Department of Applied Mathematics.

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Exercises on Numerical Analysis-Chapter 1+2

Note for students

- These are topics that you **must** prepare for your midterm and endterm test, and the exercises accompanying them below are examples.
- Some of them will be explained in hangout time by you lecturer. However, please do not fully depend on him/her, you should solve them yourself first and prepare your question to ask your lecturer in hangout time.(You absolutely can do this if you watch video lectures and livestream carefully).

I) ERROR

1. Given absolute error of an approximate quantity, find relative error and vice versa.
2. Find error(absolute+relative) of multivariable functions.

II) NONLINEAR EQUATION

i) Bisection method

A. Given equation $f(x) = 0$ with containing root interval $[a, b]$. Using bisection method to find approximate root x_n.

1. $3x - e^x = 0$ on $[1, 2]$, find x_5.
2. $2x + 3\cos x - e^x = 0$ on $[0, 1]$, find x_5.
3. $x^2 - 4x + 4 - \ln x = 0$ on $[1, 2]$, find x_5.

B. Given equation $f(x) = 0$, by bisection method, find approximate solution x_n on $[a, b]$ such that $\Delta x_n < \varepsilon$, where ε is a given positive value.

1. $x\cos x - 2x^2 + 3x - 1 = 0$, $[a, b] = [0.2,\ 0.3]$, $\varepsilon = 10^{-3}$.
2. $e^x - x^2 + 3x - 2 = 0$, $[a, b] = [0,\ 1]$, $\varepsilon = 10^{-4}$.
3. (Applied exercise) A particle stats at rest on a smooth inclined plane whose angle θ is changing at a constant rate $\omega < 0$

   At the end of t seconds, the position of object is given by

$$x(t) = -\frac{-g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin \omega t\right)$$

   Suppose that the particle has moved $0.52(m)$ in 1 second, $g \approx 9.8m/s^2$. Find ω with $\varepsilon = 10^{-3}$, $[a, b] = [-0.5,\ -0.1]$.

ii) Iterative fixed point method

A. Given equation $x = g(x)$, initial value x_0, by iterative fixed point method, compute x_n.

1. $g = \dfrac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$, $x_0 = 1$, compute x_4.

2. $g = \sqrt{2 - \dfrac{1}{x}}$, $x_0 = 1/2$, compute x_4.
3. $g = \sqrt[4]{2 - 3x^2}$, $x_0 = 1$, compute x_4.

B. Given equation $x = g(x)$, initial value x_0 on $[a, b]$, estimate priori or postepriori error of approximate root x_n.

1. $g = \dfrac{2 - e^x + x^2}{3}$, $[a, b] = [0,\ 1]$, $x_0 = 0$, compute priori error x_3.
2. $g = \dfrac{5}{x^2} + 2$, $[a, b] = [2.5,\ 3]$, $x_0 = 2.5$, compute posteriori error x_2.
3. $g = 0.5(\sin x + \cos x)$,$[a, b] = [0,\ 1]$, $x_0 = 0.5$, compute priori error x_3.

iii) Newton's method

A. Given equation $f(x) = 0$, containing root interval $[a, b]$, choose $x_0 = a$ or $x_0 = b$ appropriately. Apply Newton's method to find approximate root x_n and its error.

1. $x^3 - 2x^2 - 5 = 0, [a, b] = [1, 4]$, compute x_2 and its error.
2. $x - \cos x = 0, [a, b] = [0, \pi/2]$, compute x_2 and its error.
3. $x - 0.8 - 0.2 \sin x = 0, [a, b] = [0, \pi/2]$, compute x_2 and its error.