discrete model for a simple problem           | Deterministic finite           |
|       | ۸ ۸ . ۸   | automata                       |
| L.O.3 | Be able to compute solutions, parameters of models based on data  | Recognized languages           |
|       | L.O.3.1 - Compute/Determine optimal/feasible solutions of integer | Determinisation                |
|       | linear programming models, possibly utilizing adequate libraries  | Minimization                   |
|       | L.O.3.2 - Compute/ optimize solution models based on automata,    | DFAs combination               |
|       | , possibly utilizing adequate libraries                           | _Some applications             |
|       |   | _                              |

#### Introduction

#### Standard states of a process in operating system

• O with label: states

• →: transitions



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## Why study automata theory?

 $\Delta C M$ 

#### A useful model

for many important kinds of software and hardware

- designing and checking the behaviour of digital circuits
- 2 lexical analyser of a typical compiler: a compiler component that breaks the input text into logical units
- 3 scanning large bodies of text, such as collections of Web pages, to find occurrences of words, phrases or other patterns
- 4 verifying pratical systems of all types that have a finite number of distinct states, such as communications protocols and other protocols for securely information exchange, etc.

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#### Alphabets, symbols

#### Definition

Alphabet  $\Sigma$  (bảng chữ cái) is a finite and non-empty set of symbols (or characters).

## For example:

- $\Sigma = \{a, b\}$
- The binary alphabet:  $\Sigma = \{0, 1\}$
- The set of all lower-case letters:  $\Sigma = \{a, b, \dots, z\}$
- The set of all ASCII characters.

#### Remark

 $\Sigma$  is almost always all available characters (lowercase letters, capital letters, numbers, symbols and special characters such as space or newline).

But nothing prevents to imagine other sets.

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## Strings (words)

#### Definition

- A string/word u ( $chu\tilde{\delta}i/t\tilde{u}$ ) over  $\Sigma$  is a finite sequence (possibly empty) of symbols (or characters) in  $\Sigma$ .
- A empty string is denoted by ε.
- The length of the string u, denoted by |u|, is the number of characters.
- All the strings over Σ is denoted by Σ\*.
- A language L over  $\Sigma$  is a sub-set of  $\Sigma^*$ .

#### Remark

The purpose aims to analyze a string of  $\Sigma^*$  in order to know whether it belongs or not to E.

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Let  $\Sigma = \{0, 1\}$ 

- $\varepsilon$  is a string with length of 0.
- 0 and 1 are the strings with length of 1.
- 00, 01, 10 and 11 are the strings with length of 2.
- ullet  $\emptyset$  is a language over  $\Sigma$  . It's called the empty language.

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## **Let** $\Sigma = \{0, 1\}$

- $\varepsilon$  is a string with length of 0.
- 0 and 1 are the strings with length of 1.
- 00, 01, 10 and 11 are the strings with length of 2.
- $\emptyset$  is a language over  $\Sigma$  . It's called the empty language.
- $\Sigma^*$  is a language over  $\Sigma$  . It's called the universal language.
- $\{\varepsilon\}$  is a language over  $\Sigma$  .
- $\{0,00,001\}$  is also a language over  $\Sigma$
- The set of strings which contain an odd number of 0 is a language over Σ.
- The set of strings that contain as many of 1 as 0 is a language over Σ.

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#### String concatenation

Intuitively, the concatenation of two strings 01 and 10 is 0110. Concatenating the empty string s and the string 110 is the string 110.

#### **Definition**

String concatenation is an application of  $\Sigma^* \times \Sigma^*$  to  $\Sigma^*$ . Concatenation of two strings u and v in  $\Sigma$  is the string u.v.

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#### Languages

#### **Specifying languages**

A language can be specified in several ways:



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#### Languages

#### **Specifying languages**

A language can be specified in several ways:

- a enumeration of its words, for example:
  - $L_1 = \{\varepsilon, 0, 1\}$
  - $L_2 = \{a, aa, aaa, ab, ba\}$ ,
  - $L_3 = \{\varepsilon, ab, aabb, aaabbb, aaaabbbb, \ldots\}$



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#### Languages

#### **Specifying languages**

A language can be specified in several ways:

- a enumeration of its words, for example:
  - $L_1 = \{\varepsilon, 0, 1\},$
  - $L_2 = \{a, aa, aaa, ab, ba\}$ ,
  - $L_3 = \{\varepsilon, ab, aabb, aaabbb, aaaabbbb, \ldots\}$
- **b** a property, such that all words of the language have this property but other words have not, for example:
  - $L_4 = \{a^n b^n | n = 0, 1, 2, \ldots\},\$
  - $L_5 = \{uu^{-1} | u \in \Sigma^*\}$  with  $\Sigma = \{a, b\}$ ,
  - $L_6 = \{u \in \{a,b\}^* | n_a(u) = n_b(u)\}$  where  $n_a(u)$  denotes the number of letter 'a' in word u.

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#### **Specifying languages**

A language can be specified in several ways:

- a enumeration of its words, for example:
  - $L_1 = \{\varepsilon, 0, 1\}$
  - $L_2 = \{a, aa, aaa, ab, ba\},$
  - $L_3 = \{\varepsilon, ab, aabb, aaabbb, aaaabbb, \dots \}$
- b a property, such that all words of the language have this property but other words have not, for example:
  - $L_A = \{a^n b^n | n = 0, 1, 2, \ldots \}$
  - $L_5 = \{uu^{-1} | u \in \Sigma^*\}$  with  $\Sigma = \{a, b\}$ ,
  - $L_6 = \{u \in \{a,b\}^* | n_a(u) = n_b(u)\}$  where  $n_a(u)$  denotes the number of letter 'a' in word u.
- c its grammar, for example:
  - Let G=(N,T,P,S) where  $N=\{S\},\ T=\{a,b\},\ P=\{S\rightarrow aSb,S\rightarrow ab\}$ i.e.  $L(G) = \{a^n b^n | n \ge 1\}$  since  $S \Rightarrow aSb \Rightarrow a^2Sb^2 \Rightarrow \dots \Rightarrow a^nSb^n$

## **Operations on languages**

## L, $L_1$ , $L_2$ are languages over $\Sigma$

union

$$L_1 \cup L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ or } u \in L_2 \},$$

intersection

$$L_1 \cap L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ and } u \in L_2 \},$$

difference

$$L_1 \setminus L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ and } u \not\in L_2 \},$$

• complement

$$\overline{L} = \Sigma^* \setminus L$$
,

multiplication

$$L_1L_2 = \{uv \mid u \in L_1, v \in L_2\},\$$

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#### **Operations on languages**

