ROBOTICS

CHAPTER 3: SPATIAL DESCRIPTIONS AND TRANSFORMATIONS

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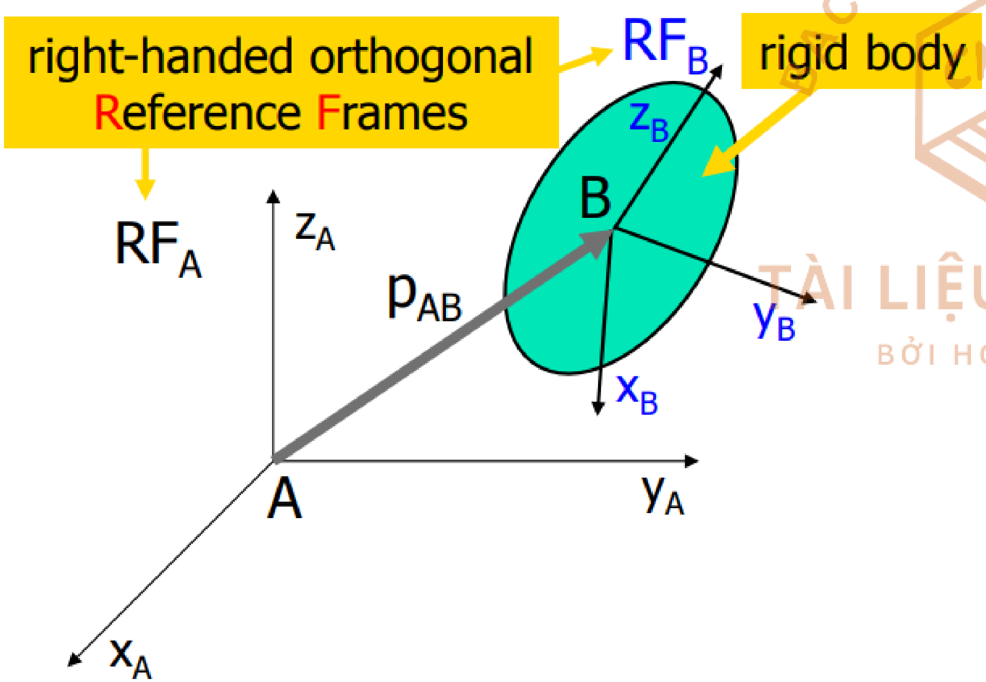
CONTENTS

3.1 POSITION AND ORIENTATION OF RIGID BODIES

POSE OF RIGID BODY

- A rigid body is completely described in space by its **position** and **orientation** (in brief **pose**) with respect to a reference frame.



- **Position** of a point B on the rigid body with respect to the coordinate frame A (RF_A):

$$ {}^{A}P_{AB} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} $$

- **Orientation:** orthonormal 3x3 matrix

$$ {}^{A}R_{B} = \begin{bmatrix} {}^{A}X_{B} & {}^{A}Y_{B} & {}^{A}Z_{B} \end{bmatrix} = \begin{bmatrix} \boxed{X_B.X_A} & Y_B.X_A & Z_B.X_A \\ X_B.Y_A & Y_B.Y_A & Z_B.Y_A \\ X_B.Z_A & Y_B.Z_A & Z_B.Z_A \end{bmatrix} $$

Direction cosine of X_B w.r.t X_A

- $X_A, Y_A, Z_A (X_B, Y_B, Z_B)$: unit vectors(with unitary norm) of $RF_A (RF_B)$

3.1 POSITION AND ORIENTATION OF RIGID BODIES

ROTATION MATRIX

$$ {}^{A}R_{B} = \begin{bmatrix} {}^{A}X_{B} & {}^{A}Y_{B} & {}^{A}Z_{B} \end{bmatrix} = \begin{bmatrix} {}^{B}X_{A}^{T} \\ {}^{B}Y_{A}^{T} \\ {}^{B}Z_{A}^{T} \end{bmatrix} = {}^{B}R_{A}^{T} $$

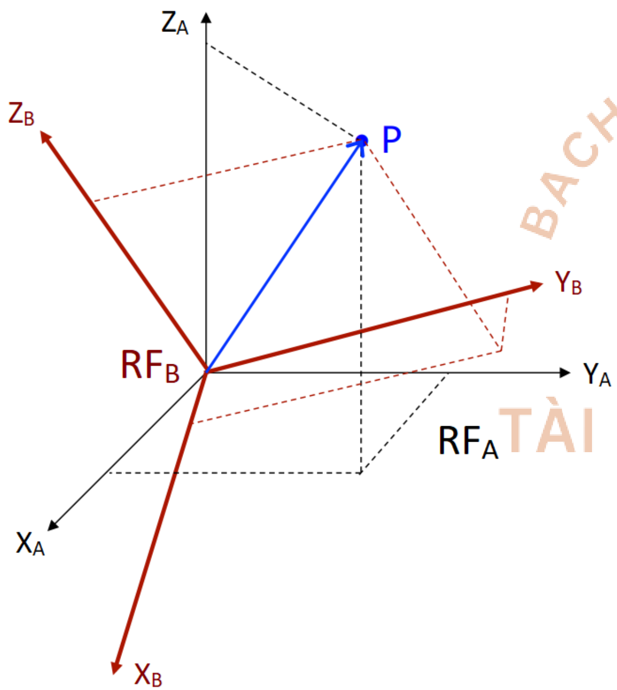
$$ {}^{A}R_{B}^{T}\,{}^{A}R_{B} = \begin{bmatrix} {}^{A}X_{B}^{T} \\ {}^{A}Y_{B}^{T} \\ {}^{A}Z_{B}^{T} \end{bmatrix} \begin{bmatrix} {}^{A}X_{B} & {}^{A}Y_{B} & {}^{A}Z_{B} \end{bmatrix} = I_{3} $$

I_3 is the 3x3 identity matrix. Hence:

$$ {}^{A}R_{B} = {}^{B}R_{A}^{T} = {}^{B}R_{A}^{-1} $$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

