Artificial Intelligence

Support Vector Machine

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Support Vector Machine

- Distance from a point to a line/plane/hyperplane

$$\frac{|w_1 x_0 + w_2 y_0 + b|}{\sqrt{w_1^2 + w_2^2}}$$

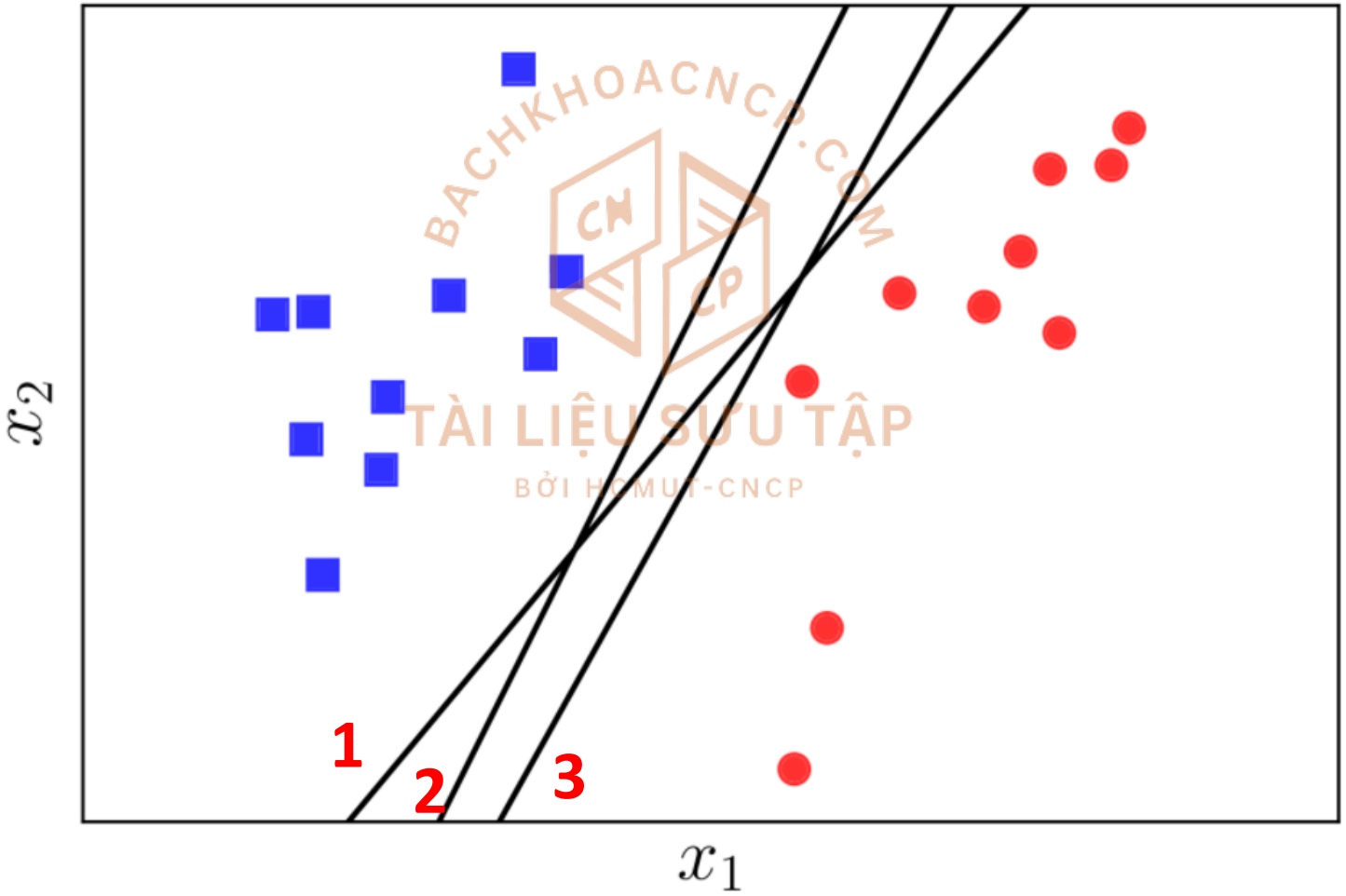
$$\frac{|w_1 x_0 + w_2 y_0 + w_3 z_0 + b|}{\sqrt{w_1^2 + w_2^2 + w_3^2}}$$