## L, $L_1$ , $L_2$ are languages over $\Sigma$

union

$$L_1 \cup L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ or } u \in L_2 \},$$

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$$L_1 \cap L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ and } u \in L_2 \},$$

difference

$$L_1 \setminus L_2 = \{ u \in \Sigma^* \mid u \in L_1 \text{ and } u \notin L_2 \},$$

• complement

$$\overline{L} = \Sigma^* \setminus L$$
,

• multiplication

$$L_1L_2 = \{uv \mid u \in L_1, v \in L_2\},\$$

power

$$L^0=\{\varepsilon\}, \qquad L^n=L^{n-1}L$$
 , if  $n\geq 1$  ,

• iteration or star operation

$$L^* = igcup_{i=0}^\infty L^i = L^0 \cup L \cup L^2 \cup \dots \cup L^i \cup \dots$$
 ,

We will use also the notation  $L^+$ 

$$L^+ = \bigcup_{i=1}^{n} L^i = L \cup L^2 \cup \dots \cup L^i \cup \dots.$$
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The union, product and iteration are called regular operations.

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Let  $\Sigma = \{a, b, c\}$ ,  $L_1 = \{ab, aa, b\}$ ,  $L_2 = \{b, ca, bac\}$ 

- a  $L_1 \cup L_2 =?$ , Q
- **b**  $L_1 \cap L_2 = ?$ ,
- **c**  $L_1 \setminus L_2 = ?$ ,
- **d**  $L_1L_2 = ?$ ,
- e  $L_2L_1 = ?$ .

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Let  $\Sigma = \{a, b, c\}$ ,  $L_1 = \{ab, aa, b\}$ ,  $L_2 = \{b, ca, bac\}$ ①  $L_1 \cup L_2 = \{ab, aa, b, ca, bac\}$ ,

- **b**  $L_1 \cap L_2 = \{b\},\$
- **c**  $L_1 \setminus L_2 = \{ab, aa\},\$
- $\mathbf{d}$   $L_1L_2 = \{abb, aab, bb, abca, aaca, bca, abbac, aabac, bbac\},$

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# HOACNO

Let 
$$\Sigma = \{a, b, c\}$$
,  $L_1 = \{ab, aa, b\}$ ,  $L_2 = \{b, ca, bac\}$ 

- a  $L_1 \cup L_2 = \{ab, aa, b, ca, bac\},\$
- **b**  $L_1 \cap L_2 = \{b\}$
- **c**  $L_1 \setminus L_2 = \{ab, aa\},$
- **d**  $L_1L_2 = \{abb, aab, bb, abca, aaca, bca, abbac, aabac, bbac\}$ ,

Let 
$$\Sigma = \{a, b, c\}$$
 and  $L = \{ab, aa, b, ca, bac\}$   $L^2 = ?$ 

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Let 
$$\Sigma=\{a,b,c\}$$
,  $L_1=\{ab,aa,b\}$ ,  $L_2=\{b,ca,bac\}$ 

- **a**  $L_1 \cup L_2 = \{ab, aa, b, ca, bac\},$
- **b**  $L_1 \cap L_2 = \{b\},\$
- **c**  $L_1 \setminus L_2 = \{ab, aa\},\$

## Let $\Sigma = \{a,b,c\}$ and $L = \{ab,aa,b,ca,bac\}$

 $L^2 = u.v$ , with  $u, v \in L$  including the following strings:

- abab, abaa, abb, abca, abbac,
  - aaab, aaaa, aab, aaca, aabac,
  - bab, baa, bb, bca, bbac, B ♂ I H C M U T C N C P
  - caab, caaa, cab, caca, cabac,
- bacab, bacaa, bacb, bacca, bacbac.

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## Let $\Sigma = \{a, b, c\}$

#### Give at least 5 strings for each of the following languages

- 1 all strings with exactly one 'a'.
- 2 all strings of even length.
- 3 all strings which the number of appearances of 'b' is divisible by 3.
- 4 all strings ending with 'a'.
- all non-empty strings not ending with 'a'.
- 6 all strings with at least one 'a'.
- 7 all strings with at most one 'a'.
- 8 all strings without any 'a'.
- 9 all strings including at least one 'a' and whose the first appearance of 'a' is not followed by 'c'.

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## Let $\Sigma = \{a, b, c\}$ and $L = \{ab, aa, b, ca, bac\}$

Which of the following strings are in  $L^*$ ?

- $aaa = a^3$ .
- $\mathfrak{Z}$   $abaabaaabaa = aba^2ba^3ba^2$ ,
- € bbb.
- $\mathbf{S} = aab$ .
- **ॐ** cc,
- $30 \ aaaabaaaa = a^4ba^4$ .
- **3**  $cabbbbaaaaaaaaab = cab^4a^9b$ ,  $cabbbbaaaaabaaaab = ba^5ba^4b$ ,  $cabbbbaaaaabaaaab = ba^5ba^4b$ ,
- $\mathfrak{F}$  baaaaabaa $c=ba^5ba^2c$ , B  $\mathfrak{O}I$  H C M U T C N C P
- 1 baca.

## Regular expressions (biểu thức chính quy)

Permit to specify a language with strings consist of letters and  $\varepsilon$ , parentheses (), operating symbols +, ., \*. This string can be empty, denoted  $\emptyset$ .

## Regular operations on the languages

- union ∪ or +
- product of concatenation
- transitive closure \*



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## Regular expressions (biểu thức chính quy)

Permit to specify a language with strings consist of letters and  $\varepsilon$ , parentheses (), operating symbols +, ., \*. This string can be empty, denoted  $\emptyset$ .

## Regular operations on the languages

- union ∪ or +
- product of concatenation
- transitive closure \*

## Example on the aphabet set $\Sigma = \{a, b\}$

- $(a+b)^*$  represent all the strings MUT-CNCP
- $a^*(ba^*)^*$  represent the same language
- $(a+b)^*aab$  represent all strings ending with aab.

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- ullet is a regular expression representing the empty language.
- ullet arepsilon is a regular expression representing language  $\{arepsilon\}_{\sim}$
- If  $a \in \Sigma$ , then a is a regular expression representing language  $\{a\}$ .
- If x, y are regular expressions representing languages X and Y respectively, then (x+y), (xy),  $x^*$  are regular expression representing languages  $X \cup Y$ , XY and  $X^*$  respectively.