MAPPING INVOLVING ROTATED FRAME



$$^{A}p_{x} = {}^{B}X_{A} . {}^{B}P$$

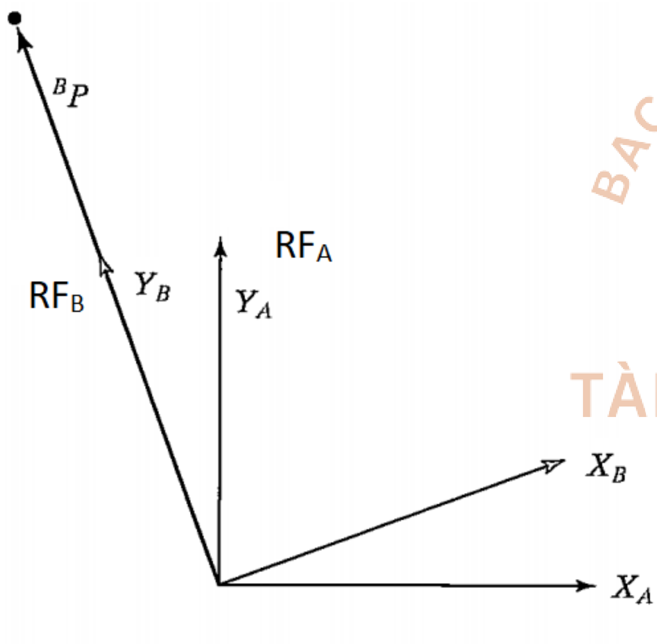
$$^{A}p_{y} = {}^{B}Y_{A} . {}^{B}P$$

$$^{A}p_{z} = {}^{B}Z_{A} . {}^{B}P$$

$$\Rightarrow {}^{A}P = {}^{A}R_{B} {}^{B}P$$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

MAPPING INVOLVING ROTATED FRAME



RF$_B$ rotated 30 degrees about Z.

Example 1:

RF$_B$ is rotated relative to RF$_A$ about Z$_A$ by 30 degree.

Given ${}^{B}P = [0.0 \quad 3.0 \quad 0.0]^T$

Question:

- Determine the direction of Z$_B$
- Calculate ${}^{A}P$

Solution:

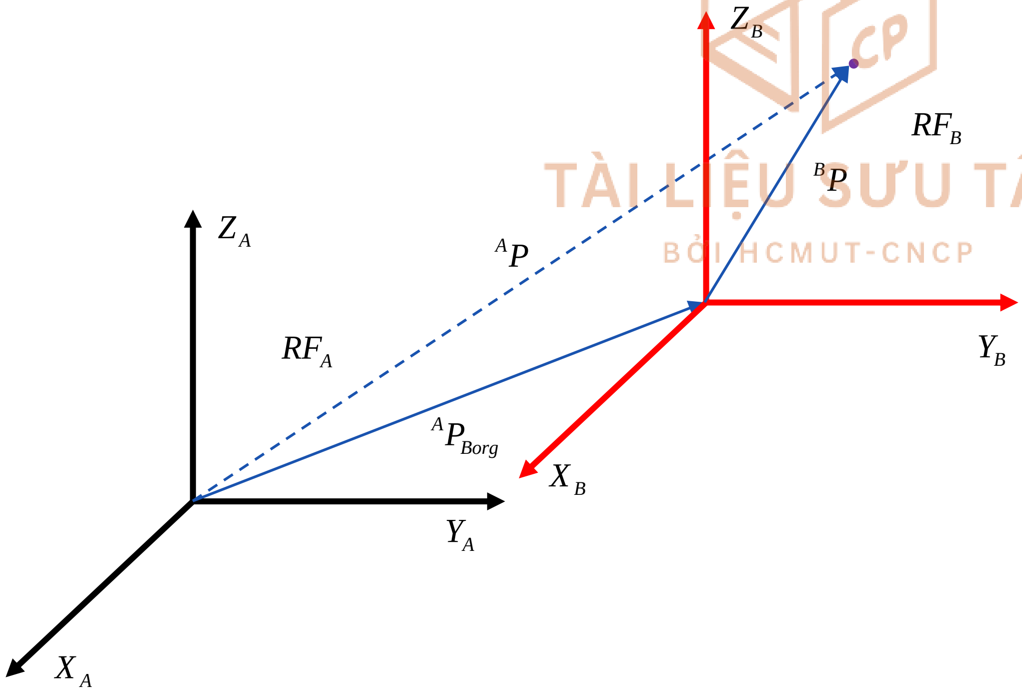
$$ {}^{A}R_B = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} $$

$$ {}^{A}P = {}^{A}R_B\, {}^{B}P = \begin{bmatrix} -1.500 \\ 2.598 \\ 0.000 \end{bmatrix} $$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

MAPPING INVOLVING TRANSLATED FRAME

- A position defined by the vector ${}^{B}P$.
- RF_A has the same orientation as RF_B.
- RF_B differs from RF_A only by a translation, which is given by ${}^{A}P_{Borg}$, a vector that locates the origin of RF_B relative to RF_A.

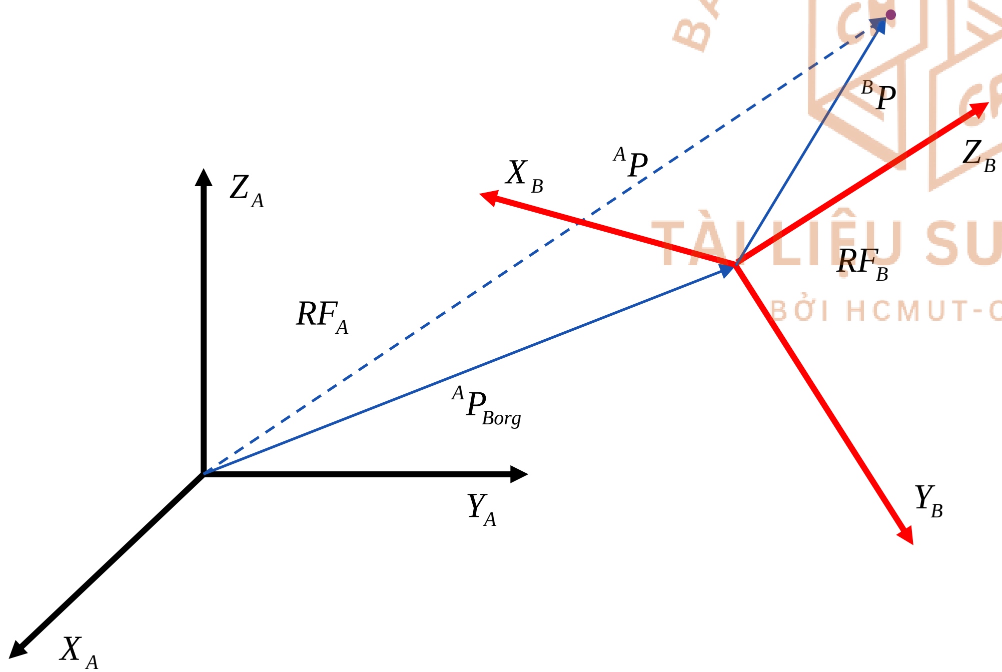


$$ {}^{A}P = {}^{B}P + {}^{A}P_{Borg} $$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