$$\frac{|\mathbf{w}^T \mathbf{x}_0 + b|}{||\mathbf{w}||_2}$$

$$||\mathbf{w}||_2 = \sqrt{\sum_{i=1}^{d} w_i^2}$$

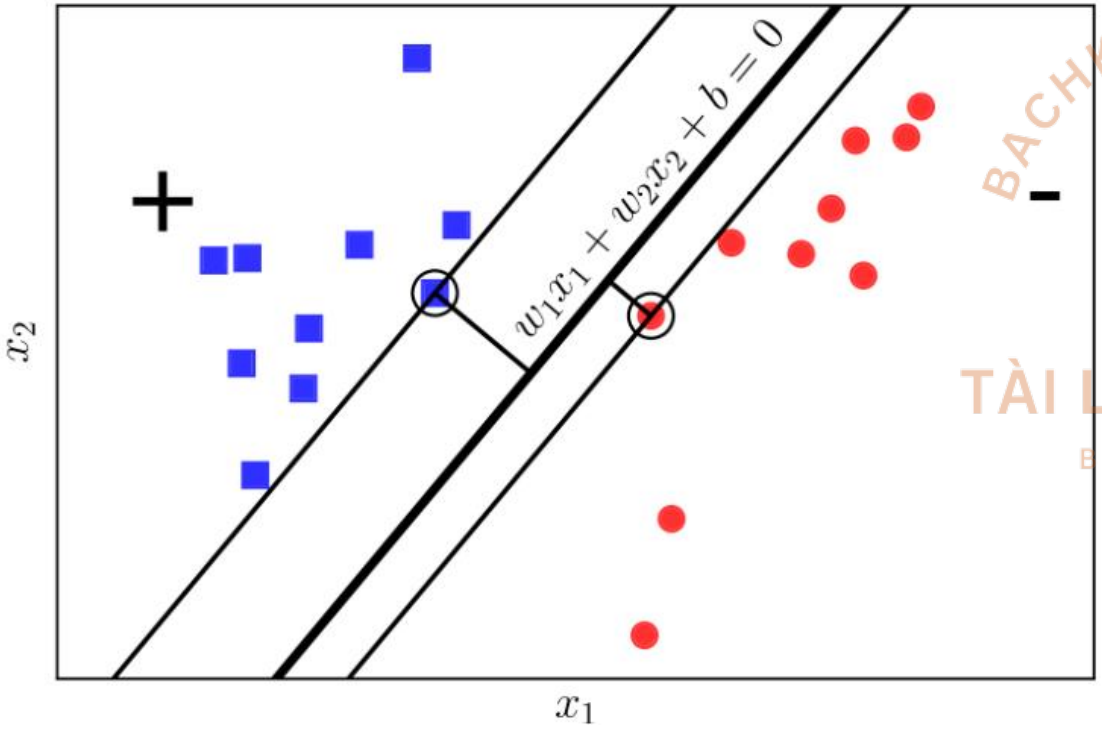
Support Vector Machine

