Q1

1.5

The reason why the C.I, is so wide is because of the small sample size and considerably large variations among numbers.

1.6

the bootstrap confidence interval is not symmetric . This is due to the fact that the random sample from the population indicated that the population distribution was not symmetric.(This is obvious if you draw a histogram)

Q2

Step 0: **Check Assumptions**

Step 1: **Hypotheses**

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ (*The mean lifetimes are equal.*)

H_a: *Not all of the means are equal.*

Step 2: **Significance Level**

$\alpha = 0.05$

Step 3: **Critical Value and Rejection Region**

$F_{\alpha,(df_1=k-1, df_2=N-k)} = F_{0.05,(df_1=4-1, df_2=20-4)} = F_{0.05,(df_1=3, df_2=16)} = 3.24$

Reject the null hypothesis if $F \geq 3.24$ (P-value ≤ 0.05).

Step 4: **Construct the One-way ANOVA Table**

$T_1 = 168;\ T_2 = 154;\ T_3 = 149;\ T_4 = 143$

$$T = 614;\ \sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}^2 = \sum_{j=1}^{n_i} y_{1j}^2 + \sum_{j=1}^{n_i} y_{2j}^2 + \sum_{j=1}^{n_i} y_{3j}^2 + \sum_{j=1}^{n_i} y_{4j}^2 = 19{,}410$$

$$SSTo = \sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}^2 - \frac{(T)^2}{N} = 19{,}410 - \frac{(614)^2}{20} = 560.2$$

$$SSTr = \left(\frac{(T_1)^2}{n_1} + \frac{(T_2)^2}{n_2} + \frac{(T_3)^2}{n_3} + \frac{(T_4)^2}{n_4}\right) - \frac{(T)^2}{N}$$

$$= \left(\frac{(168)^2}{5} + \frac{(154)^2}{5} + \frac{(149)^2}{5} + \frac{(143)^2}{5}\right) - \frac{(614)^2}{20} = 68.2$$

$SSE = SSTo - SSTr = 560.2 - 68.2 = 492.0$

Source	df	SS	MS = SS/df	F-statistic	p-value
Treatments	3	68.2	22.7333	0.7393	p-value > 0.10
Error	16	492.0	30.75		
Total	19	560.2			

Step 5: **Decision**

Since 0.7393 < 3.24 (p-value > 0.05), fail to reject the null hypothesis.

Step 6: **State conclusion in words**

At the $\alpha = 0.05$ level of significance, there is not enough evidence to conclude that the mean lifetimes of the brands of batteries differ.

Q3

3.1

The objective of this part is to state the hypotheses.

The given test involves comparison of means for more than two groups. Hence ANOVA test is used.

The null hypothesis is that the average score difference is the same for all treatments. The alternative hypothesis is that there is at least one pair of means are different.

The null and alternate hypotheses can be stated as follows:

H_0 : Average score difference is the same for all treatments.

H_A :At least one pair of means are different.

Thus, the null and alternative hypotheses are formulated.

3.2

The decision rule using *P*-value approach is that if *P*-value is less than or equal to the level of significance, reject the null hypothesis. Otherwise the null hypothesis fails to be rejected.

Because the obtained *P*-value of 0.0461 is less than the level of significance of $\alpha = 0.05$, the null hypothesis is rejected.

Therefore, the data provides convincing evidence that there is a difference between the average reductions in score among treatments.

3.3

we now conduct K = 3 × 2/2 = 3 pairwise t-tests that each use α = 0.05/3 = 0.0167 for a significance level. Use the following hypotheses for each pairwise test.
H0 : The two means are equal. HA: The two means are different.

The given information shows that the sample sizes are equal and hence the pooled standard deviation can be used. The given ANOVA output shows that the pooled estimate of 9.793 is obtained for 39 degrees of freedom.

Treatment 1 versus Treatment 2:

The estimated difference and the standard error are calculated as follows:

$$\begin{aligned} \bar{x}_1 - \bar{x}_2 &= 6.21 - 2.86 \\ &= 3.35 \end{aligned}$$

$$\begin{aligned} SE &= \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \\ &= \sqrt{\frac{12.3^2}{14} + \frac{7.94^2}{14}} \\ &\approx 3.913 \end{aligned}$$

Using the obtained estimated difference and the standard error, the *T*-score is calculated as follows:

$$\begin{aligned} T &= \frac{\text{Estimate difference} - \text{null value}}{SE} \\ &= \frac{3.35 - 0}{3.913} \\ &\approx 0.856 \end{aligned}$$

Use the degrees of freedom for pooled standard deviation of 39 degrees of freedom. Use Excel to find the *P*-value.

fx =T.DIST.2T(0.856,39)

F	G	H
	0.39723	

Thus, the *P*-value is obtained as 0.3972.

