Particle Swarm Optimization

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Introduction

- What is a ***swarm*** ?
  - A ***group*** of insects (typically bees or flies)
  - ***Pseudo-random*** behavior
  - Ending up on the same target

http://en.wikipedia.org/wiki/Swarm

- Developed by ***James Kennedy*** and ***Russel Eberhart*** in 1995
  - ***Unpredictable behavior*** of bird flocks
  - A ***social behavior*** model
  - ***Optimization*** Algorithm

The Basic Concepts

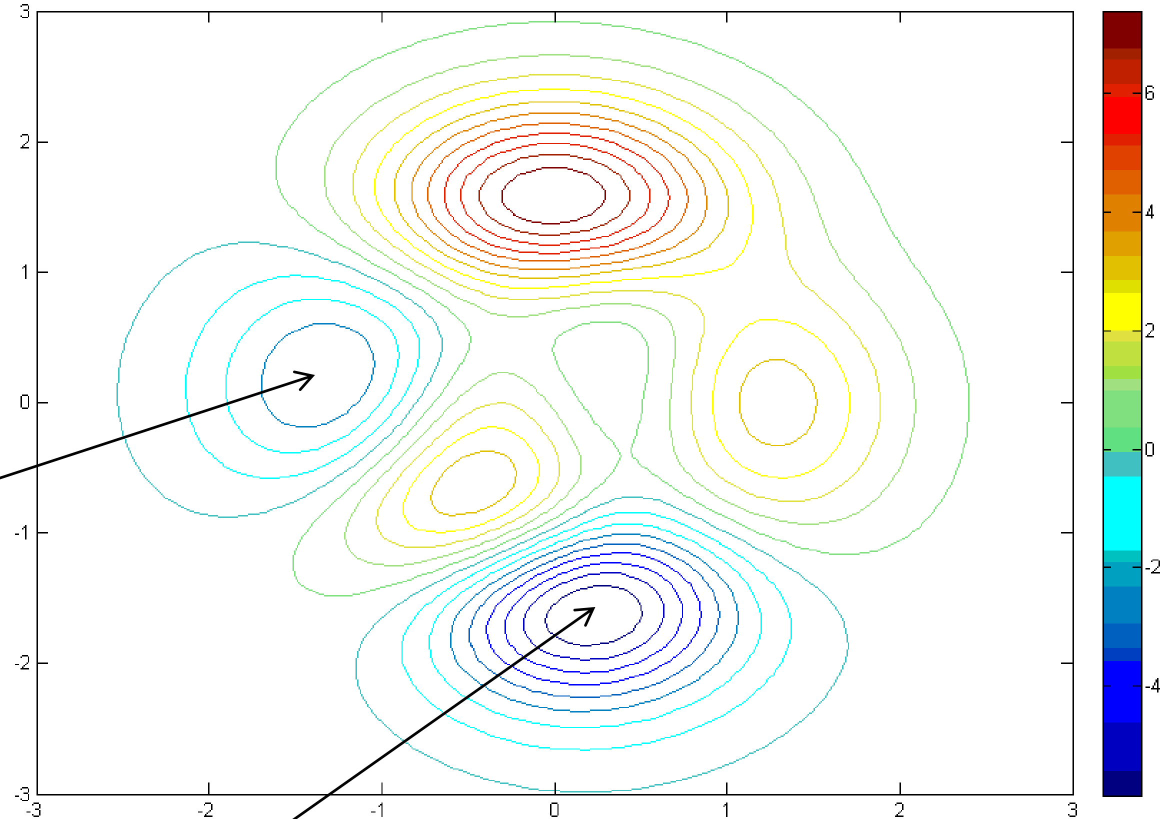
- ***N particles*** in a ***D-dimensional search-space*** looking for the ***global optima***
- Each particle changes its location according to two concepts:
  - Remembers the best location from entire flying history (***pbest***)
  - "***Communication***" between particles to obtain the global best (***gbest***)
- Particles accelarates towards pbest and gbest → velocity changes with each iteration step
- Mathematically can be modeled according to following equations:

$$v_i^d(k+1) = v_i^d(k) + c_1 \cdot r_1(k) \cdot (pBest_i^d(k) - x_i^d(k)) + c_2 \cdot r_2(k) \cdot (gBest_i^d(k) - x_i^d(k))$$

$$x_i^d(k+1) = x_i^d(k) + v_i^d(k+1)$$

The Original PSO Algorithm

- Define the limits of the search-space.

