Session 09 - Hash Table

Instructor:

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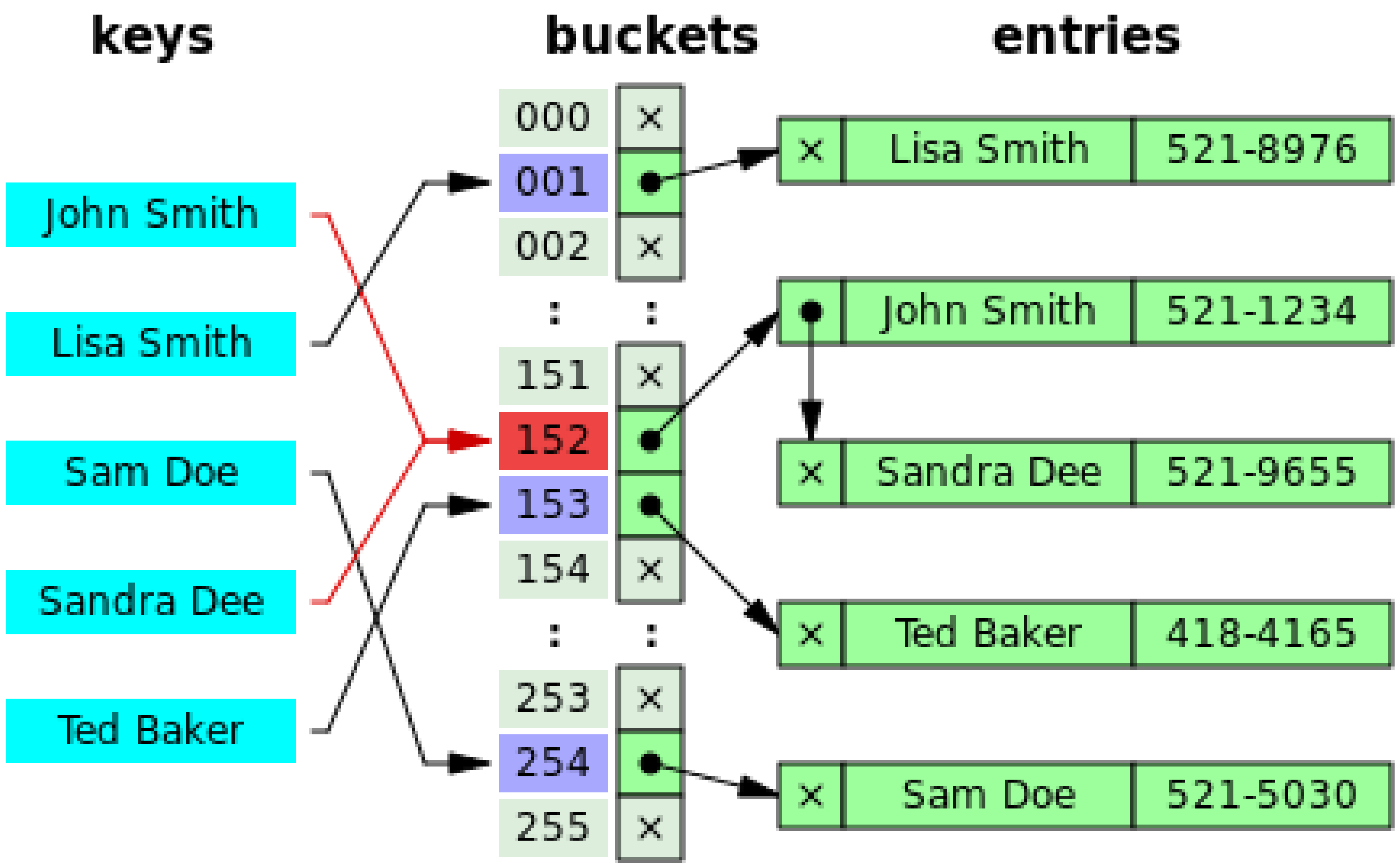
Introduction

