Probability

Problem 1: A fair coin is tossed two times. Construct the sample space of the experiment. What is the probability of getting:

(i) All head
(ii) At least one head
(iii) At best one head
(iv) A head and a tail

Soln: A fair coin is tossed two times. The sample space of the experiment

$S: \{HH, HT, TH, TT\}$

The number of sample points, $n(S) = 4$

Let, the event A: All head

$A: \{HH\}$

$\therefore n(A) = 1$

Required probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

Let, the event B: At least one head

$\therefore B: \{HH, HT, TH\}$

$\therefore n(B) = 3$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

Let event C: At best one head

$\therefore C: \{HT, TH, TT\}$

$n(C) = 3$

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{4}$$

Let event D: A head and a tail

$\therefore D: \{HT, TH\}$

$\therefore n(D) = 2$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

Problem 2: An unbaised coin is tossed four times. What is the probability of getting:

(i) At least 3 heads

(ii) At best 1 head

Soln:

If an unbaised coin is tossed four times; then the sample space is given below:

S	HH	HT	TH	TT
HH	HHHH	HHHT	HHTH	HHTT
HT	HTHH	HTHT	HTTH	HTTT
TH	THHH	THHT	THTH	THTT
TT	TTHH	TTHT	TTTH	TTTT

Total number of possible Outcome of sample space $n(S) = 16$.

(I) Let the event A: At least 3 heads

The set of favourable cases of event

$A = \{HHHH, HHHT, HHTH, HTHH, THHH\}$

$n(A) = 5$

The required probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{16}$$

(II) Let event B: at best 1 head

The set of favourable cases of event

$B = \{HTTT, THTT, TTHT, TTTH, TTTT\}$

$\therefore n(B) = 5$

So the required probability, $P(B) = \frac{n(B)}{n(S)} = \frac{5}{16}$

Problem 3: An unbaised coin is tossed 10 times.

Find the probability of getting

(I) Just 3 heads

(II) At least one head

(III) At least two heads

(IV) At least three heads

(V) At best one head.

Soln: If an unbaised coin is tossed 10 times then the total number of sample point is $2^{10} = 1024$.

(I) The probability of getting just three heads, $P(\text{just 3 heads}) = \frac{{}^{10}C_3}{1024}$ $= 0.1172$ $\quad \Big| \; {}^nC_r = \frac{n!}{r!(n-r)!}$

(II) The probability of at least one head

$= 1 - P(\text{0 head})$

$= 1 - \frac{{}^{10}C_0}{1024}$

$= 0.999$

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(iii) The probability of at least two heads

$= 1 - P(\text{0 head}) - P(\text{1 head})$

$= 1 - \dfrac{{}^{10}C_0}{1024} - \dfrac{{}^{10}C_1}{1024}$

$= 0.9893$

(iv) The probability of at least three heads

$= 1 - P(\text{0 head}) - P(\text{1 head}) - P(\text{2 head})$

$= 1 - \dfrac{{}^{10}C_0}{1024} - \dfrac{{}^{10}C_1}{1024} - \dfrac{{}^{10}C_2}{1024}$

$= 0.9453$

(v) The probability of at best one head

$= P(\text{0 head}) + P(\text{1 head})$

$= \dfrac{{}^{10}C_0}{1024} + \dfrac{{}^{10}C_1}{1024}$

$= 0.0107.$

Problem: Two unbaised coins and an unbaised die are tossed once. Write down the sample space and find the probability of the following events:

(i) Opposite face of the coin and odd number on the die.

(ii) even number of the die.

Soln: If two unbaised coins and an unbaised die are tossed once, then the sample space is given by:

S	1	2	3	4	5	6
HH	HH1	HH2	HH3	HH4	HH5	HH6
HT	HT1	HT2	HT3	HT4	HT5	HT6
TH	TH1	TH2	TH3	TH4	TH5	TH6
TT	TT1	TT2	TT3	TT4	TT5	TT6

Total number of possible outcome of sample space $n(S) = 24$

Let event A: opposite face on the coin and odd number on the die

The set of favourable cases of event

A: {HT1, HT3, HT5, TH1, TH3, TH5}

$n(A) = 6$

Required probability

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{6}{24} = \frac{1}{4}$$

Let the event B: Even number on the die

The set of favourable cases of B:

{HH2, HH4, HH6, HT2, HT4, HT6, TH2, TH4, TH6, TT2, TT4, TT6}

$n(B) = 12$

The required probability

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{12}{24} = \frac{1}{2}$$

Some necessary formulas:

Let us consider the two events A and B

(i) If A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$

or, $P(A \cap B) = \phi$

(ii) If A and B are non mutually exclusive then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(iii) If A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