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- Ø is a regular expression representing the empty language.
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- If  $a \in \Sigma$ , then a is a regular expression representing language  $\{a\}$ .
- If x, y are regular expressions representing languages X and Y respectively, then (x+y), (xy),  $x^*$  are regular expression representing languages  $X \bigcup Y$ , XY and  $X^*$  respectively.

$$x + y \equiv y + x$$

$$(x + y) + z \equiv x + (y + z)$$

$$(xy)z \equiv x(yz)$$

$$(x + y)z \equiv xz + yz$$

$$x(y + z) \equiv xy + xz$$

$$(x + y)^* \equiv (x^* + y)^* \equiv (x + y^*)^* \equiv (x^* + y^*)^*$$

$$(x + y)^* \equiv x^*$$

$$(x^*)^* \equiv x^*$$

$$x^*x \stackrel{\text{BACH KH}}{=} xx^*$$

$$xx^* + \varepsilon \equiv x^*$$

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#### Kleene's theorem

Language  $L\subseteq \Sigma^*$  is regular if and only if there exists a regular expression over  $\Sigma$  representing language L.

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## Let $\Sigma = \{a, b, c\}$

Give at least 3 words for each language represented by the following regular expressions

- **1**  $E_1 = a^* + b^*$
- $E_2 = a^*b + b^*a$
- 3  $E_3 = b(ca + ac)(aa)^* + a^*(a+b)$
- **4**  $E_4 = (a^*b + b^*a)^*$ .

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$$a^*b - \int b \ ab \ a^2b$$

$$a^*b = \{b, ab, a^2b, a^3b, \dots, aaa \dots ab\},\$$

example 
$$a^*b = \int b \ ab \ a^2b \ a^3b$$

## Let $\Sigma = \{a, b, c\}$

Determine regular expression presenting for each of the following languages.

- 1 all strings with exactly one 'a'.
- 2 all strings of even length.
- 3 all strings which the number of appearances of b' is divisible by 3.
- 4 all strings ending with 'a'.
- 6 all non-empty strings not ending with a.
- 6 all strings with at least one 'a'.
- 7 all strings with at most one 'a'.
- 8 all strings without any 'a'.
- all strings including at least one 'a' and whose the first appearance of 'a' is not followed by a 'c'.

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Let  $\Sigma = \{a, b, c\}$  and  $L = \{ab, aa, b, ca, bac\}$ 

Which languages represented by the following regular expressions are in  $L^*$ ?

- 1  $E_1 = a^* + ba$ ,
- $E_2 = b^* + a^*aba^*$
- 3  $E_3 = aab + cab^*ac$ .
- 4  $E_4 = b(ca + ac)(aa)^* + a^*(a+b),$
- **6**  $E_5 = (a^4ba^3)^{2*}c$ ,
- **6**  $E_6 = b^+ ac \ (b^+ = bb^*),$
- $E_7 = (b+c)ab + ba(c+ab)^*$ . SUUTAP
- 8  $E_8 = (b+c)^*ab + a(c+a)^*$

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TVHoai, HTNguyen, NAKhuong, LHTrang



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Let  $\Sigma = \{a, b, c\}$  and  $L = \{ab, aa, b, ca, bac\}$ 

Which languages represented by the following regular expressions are in  $L^*$ ?

- 1  $E_1 = a^* + ba$ ,
- $2 E_2 = b^* + a^* a b a^*,$
- $3 E_3 = aab + cab^*ac,$
- $E_4 = b(ca + ac)(aa)^* + a^*(a+b),$
- **5**  $E_5 = (a^4ba^3)^{2*}c$ ,
- **6**  $E_6 = b^+ ac \ (b^+ = bb^*),$
- $E_7 = (b+c)ab + ba(c+ab)^*$ ,
- $8 E_8 = (b+c)^* ab + a(c+a)^*.$

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Define a (simple) regular expression representing the language  $L^*$ .

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## Simplify each of the following regular expressions

- **2**  $E_2 = \underline{a}^*(\underline{b} + ab^*),$
- $3 E_3 = \varepsilon + ab + abab(\underline{ab})^*,$
- $E_5 = aa(b^* + a) + a(ab^* + aa),$
- **6**  $E_6 = (a^*(ba)^*)^*(b+\varepsilon)$ ,
- 7  $E_7 = a(a+b)^* + aa(a+b)^* + aaa(a+b)^*$

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#### Finite automata

#### Finite automata (Automat hữu hạn)

- The aim is representation of a process system.
- It consists of states (including an initial state and one or several (or one) final/accepting states) and transitions (events).
- The number of states must be finite.



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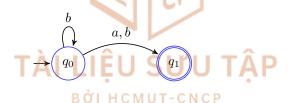
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#### Finite automata

#### Finite automata (Automat hữu hạn)

- The aim is representation of a process system.
- It consists of states (including an initial state and one or several (or one) final/accepting states) and transitions (events).
- The number of states must be finite.



## **Regular expression**

 $b^*(a+b)$ 

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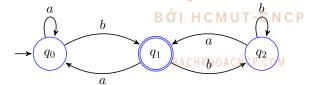
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## Let $\Sigma = \{a, b\}$

#### Which of the strings

- $\mathbf{1}$   $a^3b$ .
- $aba^2b$ ,
- $3 a^4b^2ab^3a$ ,
- $a^4ba^4$ ,
- **5**  $ab^4a^9b$ .
- **6**  $ba^5ba^4b$ ,
- $ba^5b^2$ .
- $8 bab^2a$

## are accepted by the following finite automata?



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## Let $\Sigma = \{a, b, c\}$

#### Propose FA presenting each of the following languages

- 1 all strings with exactly one 'a'.
- 2 all strings of even length.
- 3 all strings which the number of appearances of 'b' is divisible by 3.
- 4 all strings ending with 'a'.
- 5 all non-empty strings not ending with 'a'.
- 6 all strings with at least one 'a'.
- 7 all strings with at most one 'a'.
- 8 all strings without any 'a'.
- 9 all strings including at least one 'a' and whose the first appearance of 'a' is not followed by a 'c'.

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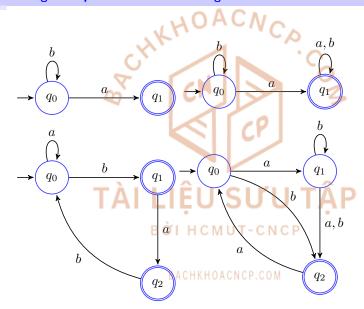
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# Give regular expression for the following finite automata.



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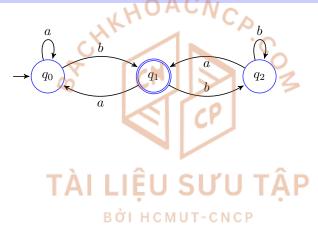
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# Give regular expression for the following finite automata.



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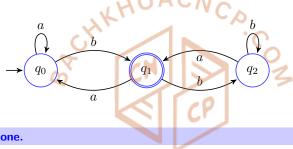
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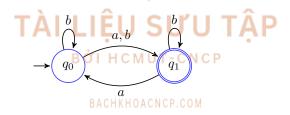
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# Give regular expression for the following finite automata.