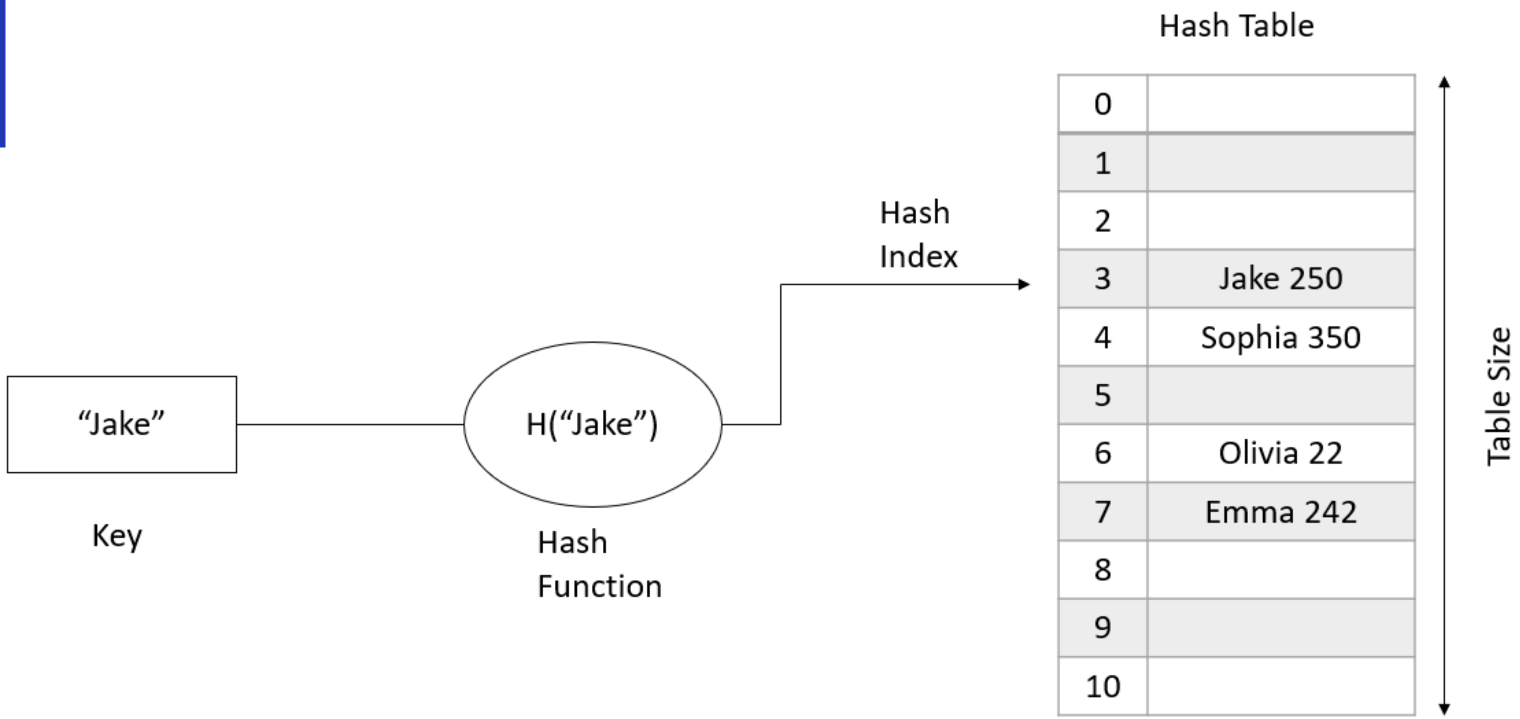
Introduction

- Binary search tree retrieval have order $O(\log_2 n)$
- Need a different strategy to locate an item
- Consider a "magic box" as an address calculator
  - Place/retrieve item from that address in an array
  - Ideally to a unique number for each key



Introduction

▪ Hashing is a technique to convert a range of key values into a range of indexes of an array.

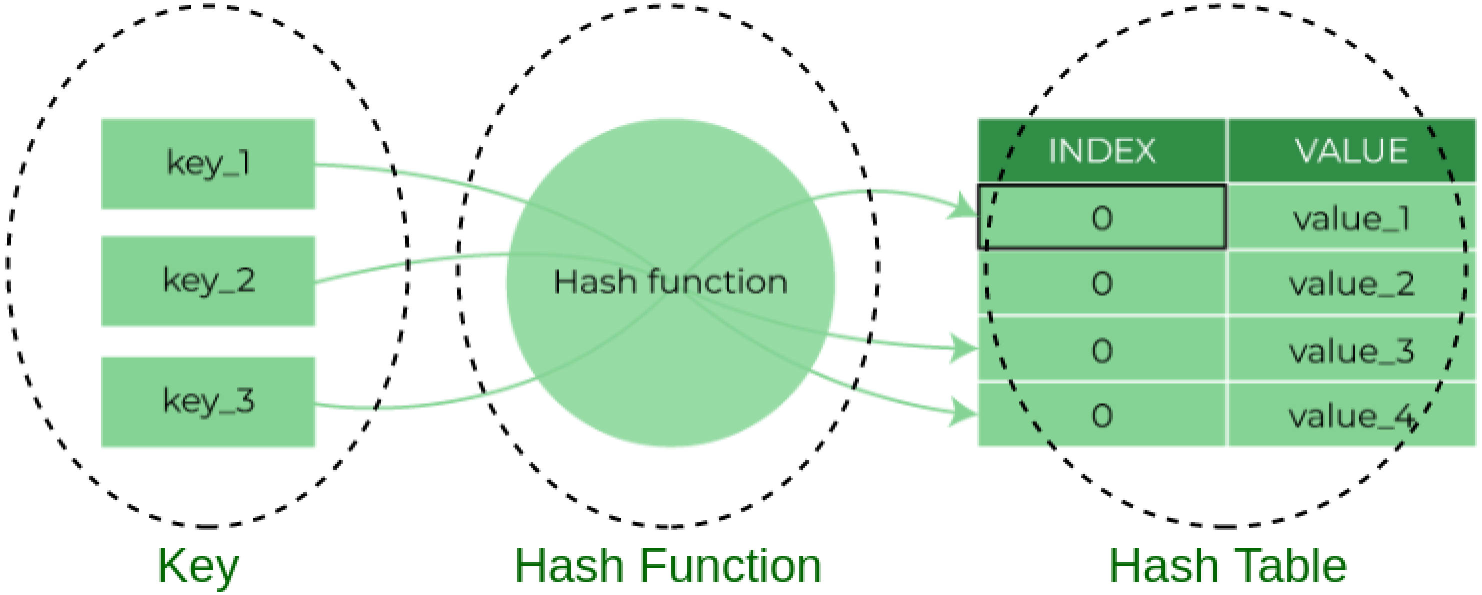


▪ Hashing is a technique to convert a range of key values into a range of indexes of an array.

▪ **Large keys** are converted into **small keys** by using **hash functions**

▪ The values are then stored in a data structure called **hash table**.

- Idea:
  - Distribute entries (key/value pairs) uniformly across an array.
  - Each element is assigned a key (converted key).