(iv) If A and B are dependent then

$$P(A \cap B) = P(A) P(B \backslash A)$$

or, $P(A \cap B) = P(B) P(A \backslash B)$

(v) $P(A) + P(\bar{A}) = 1$

Happening event

Non-happening event

(vi) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

Some necessary formulas:

Let us consider the two events A and B

(i) If A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$

or, $P(A \cap B) = \phi$

(ii) If A and B are non mutually exclusive then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(iii) If A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

(iv) If A and B are dependent then

$$P(A \cap B) = P(A) P(B \setminus A)$$

or, $P(A \cap B) = P(B) P(A \setminus B)$

(v) $\underbrace{P(A)}_{\text{Happening event}} + \underbrace{P(\bar{A})}_{\text{Non-happening event}} = 1$

(vi) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

(vii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

(viii) $P(\bar{A} \setminus \bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

Problem: A coin and a die are thrown simultaneously. If event A: head of the coin and event B: even number on the die. Then

(i) Show that the events A and B are independent.

(ii) Find the value of $P(A \cup B)$.

Soln: If a coin and a die are thrown simultaneously then the sample space S is given below:

S	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Total number of possible outcome of the sample space is $n(S)=12$.

The set of favourable cases of the event A: $\{H1, H2, H3, H4, H5, H6\}$

$n(A)=6$.

and the set of favourable cases of the event B: $\{H2, H4, H6, T2, T4, T6\}$

$n(B)=6$

The set of common elements of the events A and B: $\{H2, H4, H6\}$

$n(A\cap B)=3$

$$L.H.S = P(A\cap B) = \frac{n(A\cap B)}{n(S)}$$

$$= \frac{3}{12} = \frac{1}{4}$$

$$R.H.S = P(A)\cdot P(B)$$

$$= \frac{n(A)}{n(S)} \times \frac{n(B)}{n(S)}$$

$$= \frac{6}{12} \times \frac{6}{12} = \frac{1}{4}$$

$\because$ L.H.S = R.H.S, So the events are independent.

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$= \frac{6}{12} + \frac{6}{12} - \frac{3}{12}$$

$$= \frac{9}{12} = \frac{3}{4} \text{ (Ans)}$$

Problem: Consider two events A and B such that $P(A) = \frac{1}{8}$, $P(A \backslash B) = \frac{1}{4}$ and $P(B \backslash A) = \frac{1}{6}$. Examine the following statements and comment on the valid reason of each of these

(i) A and B are independent

(ii) A and B are mutually exclusive

(iii) $P(\bar{A} \backslash \bar{B}) = 0.5$

Soln: Given that,

$P(A) = \frac{1}{8}$, $P(A \backslash B) = \frac{1}{4}$ and

$P(B \backslash A) = \frac{1}{6}$.

We know, $P(A \cap B) = P(A)\, P(B \backslash A)$

$$= \frac{1}{8} \times \frac{1}{6} = \frac{1}{48}$$

And $P(A \cap B) = P(B)\, P(A \backslash B)$

$$\Rightarrow \frac{1}{48} = P(B) \times \frac{1}{4}$$

$$\Rightarrow P(B) = \frac{1}{12}$$

(i) We know A and B are independent if and only if

$P(A \cap B) = P(A)P(B)$

Here, $P(A \cap B) = \frac{1}{48}$ ... (i)

and $P(A)\,P(B) = \frac{1}{8} \times \frac{1}{12}$

$= \frac{1}{96}$ ... (ii)

From equations (i) and (ii) we have

$P(A \cap B) \neq P(A)\,P(B)$.

So the events A and B are not independent.

(ii) We know that A and B are mutually exclusive if and only if $P(A \cap B) = 0$

Here, $P(A \cap B) = \frac{1}{48} \neq 0$

$\therefore$ A and B are not mutually exclusive.

(iii) $L.H.S = P(\bar{A} \setminus \bar{B})$

$$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

$$= \frac{1 - [\frac{1}{8} + \frac{1}{12} - \frac{1}{48}]}{1 - \frac{1}{12}}$$

$$= 0.887$$

But given, $P(\bar{A} \setminus \bar{B}) = 0.5$, which is not true.

H.W Given that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P(A \setminus B)$, $P(B \setminus A)$.

Are A and B independent?

Some formulas:

In case of general events:

(I) At least one / any one / A or B / either A or B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(II) Both A and B

$$P(A \cap B) = P(A) \cdot P(B)$$

(III) None of two / neither A nor B

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

(iv) Only A / Just A / A but not B

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

(v) Only B / just B / B but not A

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

Problem: The probability that a husband will live 12 more years is 0.25 and the probability that his wife will live 12 more years is 0.33. Find the probability that:

(i) both will be alive in 12 years

(ii) At least one will be alive in 12 years.