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# Nondeterministic finite automata

# Definition

A nondeterministic finite automata (NFA,  $Automat\ h\tilde{w}u\ han\ phi\ don\ dinh)$  is mathematically represented by a 5-tuples  $(Q,\Sigma,q_0,\delta,F)$  where

- Q a finite set of states.
- Σ is the alphabet of the automata.
- $q_0 \in Q$  is the initial state.
- $\delta: Q \times \Sigma \to Q$  is a transition function.
- $F \subseteq Q$  is the set of final/accepting states.

## Remark

According to an event, a state may go to one or more states.

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# NFA with empty symbol $\varepsilon$

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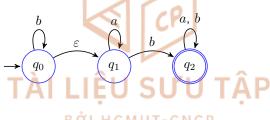
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# KHOACNCD

# Other definition of NFA

Finite automaton with transitions defined by character x (in  $\Sigma$ ) or empty character  $\varepsilon$ .



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# CHKHOACNCO

Consider the set of strings on  $\{a,b\}$  in which every aa is followed immediately by b.

For example *aab*, *aaba*, *aabaabbaab* are in the language, but *aaab* and *aabaa* are not.

Construct an accepting NFA.

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CHKHOACNCD

Let  $\Sigma = \{a, b, c\}$ 

# Propose NFA presenting each of the following languages

- 1 all strings with exactly one 'a'.
- 2 all strings of even number of appearances of b'.
- 3 all strings which the number of appearances of b is divisible by 3.

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# Let $\Sigma = \{a, b, c\}$

Construct an accepting finite automata for languages represented by the following regular expressions.

- $\bullet \ E_1 = a^*c + b^*a,$
- $\bullet \ E_2 = b^*ab + a^*aba^*,$
- $E_3 = aab + cab^*ac$ ,
- $E_4 = b(ca + ac)(aa)^* + a^*(a+b)$ ,
- $E_5 = (ab)^{2*}c + bac$ ,
- $E_6 = bb^*ac + b^*a$ ,
- $E_7 = (b+c)ab + ba(c+ab)^*$ ,
- $E_8 = (b+c)^*ba + a(c+a)^*$ ,
- $E_9 = [a(b+c)^* + bc^*]^* B \circ I H C M U T C N C P$
- $E_{10} = b^*ac + bb^*a$ .

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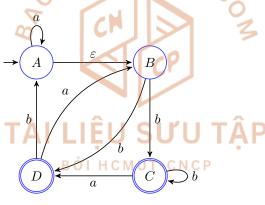
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# Let $\Sigma = \{a, b\}$

Give 3 valid strings & 5 invalid strings in language  $L^2$ , with L represented by the following finite automata.



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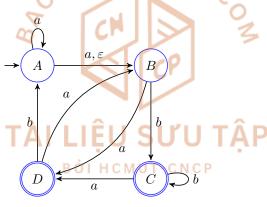
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# Let $\Sigma = \{a, b\}$

Give 3 valid strings & 5 invalid strings in language  $L^2$ , with L represented by the following finite automata.



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# **Deterministic finite automata**

# Automata TVHoai, HTNguyen, NAKhuong, LHTrang



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### **Definition**

A deterministic finite automata (**DFA**, Automat hữu hạn đơn định) is given by a 5-tuplet  $(Q, \Sigma, q_0, \delta, F)$  with

- Q a finite set of states.
- Σ is the input alphabet of the automata.
- $q_0 \in Q$  is the initial state.
- $\delta: Q \times \Sigma \to Q$  is a transition function.
- $F \subseteq Q$  is the set of final/accepting states.

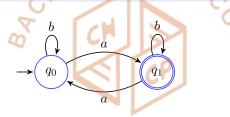
# **Condition**

Transition function  $\delta$  is an application  $\Lambda$  UT-CNCP

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# Let $\Sigma = \{a, b\}$

Hereinafter, a deterministic and complete automata that recognizes the set of strings which contain an odd number of a.



- $Q = \{q_0, q_1\},$   $\delta(q_0, a) = q_1, \ \delta(q_0, b) = q_0, \ \delta(q_1, a) = q_0, \ \delta(q_1, b) = q_1,$
- $F = \{q_1\}.$

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# Configurations and executions

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Let  $A = (Q, \Sigma, q_0, \delta, F)$ 

A configuration ( $c\hat{a}u \ hinh$ ) of automata A is a couple (q,u) where  $q \in Q$  and  $u \in \Sigma^*$ .

We define the relation  $\rightarrow$  of derivation between configurations:  $(q, a.u) \rightarrow (q', u)$  iif  $\delta(q, a) = q'$ 

An execution (thực thi) of automata A is a sequence of configurations

 $(q_0, u_0) \dots (q_n, u_n)$  such that  $(q_i, u_i) \to (q_{i+1}, u_{i+1})$ , for  $i = 0, 1, \dots, n-1$ .

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# KHOACNCX

**Let**  $\Sigma = \{0, 1\}$ 

- Give a DFA that accepts all words that contain a number of 0 multiple of 3.
- Give an execution of this automata on 1101010.

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# KHOACNCX

# **Let** $\Sigma = \{0, 1\}$

- Give a DFA that accepts all words that contain a number of 0 multiple of 3.
- Give an execution of this automata on 1101010.

# Let $\Sigma = \{a, b\}$

- Give a DFA that accepts all strings containing 2 characters a.
- Give an execution of this automata on aabb, ababb and bbaa.

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# **Recognized languages**

# MOACNCA

## **Definition**

A language L over an alphabet  $\Sigma$ , defined as a sub-set of  $\Sigma^*$ , is recognized if there exists a finite automata accepting all strings of L.

# **Proposition**

If  $L_1$  and  $L_2$  are two recognized languages, then

- $L_1 \cup L_2$  and  $L_1 \cap L_2$  are also recognized;
- ullet  $L_1.L_2$  and  $L_1^*$  are also recognized.

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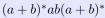
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# HOACN

# **Sub-string** ab

Construct a DFA that recognizes the language over the alphabet  $\{a,b\}$  containing the sub-string ab.

# **Regular expression**





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# **Sub-string** *ab*

Construct a DFA that recognizes the language over the alphabet  $\{a,b\}$  containing the sub-string ab.