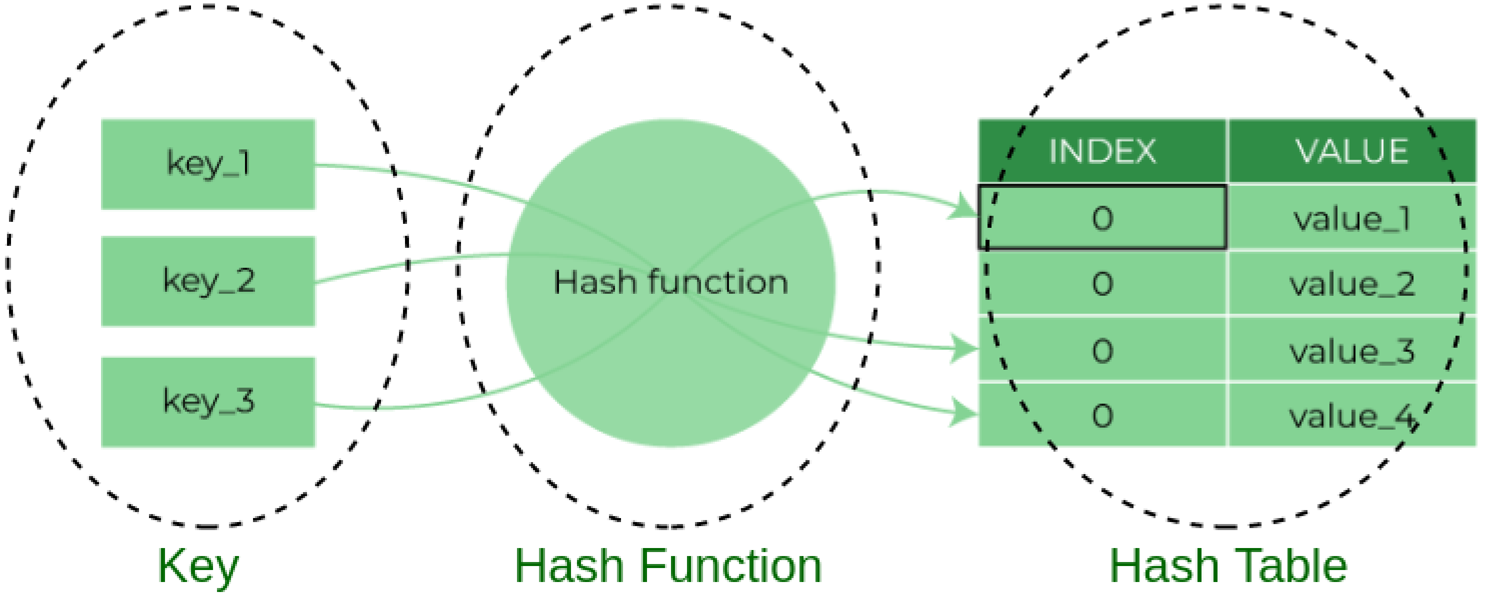


- Using that key to access the element in O(1) time. (The hash function computes an index suggesting where an entry can be found or inserted.)

Introduction

- A hash table is a data structure that is used to store keys/value pairs.

- It uses a hash function to compute an index into an array in which an element will be inserted or searched.



Hash Function

Hash Function

- Hash function is a mathematical function that can be used to map/converts a key to an integer value (an array index).

- The values returned by a hash function
  - hash values
  - hash codes
  - hash sums
  - digests
  - hashes.

Some Hash Functions

- Possible functions
  - Selecting digits
  - Folding
  - Modulo arithmetic
  - Converting a character string to an integer
    - Use ASCII values
    - Factor the results, Horner’s rule

Some Hash Functions

- Digit-selection:
  - Select some digits in the keys to create the hash value.
    - h(001**3**6482**5**) = 35
- Folding
  - h(001364825) = 0 + 0 + 1 + 3 + 6 + 4 + 8 + 2 + 5 = 29
  - h(**001**364**825**) = 001 + 364 + 825 = 1190
- Modulo arithmetic
  - h(Key) = Key mod 101
    - h(001364825) = 12

Some Hash Functions

- A string key hash function

$$h = \sum_{i=0}^{keylength} 128^i \times char(key[i])$$

Modulo Arithmetic

- Assume all keys are integers, and define

$$h(k) = k \; mod \; m$$

  - Where k is the key and m is the size of hash table
- Extreme deficiency: if $m = 2^r$, then the hash value doesn't even depend on all the bits of k
  - Ex: if k = 1011000111**011010** and $\mathrm{r} = 6$
  - then $h(k) = 011010$
- Size of hash table array should be a **prime** number