The decision rule using *P*-value approach is that if *P*-value is less than or equal to the level of significance, reject the null hypothesis. Otherwise the null hypothesis fails to be rejected.

Because the obtained *P*-value of 0.3972 is greater than the level of significance of 0.0167, the null hypothesis fails to be rejected. Hence the difference between treatment 1 and treatment 2 is not significant.

Treatment 2 versus Treatment 3:

The estimated difference and the standard error are calculated as follows:

$$\begin{aligned} \bar{x}_2 - \bar{x}_3 &= 2.86 - (-3.21) \\ &= 2.86 + 3.21 \\ &= 6.07 \end{aligned}$$

$$\begin{aligned} SE &= \sqrt{\frac{s_2^2}{n_2} + \frac{s_3^2}{n_3}} \\ &= \sqrt{\frac{7.94^2}{14} + \frac{8.57^2}{14}} \\ &\approx 3.1224 \end{aligned}$$

Using the obtained estimated difference and the standard error, the T-score is calculated as follows:

$$\begin{aligned} T &= \frac{\text{Estimate difference} - \text{null value}}{SE} \\ &= \frac{6.07 - 0}{3.1224} \\ &\approx 1.944 \end{aligned}$$

Use the degrees of freedom for pooled standard deviation of 39 degrees of freedom. Use Excel to find the P-value.

f_x	=T.DIST.2T(1.944,39)	
F	G	H
	0.05914	

Thus, the P-value is obtained as 0.0591.

The decision rule using P-value approach is that if P-value is less than or equal to the level of significance, reject the null hypothesis. Otherwise the null hypothesis fails to be rejected.

Because the obtained P-value of 0.0591 is greater than the level of significance of 0.0167, the null hypothesis fails to be rejected. Hence the difference between treatment 2 and treatment 3 is not significant.

Treatment 1 versus Treatment 3:

The estimated difference and the standard error are calculated as follows:

$$\begin{aligned} \bar{x}_1 - \bar{x}_3 &= 6.21 - (-3.21) \\ &= 6.21 + 3.21 \\ &= 9.42 \end{aligned}$$

$$\begin{aligned} SE &= \sqrt{\frac{s_1^2}{n_1} + \frac{s_3^2}{n_3}} \\ &= \sqrt{\frac{12.3^2}{14} + \frac{8.57^2}{14}} \\ &\approx 4.0066 \end{aligned}$$

Using the obtained estimated difference and the standard error, the T-score is calculated as follows:

$$
\begin{aligned}
T &= \frac{\text{Estimate difference} - \text{null value}}{SE} \\
&= \frac{9.42 - 0}{4.0066} \\
&\approx 2.3511
\end{aligned}
$$

Use the degrees of freedom for pooled standard deviation of 39 degrees of freedom. Use Excel to find the P-value.

f_x	=T.DIST.2T(2.3511,39)	
F	G	H
	0.02387	

Thus, the P-value is obtained as 0.0239.

The decision rule using P-value approach is that if P-value is less than or equal to the level of significance, reject the null hypothesis. Otherwise the null hypothesis fails to be rejected.

Because the obtained P-value of 0.0239 is greater than the level of significance of 0.0167, the null hypothesis fails to be rejected. Hence the difference between treatment 1 and treatment 3 is not significant.

Therefore, it is not possible identify the pair of groups which are actually different.

3.4

One advantage of the Bonferroni correction method is its simplicity and ease of implementation. It is a straightforward and widely used method for adjusting p-values in multiple hypothesis testing scenarios. The Bonferroni correction divides the desired significance level (usually 0.05) by the number of comparisons being made, providing a conservative adjustment to control the family-wise error rate.