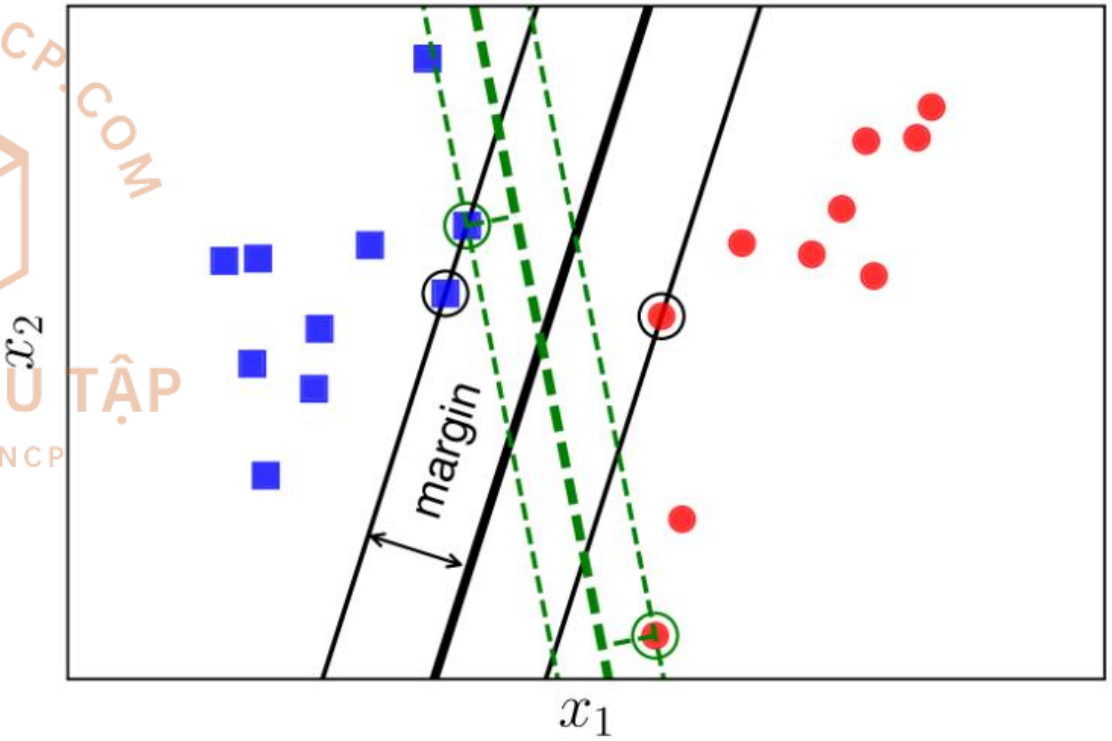
- ✓ Two-class classification problem with linearly separable data



Support Vector Machine

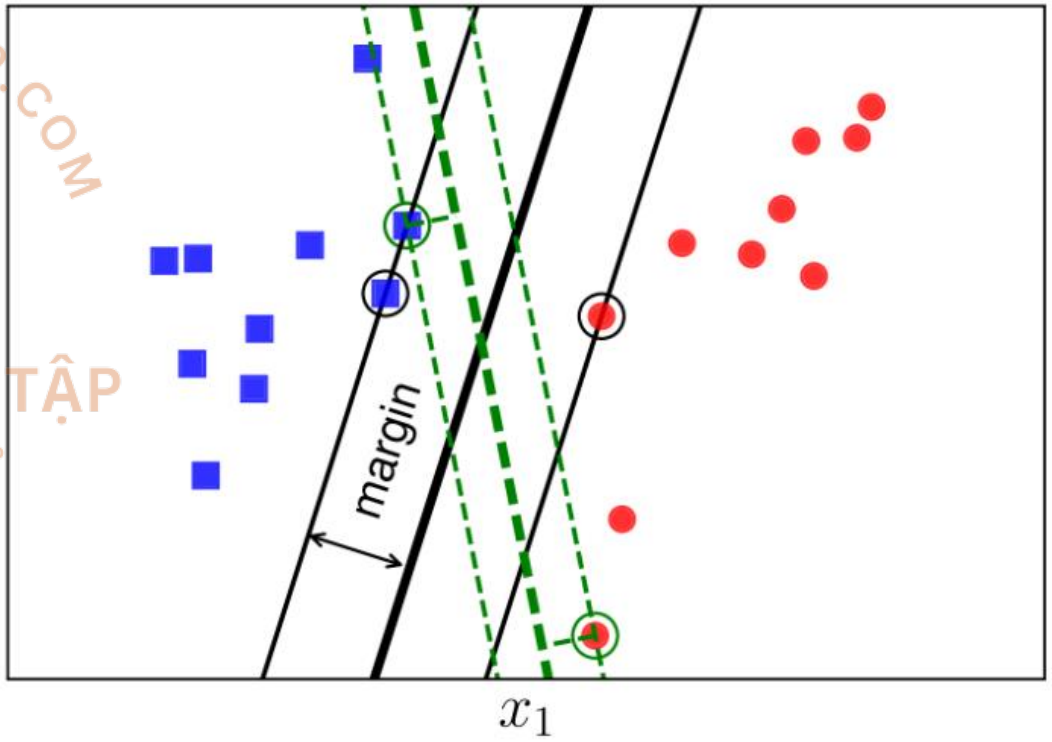
- ✓ Best classification?