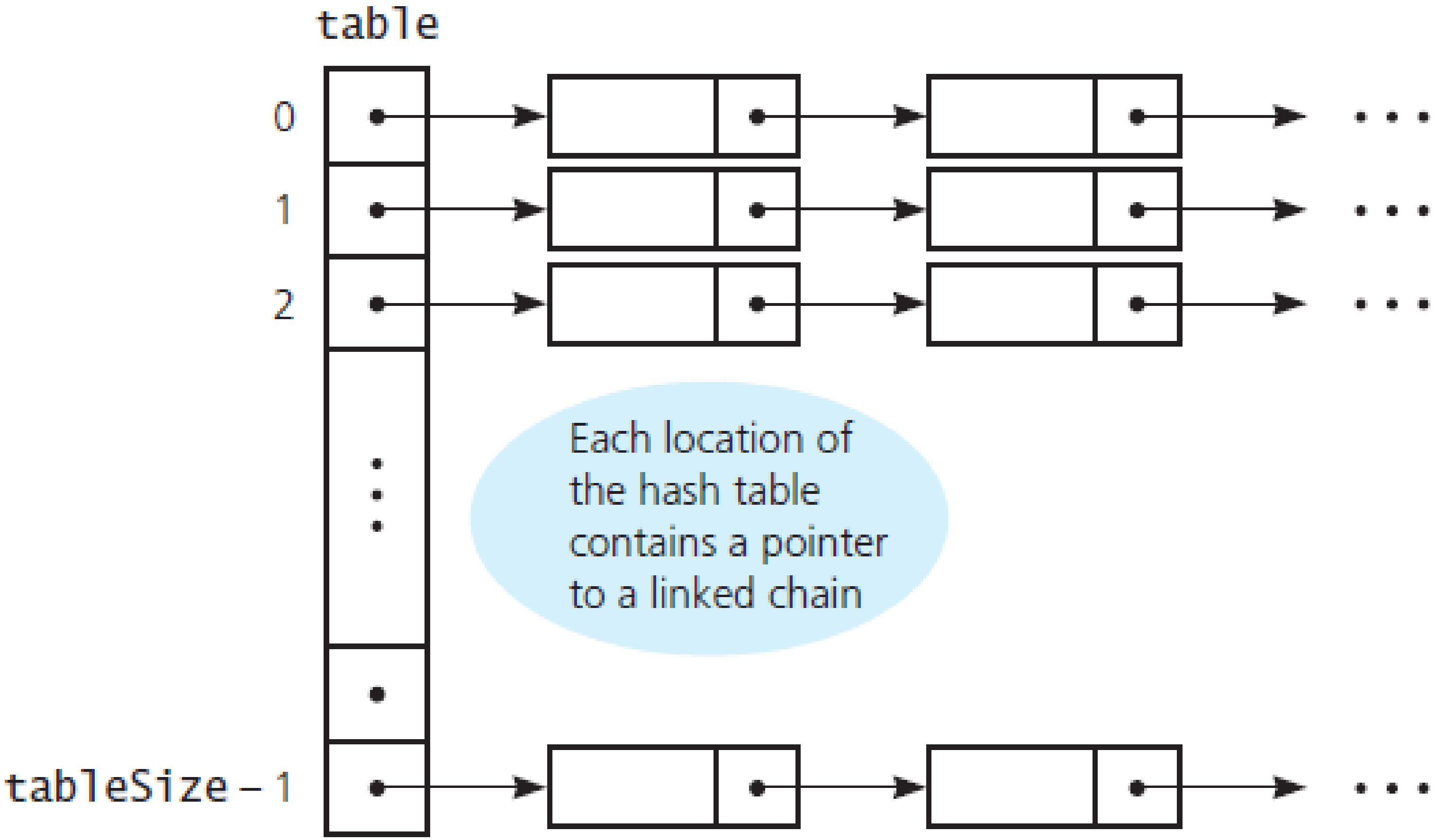
- Properties of good hash functions

Resolving Collisions

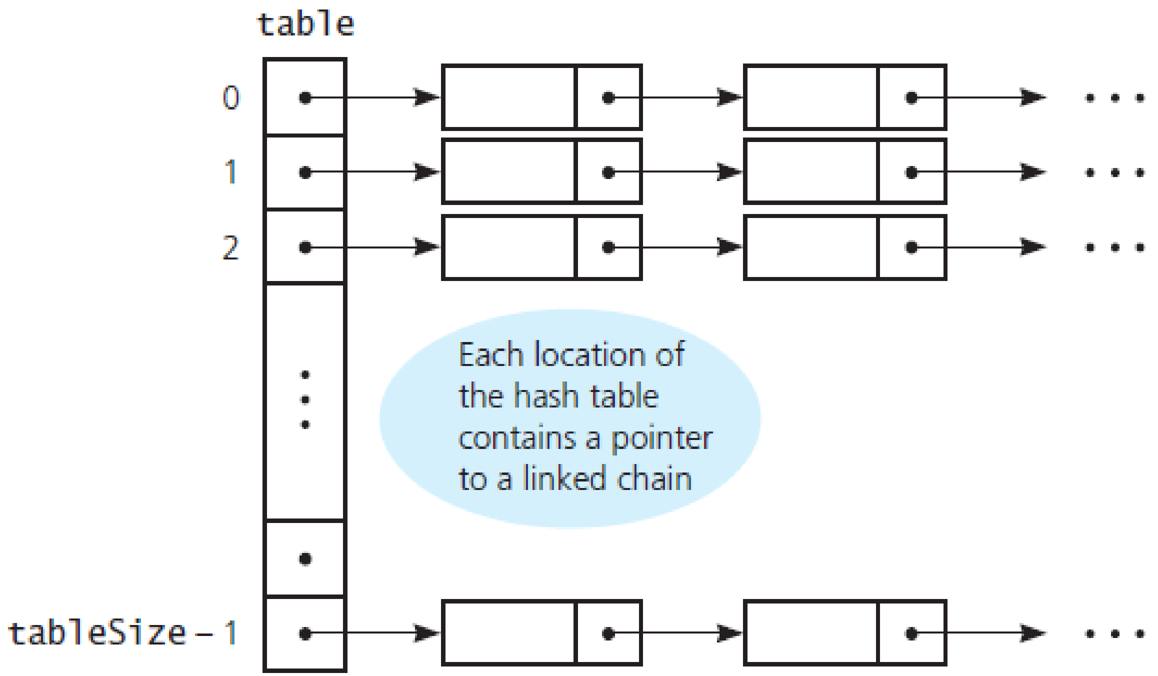
Collisions

- **Collision**: when two keys map to the **same location** in the hash table.
- Two ways to resolve collisions:
  - Separate Chaining – open hashing
  - Open Addressing – closed hashing
    - Linear probing
    - Quadratic probing
    - Double hashing

- Separate chaining: All keys that map to the same hash value are kept in a list (or "bucket")



- Each hash location can accommodate more than one item
- Each location is a "bucket" or an array itself
- Alternatively, design the hash table as an array of linked chains ("separate chaining").