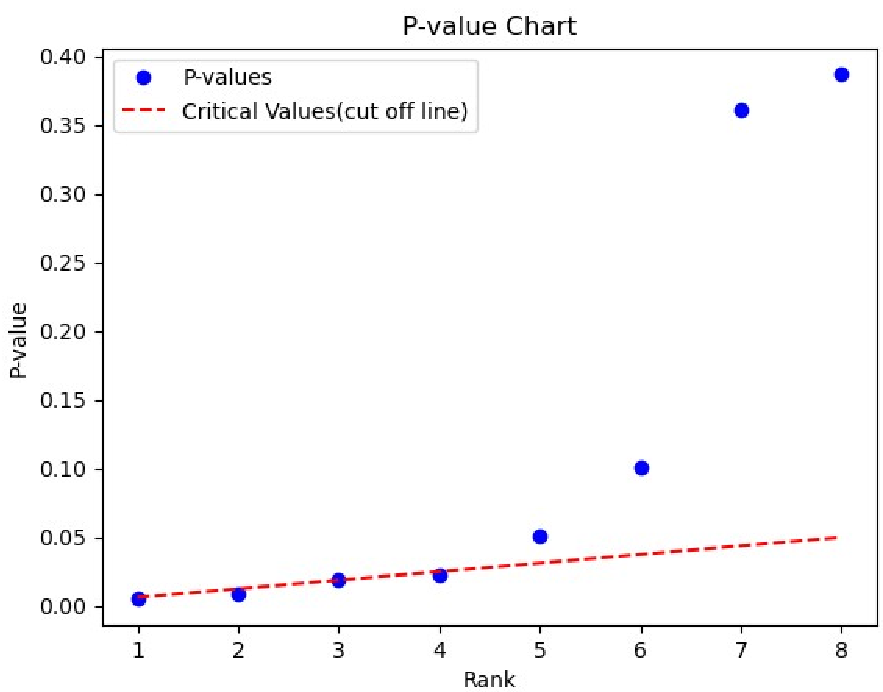
Disadvantage of the Bonferroni correction method:
One disadvantage of the Bonferroni correction method is its conservative nature, which can lead to an increased risk of type II errors.

4.1

steps :
1. sort p-values
2.compute BH-critical value for each row in the table ((k/m).siginicant level)
3. find the largest k for which the p-value is less than the corresponding critical value
(in this case k = 4)
4. reject the p values that are below the k value in previous step

Rank (k)	Sorted p-values	(k/m)*alpha	reject(yes/no)?
1	0.005	(1/8)*0.05 = 0.0062	yes
2	0.009	(2/8)*0.05 = 0.0125	yes
3	0.019	(3/8)*0.05 = 0.0187	yes
4	0.022	(4/8)*0.05 = 0.0250	yes
5	0.051	(5/8)*0.05 = 0.0312	no
6	0.101	(6/8)*0.05 = 0.0375	no
7	0.361	(7/8)*0.05 = 0.0437	no
8	0.387	(8/8)*0.05 = 0.05	no

4.2



4.3
FWER control methods, such as the Bonferroni correction, aim to control the family-wise error rate. The family-wise error rate is the probability of making at least one type I error (rejecting a null hypothesis when it is true) among all the comparisons being made. The Bonferroni correction is a conservative method that adjusts the significance level for each individual test by dividing it by the number of comparisons being made. This adjustment ensures that the overall family-wise error rate remains below a specified threshold (e.g., 0.05). FWER control methods prioritize the avoidance of any false positives across all tests, making them suitable when the focus is on maintaining a low overall error rate.

FDR control methods, such as the Benjamini-Hochberg procedure, aim to control the false discovery rate. The false discovery rate is the expected proportion of false discoveries (rejecting a null hypothesis when it is true) among all the hypotheses that are rejected. FDR control methods allow for a higher rate of false positives compared to FWER control methods, but they still aim to limit the proportion of false discoveries. These methods adjust the individual test p-values to control the FDR at a specified level (e.g., 0.05). FDR control methods are commonly used in exploratory analyses or situations where a higher tolerance for false positives is acceptable.

Q5 - 5.1 False, the corrected significance level should decrease.

5.2 True, df(E) = df(total) – df(group), df(total)=n-1

5.3 False, The F distribution function is a one-way distribution function.

5.4 False, We can only say that there are at least two different groups

5.5 True. In order to be able to reject H0 , we need a small p-value, which requires a large F statistic. Obtaining a large F statistic requires that the variability between sample means is greater than the variability within the samples.

Q6

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*
Between Groups	78	3	26	13
Within Groups	16	8	2	
Total	94	11		

c) Hypothesis H_0 : All means are equal

H_1 : Atleast one mean is different

Critical value $F_{0.05,3,8} = 4.066$

conclusion: Since the test statistic is greater than critical value we reject null hypothesis and there is a significant evidence to conclude that atleast one mean is different from the other.

d) Tukeys multiple comparison

$q_{0.05,4,8} = 4.53$

$q_{0.05,4,8} * \sqrt{\frac{MSE}{n}} = 4.53*(2/3)^{0.5} = 3.699$

difference in means

group comparisons	*| Mean Difference |*
1 - 2	1
1 - 3	5
1 - 4	4
2 - 3	6
2 - 4	5
3 - 4	1

The mean difference which is greater than $q_{0.05,4,8} * \sqrt{\frac{MSE}{n}}$ are significantly different.

hence the groups 1-3 , 1 - 4 , 2 - 3 , 2 -4 are significantly different.

