Moment, Skewness and Kurtosis

Moment: $x_1, x_2, \dots \dots, x_n$ **be n times of observation**

i) **Raw moment**

$$\mu_r{}' = \frac{\sum_{i=1}^{n} f_i (x_i - A)^r}{N}$$

A= any arbitrary value;

$r = 1, 2, 3, \dots, n$

ii) **Central moment**

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^r}{N}$$

$\overline{x}$**=arithmetic mean;**

$r = 1, 2, 3, \dots, n$

Relation between raw moment and central moment:

$$\mu_2 = \mu_2{}' - {\mu_1{}'}^2$$

$$\Rightarrow 2nd\ central\ moment = 2nd\ raw\ moment - (1st\ raw\ moment)^2$$

Skewness:

Skewness means lack of symmetry or departure from symmetry.

A distribution which is not symmetrical is called skew symmetrical distribution.

i) **Positive skewness**
$(mean > median > mode)$

ii) **Negative skewness**
$(mean < median < mode)$

Measure of skewness:

i) **Pearson's 1st measure of skewness**

$$Skewness = \frac{mean - mode}{standard\ deviation}$$

ii) **Pearson's 2nd measure of skewness**

$$Skewness = \frac{3(mean - median)}{standard\ deviation}$$

iii) **Bowley's measure of skewness**

$$Skewness = \frac{Q_3 + Q_1 - 2M_e}{Q_3 - Q_1}$$

iv) **Skewness based on moments**

$$Skewness, \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{{\mu_2}^3}}$$

$$= \frac{3rd\ central\ moment}{\sqrt{(2nd\ central\ moment)^3}}$$

Kurtosis:

Kurtosis measures the flatness or peakedness of the curve of a distribution.

i) **Laptokurtic distribution (highly peaked) $\beta_2 > 3$**
ii) **Platykurtic distribution (flat topped) $\beta_2 < 3$**
iii) **Mesokurtic distribution (neither peaked or flat) $\beta_2 = 3$**

Measure of kurtosis:

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2} = \frac{4th\ central\ moment}{(2nd\ central\ moment)^2}$$

$$\mu_4 = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^4}{N}$$

$$\mu_2 = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{N} = \sigma^2$$