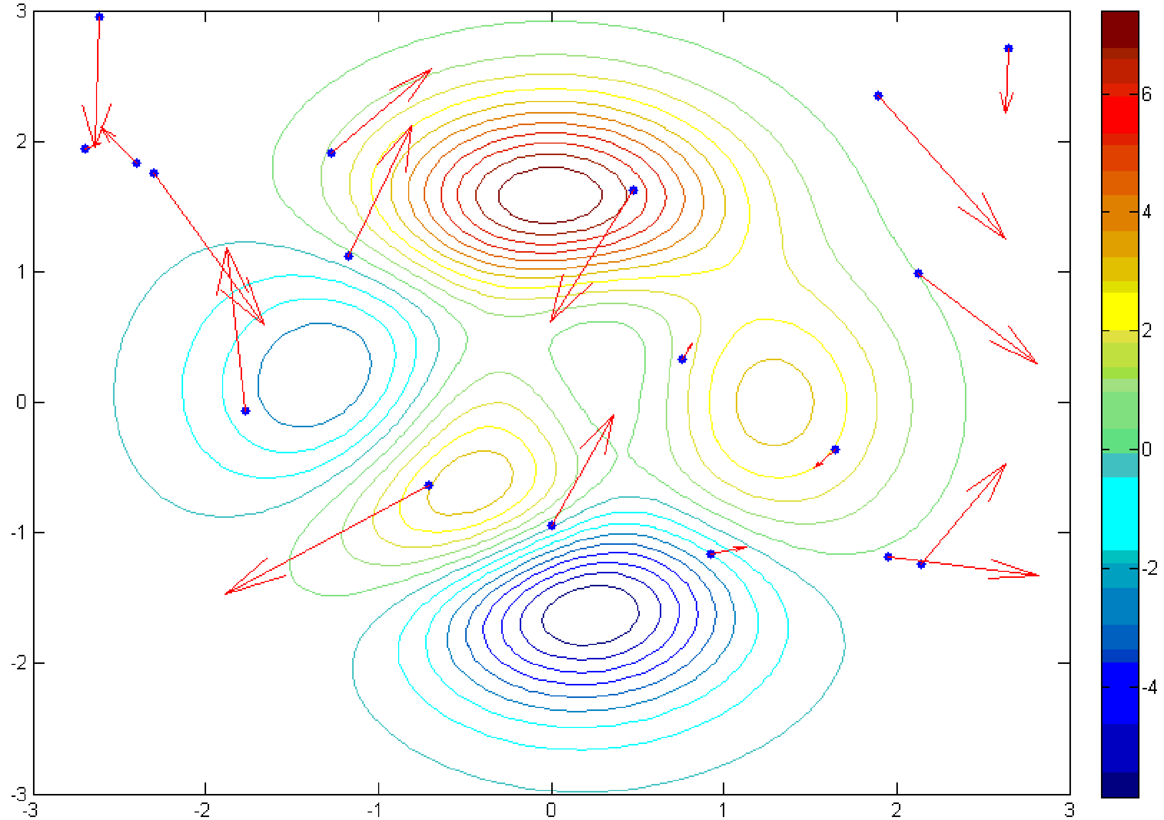


Local minima

Global minima

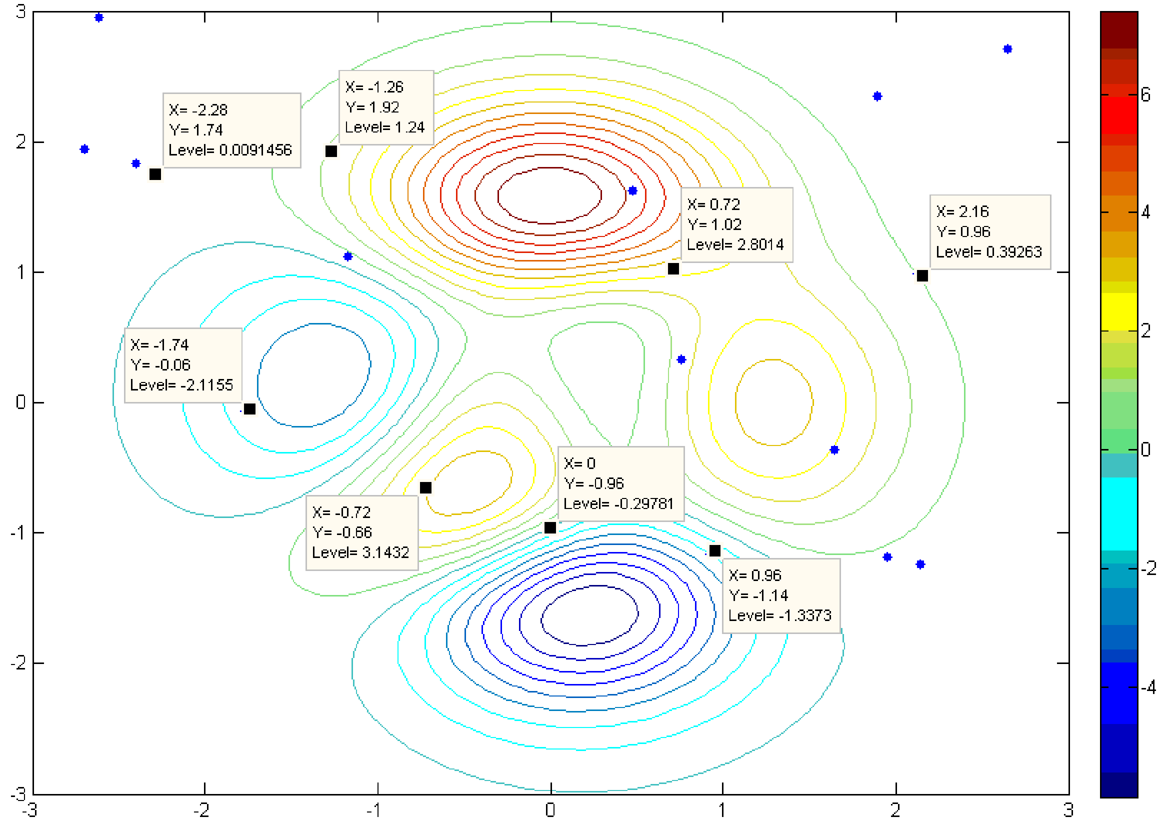
The Original PSO Algorithm

- Initialize a population on N particles. Randomly select the position and velocity for each particle in all dimensions.



The Original PSO Algorithm

- Evaluate the fitness function for each particle.



The Original PSO Algorithm

- Compare fitness value with pbest. If fitness is better than pbest, update pbest as the current fitness value.

- Example: In case of a minimization problem

    - at the k-th iteration:

      fitness(k) = 2.6 , pbest(k-1) = -0.6

      fitness(k) > pbest(k-1) ➔ pbest = -0.6 (no change)