MAPPING INVOLVING GENERAL FRAME

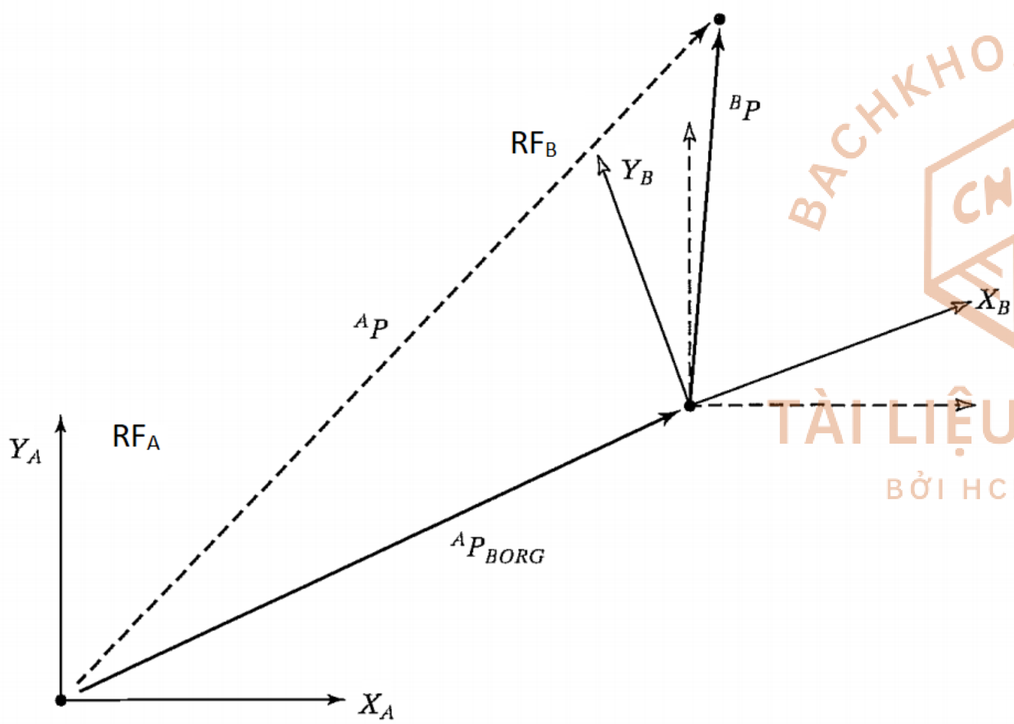
- A position defined by the vector ${}^{B}P$.
- RF_B is rotated with respect to RF_A , as described by ${}^{A}R_{B}$
- ${}^{A}P_{Borg}$ is the vector that locates RF_B 's origin



$$ {}^{A}P = {}^{A}R_{B}\,{}^{B}P + {}^{A}P_{Borg} $$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

MAPPING INVOLVING GENERAL FRAME



Example 2:

RF_B is rotated relative to RF_A about Z_A by 30 degree, translated 12 units in X_A , and translate 8 units in Y_A

Given $^{B}P = \begin{bmatrix} 5.0 & 9.0 & 0.0 \end{bmatrix}^T$

Question:

- Calculate ^{A}P

$$^{A}R_B = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

$$^{A}P_{Borg} = \begin{bmatrix} 12 & 8 & 0 \end{bmatrix}^T$$

$$^{A}P = {}^{A}R_B\, {}^{B}P + {}^{A}P_{Borg} = \begin{bmatrix} 11.830 & 18.294 & 0.000 \end{bmatrix}^T$$

3.2 MAPPINGS: CHANGING FROM FRAME TO FRAME

HOMOGENEOUS TRANSFORM

$$^{A}P = {}^{A}R_{B}\,{}^{B}P + {}^{A}P_{Borg}$$

$$\Rightarrow \begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & ^{A}R_{B} & & ^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix}$$

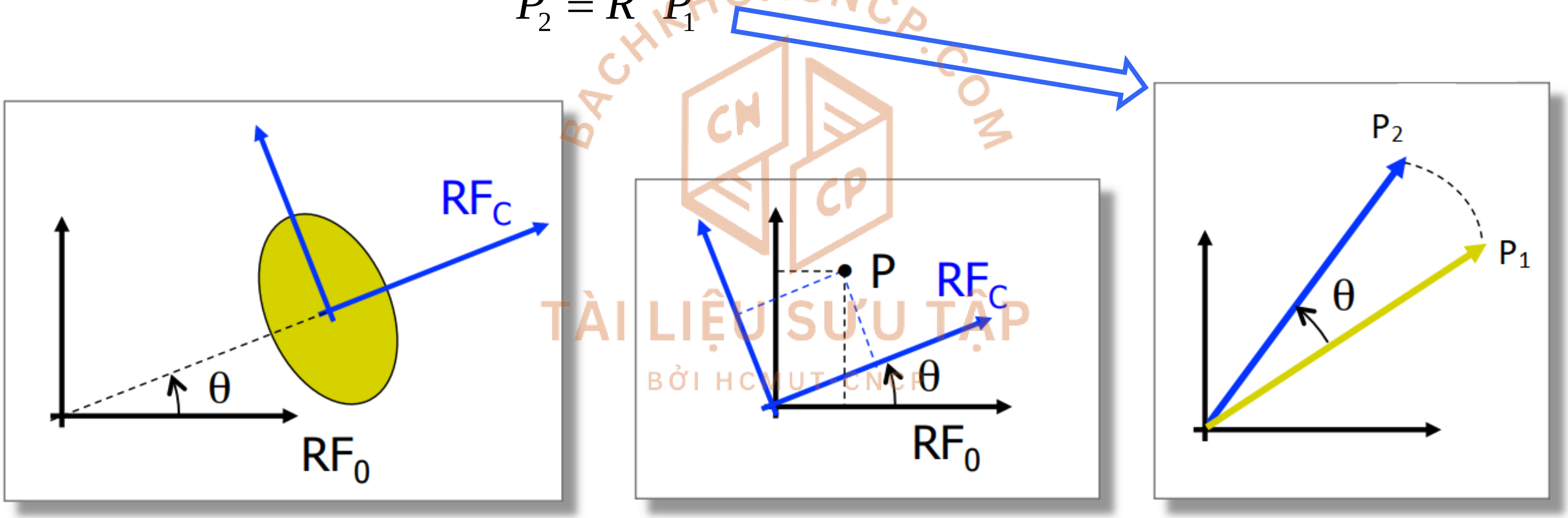
$$\Rightarrow {}^{A}T_{B} = \left[\begin{array}{ccc|c} & ^{A}R_{B} & & ^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$^{A}T_{B}$ is called a homogeneous transform

