PROPERTIES OF DOUBLE INTEGRALS

We list here three properties of double integrals that can be proved in the same manner as in Section 5.2. We assume that all of the integrals exist. Properties 7 and 8 are referred to as the *linearity* of the integral.

7 $$\iint_R [f(x, y) + g(x, y)]\,dA = \iint_R f(x, y)\,dA + \iint_R g(x, y)\,dA$$

■ Double integrals behave this way because the double sums that define them behave this way.

8 $$\iint_R c f(x, y)\,dA = c \iint_R f(x, y)\,dA \qquad \text{where } c \text{ is a constant}$$

If $f(x, y) \geqslant g(x, y)$ for all (x, y) in R, then

9 $$\iint_R f(x, y)\,dA \geqslant \iint_R g(x, y)\,dA$$

15.1 EXERCISES

1. (a) Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle

$$R = \{(x, y) \mid 0 \leqslant x \leqslant 6,\ 0 \leqslant y \leqslant 4\}$$

Use a Riemann sum with $m = 3$, $n = 2$, and take the sample point to be the upper right corner of each square.
(b) Use the Midpoint Rule to estimate the volume of the solid in part (a).

2. If $R = [-1, 3] \times [0, 2]$, use a Riemann sum with $m = 4$, $n = 2$ to estimate the value of $\iint_R (y^2 - 2x^2)\,dA$. Take the sample points to be the upper left corners of the squares.

3. (a) Use a Riemann sum with $m = n = 2$ to estimate the value of $\iint_R \sin(x + y)\,dA$, where $R = [0, \pi] \times [0, \pi]$. Take the sample points to be lower left corners.
(b) Use the Midpoint Rule to estimate the integral in part (a).

4. (a) Estimate the volume of the solid that lies below the surface $z = x + 2y^2$ and above the rectangle $R = [0, 2] \times [0, 4]$. Use a Riemann sum with $m = n = 2$ and choose the sample points to be lower right corners.
(b) Use the Midpoint Rule to estimate the volume in part (a).

5. A table of values is given for a function $f(x, y)$ defined on $R = [1, 3] \times [0, 4]$.
(a) Estimate $\iint_R f(x, y)\,dA$ using the Midpoint Rule with $m = n = 2$.
(b) Estimate the double integral with $m = n = 4$ by choosing the sample points to be the points farthest from the origin.

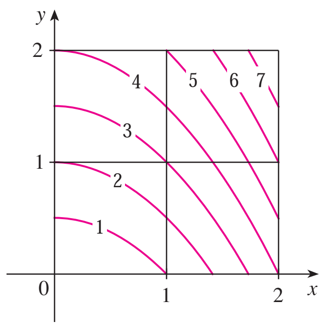
x \ y	0	1	2	3	4
1.0	2	0	−3	−6	−5
1.5	3	1	−4	−8	−6
2.0	4	3	0	−5	−8
2.5	5	5	3	−1	−4
3.0	7	8	6	3	0

6. A 20-ft-by-30-ft swimming pool is filled with water. The depth is measured at 5-ft intervals, starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water in the pool.

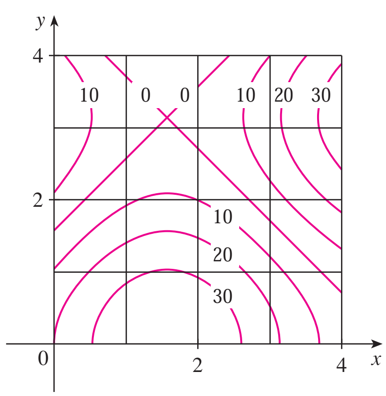
	0	5	10	15	20	25	30
0	2	3	4	6	7	8	8
5	2	3	4	7	8	10	8
10	2	4	6	8	10	12	10
15	2	3	4	5	6	8	7
20	2	2	2	2	3	4	4

7. Let V be the volume of the solid that lies under the graph of $f(x, y) = \sqrt{52 - x^2 - y^2}$ and above the rectangle given by $2 \leqslant x \leqslant 4$, $2 \leqslant y \leqslant 6$. We use the lines $x = 3$ and $y = 4$ to divide R into subrectangles. Let L and U be the Riemann sums computed using lower left corners and upper right corners, respectively. Without calculating the numbers V, L, and U, arrange them in increasing order and explain your reasoning.