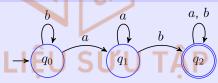
# Regular expression

$$(a+b)^*ab(a+b)^*$$

# Transition table

|                   | a     | b     | 7 |
|-------------------|-------|-------|---|
| $\rightarrow q_0$ | $q_1$ | $q_0$ | / |
| $q_1$             | $q_1$ | $q_2$ | _ |
| $q_2*$            | $q_2$ | $q_2$ |   |





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# Let $\Sigma = \{a, b, c\}$

# Propose DFA presenting each of the following languages

- 1 all strings which the number of appearances of 'aa' and the one of 'b' are the same.
- 2 all strings which the number of appearances of 'a' is equal to the one of 'b' plus the one of 'a'.
- 3 all strings including at least one a and whose the first appearance of a is not followed by a c.
- 4 all strings which the difference between number of appearances of 'a' and the one of 'c' is less than 1.
- $\mathbf{6}$  all strings which there is at least  $\mathbf{6}$  or  $\mathbf{6}$  after  $\mathbf{6}$  or  $\mathbf{6}$  or  $\mathbf{6}$

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# MOACNCA

Let 
$$\Sigma = \{a, b, c\}$$

Construct DFAs that recognize the languages represented by the following regular expressions.

- $E_1 = a^* + b^*a$ ,
- $\bullet E_2 = b^* + a^*aba^*,$
- $E_3 = aab + cab^*ac$ ,
- $E_4 = bb^*ac + b^*a$ ,
- $\bullet E_5 = b^*ac + bb^*a.$

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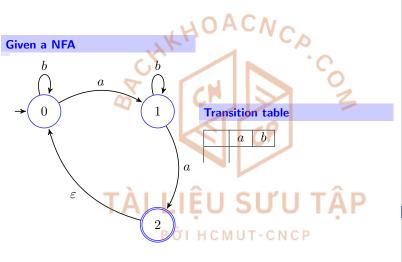
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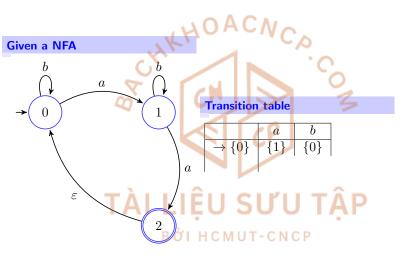
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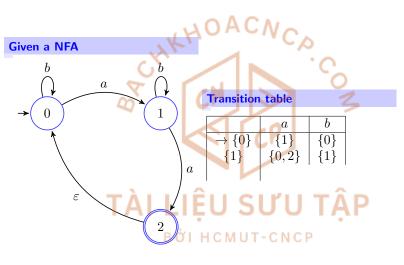
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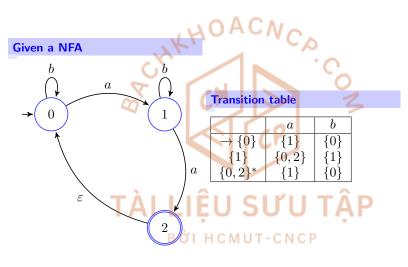
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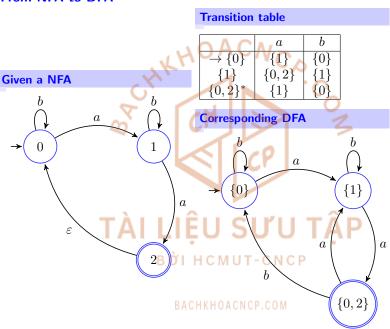
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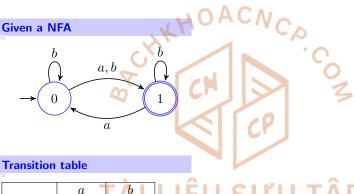
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# Other example of determinisation



|                     | a          | b          |
|---------------------|------------|------------|
| $\rightarrow \{0\}$ | {1}        | $\{0, 1\}$ |
| {1}*                | {0}        | {1}        |
| $\{0,1\}^*$         | $\{0, 1\}$ | $\{0,1\}$  |

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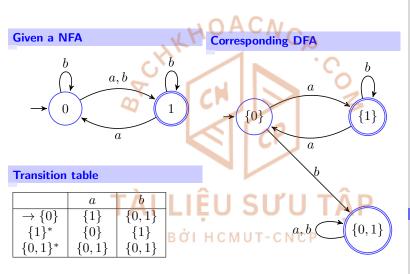
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# Other example of determinisation



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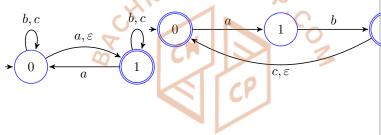
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Let  $\Sigma = \{a, b, c\}$ 

Determine DFAs which corresponds to the following NFAs:



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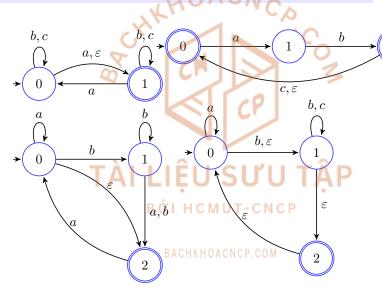
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 ${\sf DFAs\ combination}$ 

# Let $\Sigma = \{a, b, c\}$

# Determine DFAs which corresponds to the following NFAs:



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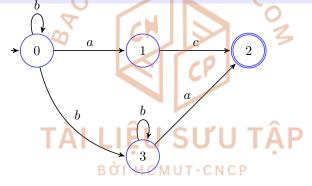
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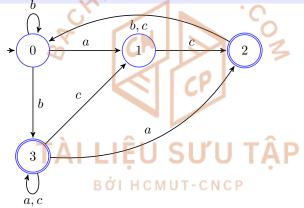
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# Let $\Sigma = \{a, b, c\}$

# Determine DFAs which corresponds to the following NFAs:



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# DACNCA

Let 
$$\Sigma = \{a, b, c\}$$

Determine finite automata, not necessarily deterministic, recognizing the following languages:

- $L_1 = \{a, ab, ca, cab, acc\}$ ,
- $L_2 = \{$  set of words of even number of  $a\}$ ,
- $L_3 = \{$  set of words containing ab and ending with  $b\}$ .

Then, determine the corresponding complete DFAs.

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# Let $\Sigma = \{a, b, c\}$

# Construct DFAs for languages represented by following expressions.

- $E_1 = a^* + b^*a$ ,
- $E_2 = b^* + a^*aba^*$
- $E_3 = (aab + ab^*)^*$ ,
- $E_4 = b(ca + ac)(aa)^* + a^*(a+b)$ ,
- $E_5 = ba^*b + baa + baba$ ,
- $E_6 = (ba^*b + baa + baaba)^*$ ,
- $E_7 = ba^*b + baa + aba(a+b)^*$ ,
- $E_9 = [a(b+c)^* + bc^*]^*$ ,
- $\bullet E_{10} = bb^*ac + ba^*b.$
- $E_{11} = bb^*ac + b^*a$ , BÖI HCMUT-CNCP
- $E_{12} = (b+c)ab + ba(c+ab)^*$ ,
- $E_{13} = (b+c)^*ba + a(c+a)^*$

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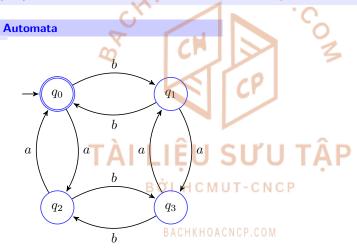
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Determine a DFA that recognizes the language over the alphabet  $\{a,b\}$  with an even number of a and an even number b.