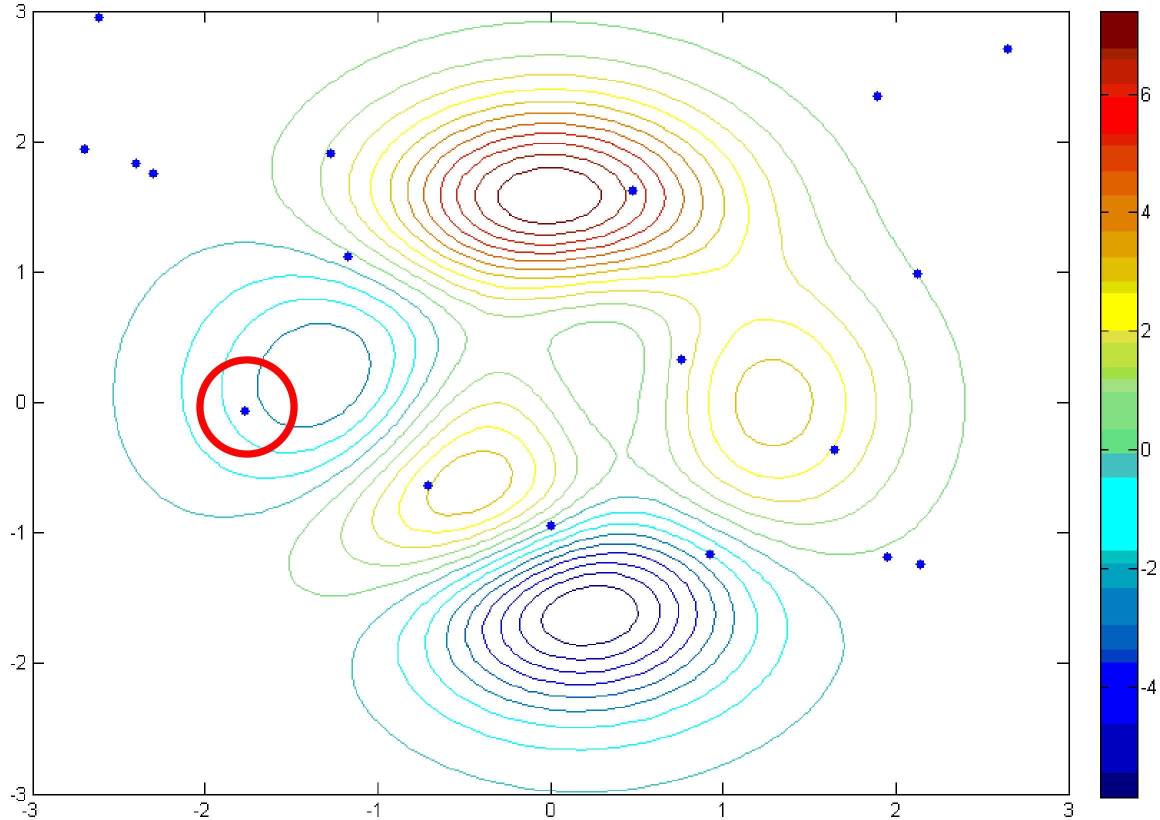
    - If :

      fitness(k) = -0.6 , pbest(k-1) = 2.6

      fitness(k) < pbest(k-1) ➔ pbest = fitness(k) = -0.6 (update)