ROTATIONAL OPERATORS

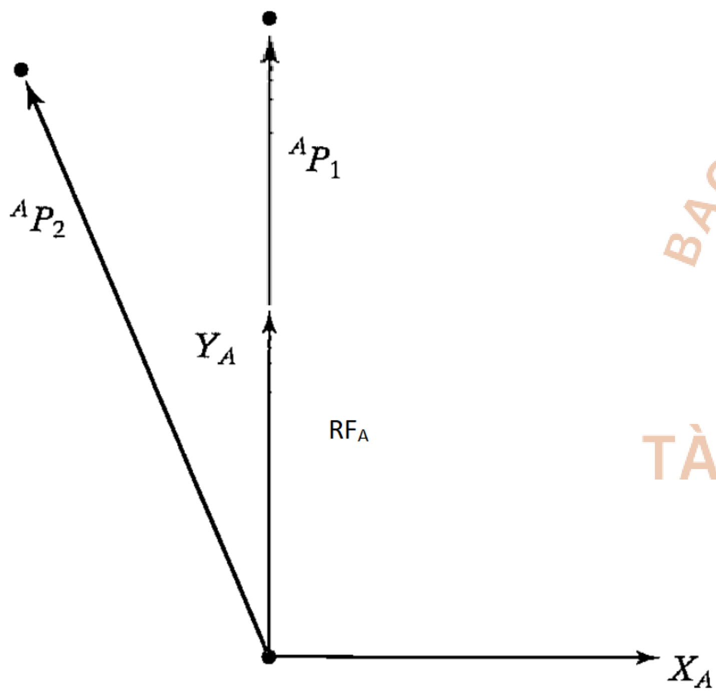
- Operates on a vector ${}^{A}P_1$ and changes that vector to ${}^{A}P_2$, by means of a rotation R

$${}^{A}P_2 = R\,{}^{A}P_1$$



The rotation matrix that rotates vectors through some rotation, R, is the same as the rotation matrix that describes a frame rotated by R relative to the reference frame

ROTATIONAL OPERATORS



Example 3:

$^AP_1$ is rotated about Z_A by 30 degree.

Given $^AP_1 = [0.0 \quad 3.0 \quad 0.0]^T$

Question:

- Determine the direction of Z_A
- Calculate $^AP_2$

Solution:

$$R_Z(30.0) = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

$$^AP_2 = R_Z(30.0)\,{}^AP_1 = \begin{bmatrix} -1.500 \\ 2.598 \\ 0.000 \end{bmatrix}$$

TRANSLATIONAL OPERATORS

- ${}^{A}P_1$ is translated by a vector ${}^{A}Q$
- The result of the operation is a new vector ${}^{A}P_2$

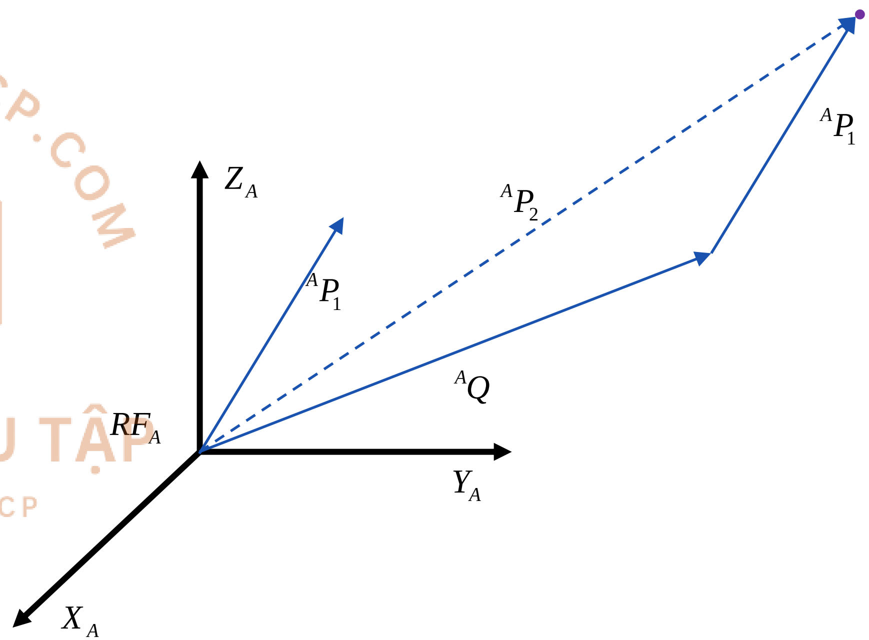
$$ {}^{A}P_2 = {}^{A}P_1 + {}^{A}Q $$

- Translation operation as a matrix operator:

$$ {}^{A}P_2 = D_Q(q)\,{}^{A}P_1 $$

- D_Q is homogeneous transform of a simple form:

$$ D_Q(q) = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



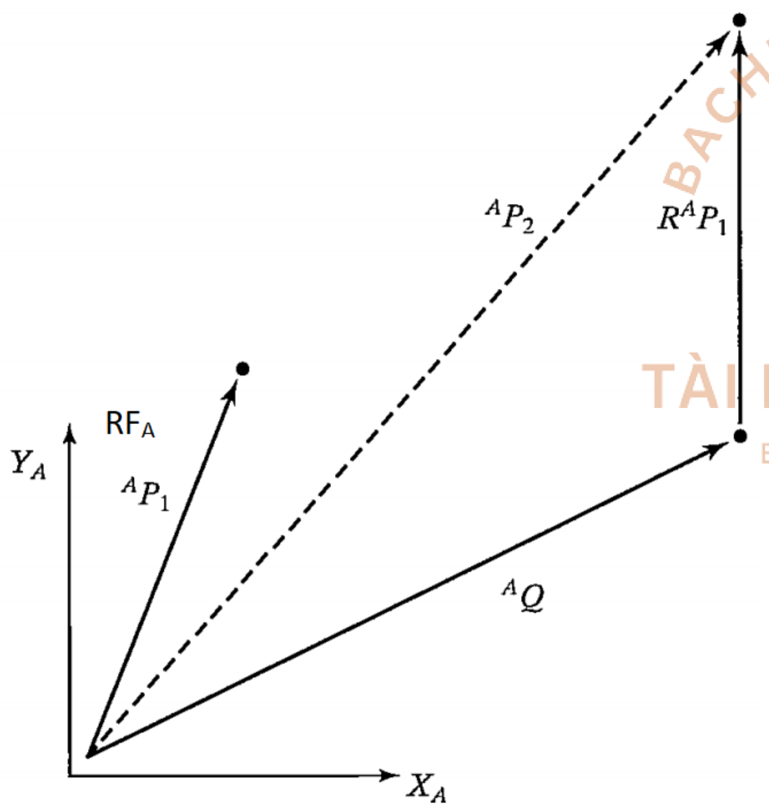
TRANSFORMATION OPERATORS

- The operator T rotates and translates a vector ${}^{A}P_1$ to compute a new vector:

$$ {}^{A}P_2 = T\,{}^{A}P_1 $$

The transform that rotates by R and translates by Q is the same as the transform that describes a frame rotated by R and translated by Q relative to the reference frame.