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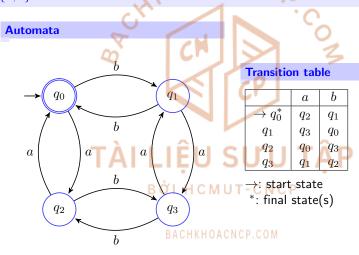
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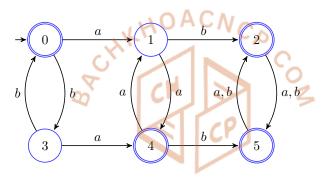
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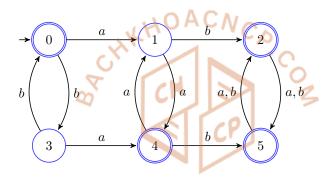
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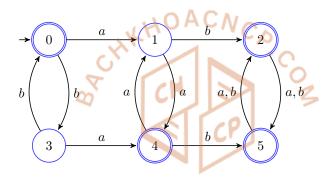
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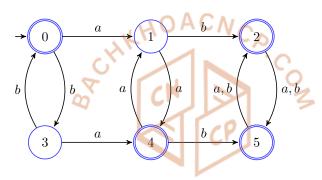
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## equivalence relationships

| s             | 0 | 4 | 2     | 3   | 4  | 5   |
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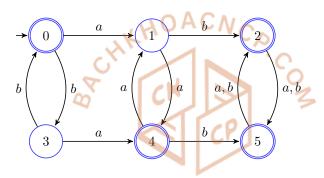
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## equivalence relationships

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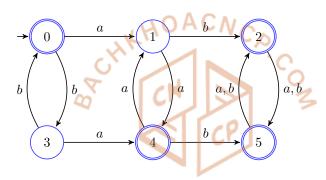
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## equivalence relationships

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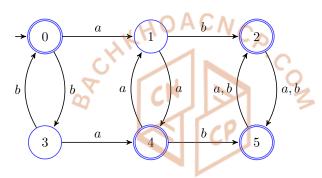
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## equivalence relationships

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| cl(s.b) | П  | I   | I     |    |       |    |

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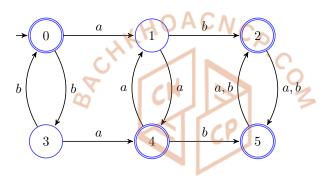
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| s             | 0 | 1   | 2     | 3   | 4     | 5   |
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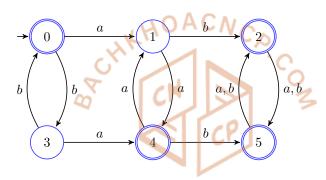
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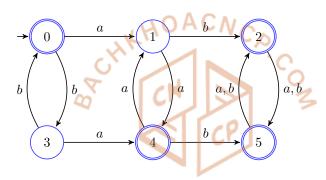
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| s             | 0    | 1  | 2     | 3  | 4   | 5     |
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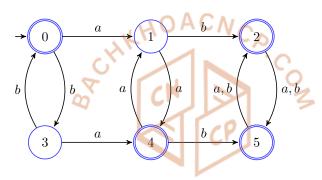
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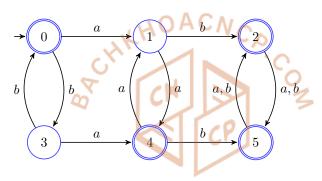
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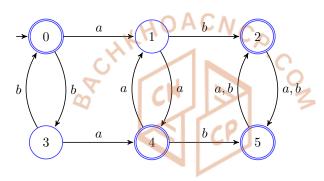
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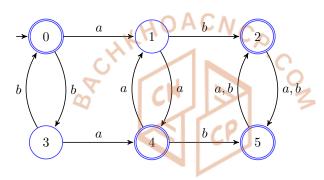
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| s       | 0 | 1 / | 2 | 3 | 4   | 5  |
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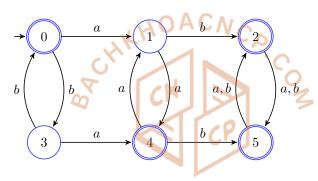
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| s       | 0 | 1 / | 2 | 3  | 4   | 5   | 0     | V    | 2     | 3    | 4  | 5 |
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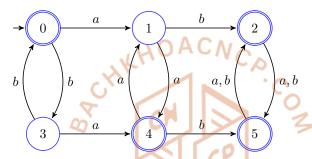
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## equivalence relationships

| s       | 0   | 1   | 2          | 3 | 4  | 5   | 0 | 1  | 2   | 3    | 4             | 5       |
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| cl(s.b) | Ш   | 1 / |            |   |    | -1- | 1 | Ш  | 111 | 17   | 111           | Ш       |

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|---------|---|-----------------|------|--------------|-------|-----|
| s       | 0 | 1               | 2    | 3            | 4     | 5   |
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| cl(s.b) |   | III             | III  |              | III   | III |

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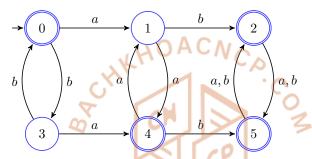
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## equivalence relationships

| s       | 0 | 1   | 2          | 3 | 4 | 5   | 0 | 1  | 2   | 3    | 4             | 5     |
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| cl(s.b) |   | III | Ш    |          | III    | III  |

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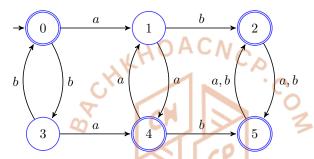
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## equivalence relationships

| s       | 0 | 1   | 2          | 3 | 4 | 5   | 0 | 1    | 2   | 3    | 4             | 5     |
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| cl(s.b) | V | III | Ш    |                   | 111   | III  |

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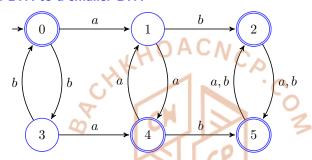
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## Recognized languages