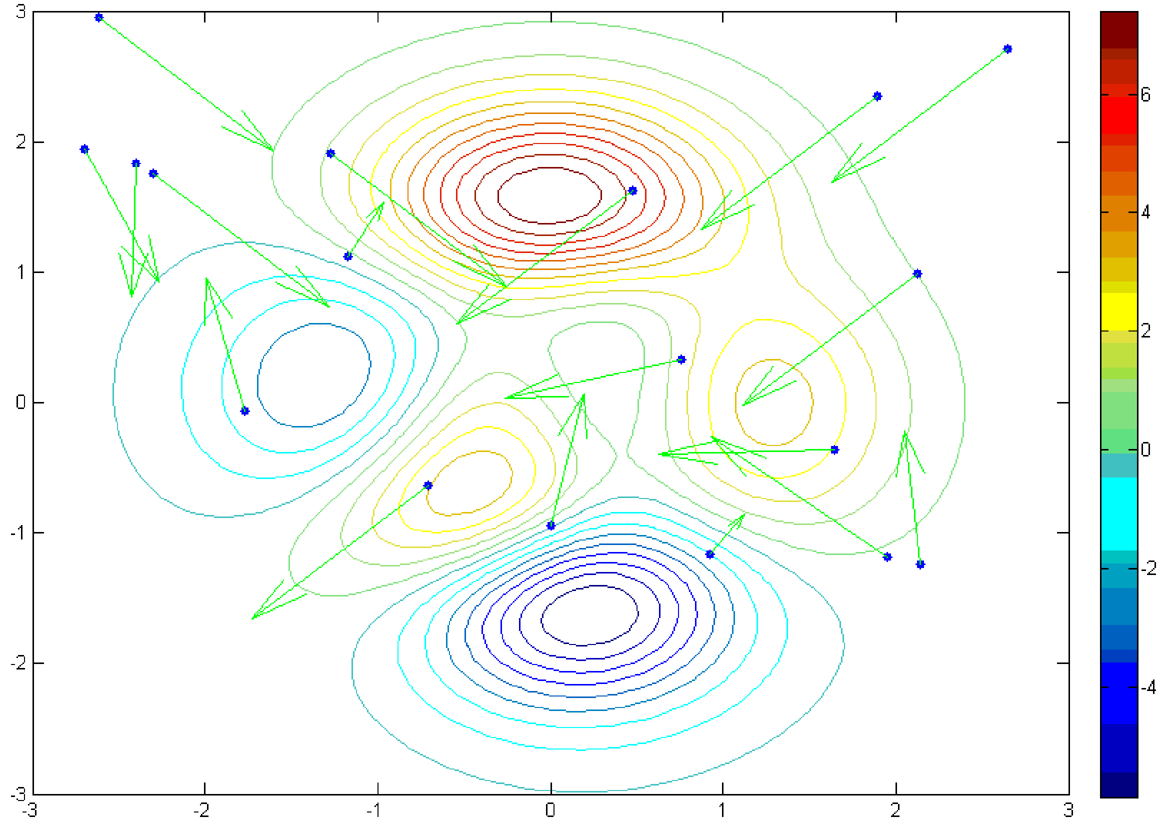
The Original PSO Algorithm

- Find the best pbest of the entire population. If it is better than gbest , update gbest as the current best pbest.



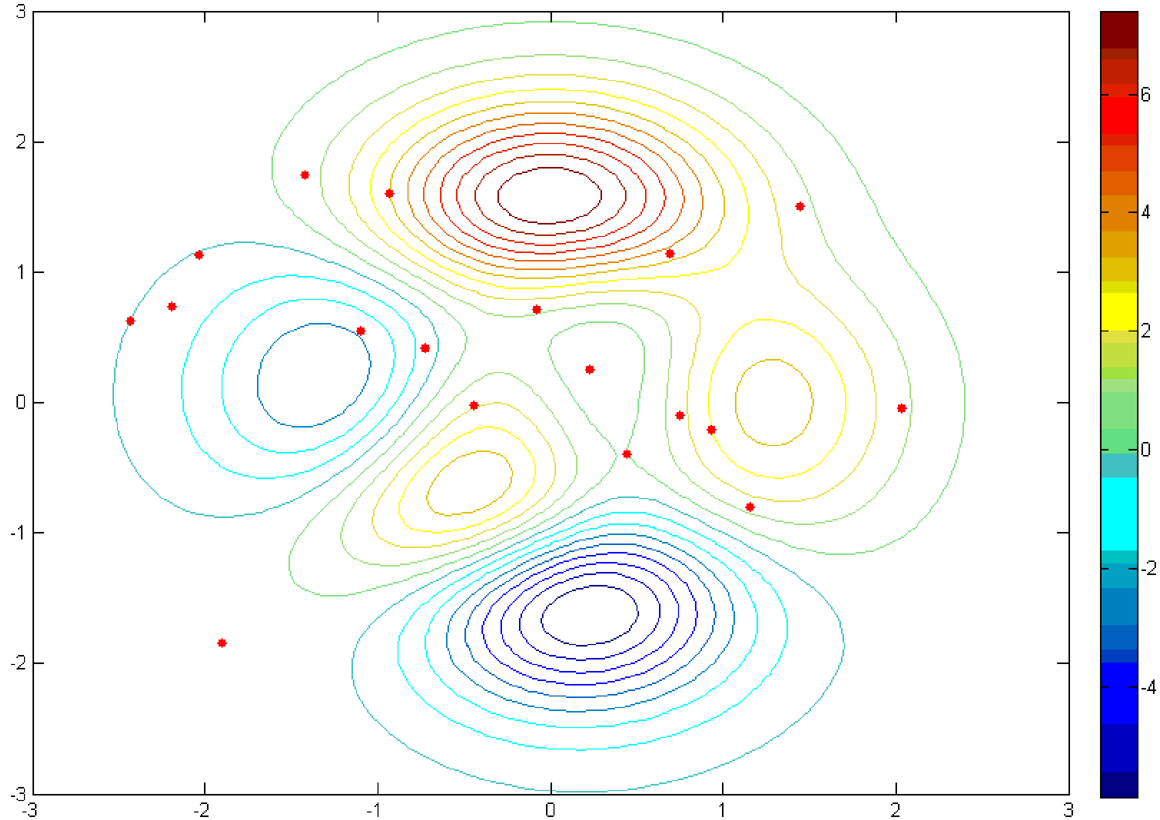
The Original PSO Algorithm

- Update velocities and positions of particles according to the iterative equations.