TRANSFORMATION OPERATORS



Example 4:

$^{A}P_1$ is rotated about Z_A by 30 degree, translated 12 units in X_A, and translate 8 units in Y_A

Given $^{A}P_1 = [5.0 \quad 9.0 \quad 0.0]^T$

Question:

- Calculate $^{A}P_2$

$$T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 12.0 \\ 0.500 & 0.866 & 0.000 & 8.0 \\ 0.000 & 0.000 & 1.000 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{A}P_2 = T\,^{A}P_1 = [11.830 \quad 18.294 \quad 0.000]^T$$

3.4 TRANSFORMATION ARITHMETIC

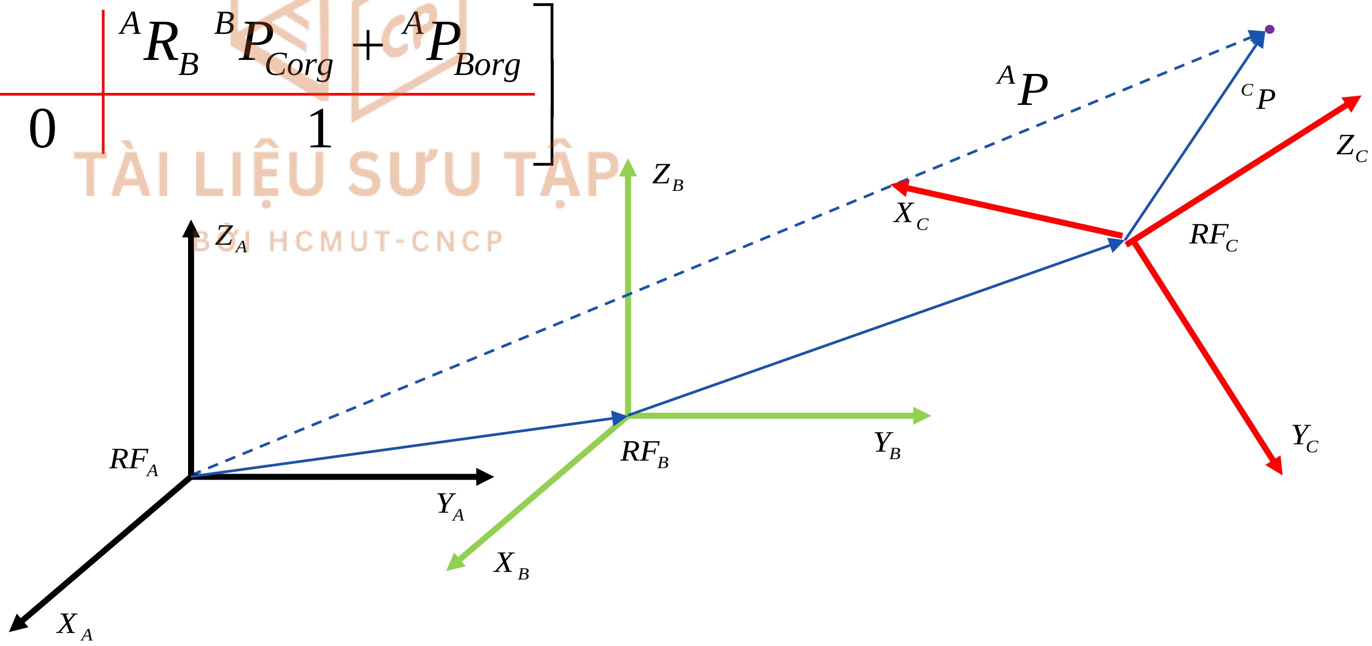
COMPOUND TRANSFORMATIONS

RF_C is known relative to RF_B, and RF_B is known relative to RF_A

Transform ${}^{C}P$ into ${}^{B}P$: ${}^{B}P = {}^{B}T_C\,{}^{C}P$

Transform ${}^{B}P$ into ${}^{A}P$: ${}^{A}P = {}^{A}T_B\,{}^{B}P = {}^{A}T_B\,{}^{B}T_C\,{}^{C}P = {}^{A}T_C\,{}^{C}P$

$${}^{A}T_C = \left[\begin{array}{ccc|c} & {}^{A}R_B\,{}^{B}R_C & & {}^{A}R_B\,{}^{B}P_{Corg} + {}^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$



3.4 TRANSFORMATION ARITHMETIC

INVERTING A TRANSFORM

RF_B is known with respect to RF_A- that is, we know ${}^{A}T_{B}$

$$
{}^{A}T_{B}=\left[\begin{array}{ccc|c} & {}^{A}R_{B} & & {}^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

To find ${}^{B}T_{A}$, we must compute ${}^{B}R_{A}$ and ${}^{B}P_{Aorg}$ from ${}^{A}R_{B}$ and ${}^{A}P_{Borg}$

