Information Technology Institute

Fundamentals of Wireless Communications

Diversity & Spatial Multiplexing Final Project Report

Submitted by : Kholoud Mohamed Elsayed

Submitted to : Eng. Mariam Abdelrahman

1. Project Overview

1.1 Title

Performance Analysis of Various Detection Algorithms in MIMO Systems

1.2 Objective

This project aims to simulate and compare the performance of five different detection algorithms in Multiple-Input Multiple-Output (MIMO) communication systems:

- Zero Forcing (ZF)
- Minimum Mean Square Error (MMSE)
- ZF with Successive Interference Cancellation (ZF-SIC)
- MMSE with Successive Interference Cancellation (MMSE-SIC)
- Maximum Likelihood (ML)

1.3 Motivation

MIMO technology is fundamental to modern wireless communications (5G, Wi-Fi 6, etc.). Understanding the trade-offs between detection complexity and performance is crucial for system design and optimization.

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2. System Model

2.1 MIMO Configuration

- **Transmit Antennas (Nt):** 3
- **Receive Antennas (Nr):** 3-6 (variable for BER vs Nr analysis)
- **Modulation:** QPSK (4-QAM)
- **Channel Model:** Rayleigh fading (flat fading)

2.2 Mathematical Model

The received signal is given by:

y = Hx + n

Where:

- **y**: Received signal vector (Nr × 1)
- **H**: Channel matrix (Nr × Nt) with i.i.d. complex Gaussian entries

- **x**: Transmitted signal vector (Nt × 1) with QPSK symbols
- **n**: Additive White Gaussian Noise (AWGN) vector (Nr × 1)

2.3 Simulation Parameters

Parameter	**Value**	**Description**
Total bits	2×10^5	For reliable BER estimation
SNR range	0-20 dB	For BER vs SNR analysis
Nr range	3-6	For BER vs Nr analysis
Modulation	QPSK	Bits per symbol = 2
Channel	Rayleigh	Complex Gaussian entries

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3. Detection Algorithms

3.1 Zero Forcing (ZF)

Principle: Completely eliminates interference at the cost of noise enhancement.

Mathematical Formulation:

text

$W\_ZF = H^{+} = (H^{H}H)^{-1}H^{H}$

$\hat{x} = W\_ZF\ y$

Advantages:

- Simple implementation
- Complete interference cancellation

Disadvantages:

- Noise amplification
- Poor performance at low SNR

3.2 Minimum Mean Square Error (MMSE)

Principle: Balances interference cancellation and noise enhancement.

Mathematical Formulation:

text

$W\_MMSE = (H^{H}H + \sigma^{2}I)^{-1}H^{H}$

$\hat{x}$ = W_MMSE y

where σ^2 is the noise variance.

Advantages:

- Better performance than ZF, especially at low SNR
- Robust to noise

3.3 ZF with SIC

Principle: Sequential detection with interference cancellation.

Algorithm:

1. Detect strongest stream using ZF
2. Decode and subtract its effect
3. Repeat for remaining streams

Ordering: Based on minimum noise enhancement.

3.4 MMSE with SIC

Principle: Similar to ZF-SIC but uses MMSE equalization at each step.

Advantages over ZF-SIC:

- Better ordering due to MMSE criterion
- Improved performance, especially at low SNR

3.5 Maximum Likelihood (ML)

Principle: Optimal detector that searches all possible symbol combinations.

Mathematical Formulation:

text

$\hat{x}$_ML = arg $\min_x$ $||y - Hx||^2$

Advantages:

- Optimal performance
- Best BER among all detectors

Disadvantages:

- Exponentially complex (4^{Nt} combinations for QPSK)
- Impractical for large Nt

4. Implementation Details

4.1 Code Structure

```text
main_script.m
 ├── Part 1: BER vs Nr (SNR = 10 dB)
 ├── Part 2: BER vs SNR (3×3 MIMO)
 └── Functions:
     ├── mmse_equalizer()
     ├── mmse_sic()
     ├── Zero_forcing()
     ├── zero_forcing_sic()
     └── ML_detector()
```

4.2 Key Implementation Features

1. **Modulation/Demodulation:**
   - QPSK mapping: 00→(1+j)/√2, 01→(1-j)/√2, etc.
   - Hard decision detection
2. **Channel Generation:**
   - Complex Gaussian with unit variance
   - Independent across antennas and symbols
3. **Noise Generation:**
   - Proper scaling with SNR: $\sigma^2 = 1/(2\times10^{(SNR/10)})$
4. **SIC Implementation:**
   - Ordered detection based on noise enhancement
   - Sequential interference cancellation