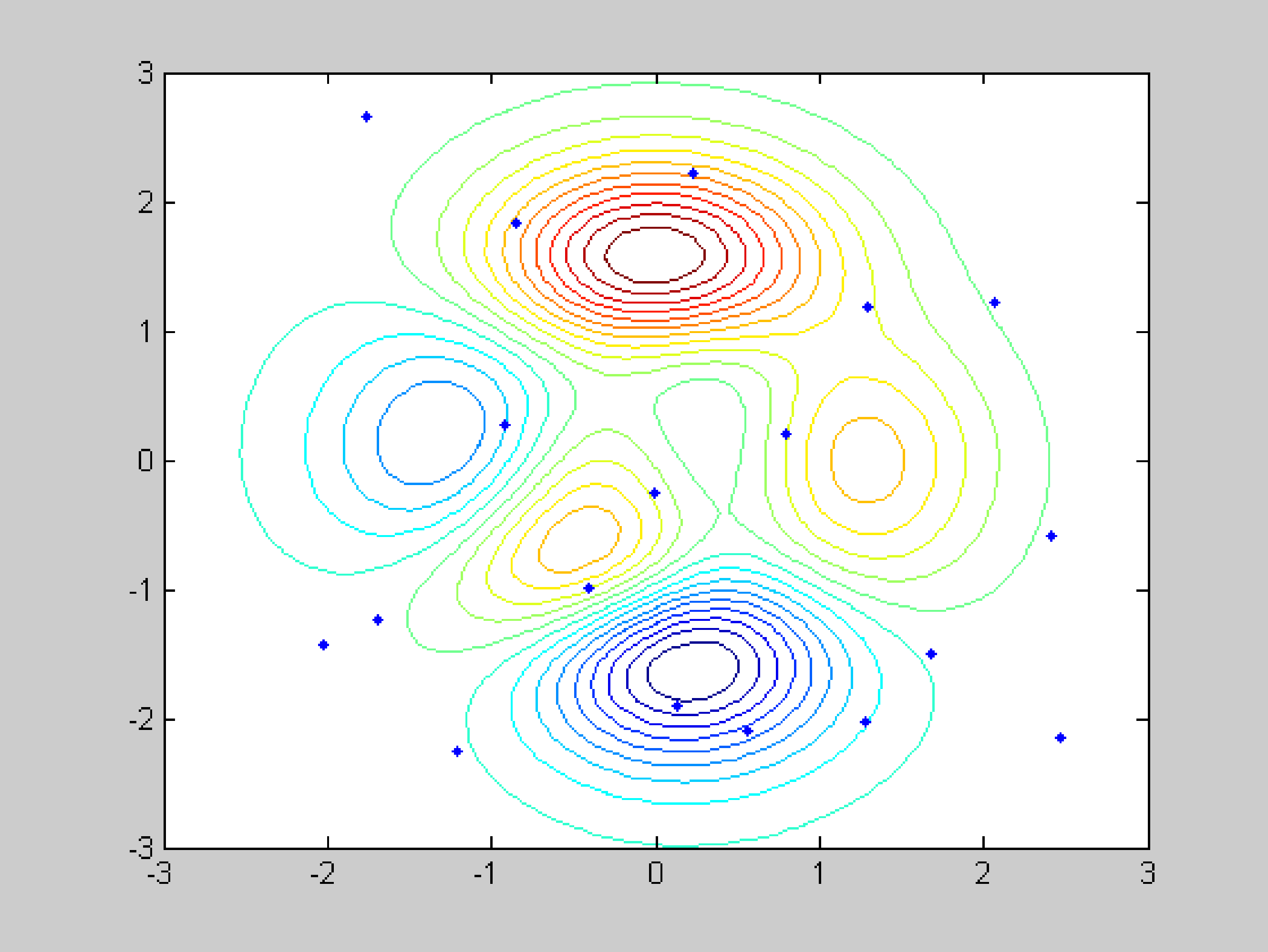


The Original PSO Algorithm

- Update velocities and positions of particles according to the iterative equations.