$$
{}^{B}R_{A}={}^{A}R_{B}^{T}
$$

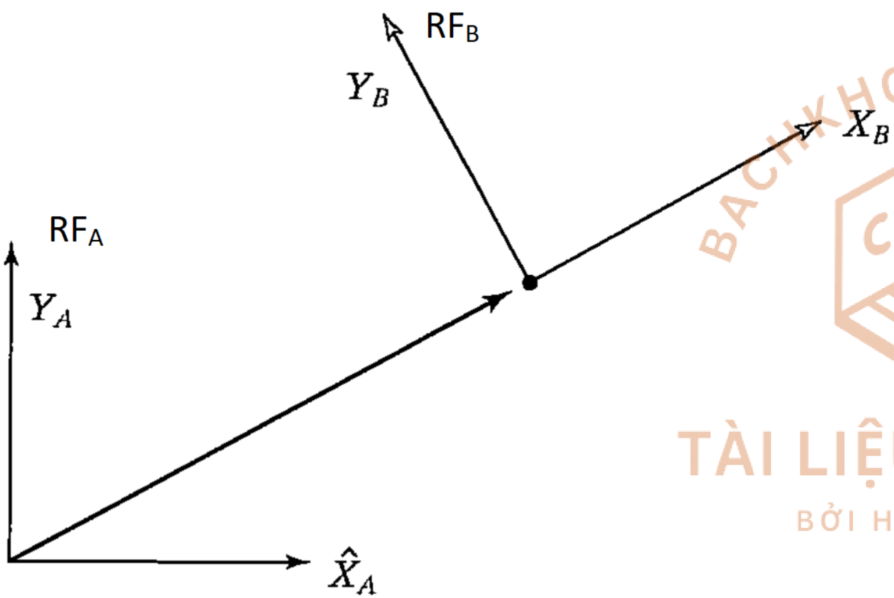
$$
{}^{B}\left({}^{A}P_{Borg}\right)={}^{B}R_{A}\,{}^{A}P_{Borg}+{}^{B}P_{Aorg}
$$

$$
\Rightarrow {}^{B}P_{Aorg}=-{}^{B}R_{A}\,{}^{A}P_{Borg}=-{}^{A}R_{B}^{T}\,{}^{A}P_{Borg}
$$

$$
\boxed{{}^{B}T_{A}=\left[\begin{array}{ccc|c} & {}^{A}R_{B}^{T} & & -{}^{A}R_{B}^{T}\,{}^{A}P_{Borg} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]}
$$

3.4 TRANSFORMATION ARITHMETIC

INVERTING A TRANSFORM



Example 5:

RF$_B$ is rotated to RF$_A$ about Z_A by 30 degrees

And translated 5 units in X_A and 4 units in Y_A

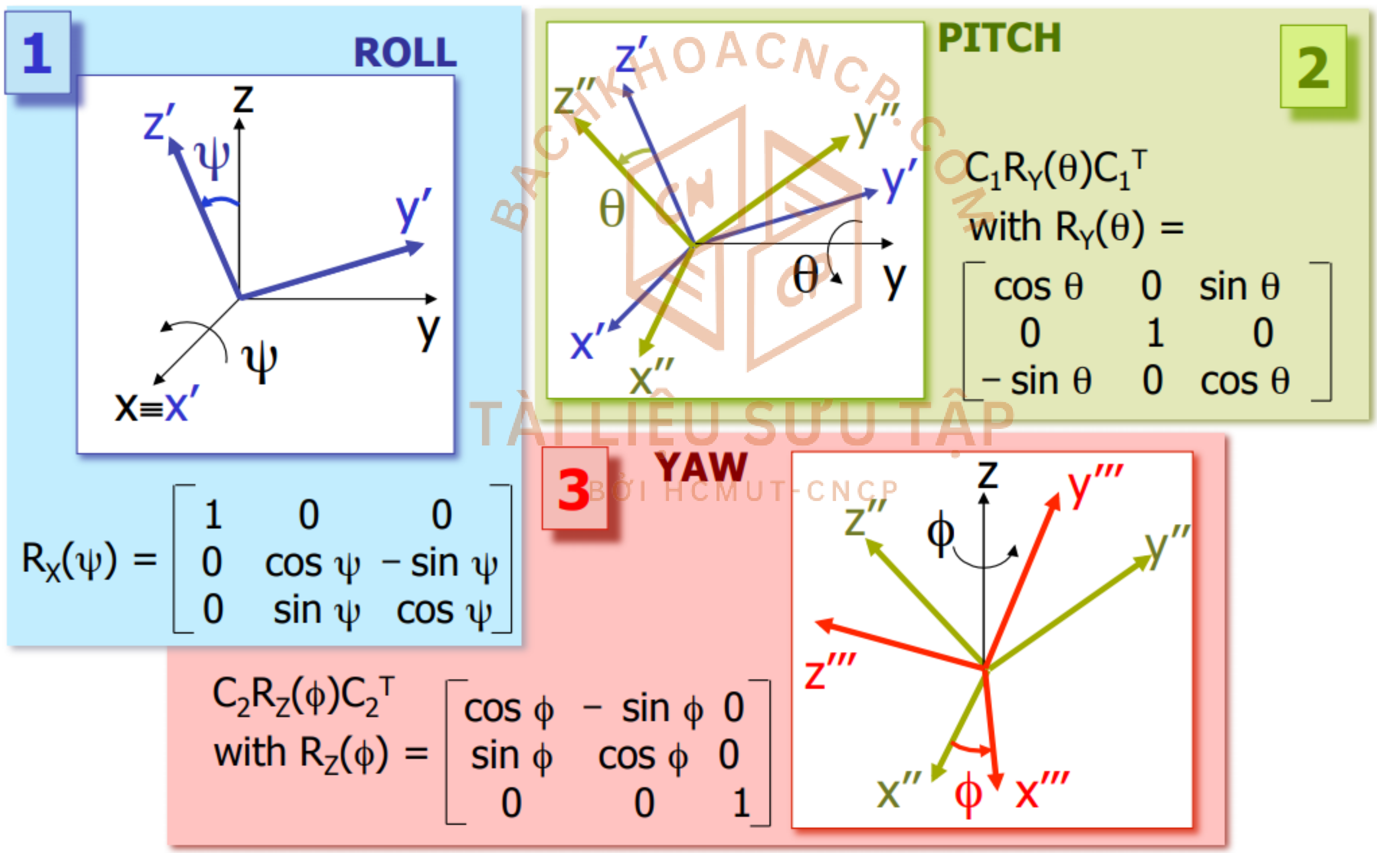
Question: find ${}^{B}T_{A}$

$$ {}^{A}T_{B} = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 5.0 \\ 0.500 & 0.866 & 0.000 & 4.0 \\ 0.000 & 0.000 & 1.000 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \Rightarrow {}^{B}T_{A} = \begin{bmatrix} 0.866 & 0.500 & 0.000 & -6.330 \\ -0.500 & 0.866 & 0.000 & -0.964 \\ 0.000 & 0.000 & 1.000 & 0.000 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3.5 MORE ON REPRESENTATION OF ORIENTATION

X-Y-Z FIXED ANGLES(ROLL-PITCH-YAW ANGLES)



Each of the three rotations takes place about an axis in the fixed reference frame

3.5 MORE ON REPRESENTATION OF ORIENTATION

X-Y-Z FIXED ANGLES(ROLL-PITCH-YAW ANGLES)

