University of Technology - HCMC
Faculty of Applied Mathematics

Numerical Method Project
Semester 1 (2021 - 2022)

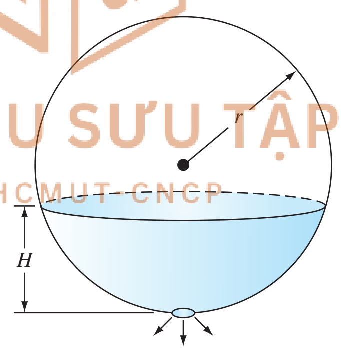
Problem 1. A spherical tank has a circular orifice in its bottom through which the liquid flows out. The following data is collected for the flow rate through the orifice as a function of time:

t (s)	0	500	1000	1500	2200	2900
Q (m^3/hr)	10.55	9.576	9.072	8.640	8.100	7.560
t (s)	3600	4300	5200	6500	7000	7500
Q, (m^3/hr)	7.020	6.480	5.688	4.752	3.348	1.404

Write a script with supporting functions

(a) to estimate the volume of fluid (in liters) drained over the entire measurement period

(b) to estimate the liquid level in the tank at t = 0 s. Note that r = 1.5 m.



Problem 2. Let R be the rectangle $[0;2] \times [1;4]$.

(a) Let $f(x;y) = x\cos(x^2+y)$. Calculate the integral $\iint\limits_R f(x,y)dA$.

(b) Study the Simpson formula. Develop a function to estimate the integral in R using Simpson formula.

(c) Let n and m be the number of sub-interval in x and y components, respectively. Estimate the integral with $[n,m] = [40,60]$ and $[n,m] = [80,120]$ and estimate the errors.

Problem 3. Heat is conducted along a metal rod positioned be- tween two fixed temperature walls. Aside from conduction, heat is transferred between the rod and the surrounding air by convection. Based on a heat balance, the distribution of temperature along the rod is described by the following sec- ond-order differential equation

$$0 = \frac{d^2T}{dx^2} + h(T_\infty - T)$$

where T = temperature (K), h = a bulk heat transfer coefficient reflecting the relative importance of convection to conduction m^{-2}, x =distance along the rod (m), and T_∞ = temperature of the surrounding fluid (K).

(a) Convert this differential equation to a equivalent system of simultaneous algebraic equations using a centered difference approximation for the second derivative.

(b) Develop a function to solve these equations from $x = 0$ to L and return the resulting distances and temperatures, in which, the algebraic equations must be solved by tridiagonal matrix.

(c) Develop a script that invokes this function and then plots the results.

(d) Test your script for the following parameters: $h = 0.0425$ m^{-2}, $L = 12$ m, $T_\infty = 220$ K, $T(0) = 320$ K, $T(L) = 450$ K, and $\Delta x = 0.5$ m.