Separate Chaining

- Give the hash function h(k) = k mod 10, distribute the list of integers

  10, 22, 107, 12, 42

  into the hash table

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Separate Chaining

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0	10
1	
2	22 → 12 → 42
3	
4	
5	
6	
7	107
8	
9	

Open Addressing

- Probe for another available location
- Some techniques:
  - Linear probing
  - Quadratic probing
  - Double hashing

Linear Probing

- In linear probing, the hash table is searched sequentially that starts from the original location of the hash.
- If in case the location that we get is **already occupied**, then we check for the next location
- Specifically,

$$\boldsymbol{H(k, step) = (h(k) + step) \bmod M}$$

Where:

- $step$ = 0, 1,...
- M : size of hash table

Linear Probing

fit@hcmus

- Given the hash function h(k) = k mod 10, we will distribute a list of integers

  10, 22, 107, 12, 42

  into a hash table. Additionally, we will employ linear probing to address any collisions that may arise during the process, using a step size of 1.

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

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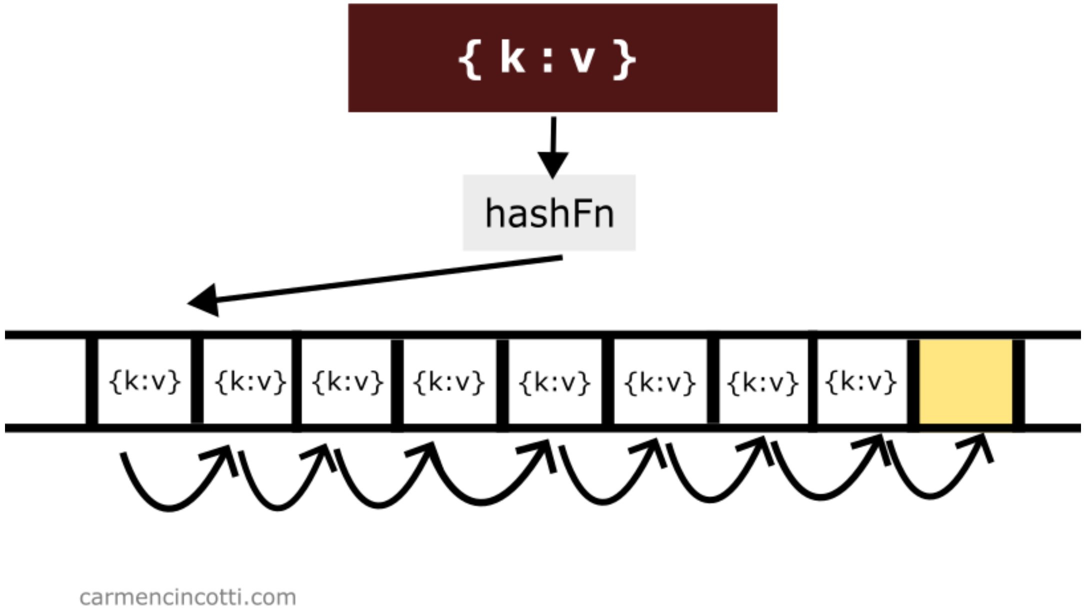
0	10
1	
2	22
3	12
4	42
5	
6	
7	107
8	
9	

- Clustering is a phenomenon that occurs as elements are added to a hash table. Elements may have a tendency to clump together, forming clusters, which over time will significantly impact performance for searching and adding elements



Quadratic Probing

- Specifically,

$$H(k, step) = (h(k) + step^2) \bmod M$$

Where:

- $step$ = 0, 1,...
- M : size of hash table

Quadratic Probing

- Given the hash function h(k) = k mod 7, we will distribute a list of integers

  22, 30, 50, 37, 44

  into a hash table. Additionally, we will employ quadratic probing to address any collisions that may arise during the process

0	
1	
2	
3	
4	
5	
6	

Quadratic Probing

- Given the hash function h(k) = k mod 7, we will distribute a list of integers

  22, 30, 50, 37, 44

  into a hash table. Additionally, we will employ quadratic probing to address any collisions that may arise during the process

0	
1	22
2	30
3	37
4	
5	50
6	44

- Specifically,

$$H(k, step) = \left(h(k) + step * h_2(k)\right) mod\ M$$

Where:

  - $step$ = 0, 1,…
  - M : size of hash table
  - $h_2(.)$: the second hash function

Double Hashing

- Given the hash function $h(k) = k \ mod \ 7$, we will distribute a list of integers

  27, 43, 692, 72

  into a hash table. Additionally, we will employ double hashing to address any collisions that may arise during the process with the second hash function of $h_2(k) = 1 + (k \ mod \ 5)$

0	
1	
2	
3	
4	
5	
6	

Double Hashing

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  into a hash table. Additionally, we will employ double hashing to address any collisions that may arise during the process with the second hash function of $h_2(k) = 1 + (k \; mod \; 5)$

0	
1	43
2	692
3	
4	
5	72
6	27

- Some functions recommended in the literature:
  - $h_2(Key) = m - 2 - Key \, mod \, (m - 2)$
  - $h_2(Key) = 8 – (Key \, mod \, 8)$
  - $h_2(Key) = Key \, mod \, 97 + 1$

- **Advantages:**
  - Simple to implement.
  - Hash table never fills up, we can always add more elements to the chain.
  - Less sensitive to the hash function or load factors.
  - It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.

Separate Chaining

Disadvantages:

- Cache performance of chaining is not good as keys are stored using a linked list. Wastage of space (Some parts of hash table are never used)
- If the chain becomes long, then search time can become O(n) in the worst case.
- Uses extra space for links.

Open Addressing

- Removal requires specify state of an item
  - Occupied, emptied, removed → **Why?**
- Clustering is a problem
  - 2 keys have the same collision chain if their initial position is the same
- Double hashing can reduce clustering

Open Addressing

- Linear probing has the best cache performance but suffers from clustering. One more advantage of Linear probing is easy to compute.
- Quadratic probing lies between the two in terms of cache performance and clustering.
- Double hashing has poor cache performance but no clustering. Double hashing requires more computation time as two hash functions need to be computed.