- Direct problem: Given ψ, θ, ϕ. Find R

$$
{}^{A}_{B}R_{XYZ}(\psi,\theta,\phi) = R_Z(\phi)R_Y(\theta)R_X(\psi)
$$

$$
= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}
$$

$$
= \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix}
$$

3.5 MORE ON REPRESENTATION OF ORIENTATION

X-Y-Z FIXED ANGLES(ROLL-PITCH-YAW ANGLES)

- Inverse problem: Given R. Find ψ, θ, ϕ

$$ {}^{A}_{B}R_{XYZ}(\psi,\theta,\phi)=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}=\begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} $$

- $$\left.\begin{aligned} & r_{32}^2+r_{33}^2=(c\theta)^2 \\ & r_{31}=-s\theta \end{aligned}\right\} \Rightarrow \theta = ATAN2\left(-r_{31}, \pm\sqrt{r_{32}^2+r_{33}^2}\right)$$

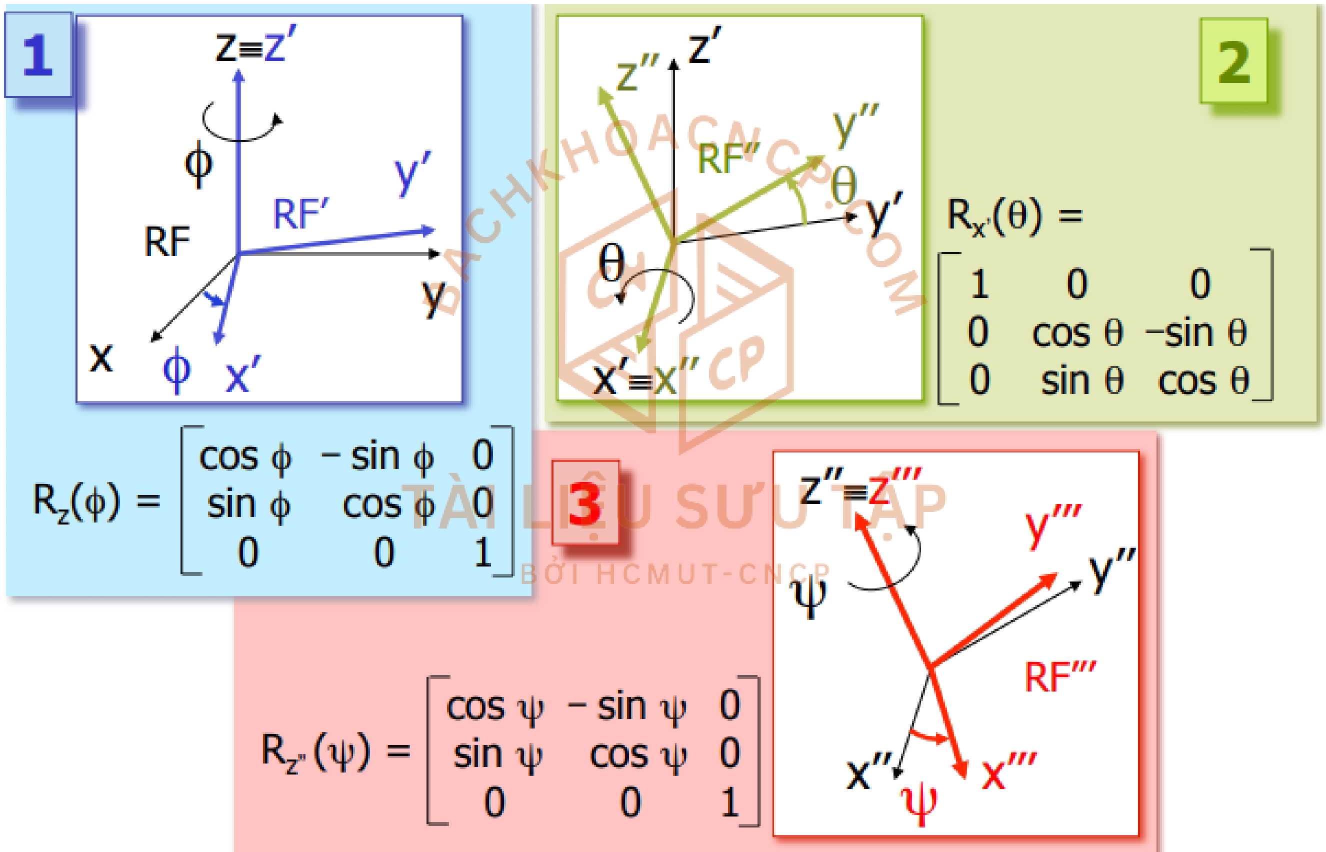
- $$\left.\begin{aligned} & \text{If } r_{32}^2+r_{33}^2 \neq 0 \text{ (i.e., } c\theta \neq 0) \\ & r_{32}/c\theta = s\psi, r_{33}/c\theta = c\psi \end{aligned}\right\} \Rightarrow \psi = ATAN2\left(r_{32}/c\theta, r_{33}/c\theta\right)$$

- $$\phi = ATAN2\left(r_{21}/c\theta, r_{11}/c\theta\right)$$

  - If $\theta = 90^0$: $\phi = 0.0$; $\psi = ATAN2(r_{12}, r_{22})$
  - If $\theta = -90^0$: $\phi = 0.0$; $\psi = -ATAN2(r_{12}, r_{22})$

3.5 MORE ON REPRESENTATION OF ORIENTATION

Z-X'-Z'' EULER ANGLES



$$R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{x'}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_{z''}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Each rotation is performed about an axis of the moving system rather than one of the fixed reference

3.5 MORE ON REPRESENTATION OF ORIENTATION

Z-X'-Z'' EULER ANGLES

- Direct problem: Given ψ, θ, ϕ. Find R

$$
{}^{A}_{B}R_{ZX'Z''}(\phi,\theta,\psi) = R_Z(\phi)R_{X'}(\theta)R_{Z''}(\psi)
$$

$$
= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}\begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c\phi c\psi - s\phi c\theta s\psi & -c\phi s\psi - s\phi c\theta c\psi & s\phi s\theta \\ s\phi c\psi + c\phi c\theta s\psi & -s\phi s\psi + c\phi c\theta c\psi & -c\phi s\theta \\ s\theta s\psi & s\theta c\psi & c\theta \end{bmatrix}
$$

The orientation of RF''' is the same that would be obtained with the sequence of rotations: ψ around Z, θ around X (fixed), ϕ around Z (fixed)