## Determinisation

#### /linimization

## DFAs combination



## equivalence relationships

| s       | 0 | 1   | 2          | 3 | 4 | 5   | 0 | 1    | 2   | 3    | 4             | 5     |
|---------|---|-----|------------|---|---|-----|---|------|-----|------|---------------|-------|
| cl(s)   | ı | Ш   | <u>, I</u> | Ш | L | L   | I | ll _ | Ш   | Ш    | IV            | Ш     |
| cl(s.a) | Ш | T,  | ΔĻ         | I | Ш | - 1 | I | IV   | Ш   | TV / | $\Delta\Pi =$ | ) III |
| cl(s.b) | Ш | 1.1 |            | - | 1 |     | 1 | Ш    | 111 | 1 "  | 111           | Ш     |

| s       | 0 | 1   | 2    | 3    | 4    | 5      |
|---------|---|-----|------|------|------|--------|
| cl(s)   | ı | II  | Ш    | V    | IV   | Ш      |
| cl(s.a) | Ш | ΙVΔ | сикн | OACN | CP.C | 0 1/11 |
| cl(s.b) | V | III | Ш    | ĺ    | III  | Ш      |

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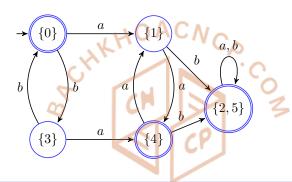
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## equivalence relationships

| - I Δ   |     |       | - 1 |    |    |   |
|---------|-----|-------|-----|----|----|---|
| s       | 0   | 1     | 2   | 3  | 4  | 5 |
| cl(s)   | I p | 8 dFT | HIC | M  |    | Ν |
| cl(s.a) | Ш   | IV    | THE | IV | ll | Ш |
| cl(s.b) | V   | Ш     | Ш   | ı  | Ш  | Ш |

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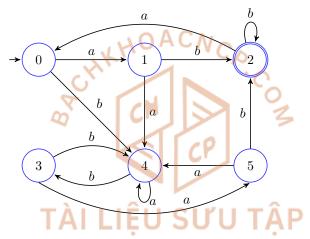
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## 

| s       | 0 | 1 | 2 | 3 | 4  | 5   |
|---------|---|---|---|---|----|-----|
| cl(s)   | I | ı | Ш | Т | I  | ı   |
| cl(s.a) | ı | ı | ı | I | ΒA | CHK |
| cl(s.b) | ı | Ш | Ш | 1 | ı  | Ш   |

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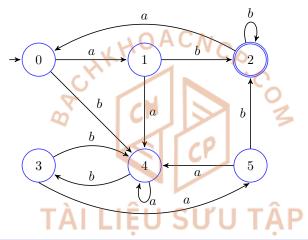
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| s       | 0 | 1 | 2 | 3 | 4  | 5   |
|---------|---|---|---|---|----|-----|
| cl(s)   | ı | ı | Ш | I | ı  | ı   |
| cl(s.a) | ı | ı | ı | I | ΒA | CHK |
| cl(s.b) | ı | Ш | Ш | I | ı  | П   |

| ė |         | 1-6   | NO  |   |   |   |
|---|---------|-------|-----|---|---|---|
|   | 0       | 1     | 2   | 3 | 4 | 5 |
|   | ı       | Ш     | Ш   | ı | ı | Ш |
|   | O AIC N | CPI.C | 0 M | Ш | ı | ı |
|   | -       | П     | Ш   | ı | ı | Ш |

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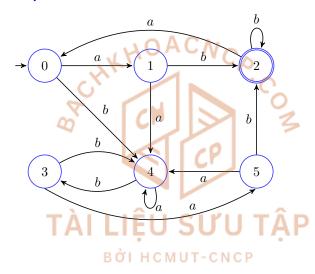
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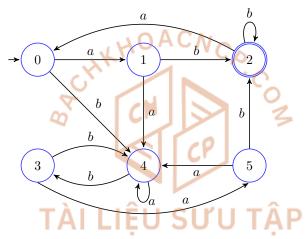
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## equivalence relationships BOI HCMUT-CN

| s       | 0 | 1 | 2 | 3 | 4    | 5   |     |
|---------|---|---|---|---|------|-----|-----|
| cl(s)   | ı | Ш | Ш | ı | ı    | Ш   |     |
| cl(s.a) | Ш | ı | ı | Ш | BIAC | HMH | ) A |
| cl(s.b) |   | Ш | Ш |   | -    | Ш   |     |

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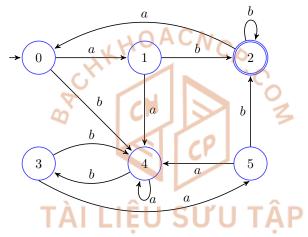
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## equivalence relationships BOI HCMUT-CNCP

| s       | 0 | 1 | 2 | 3 | 4    | 5    | 0       | 1    | 2   | 3  | 4  | 5  |
|---------|---|---|---|---|------|------|---------|------|-----|----|----|----|
| cl(s)   | I | Ш | Ш | I | I    | III  | I       | Ш    | Ш   | I  | IV | Ш  |
| cl(s.a) | Ш | ı | ı | Ш | BIAC | HKH( | A CIN C | PIV0 | M I | Ш  | IV | IV |
| cl(s.b) | ı | Ш | Ш |   |      | II   | IV      | II   | Ш   | IV | I  | Ш  |

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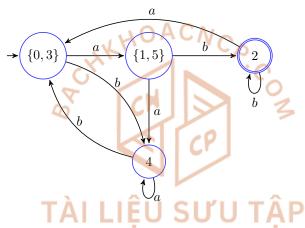
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## equivalence relationships

| equivalen | ce re | lation | ship | s <sub>B</sub> | ďΙ | HCN  | AUT-CI   |
|-----------|-------|--------|------|----------------|----|------|----------|
| s         | 0     | 1      | 2    | 3              | 4  | 5    |          |
| cl(s)     | ı     | III    | Ш    | I              | IV | Ш    |          |
| cl(s.a)   | Ш     | IV     | I    | Ш              | Mc | нКНО | ACNCP.CC |
| cl(s.b)   | IV    | II     | Ш    | IV             | ı  | Ш    |          |

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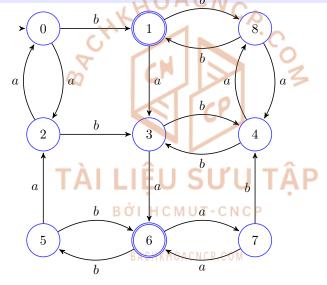
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## Determine minimal DFA which corresponds to the following DFA:



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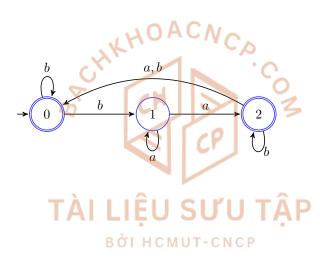
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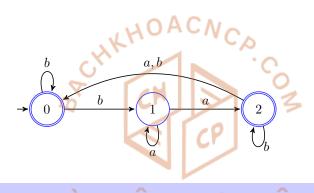
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## Hint

Two-steps approach: (NFA  $\rightarrow$  DFA); min (DFA).