The Efficiency of Hashing

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- Efficiency of hashing involves the load factor alpha (α)

$$\alpha = \frac{\text{Current number of table items}}{tableSize}$$

The Efficiency of Hashing

- Efficiency of hashing involves the load factor alpha (α)
- The load factor is the average number of key-value pairs per bucket.

$$\alpha = \frac{\textit{Current number of table items}}{\textit{tableSize}}$$

- The higher the load factor, the slower the retrieval
- With open addressing, the load factor cannot exceed 1.
- With chaining, the load factor often exceeds 1

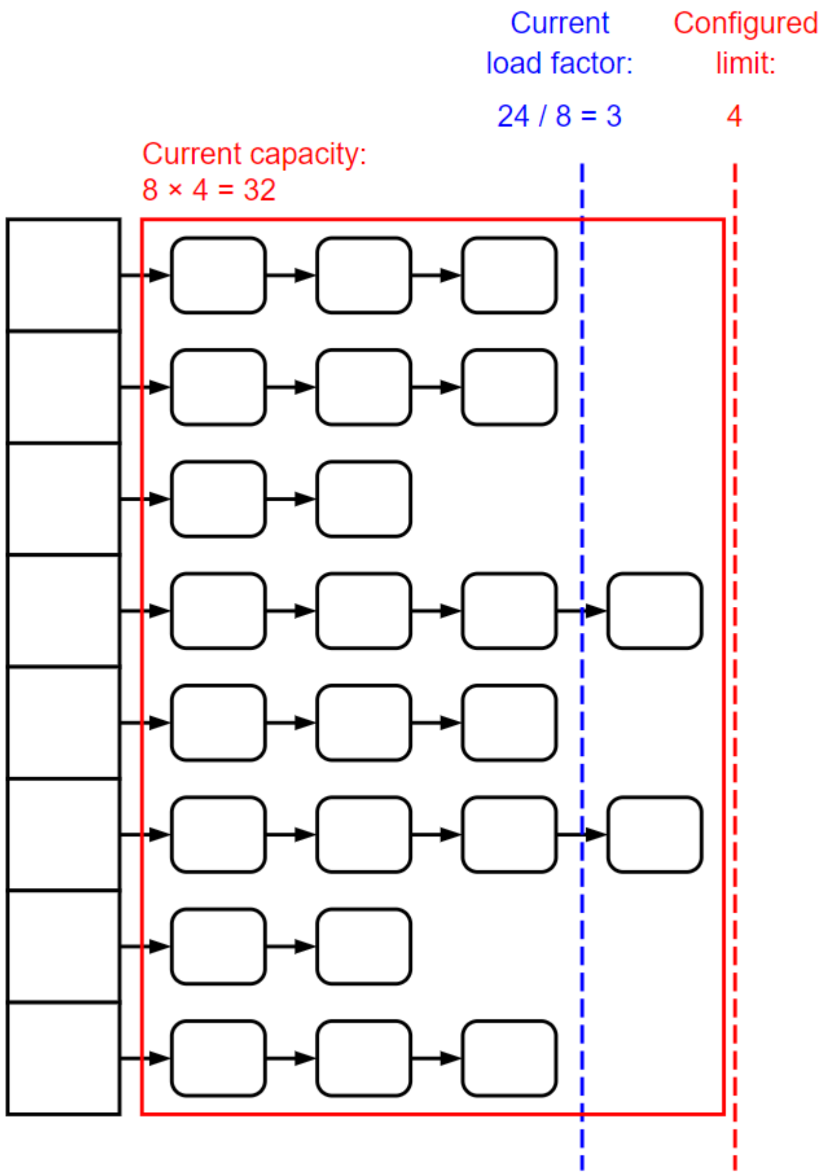
Capacity

- The capacity is the maximum number of key-value pairs for the given load factor limit and current bucket count

$$\text{capacity} = \text{number of buckets} \times \text{load factor limit}$$

The Efficiency of Hashing

- It is when the load factor reaches a given limit that rehashing kicks in.
- Since rehashing increases the number of buckets, it reduces the load factor
- The load factor limit is usually configurable and offers a **tradeoff** between **time** and **space** costs



The Efficiency of Hashing

- Linear probing – average value for α

$$\frac{1}{2}\left[1 + \frac{1}{1-\alpha}\right] \quad \text{for a successful search, and}$$

$$\frac{1}{2}\left[1 + \frac{1}{(1-\alpha)^2}\right] \quad \text{for an unsuccessful search}$$