(iii) Neither will be alive in 12 years

(iv) Only the wife will be alive in 12 years.

Soln: Let, the event

H = The husband is alive in 12 years

W = His wife is alive in 12 years

Here, $P(H) = 0.25$

$P(W) = 0.33$

(i) Since H and W are independent, so the probability that both will be alive in 12 years

$$P(H \cap W) = P(H)\,P(W)$$
$$= 0.25 \times 0.33$$
$$= 0.0825$$

(ii) The probability that at least one will be alive in 12 years

$$P(H \cup W) = P(H) + P(W) - P(H \cap W)$$
$$= 0.25 + 0.33 - 0.0825$$
$$= 0.4975$$

(iii) Neither will be alive in 12 years

$$P(\bar{H} \cup \bar{W}) = 1 - P(H \cup W)$$
$$= 1 - 0.4975$$
$$= 0.5025$$

(iv) Only the wife will be alive in 12 years.

$$P(\bar{H} \cup W) = P(W) - P(H \cap W)$$
$$= 0.33 - 0.0825$$
$$= 0.2475$$

Problem: A problem of statistics is given to three students Nafiza, Nazma and Asma whose chances of solving it are $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{15}$ respectively. What is the probability that the problem will be solved?

Soln: Let us define the events:

A: The problem will be solved by Nafiza

B: The problem will be solved by Nazma

C: The problem will be solved by Asma

Normally, events A and B and C are independent as they are not affected with each other.

The probability that the problem will be solved is

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$$
$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}).$$

[Since A, B, C are independent]

$= 1 - \{1-P(A)\}\{1-P(B)\}\{1-P(C)\}$

$= 1 - (1-\frac{2}{5})(1-\frac{3}{10})(1-\frac{7}{15})$

$= 0.776$ (Ans)

Problems: In a survey on the shampoo use of women, it is found that 50% women use Sunsilk Shampoo, 45% all clear Shampoo, 40% meril shampoo, 25% Sunsilk and all clear Shampoo, 10% all clear and meril, 16% meril and Sunsilk and 8% all the above three brands. A woman is selected at random, find the probability that among the above mentioned Shampoo the women

(i) Use at least one

(ii) Use none

(iii) Use only Sunsilk

Soln: Let us define three events A, B and C as follows:

A: The women use Sunsilk Shampoo

B: The women use all clear.

C: The women use meril Shampoo

From the given information we have

$P(A) = 50\% = 0.\cancel{0}5$

$P(B) = 45\% = 0.45$

$P(C) = 40\% = 0.4$

$P(A \cap B) = 25\% = 0.25$

$P(B \cap C) = 10\% = 0.10$

$P(C \cap A) = 16\% = 0.16$

$P(A \cap B \cap C) = 8\% = 0.08$

(i) The required probability

$$= P(A \cup B \cup C)$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= 0.5 + 0.45 + 0.4 - 0.25 - 0.1 - 0.16 + 0.08$$

$$= 0.92$$

(ii) The required probability $= P(\overline{A \cup B \cup C})$

$$= 1 - P(A \cup B \cup C)$$

$$= 1 - 0.92$$

$$= 0.08$$

(iii) The required probability $= P(A\cap B\cap \bar{C})$

$$= P(A) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C)$$

$$= 0.5 - 0.25 - 0.16 + 0.08$$

$$= 0.17.$$

Problem: The HR department of a company has records which show the following analysis of its 800 employees:

Age	B.Sc degree	M.Sc degree	Total
Under 30	360	40	400
30 to 40	80	120	200
Over 40	160	40	200
Total	600	200	800

If one employee is selected at random find

(i) The probability he has a B.Sc degree

(ii) The probability he has an M.Sc degree, given that he is over 40

(iii) The probability he is under 30 given that he has only a B.Sc degree

Soln:

(i) From the given information, the required probability = P (Only a B.Sc degree)

$$= \frac{600}{800} = 0.75$$

(ii) P (M.Sc degree / Age over 40)

$$= \frac{P(\text{M.Sc degree and age over } 40)}{P(\text{Age over } 40)}$$

$$= \frac{\frac{40}{800}}{\frac{200}{800}} = \frac{1}{5}$$

(iii) P (Age $\overset{\text{under}}{\cancel{\text{over}}}$ 30 / Only a B.Sc degree)

$$= \frac{P(\text{Age under 30 and only a B.Sc degree})}{P(\text{Only a B.Sc degree})}$$

$$= \frac{\frac{360}{800}}{\frac{600}{800}} = 0.6$$