The Original PSO Algorithm - Example



Advantages and Disadvantages (Basic PSO)

- **Advantages**
  - Easy to implement and ***low computational effort***.
  - Feasible for ***multi-dimensional*** problems (with a relatively small number of particles) .
- **Disadvantages**
  - A ***premature convergence*** to a local optima.
  - No convergence / slow convergence → long ***run-time***
  - Not adaptive to ***time-variant*** problems

The Exploration-Exploitation Trade-off

Search Method	Meaning	Aim	Disadvantage
Exploration	The desire of the swarm to explore ***as many regions as possible*** of the search space	Avoiding a premature convergence to a local optima	Slow convergence → long run-time
Exploitation	A search being held in a smaller region of search space, in order to ***pin-point*** the optimal solution	To increase the rate of convergence	A premature convergence to a local optima

An unavoidable Trade-off?

The Evolution of Control Parameters

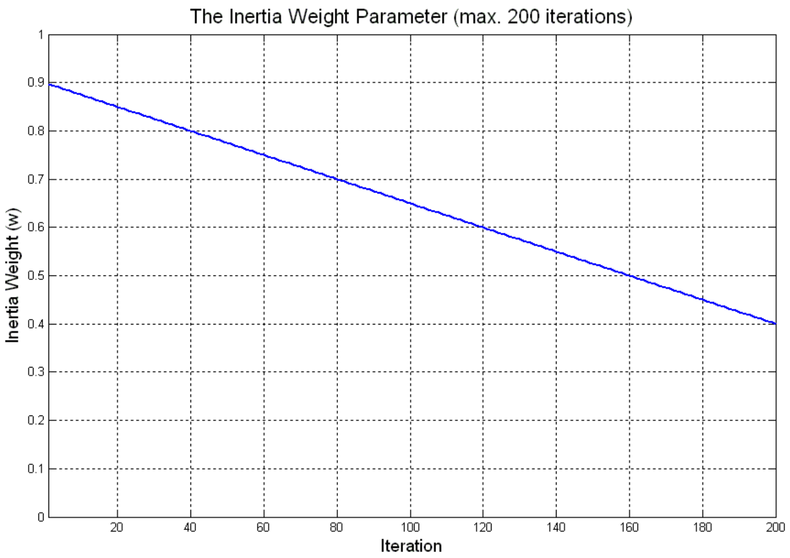
- **The Original Equation**

$$v_i^d(k+1) = v_i^d(k) + c_1 \cdot r_1(k) \cdot (pBest_i^d(k) - x_i^d(k)) + c_2 \cdot r_2(k) \cdot (gBest_i^d(k) - x_i^d(k))$$

Acceleration Coefficients

- **The Inertia Weight**

$w(k)$



- **A smooth transition between exploration and exploitation**

A State-Space Model for Parameter Selection

- The Original Equations

$$\begin{cases} v(k+1) = a \cdot v(k) + b_1 \cdot r_1(k) \cdot (pBest(k) - x(k)) + b_2 \cdot r_2(k) \cdot (gBest(k) - x(k)) \\ x(k+1) = c \cdot x(k) + d \cdot v(k+1) \end{cases}$$

- Deterministic Algorithm
  - $r_1(k) = r_2(k) = 1/2, \ \forall k$

- Manipulating the original equations

$$b = \frac{b_1 + b_2}{2} \quad , \quad p = \frac{b_1}{b_1 + b_2} p_1 + \frac{b_2}{b_1 + b_2} p_2$$

A State-Space Model for Parameter Selection

- Modified Equations

$$\begin{cases} v(k+1) = a \cdot v(k) + b \cdot (p(k) - x(k)) \\ x(k+1) = c \cdot x(k) + d \cdot v(k+1) \end{cases}$$