3.5 MORE ON REPRESENTATION OF ORIENTATION

Z-X’-Z’’ EULER ANGLES

- Inverse problem: Given R. Find ψ, θ, ϕ

$$
{}^{A}_{B}R_{ZX'Z''}(\phi,\theta,\psi)=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}=\begin{bmatrix} c\phi c\psi - s\phi c\theta s\psi & -c\phi s\psi - s\phi c\theta c\psi & s\phi s\theta \\ s\phi c\psi + c\phi c\theta s\psi & -s\phi s\psi + c\phi c\theta c\psi & -c\phi s\theta \\ s\theta s\psi & s\theta c\psi & c\theta \end{bmatrix}
$$

- $$\left.\begin{aligned} r_{13}^2 + r_{23}^2 &= (s\theta)^2 \\ r_{33} &= c\theta \end{aligned}\right\} \Rightarrow \theta = ATAN2\left(\pm\sqrt{r_{11}^2 + r_{21}^2}, r_{33}\right)$$

- $$\left.\begin{aligned} &\text{If } r_{13}^2 + r_{23}^2 \neq 0 \text{ (i.e., s}\theta \neq 0) \\ &r_{31}/s\theta = s\psi, r_{32}/s\theta = c\psi \end{aligned}\right\} \Rightarrow \psi = ATAN2\left(r_{31}/s\theta, r_{32}/s\theta\right)$$

- $$\phi = ATAN2\left(r_{13}/s\theta, -r_{23}/s\theta\right)$$

  - If $\theta = 0^0$: $\phi = 0.0$; $\psi = ATAN2(r_{21}, r_{11})$
  - If $\theta = 180^0$: $\phi = 0.0$; $\psi = -ATAN2(-r_{21}, r_{11})$

EXERCISE

EXERCISE 1:

RF_B is rotated relative to RF_A about X_A by 60 degree.

Given

$$^{B}P = \begin{bmatrix} 2.0 & 3.0 & 5.0 \end{bmatrix}^T$$

Question:

- Calculate ^{A}P

EXERCISE 2:

RF_B is rotated relative to RF_A about Y_A by 60 degree, translated 7 units in X_A , and translate 4 units in Y_A

Given $^{B}P = \begin{bmatrix} 3.0 & 5.0 & 8.0 \end{bmatrix}^T$

Question:

- Calculate ^{A}P

EXERCISE

EXERCISE 3:

${}^{A}P_1$ is rotated about X_A by 30 degree.

Given ${}^{A}P_1 = \begin{bmatrix} 6.0 & 5.0 & 7.0 \end{bmatrix}^T$

Question:

- Calculate ${}^{A}P_2$

EXERCISE 4:

${}^{A}P_1$ is rotated about Y_A by 60 degree, translated 5 units in X_A , translate 3 units in Y_A , and translate 2 units in Z_A

Given ${}^{A}P_1 = \begin{bmatrix} 4.0 & 9.0 & 7.0 \end{bmatrix}^T$

Question:

- Calculate ${}^{A}P_2$

EXERCISE 5:

RF_B is rotated to RF_A about X_A by 45 degrees

And translated 2 units in X_A , 3 units in Y_A , and 4 units in Z_A

Question: find ${}^{B}T_{A}$

EXERCISE

EXERCISE 1:

RF_B is rotated relative to RF_A about X_A by 60 degree.

Given

$$^{B}P = \begin{bmatrix} 2.0 & 3.0 & 5.0 \end{bmatrix}^T$$

Question:

- Calculate ^{A}P

Solution:

$$^{A}R_B = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 0.5000 & -0.866 \\ 0.000 & 0.866 & 0.500 \end{bmatrix}$$

$$^{A}P = {}^{A}R_B \, {}^{B}P = \begin{bmatrix} 2.000 \\ -2.830 \\ 5.098 \end{bmatrix}$$

EXERCISE

EXERCISE 2:

RF_B is rotated relative to RF_A about Y_A by 60 degree, translated 7 units in X_A , and translate 4 units in Y_A

Given $^{B}P = \begin{bmatrix} 3.0 & 5.0 & 8.0 \end{bmatrix}^T$

Question:

- Calculate ^{A}P

Solution:

$$^{A}R_B = \begin{bmatrix} 0.500 & 0.000 & 0.866 \\ 0.000 & 1.000 & 0.000 \\ -0.866 & 0.000 & 0.500 \end{bmatrix}$$

$$^{A}P_{Borg} = \begin{bmatrix} 7.0 & 4.0 & 0.0 \end{bmatrix}^T$$

$$^{A}P = {}^{A}R_B\,{}^{B}P + {}^{A}P_{Borg} = \begin{bmatrix} 15.428 & 9.000 & 1.402 \end{bmatrix}^T$$

EXERCISE

EXERCISE 3:

${}^{A}P_1$ is rotated about X_A by 30 degree.

Given ${}^{A}P_1 = \begin{bmatrix} 6.0 & 5.0 & 7.0 \end{bmatrix}^T$

Question:

- Calculate ${}^{A}P_2$

Solution:

$$R_X(30.0) = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 0.866 & -0.500 \\ 0.000 & 0.500 & 0.866 \end{bmatrix}$$

$${}^{A}P_2 = R_Z(30.0)\,{}^{A}P_1 = \begin{bmatrix} 6.000 \\ 0.830 \\ 8.562 \end{bmatrix}$$

EXERCISE

EXERCISE 4:

${}^{A}P_1$ is rotated about Y_A by 60 degree, translated 5 units in X_A , translate 3 units in Y_A , and translate 2 units in Z_A

Given ${}^{A}P_1 = \begin{bmatrix} 4.0 & 9.0 & 7.0 \end{bmatrix}^T$

Question:

- Calculate ${}^{A}P_2$

Solution:

$$T = \begin{bmatrix} 0.500 & 0.000 & 0.866 & 5.0 \\ 0.000 & 1.000 & 0.000 & 3.0 \\ -0.866 & 0.000 & 0.500 & 2.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}P_2 = T\,{}^{A}P_1 = [13.062 \quad 12.000 \quad 2.036]^T$$

EXERCISE

EXERCISE 5:

RF_B is rotated to RF_A about X_A by 45 degrees

And translated 2 units in X_A , 3 units in Y_A , and 4 units in Z_A

Question: find ${}^{B}T_{A}$

Solution:

$$ {}^{A}T_{B} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 2.0 \\ 0.000 & 0.707 & -0.707 & 3.0 \\ 0.000 & 0.707 & 0.707 & 4.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \Rightarrow {}^{B}T_{A} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & -2.0 \\ 0.000 & 0.707 & 0.707 & -4.94 \\ 0.000 & -0.707 & 0.707 & -0.707 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$