- Quadratic probing and double hashing – efficiency for given α

$$\frac{-\log_e(1-\alpha)}{\alpha} \quad \text{for a successful search, and}$$

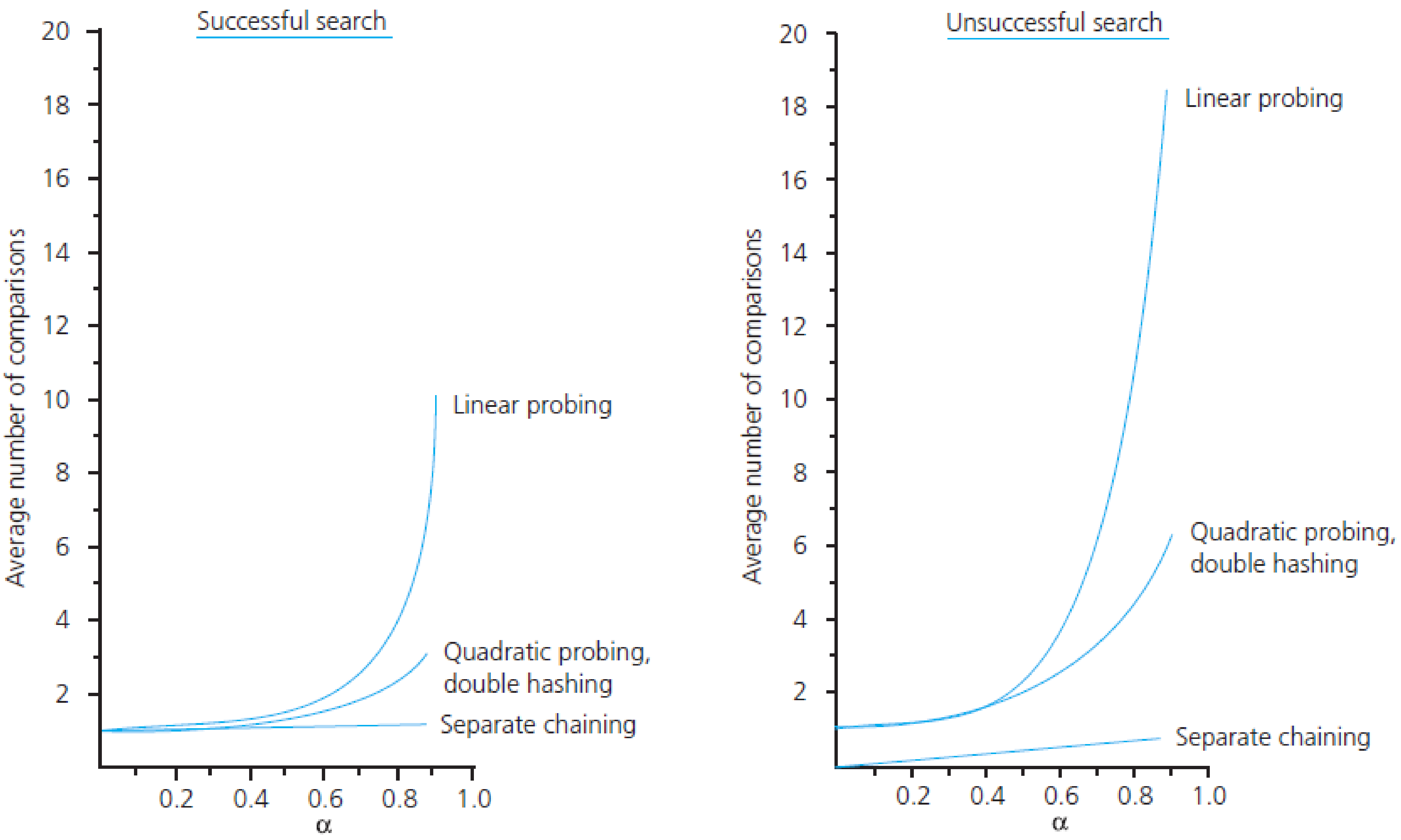
$$\frac{1}{1-\alpha} \quad \text{for an unsuccessful search}$$

- Separate chaining – efficiency for given α

$$1 + \frac{\alpha}{2} \qquad \text{for a successful search, and}$$

$$\alpha \qquad \text{for an unsuccessful search}$$

The Efficiency of Hashing



Maintaining Hashing Performance

- Collisions and their resolution typically cause the load factor α to increase
- To maintain efficiency, restrict the size of α
  - $\alpha \leq 0.5$ for open addressing
  - $\alpha \leq 1.0$ for separate chaining
- If load factor exceeds these limits
  - Increase size of hash table
  - Rehash with new hashing function

Exercise

- Given a hash table with m = 13 entries and the hash function

$$h(key) = key \ mod \ m$$

- Insert the keys {**10, 22, 31, 4, 15, 28, 17, 88, 59**} in the given order (from left to right) to the hash table. If there is a collision, use each of the following open addressing resolving methods:

  A. Linear probing

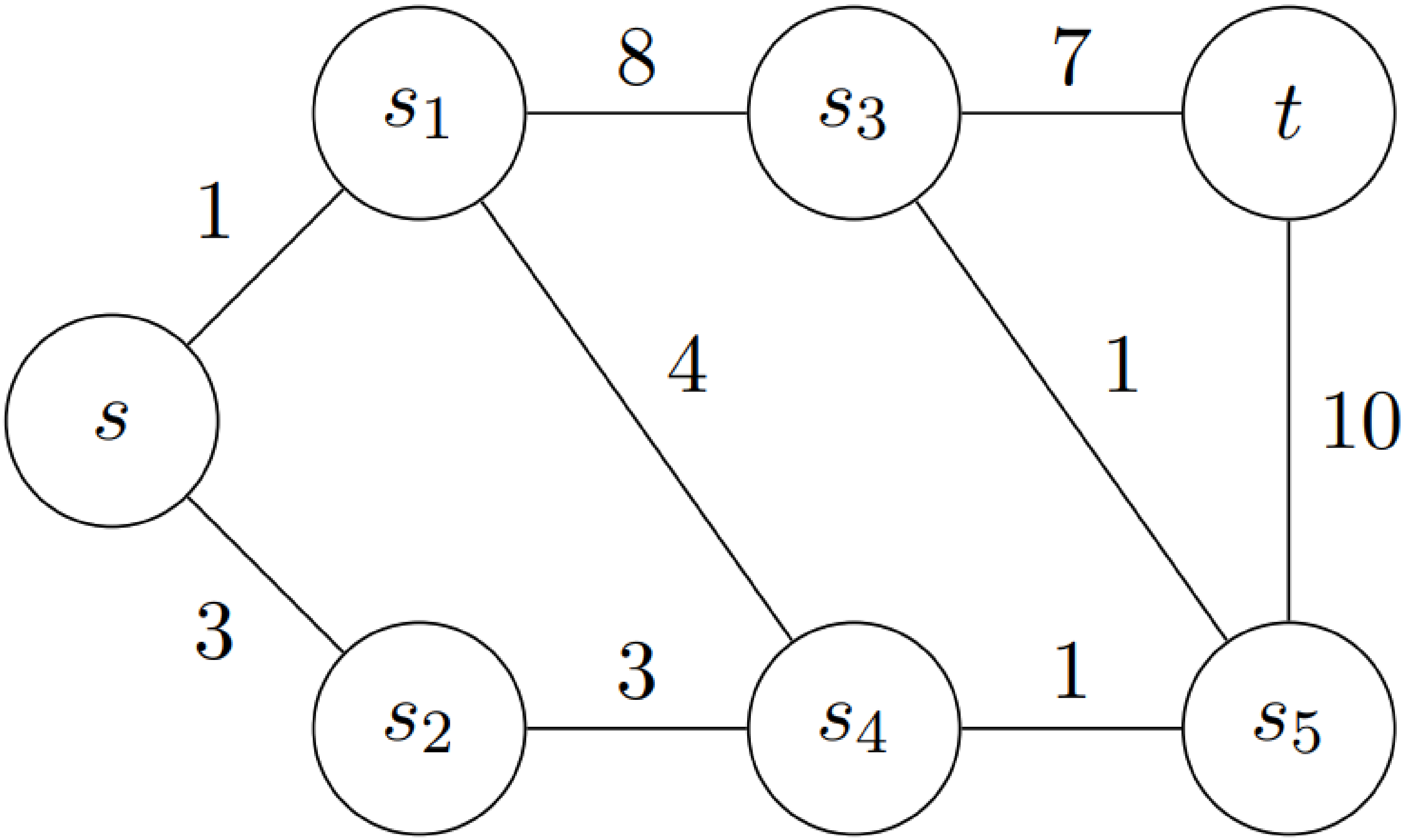
  B. Quadratic probing

  C. Double hashing with $h_2(key) = (key \ mod \ 7) + 1$

THANK YOU
for YOUR ATTENTION

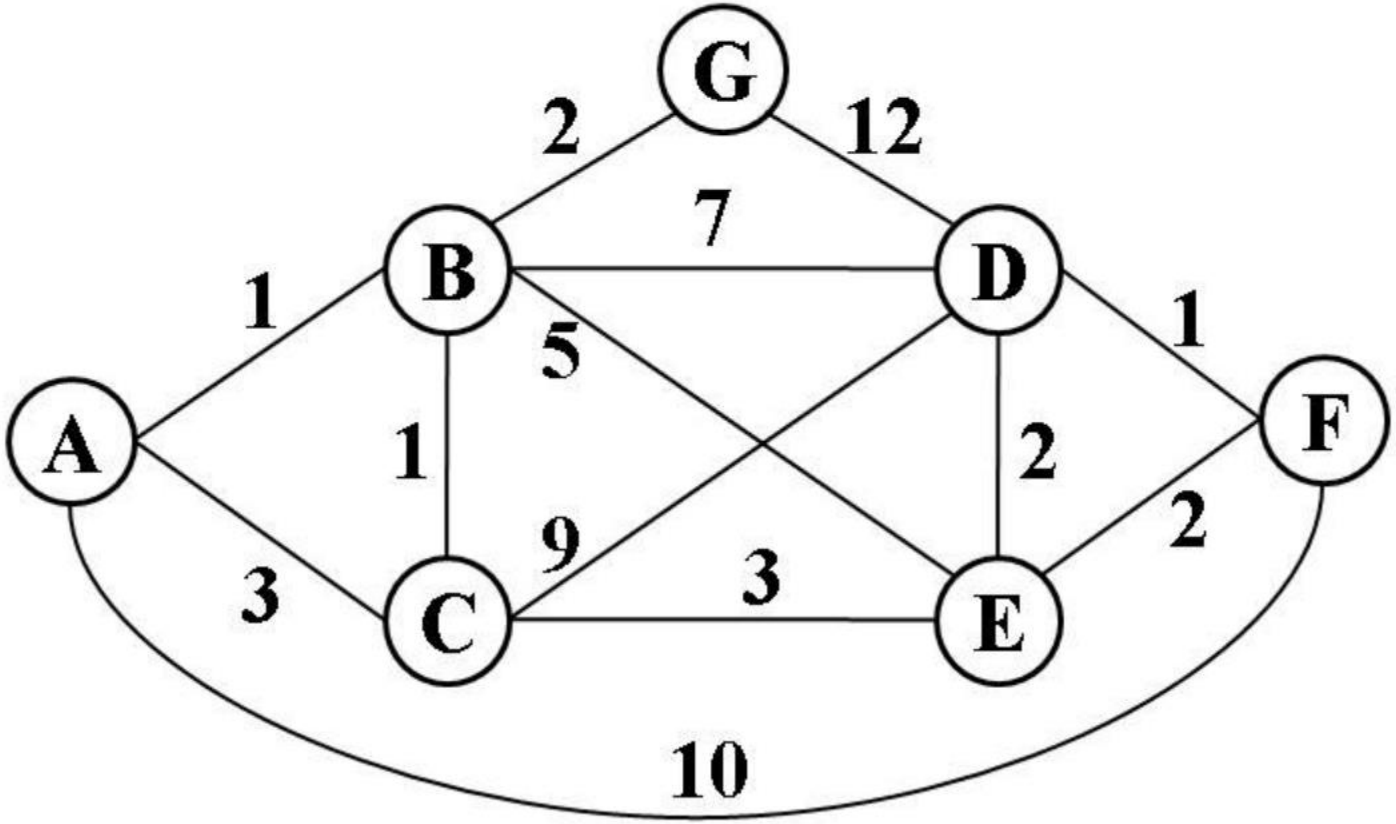
Exercise

- Apply Dijkstra's algorithm to the following graph to find the shortest path (and its cost) from s to the other vertices. Write down all intermediate steps



Exercise

- Apply Dijkstra's algorithm to the following graph to find the shortest path (and its cost) from **G** to the other vertices. Write down all intermediate steps



Exercise

- Apply Prim/ Kruskal's algorithm to find the minimum spanning tree of the graph