- Can be proven: $c = d = 1$

- Matrix Form

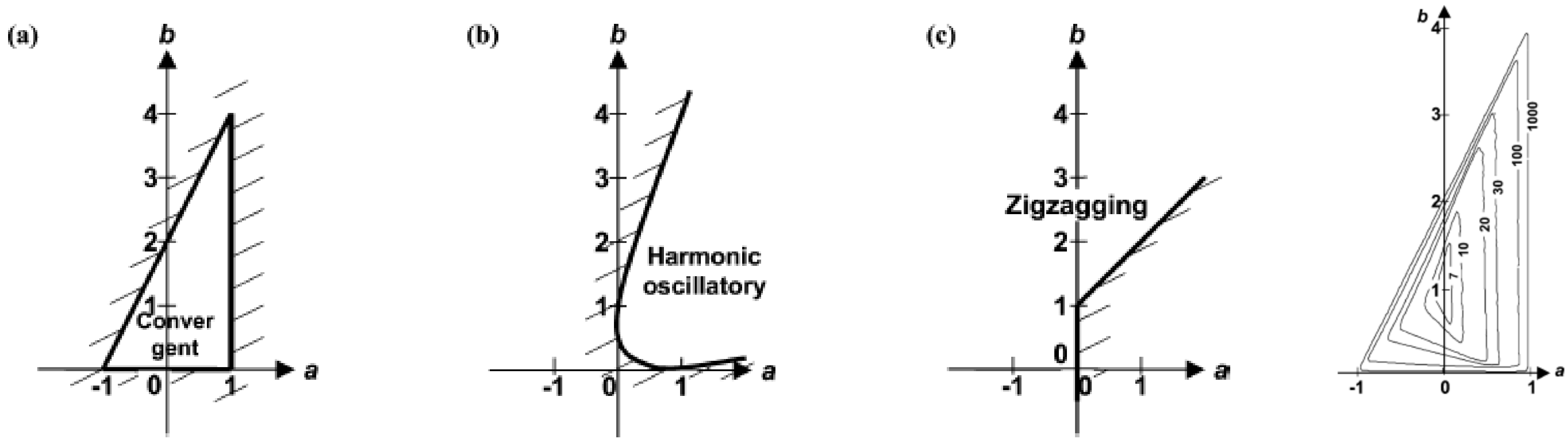
$$\underbrace{\begin{bmatrix} v(k+1) \\ x(k+1) \end{bmatrix}} = \underbrace{\begin{bmatrix} 1-b & a \\ -b & a \end{bmatrix}} \cdot \begin{bmatrix} v(k) \\ x(k) \end{bmatrix} + \underbrace{\begin{bmatrix} b \\ b \end{bmatrix}} \cdot p(k)$$

$$\underline{x}(k+1) = \underline{A} \cdot \underline{x}(k) + \underline{B} \cdot p(k)$$

State Vector System Matrix Steering Matrix

A State-Space Model for Parameter Selection

- Convergence Analysis
  - Find ***Eigen values*** $\lambda_i \in \mathbb{C}$ of $\underline{A}$: $\det\left(\underline{A} - \lambda \underline{I}\right) = 0$
- Condition for convergence: $|\lambda_i| < 1, \forall i$



Taken from: I. C. Trelea, “The particle swarm optimization algorithm: convergence analysis and parameter selection,” Information Processing Letters vol. 85, no. 6, pp. 317–325, Mar. 2003

Adaptive PSO

- **Aims to eliminate the *exploration-exploitation trade-off***
  - Adaptive parameter control
  - Evolutionary State Estimation (ESE)
  - Elitist Learning Strategy (ELS)
  - For more information: see corresponding paper

Applications

- **Optimization Problems**
  - PSO for adaptive IIR filter structures
  - PSO for virtual MIMO communication protocol
  - PSO for Space Alternating Generalized Expectation maximization (SAGE) algorithm for estimation of channel parameters

Summary

- PSO - an ***Optimizer***
  - Very easy for implementation

- An ongoing search to improve its robustness

- *" Physicists have been working for years to discover the properties that guide the movement of birds...*
  This project will never succeed, however, because there is no analogy that can be brought to the mechanical world "

  Deepak Chopra