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## $\sigma = \{a, b\}$

Determine minimimal DFA regconized the languages represented by the following regular expressions:

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# HOACNCA

$$\sigma = \{a, b\}$$

Determine minimimal DFA regconized the languages represented by the following regular expressions:

$$1 E_1 = (a+b)^*b(a+b)^*$$

**2** 
$$E_2 = ((a+b)^2)^* + ((a+b)^3)^*$$

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# MOACNCA

$$\sigma = \{a, b\}$$

Determine minimimal DFA regconized the languages represented by the following regular expressions:

$$1 E_1 = (a+b)^*b(a+b)^*$$

**2** 
$$E_2 = ((a+b)^2)^* + ((a+b)^3)^*$$

3 
$$E_3 = ((a+b)^2)^+ + ((a+b)^3)^+$$

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# KHOACNCD

$$\sigma = \{a, b\}$$

Determine minimimal DFA regconized the languages represented by the following regular expressions:

$$\mathbf{1} E_1 = (a+b)^*b(a+b)^*$$

**2** 
$$E_2 = ((a+b)^2)^* + ((a+b)^3)^*$$

3 
$$E_3 = ((a+b)^2)^+ + ((a+b)^3)^+$$

$$4 E_4 = baa^* + ab + (a+b)ab^*.$$

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# KHOACNCD

 $\sigma = \{a, b, c, d\}$ 

Determine minimimal complete DFA regconized the languages consisting of all strings where all a' is followed by a b' and all c' is followed by a b'.

Then, deduce the corresponding regular expressions.

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# KHOACNCD

$$\sigma = \{a, b, c, d\}$$

Determine minimimal complete DFA regconized the languages consisting of all strings where all a' is followed by a b' and all c' is followed by a b'.

Then, deduce the corresponding regular expressions.

## $\sigma = \{a,b\}$

Give a NFA (as simple as possible) for the language defined by the regular expression  $ab^* + a(ba)^*$ . Then determine the equivalent DFA.

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## Let $\Sigma = \{a, b, c\}$

Determine minimimal DFA regconized the languages represented by the following regular expressions:

1 
$$a^* + b^*$$
,

$$a^*b + b^*a$$
,

3 
$$b(ca+ac)(aa)^* + a^*(a+b)$$
,

**4** 
$$(a^*b + b^*a)^*$$
.

**5** 
$$a^*bc + bca^*$$
,

**6** 
$$b(c+c)(aa)^* + (a+c)a^*$$
,

$$\mathbf{0}$$
  $aab + cab^*ac$ ,  $\mathbf{A}$ 

**8** 
$$b(ca + ac)(a)^* + a(a + b)^*$$

**3** 
$$b(ca+ac)(a)^* + a(a+b)^*$$
,  
**9**  $ab(b+c)ab + ba(c+b)^* + (b+c)ab(b+c)$ .

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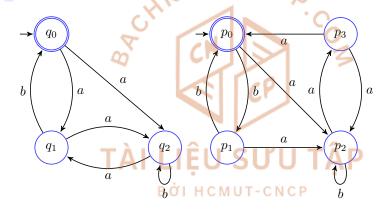
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## **Equivalent automatons**

## Two following automatas are equivalent



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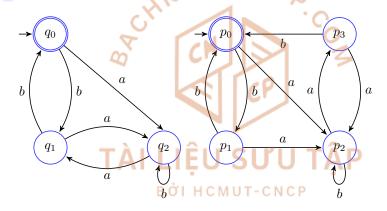
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## **Equivalent automatons**

## Two following DFAs are equivalent?



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## Combination of two automata

## Automata

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## $\Sigma = \{a, b\}$

- **a** Given two languages  $L_a$ ,  $L_b$  defined by regular expressions  $E_a=a(a+b)^*$  and  $E_b=a^*(ba)^*$
- **6)** Give a DFA for the language  $L_a$ ,  $L_b$ .
- Then, determine a (minimized) DFA for the following languages.
  - $1 L_1 = L_a \circ L_b$
  - $2 L_2 = L_a \cap L_b$
  - $3 L_3 = L_a \bigcup L_b$
  - $4 L_4 = L_a \setminus L_b$

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 $\Sigma = \{a,b\}$ 

$$\Sigma = \{a, b\}$$

- **a** Given two languages  $L_a$ ,  $L_b$  defined by regular expressions  $E_a = (a^*b + b^*a)^+$  and  $E_b = (a+b)^*b(a+b)^*a$
- **6)** Give a DFA for the language  $L_a$ ,  $L_b$ .
- (a) Then, determine a (minimized) DFA for the following languages.
  - $1 L_1 = L_a \circ L_b$
  - $2 L_2 = L_a \cap L_b$
  - $3 L_3 = L_a \bigcup L_b$
  - $4 L_4 = L_a \setminus L_b$

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# KHOYCIVC

## $\Sigma = \{a,b\}$

- **a**) Given two languages  $L_a$ ,  $L_b$  defined by regular expressions  $E_a = ab^* + a(ba)^*$  and  $E_b = baa^* + ab + (a+b)ab^*$
- **b** Give a DFA for the language  $L_a$ ,  $L_b$ .
- 1 Then, determine a (minimized) DFA for the following languages.
  - $1 L_1 = L_a \circ L_b$
  - $2 L_2 = L_a \cap L_b$
  - $3 L_3 = L_a \bigcup L_b$
  - $4 L_4 = L_a \setminus L_b$

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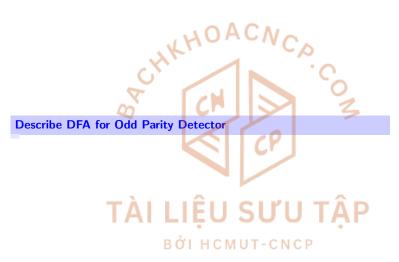
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## **Odd Parity Detector**



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## TCP/IP protocol



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## **Application**

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Propose an automata to describe a vehicular multi-information display system with a given number of buttons.

For example, digital speedo meter of Honda Lead motor with only one button can display information about: petroleum level, speed, trip, date, time, engine oil life.

(Hint: we distinguish two different actions: quickly press the button; press the button and hold-down over two seconds.)

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