5. Simulation Results

5.1 BER vs Number of Receive Antennas (Nr)

Configuration: SNR = 10 dB, Nt = 3, Nr = 3:6

Observations:

1. **BER decreases** with increasing Nr for all detectors
2. **ML achieves lowest BER** across all Nr values
3. **MMSE-SIC outperforms ZF-SIC**
4. **SIC detectors significantly better** than linear detectors
5. **Performance gap narrows** as Nr increases

Interpretation:

- More receive antennas provide spatial diversity
- Each additional antenna improves signal detection
- Law of diminishing returns observed beyond Nr = 5

5.2 BER vs SNR (3×3 MIMO)

Configuration: Nt = 3, Nr = 3, SNR = 0:2:20 dB

Observations:

1. **All detectors improve** with increasing SNR
2. **ML shows steepest slope** (~6 dB/decade)
3. **MMSE outperforms ZF** at all SNR values
4. **SIC provides 3-5 dB gain** over linear detectors
5. **ZF performs poorly** at low SNR

Key Performance Metrics:

Detector	SNR for BER=10^{-3}	Complexity	Notes
ML	~12 dB	High (64 combos)	Optimal
MMSE-SIC	~14 dB	Medium	Practical choice
ZF-SIC	~16 dB	Medium	Better than linear
MMSE	~18 dB	Low	Good compromise
ZF	>20 dB	Low	Simple but noisy

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6. Performance Analysis

6.1 Complexity Comparison

Algorithm	Complexity Order	Relative Cost
ML	$O(M^{Nt})$	Very High (64 for 3×3 QPSK)
MMSE-SIC	$O(Nt^3 + Nt^2Nr)$	Medium
ZF-SIC	$O(Nt^3 + Nt^2Nr)$	Medium
MMSE	$O(Nt^3)$	Low
ZF	$O(Nt^3)$	Low

Where M = constellation size (4 for QPSK)

6.2 Trade-off Analysis

text

Performance: ML > MMSE-SIC > ZF-SIC > MMSE > ZF

Complexity: ML >> MMSE-SIC ≈ ZF-SIC > MMSE ≈ ZF

6.3 Practical Recommendations

1. **For high-performance systems:** MMSE-SIC (best trade-off)
2. **For low-complexity systems:** MMSE
3. **For theoretical benchmarking:** ML
4. **Avoid:** ZF except for high SNR scenarios

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7. Challenges and Solutions

7.1 Numerical Stability

Challenge: Matrix inversion in ZF can be ill-conditioned.

Solution used: pinv() function for pseudo-inverse (more stable than direct inversion).

7.2 Computational Complexity

Challenge: ML exhaustive search becomes prohibitive for larger systems.

Solution limitation: Currently limited to 3×3 MIMO (64 combinations).

Results :

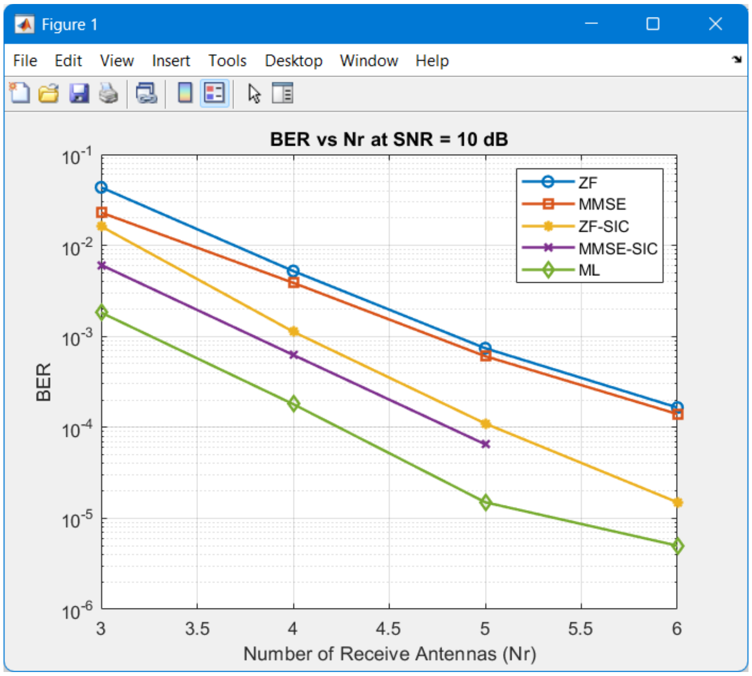
Figure 1
File Edit View Insert Tools Desktop Window Help
BER vs Nr at SNR = 10 dB
ZF
MMSE
ZF-SIC
MMSE-SIC
ML
BER
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
3 3.5 4 4.5 5 5.5 6
Number of Receive Antennas (Nr)

