2.2

Calculate the mean for the following frequency distribution.

Class interval:

0-8 8-16 16-24 24-32 32-40 40-48

Frequency:

8 7 16 24 15 7.

Solution:

Class-interval	Mid value (x_i)	frequency (f_i)	d_i	d_if_i
0 – 8	4	8	-3	-24
8 – 16	12	7	-2	-14
16 – 24	20	16	-1	-16
24 - 32	28	24	0	0
32 – 40	36	15	1	15
40 - 48	44	7	2	14
		77		-25

Here, take $A = 28$ and $h = 8$

$$\bar{x} = A + \frac{\Sigma d_if_i}{N} \times h$$

$$= 28 + \frac{(-25)}{77} \times 8$$

$$= 25.404 \text{ (Ans)}$$

2.12

Show that in finding the arithmetic mean of a set of readings on thermometer it does not matter whether we measure temperature in centigrade or faherenheit, but that in finding the geometric mean it does matter which scale we use.

Solution:

Let, $c_1, c_2 \ldots c_n$ be the n readings on the centigrade thermometter. Then their arithmetic mean $\bar{c}$ is given by: $\bar{c} = \frac{1}{n}(c_1 + c_2 + \cdots + c_n)$

If F and c be the readings in fahrenheit and centigrade respectively then we have the relation.

$$\frac{F-32}{180} = \frac{c}{100} \Rightarrow 32 + \frac{9}{5}c$$

Thus the fahrenheit equivalents of $c_1, c_2, \ldots c_n$ are $32+\frac{9}{5}c_1$, $32+\frac{9}{5}c_2 \ldots 32+\frac{9}{5}c_n$ respectively

Hence the arithmetic mean of the readings in fahrenheit it is

$$\bar{F} = \frac{1}{n}\left\{\left(32+\frac{9}{5}c_1\right) + \left(32+\frac{9}{5}c_2\right) + \cdots + \left(32+\frac{9}{5}\right)c_n\right\}$$

$$= \frac{1}{n}\left\{32n + \frac{9}{5}(c_1 + c_2 + \cdots + c_n)\right\}$$

$$= 32 + \frac{9}{5}\left(\frac{c_1 + c_2 + \cdots + c_n}{n}\right)$$

$$= 32 + \frac{9}{5}\bar{c}$$

which is the fahrenheit equivalent of $\bar{c}$ Hence in finding the arithmetic mean of a set of n readings on a thermometer it is immaterial whether we measure temperature in centigrade on Fahrenheit Geometric mean G n of readings in Centigrade is,

$$G = (c_1 c_2 \cdots c_n)^{1/n}$$

Geometric mean G_1 (say) of fahrenheit equivalents of $c_1, c_2, \cdots c_n$ is.

$$G_1 = \left\{ (32+\tfrac{9}{5}c_1)(32+\tfrac{9}{5}c_2)\cdots(32+\tfrac{9}{5}c_n) \right\}^{1/n}$$

which is not equal to fahrenheit equivalant of G vit.

$$\left\{ \tfrac{9}{5}(c_1 \cdots c_2 \cdots c_n)^{1/n} + 32 \right\}$$

Hence is finding the geometric mean of the n readings on a thermometer the scale is important.

2.13

A cyclist pedals from his house to his college at a speed of 10 mph and back from the college to his house at 15 mph. find the average speed.

Solution:

Let the distance (x-miles) from the house to the college be x miles. In going from house to college, the distance (x-miles) is covered in $\frac{x}{10}$ hours, while in coming from college to house the distance is covered in $\frac{x}{15}$ hours. Thus a total distance of $2x$ miles is covered in,

$\left(\frac{x}{10}+\frac{x}{15}\right)$ hours.

Hence average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{2x}{\left(\frac{x}{10}+\frac{x}{15}\right)}$$

$$= \frac{2}{\left(\frac{1}{10}+\frac{1}{15}\right)}$$

$= 12$ mph (Ans)

2.7

In a factory employing 3000 persons 5 per cent earn less than Rs. 3 per hours. 580 earn from Rs. 3.01 to Rs. 4.50 per hour, 30 percent earn from Rs. 4.51 to Rs. 6.00 per hour, 500 earn from Rs. 6.01 to Rs. 7.50 per hour, 20 percent earn form Rs. 7.51 to Rs. 9.00 per hour, and the rest earn Rs. 9.01 or more per hour. what is the median wage?

