CONCORDIA UNIVERSITY
Dept. of Computer Science and Software Engineering
COMP 233 – Probability and Statistics for Computer Science

Winter 2017
Assignment 1

1. A group of 5 boys and 5 girls are lined up in random order.
   (a) What is an appropriate sample space for this experiment?
   (b) What is the size of the sample space?
   (c) What is the probability that the first person in the line is a boy?
   (d) What is the probability that a particular boy is in the third position?
   (e) What is the probability that boys and girls alternate?
   (f) What is the probability that no two boys are next to each other?

2. Suppose you are given a set $S = \{x_1, x_2, \ldots, x_n\}$ of n items, and you want to choose a subset R of r items from S.
   (a) In how many ways can your choose R?
   (b) In how many ways can you choose R such that $x_1 \in R$?
   (c) In how many ways can you choose R so that $x_1 \notin R$?
   (d) Use your responses to the above questions to give a combinatorial argument for the following identity:
   $$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$
   (e) Give an algebraic proof for the same identity.

3. Let E and F be events. Prove that $(E^C F^C) = 1 - P(E) - P(F) + P(EF)$

4. Suppose you are walking in a rectangular grid from point $(0, 0)$ to the point (m, n). In every step of the path, if you are at point (x, y), you can either walk one step to the right, to $(x + 1, y)$ or a step up to $(x, y + 1)$. You are not allowed to move left or down.

(a) What is the number of possible paths you can take?

(b) If you choose a path at random, what is the probability that your path has exactly one turn?

5. Two balls, each equally likely to be red or blue are put in an urn. In each step, one of the balls is randomly chosen, its color is noted and it is returned to the urn. If the first two balls chosen are red, what is the probability that

   (a) both balls in the urn are red?

   (b) the next ball chosen will be red?

6. What is the probability that the sum of numbers obtained in three rolls of a six-sided die is at most 10?

7. A woman has n keys, of which one will open the door. If she tries all the keys at random, what is the probability that the k-th key she tries will open the door

   (a) assuming she discards keys that do not work.

   (b) assuming she does not discard keys that do not work, but instead returns them to the pile of possible keys she is trying.

8. Each of 2 cabinets identical in appearance has 2 drawers. One of the cabinets has a silver coin in each drawer. The other cabinet has a silver coin in one drawer and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is revealed. What is the probability that the other drawer of the cabinet holds a silver coin?

9. When coin A is flipped, it comes up heads with probability $1/4$ whereas when coin B is flipped, it comes up heads with probability $3/4$. Suppose that one of these coins is randomly chosen and is flipped twice. If both flips land heads, what is the probability that coin B was the one flipped?

10. The following table gives the characteristics of three types of light bulbs present in a bin in a store.

Type	chance of lifetime > 1000 hours	percentage present in bin
A	0.7	20
B	0.4	30
C	0.3	50

    (a) What is the probability that a randomly chosen bulb has a lifetime greater than 1000 hours?

    (b) Given a bulb that lasted over 1000 hours, what is the conditional probability that it was a type A bulb?