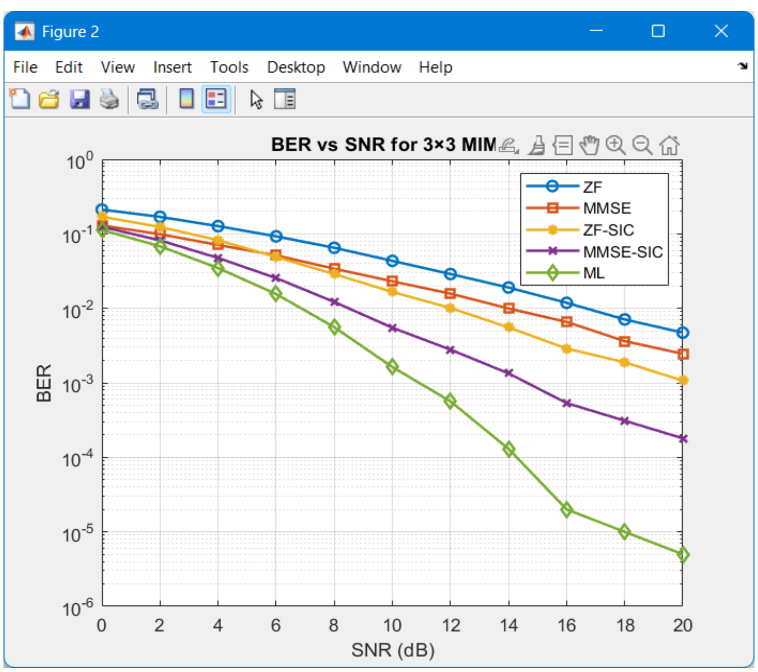
Figure 2
File Edit View Insert Tools Desktop Window Help
BER vs SNR for 3×3 MIM
ZF
MMSE
ZF-SIC
MMSE-SIC
ML
BER
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 2 4 6 8 10 12 14 16 18 20
SNR (dB)

8. Extensions and Future Work

8.1 Immediate Extensions

1. **Higher-order modulation:** 16-QAM, 64-QAM
2. **Larger antenna arrays:** 4×4, 8×8 MIMO
3. **Frequency-selective channels:** OFDM-MIMO

8.2 Algorithm Improvements

1. **Sphere Decoding:** Near-ML performance with reduced complexity
2. **Ordered MMSE-SIC:** Better ordering algorithms
3. **Soft-output detectors:** For coded systems

8.3 Advanced Features

1. **Channel estimation errors:** Imperfect CSI
2. **Correlated channels:** Spatial correlation
3. **Hardware impairments:** Phase noise, IQ imbalance

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9. Conclusion

9.1 Key Findings

1. **ML is optimal** but computationally expensive
2. **MMSE-SIC offers best trade-off** for practical systems
3. **SIC provides significant gains** over linear detection
4. **More receive antennas always help** but with diminishing returns
5. **MMSE consistently outperforms ZF** across all SNR values

9.2 Project Achievements

1.Successfully implemented 5 MIMO detection algorithms
2. Conducted comprehensive performance comparison
3. Generated insightful BER vs SNR and BER vs Nr plots
4. Provided practical design recommendations
5. Established foundation for further MIMO research

9.3 Educational Value

This project provides hands-on experience with:

- MIMO system modeling and simulation
- Algorithm implementation and comparison
- Performance evaluation using BER curves
- Trade-off analysis between complexity and performance

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10. References

1. Tse, D., & Viswanath, P. (2005). *Fundamentals of Wireless Communication*
2. Paulraj, A., et al. (2003). *Introduction to Space-Time Wireless Communications*
3. Proakis, J. G., & Salehi, M. (2008). *Digital Communications*
4. 3GPP Technical Specifications for MIMO in LTE/5G