Chapter 0
Introduction

Discrete Structures for Computing on January 5, 2022

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1. **Course description**
   - Course outline
   - Document

2. **Some applications**

Context

Global

- 12 principal chapters on 45 hours for courses & exercises.
- 10 Homeworks (0%), 1 Assignment (20%)
- 2 evaluations: mid-exam (MCQ - 90 minutes - 30%) + final exam (MCQ + writing - 90 minutes - 50%)

Aims

The content of this subject is mainly a great part of logic, set theory and graph theory.
This is the mathematical base for many topics of Computational Science

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Subjects in general discrete mathematics course

- Logic
- Set theory
- Number theory
- Combinatorics: enumerative combinatorics, graph theory
- Algorithmics
- Information theory
- Complexity theory
- Probability theory
- Proof
- Counting and Relations

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Topics relational to discrete mathematics

1. Theoretical computer science
2. Information theory
3. Logic
4. Set theory
5. Combinatorics
6. Graph theory
7. Probability
8. Number theory
9. Algebra
10. Calculus of finite differences, discrete calculus or discrete analysis
11. Geometry
12. Topology
13. Operations research: scheduling
14. Game theory, decision theory, utility theory, social choice theory
15. Discretization
16. Discrete analogues of continuous mathematics
17. . . .

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Context

Course outline

- **Proof methods**
  - modular arithmetic over integers.
  - induction, contradiction.
- **Set theory**
  - relations, functions, cardinalities, relation, equivalence equation, partial order
  - combinatorics: counting, principles of sum, multiplication, division, inclusion and exclusion.
- **Graph theory**
  - directed, undirected, isomorphism
  - weighted graphs, algorithm for finding shortest paths
  - trees: features, binary trees, minimum spanning trees in connected and weighted graphs
  - flows network
- **Probabilistics Modelling**
  - introductory random variables.

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Course learning outcomes

L.O.1	Understanding of logic and discrete structures
L.O.1.1	Describe definition of propositional and predicate logic
L.O.1.2	Define basic discrete structures: set, mapping, graphs, ..
L.O.2	Modeling and formulating generic problems in real life by the discrete mathematical structures and languages
L.O.2.1	Logically describe some problems arising in Computing
L.O.2.2	Use proving methods: direct, contrapositive, induction
L.O.2.3	Explain problem modeling using discrete structures
L.O.3	Applying learnt knowledge to compute quantities of discrete structures and probabilities
L.O.3.1	Operate/ compute on some basic problems which were modeled by discrete structures (set, graph, tree,..)
L.O.3.2	Calculate optimally based for solving basic problems in graph theory (shortest path, minimum spanning tree,..)

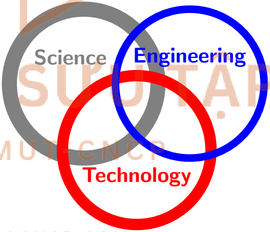
Document

Book

- Discrete mathematics and applications - Kenneth H. Rosen. (Vietnamese translation - NXB KHKT 1997)
- Discrete mathematics - Richard Johnsonbaugh, Willey, 1997
- Discrete mathematics with algorithms - Micheal O. Albertson & Joan P. Hutchinson, Willey, 1998

Application

- it concerns a wide range of disciplines in various areas: science, technology, business and commerce.
- applied mathematicians are engaged in the creation, study and application of advanced mathematical methods relevant to specific problems.
- applied mathematics has assumed a much broader meaning and embraces such diverse fields as communication theory, optimization, game theory and numerical analysis.
- today there is a remarkable variety of applications of mathematics in industry and government, such as materials processing, design, medical diagnosis, development of financial products, network management, weather prediction, etc.



Engineers use technology, mathematics and scientific knowledge to solve practical problems. (wikipedia.org)

Computing of algorithm complexity

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Know results

Size	Approximating of computational time					
n	$O(\log n)$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(2^n)$	$O(n!)$
10	3.10^{-9}s	10^{-8}s	3.10^{-8}s	10^{-7}s	10^{-6}s	3.10^{-3}s
10^2	7.10^{-9}s	10^{-7}s	7.10^{-7}s	10^{-5}s	4.10^{13}y	*
10^3	10^{-8}s	10^{-6}s	10^{-5}s	10^{-3}s	*	*
10^4	$1,3.10^{-8}$s	10^{-5}s	10^{-4}s	10^{-1}s	*	*
10^5	$1,7.10^{-8}$s	10^{-4}s	2.10^{-3}s	10s	*	*
10^6	2.10^{-8}s	10^{-3}s	2.10^{-2}s	17m	*	*