Solution:

The given information can be expressed in tabular from as follows:

Earnings (in Rs.)	percentage of workers	No of works (f)	Less than c.f	class boundaries.
	5%	$\frac{5}{100} \times 3000 = 150$	150	Below 3.005
3.01 – 4.50		580	730	3.005 – 4.505
4.51 – 6.00	30%	$\frac{30}{100} \times 3000 = 900$	1630	4.505 – 6.005
6.01 – 7.50		500	2130	6.005 – 7.505
7.51 – 9.00	20%	$\frac{20}{100} \times 3000 = 600$	2730	7.505 – 9.005
9.01 – above		$3000 - 2730 = 270$	$3000 = N$	9.005 and above

Here,

$\frac{N}{2} = 1500$. The c.f just greater than 1500 is 1630.
the corresponding class ~~1500 is 1630~~. 4.51 - 6.00.
whose class boundaries are 4.505 - 6.005

So,

$l = 4.505$
$h = 1.5$
$f = 900$
$c = 730$

$$\text{median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$
$$= 4.505 + \frac{1.5}{900}(1500 - 730)$$
$$= 4.505 + 1.233$$
$$= 5.79$$

$\therefore$ Median wage is Rs. 5.79 Ans.

2.11

The mode of the following wage distribution are known to be Rs. 33.50 and Rs. 34 respectively find the value of f_3, f_4 and f_5

Wages	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	4	16	f_3	f_4	f_5	6	4

Solution:

Calculations for mode

	Frequency(f)	less than c.f	
0-10	4	4	
10-20	16	20	
20-30	f_3	$20+f_3$	
30-40	f_4	$20+f_3+f_4$	
40-50	f_5	$20+f_3+f_4+f_5$	
50-60	6	$26+f_3+f_4+f_5$	
60-70	4	$30+f_3+f_4+f_5$	

Total $\Rightarrow$ $\Sigma f = 30+f_3+f_4+f_5$

$\Rightarrow 230 = 30+f_3+f_4+f_5$

$\Rightarrow f_3+f_4+f_5 = 200$

2.15

Eight coins were together and the number of heads resulting was noded. The operating on was repeated 256 times and the frequency (f) that were obtained for different values of x, the number of heads, are shown in the following table. calculate median quartiles, 4th decile and 27th precentile.

x	0	1	2	3	4	5	6	7	8
f	1	9	26	59	72	52	29	7	1

Solution:

x	f	c.f
0	1	1
1	9	10
2	26	36
3	59	95
4	72	167
5	52	219
6	29	248
7	7	255
8	1	256

Here, N = 256

Thus, median = 4

Q_1 Here, $\frac{N}{4} = \frac{256}{4} = 64$

c.f just greater than 64 is 95 $Q_1 = 3$

Q_3 Here, $\frac{3N}{4} = \frac{3 \times 256}{4} = 192$

c.f just greater than 192 is 219 $Q_3 = 5$

D_4: $\frac{4N}{10} = 4 \times 25.6 = 102.4$ and just greater than 102.4 is 167, Hence, $D_4 = 4$

P_{27}: $\frac{27N}{100} = 27 \times 2.56 = 69.12$ and c.f just greater than 69.12 is 95 Hence $P_{27} = 3$

3.6

The first of the two samples has 100 items with mean 15 and standard deviation 3 If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, find the standard deviation of the second group.