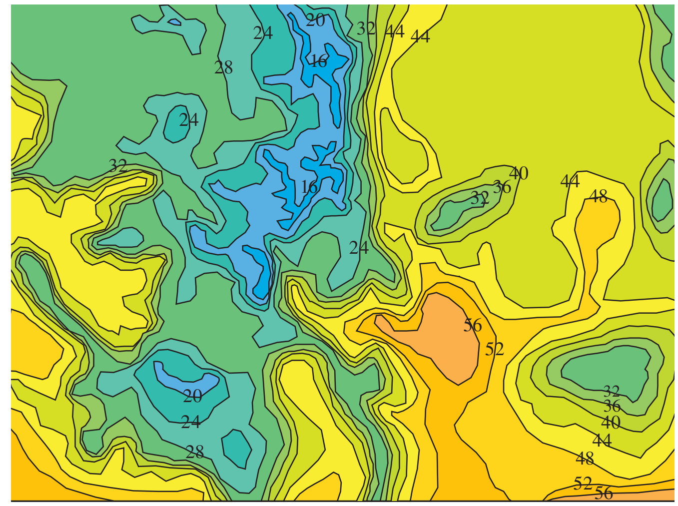
8. The figure shows level curves of a function f in the square $R = [0, 2] \times [0, 2]$. Use the Midpoint Rule with $m = n = 2$ to estimate $\iint_R f(x, y)\, dA$. How could you improve your estimate?



9. A contour map is shown for a function f on the square $R = [0, 4] \times [0, 4]$.
(a) Use the Midpoint Rule with $m = n = 2$ to estimate the value of $\iint_R f(x, y)\, dA$.
(b) Estimate the average value of f.



10. The contour map shows the temperature, in degrees Fahrenheit, at 4:00 PM on February 26, 2007, in Colorado. (The state measures 388 mi east to west and 276 mi north to south.) Use the Midpoint Rule with $m = n = 4$ to estimate the average temperature in Colorado at that time.



11–13 Evaluate the double integral by first identifying it as the volume of a solid.

11. $\iint_R 3\, dA$, $\quad R = \{(x, y) \mid -2 \leqslant x \leqslant 2,\ 1 \leqslant y \leqslant 6\}$

12. $\iint_R (5 - x)\, dA$, $\quad R = \{(x, y) \mid 0 \leqslant x \leqslant 5,\ 0 \leqslant y \leqslant 3\}$

13. $\iint_R (4 - 2y)\, dA$, $\quad R = [0, 1] \times [0, 1]$

14. The integral $\iint_R \sqrt{9 - y^2}\, dA$, where $R = [0, 4] \times [0, 2]$, represents the volume of a solid. Sketch the solid.

15. Use a programmable calculator or computer (or the sum command on a CAS) to estimate

$$\iint_R \sqrt{1 + xe^{-y}}\, dA$$

where $R = [0, 1] \times [0, 1]$. Use the Midpoint Rule with the following numbers of squares of equal size: 1, 4, 16, 64, 256, and 1024.

16. Repeat Exercise 15 for the integral $\iint_R \sin\left(x + \sqrt{y}\right) dA$.

17. If f is a constant function, $f(x, y) = k$, and $R = [a, b] \times [c, d]$, show that $\iint_R k\, dA = k(b - a)(d - c)$.

18. Use the result of Exercise 17 to show that

$$0 \leqslant \iint_R \sin \pi x \cos \pi y\, dA \leqslant \frac{1}{32}$$

where $R = \left[0, \frac{1}{4}\right] \times \left[\frac{1}{4}, \frac{1}{2}\right]$.

15.2 ITERATED INTEGRALS

Recall that it is usually difficult to evaluate single integrals directly from the definition of an integral, but the Fundamental Theorem of Calculus provides a much easier method. The evaluation of double integrals from first principles is even more difficult, but in this sec-