Mathematical model

Solver

- Simplex, GLPK
- CPLEX, MPL
- Excel, Mathlab, etc.

$$\text{maximise } z = x_1 + x_2 - 2x_3 + 2x_4$$

subject to:

$$x_1 - x_2 - x_3 - 2x_4 \geq 2$$

$$x_1 + x_2 + x_4 \leq 8$$

$$x_1 + 2x_2 - x_3 = 4$$

$$x_1, ..., x_4 \geq 0.$$

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Mathematical model

Exercise - Carpenter's decision

- A carpenter makes tables and chairs.
- Each table can be sold for a profit of \$30 and each chair for a profit of \$10.
- The carpenter can afford to spend up to 40 hours per week working.
- He takes 7 hours to make a table and 3 hours to make a chair.
- It requires that he makes at least 3 times as many chairs as tables.
- Tables take up 4 times as much storage space as chairs and there is room for at most 4 tables each week.

Question.
How many tables and chairs could be produced to maximize weekly profit?

Mathematical model

Exercise - Bookseller's decision

- A bookseller $\mathcal{A}$ buys books from two publishers $\mathcal{B}$, and $\mathcal{C}$.
- Publisher $\mathcal{B}$ offers a package of 6 mysteries and 8 romance novels for $40.
- Publisher $\mathcal{C}$ offers a package of 5 mysteries and 10 romance novels for $100.
- The bookseller $\mathcal{A}$ wants to buy at least 2.000 mysteries and 3.000 romance novels,
- $\mathcal{A}$ has promised $\mathcal{C}$ (who has influence on the Senate Textbook Committee) that at least 30% of the total number of books he purchases will come from publisher $\mathcal{C}$.

Question.
How many packages should $\mathcal{A}$ order from each publisher in order to minimize his cost and satisfy $\mathcal{C}$?

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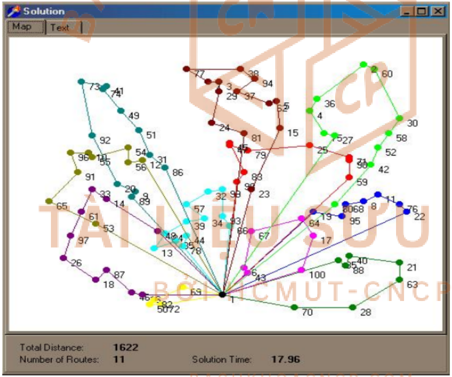
Graph

- Shortest path problem
- Min cut and maximum flow
- **V**ehicle **R**outing **P**roblem



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Scheduling

Scheduling

Exercise

Problem $1||T_{\max}$.
Given 8 jobs with processing times and due dates as follows:

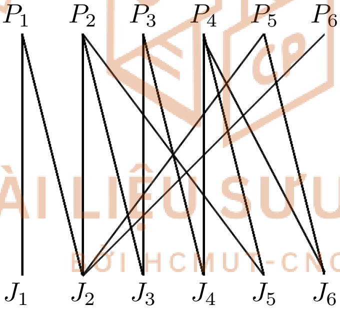
Job	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
p_i	1	2	2	3	3	4	4	3
d_i	25	16	19	7	18	22	27	8

Let C_i be completion time of job J_i and let $T_i = max(0, C_i - d_i)$ its tardiness.
Question. How to minimize $T_{max} = \max_i T_i$? What is the minimum value of T_{max} ?

Timetabling

Example

In the bipartite graph below, the vertices $P_1, \ldots, P_6$ represent workers and edges $J_1, \ldots, J_6$ of jobs. An edge connects a worker with a job if the worker has the necessary qualifications to occupy this job. Here, all the edges have an unit weight 1, mean that P_i has the skill(competence) to operate J_j if there is an edge between P_i and J_j.



Game and simulation

Sally Salon Game

Probabilistics Modelling

Calculating of Pi

Using a Monte-Carlo method to determine an approximate value of π :
randomly draw a great number of points in a square of side 2, and determine the ratio C/N where N is the total number of points, and C the number of points whose distance to the center of the square is ≤ 1).

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