Solution:

Here we given,

$n_1 = 100, \quad \bar{x}_1 = 15 \quad \sigma_1 = 3$

$n = n_1 + n_2$

$\quad = 250$

$\bar{x} = 15.6$ and $\sigma = \sqrt{13.44}$

We want σ_2

obviously $n_2 = 250 - 100 = 150$

We have, $\bar{x} = \dfrac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

$\Rightarrow 15.6 = \dfrac{100 \times 15 + 150 \times \bar{x}_2}{250}$

$\Rightarrow 150\,\bar{x}_2 = 250 \times 15.6 - 1500$

$\Rightarrow 150\,\bar{x}_2 = 2400$

$\Rightarrow \bar{x}_2 = \dfrac{2400}{150}$

$\therefore \bar{x}_2 = 16$

Hence, $d_1 = \bar{x}_1 - \bar{x} = 15 - 15.6 = -0.6$

$d_2 = \bar{x}_2 - \bar{x} = 16 - 15.6 = 0.4$

The variance σ^2 of the combined group is given by the formula.

$$(n_1 + n_2)\sigma^2 = n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)$$

$$\Rightarrow 250 \times 13.44 = 100(9 + 0.36) + 150(\sigma_2^2 + 0.16)$$

$$\Rightarrow 150\sigma_2^2 = 250 \times 13.44 - 100 \times 9.36 - 150 \times 0.16$$

$$\Rightarrow 150\sigma_2^2 = 2400$$

$$\Rightarrow \sigma_2^2 = \frac{2400}{150}$$

$$\Rightarrow \sigma_2 = \sqrt{16}$$

$$\Rightarrow \sigma_2 = 4$$

$$\therefore \sigma_2 = 4 \text{ Ans.}$$

3.7

An analysis of monthly wages paid to the workers of two firm A and B belonging to the same industry gives the following results.

	Firm A	Firm B
Numb of workers	500	600
Average monthly wage	Rs 186.00	Rs. 175.00
Variance of distribution	81	100

(1) Which firm A or B has a larger wage bill?

Solution:

(1) firm A,

No. of wage-earners (say) $n_1 = 500$

Average monthly wages (say) $\bar{x}_1 = Rs. 186$

$$\text{Average monthly wage} = \frac{\text{Total wages paid}}{\text{No. of worker}},$$

Hence, total wages paid to the workers

$\Rightarrow n_1 \bar{x}_1 = 500 \times 186$
$= Rs.\ 93{,}000.$

firm B,

No. of wage-earners (say) $n_2 = 600$

Average monthly wage (say) $\bar{x}_2 = Rs.\ 175$

$\therefore$ Total wages paid to the workers $= n_2 \bar{x}_2$
$= 600 \times 175$
$= Rs.\ 1{,}05{,}000.$

Thus we see that the firm B has larger wage bill

(ii) In which firm A or B is there greatter variability in individual wages?

Variance of distribution of wages in firm A (say) $\sigma_1^2 = 81$

Variance of distribution of wages in firm B (say) $\sigma_2^2 = 100$

c.v of distribution of wages for

$$\text{firm A} = 100 \times \frac{\sigma_1}{\bar{x}_1} = \frac{100 \times 9}{186} = 4.89$$

c.v of distribution of wages for

$$\text{firm B} = 100 \times \frac{\sigma_2}{\bar{x}_2} = \frac{100 \times 10}{175} = 5.71$$

Since, c.v for firm B is greatter than c.v for firm A firm B has greatter variability wages.

(iii) Calculate (a) the average monthly wage and (b) the variance of the distribution of wages of all the workes in the firms A and B taken together.

The average monthly wages (say) $\bar{x}$, of all the workers in the two firms A and B taken together is given by.

$$\bar{u} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$= \frac{500 \times 186 + 600 \times 175}{500 + 600}$$

$$= \frac{198000}{1100}$$

$$= \text{Rs } 180$$

(b) The combined variance σ^2 is given by the formula, $\sigma^2 = \frac{1}{n_1+n_2}\left[n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)\right]$

where $d_1 = \bar{x}_1 - \bar{x}$ and $d_2 = \bar{x}_2 - \bar{x}$

Here $d_1 = 186 - 180 = 6$

$d_2 = 175 - 180 = -5$

Hence, $\sigma^2 = \frac{500(81 + 36) + 600(100 + 25)}{500 + 600}$

$$\sigma^2 = \frac{1335000}{1100}$$

$$\sigma^2 = 121.36$$