Support Vector Machine

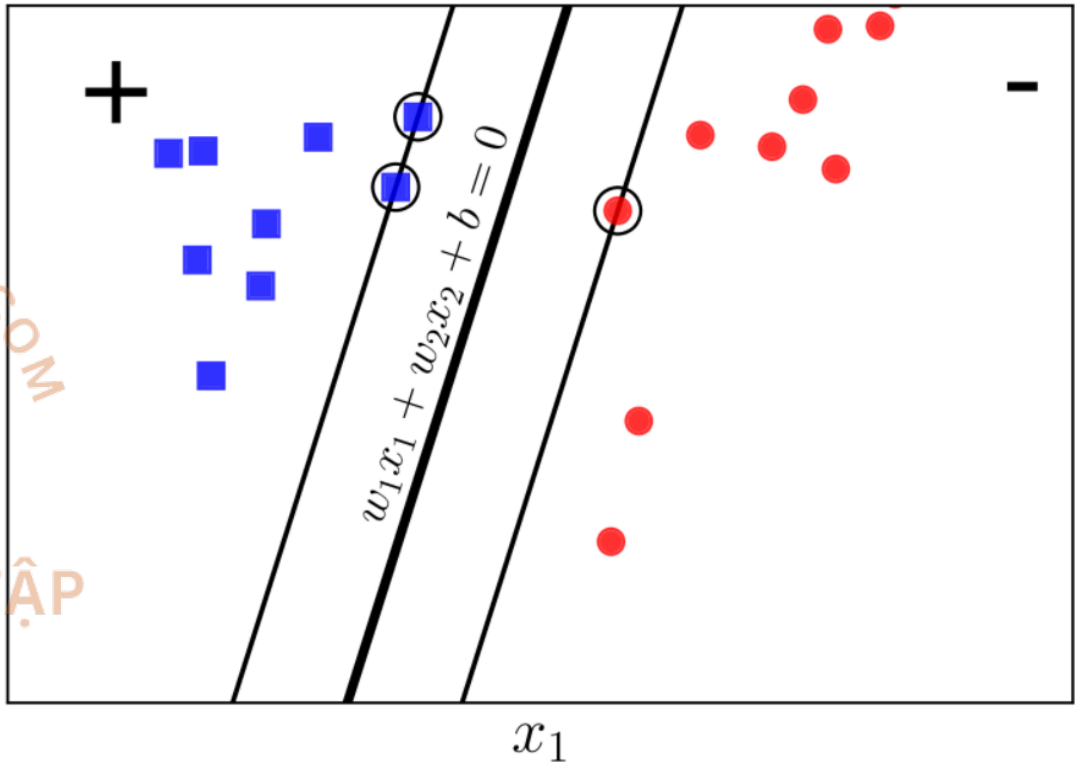
- ✓ SVM
  - ❖ Equal margins
  - ❖ Largest margins



Support Vector Machine

- Training set $(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_N, y_N)$

$$\mathbf{w}^T\mathbf{x} + b = w_1x_1 + w_2x_2 + b = 0$$



- Distance form a data point to the separation line

$$\frac{|w_1x_0 + w_2y_0 + b|}{\sqrt{w_1^2 + w_2^2}}$$

$$\frac{y_n(\mathbf{w}^T\mathbf{x}_n + b)}{||\mathbf{w}||_2}$$

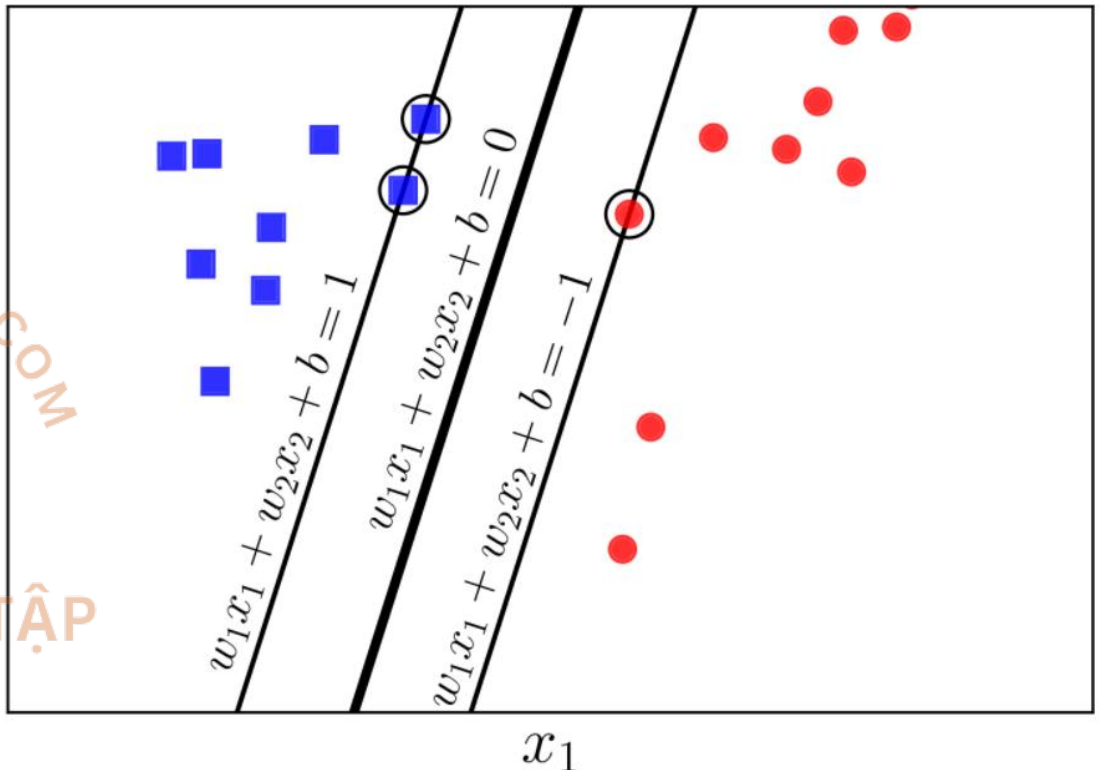
$$\text{margin} = \min_n \frac{y_n(\mathbf{w}^T\mathbf{x}_n + b)}{||\mathbf{w}||_2}$$

Support Vector Machine

- Optimization problem

$$(\mathbf{w}, b) = \arg\max_{\mathbf{w},b} \left\{ \min_n \frac{y_n(\mathbf{w}^T\mathbf{x}_n + b)}{||\mathbf{w}||_2} \right\}$$

$$\arg\max_{\mathbf{w},b} \left\{ \frac{1}{||\mathbf{w}||_2} \min_n y_n(\mathbf{w}^T\mathbf{x}_n + b) \right\}$$



- Assume that

$$\min \; y_n(\mathbf{w}^T\mathbf{x}_n + b) = 1$$

$$(\mathbf{w}, b) = \arg\max_{\mathbf{w},b} \frac{1}{||\mathbf{w}||_2} \text{ subject to: } \; y_n(\mathbf{w}^T\mathbf{x}_n + b) \geq 1, \forall n = 1, 2, \ldots, N$$

$$(\mathbf{w}, b) = \arg\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||_2^2 \text{ subject to: } \; 1 - y_n(\mathbf{w}^T\mathbf{x}_n + b) \leq 0, \forall n = 1, 2, \ldots, N$$