A limitation of Tukey's procedure is the requirement that all the sample means be based on the same number of data values. Tukey (1953) and Kramer (1956) independently proposed an approximate procedure in the case of unequal sample sizes. In place of Tukey's W, use

$$W* = \frac{q_\alpha(t, v)}{\sqrt{2}}\sqrt{s_w^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Q7

7.1 this is an experimental study because the participants were divided randomly in different groups and then treatments applied on them.

7.2 the experiment treatment is 25 grams of chia seeds twice a day.
The control treatment is placebo

7.3 yes, gender

7.4 and 7.5 : just single blind since the patients were blinded to the treatment they received.

7.6 Since this is an experiment, we can make a causal statement. However, since the sample is not random, the causal statement cannot be generalized to the population at large.

8.

a) The relationship is linear and is positive and reasonably strong.

b) The equation is as follows:
$Weight = -105.0113 + 1.0176 \times height$
The model tells us that for every centimeter in height, we expect an additional 1.02 kg in weight. The intercept doesn't have any intrinsic meaning here because if height == 0cm, the model would give us a weight of -105 kg. As such, it serves only as an adjustment.

c) $\begin{cases} H_0: \beta_1 = 0 \\ H_A: \beta_1 > 0 \end{cases}$
The p-value for this test is incredibly small as shown in the table above. We can reject the null and say that the true slope parameter is > 0.

d) $R^2 = 0.72 \times 0.72 = 0.5184$
The r-square is 0.5184 and tells us that 51.84% of the variability around the mean in weight is explained by height.

9.

a) The **M.L.E.**'s $\hat{\beta}_0$ and $\hat{\beta}_1$ are the same as the least squares estimates, so:

$$S_{xy} = \sum xy - \frac{1}{n}\left(\sum x\right)\left(\sum y\right) = 48.143$$

$$S_{xx} = \left(\sum x^2\right) - \frac{1}{n}\left(\sum x\right)^2 = 108.969$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = 0.4418\,, \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 0.1137$$

$$y = 0.1137 + 0.4418\,x$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum (y_i - \hat{y}_i)^2 = 0.02912$$

$$Var(\hat{\beta}_0) = \sigma^2\frac{\sum x_i^2}{nS_{xx}} = 0.009\,, \qquad Var(\hat{\beta}_1) = \frac{\sigma^2}{S_{xx}} = 0.0658$$

b)

i. $t_0 = \frac{\hat{\beta}_0 - \beta_0^*}{SE(\hat{\beta}_0)} = \frac{\hat{\beta}_0 - \beta_0^*}{s\sqrt{\frac{\sum x_i^2}{nS_{xx}}}} = \frac{\hat{\beta}_0 - \beta_0^*}{\sqrt{\frac{SSE}{n-2}}\sqrt{\frac{\sum x_i^2}{nS_{xx}}}} = -3.619, \qquad DOF = n - 2 = 8$

$\xrightarrow{t-table} \quad |t_0| > 2.306 \quad \longrightarrow \quad H_0\ is\ regected$

ii. $\begin{cases} H_0: \beta_0 = 0 \\ H_A: \beta_0 \neq 0 \end{cases}$

$t_0 = \frac{\hat{\beta}_0}{SE(\hat{\beta}_0)} = 1.065, \qquad DOF = n - 2 = 8$

$\xrightarrow{t-table} \quad |t_0| < 2.306 \quad \longrightarrow \quad H_0\ is\ not\ regected$

10.

The open interval between the two random variables:

$$c_0\hat{\beta}_0 + c_1\hat{\beta}_1 \pm \sigma' \left[\frac{c_0^2}{n} + \frac{(c_0\bar{x} - c_1)^2}{s_x^2}\right]^{1/2} T_{n-2}^{-1}\left(1 - \frac{\alpha_0}{2}\right)$$

$$\xRightarrow{c_0=1\,,\ \ c_1=x} (\beta_0 + \beta_1 x) \pm \sigma' \left(\sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{s_x^2}}\right) T_{n-2}^{-1}\left(1 - \frac{\alpha_0}{2}\right)$$

The length of this confidence interval is:

$$l(x) = 2\sigma' \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{s_x^2}} T_{n-2}^{-1}\left(1 - \frac{\alpha_0}{2}\right)$$

To minimize this length, we need to minimize $\frac{1}{n} + \frac{(x-\bar{x})^2}{s_x^2}$

We conclude that the same value of x which minimize $\frac{1}{n} + \frac{(x-\bar{x})^2}{s_x^2}$ also minimizes $(x-\bar{x})^2$

$$((x-\bar{x})^2)' = 0 \quad \rightarrow \quad 2(x-\bar{x}) = 0 \quad \rightarrow \quad x = \bar{x}$$

11.

a) The log likelihood function:

$$l(p) = \ln\binom{n}{x} + xln(p) + (n-x)\ln(1-p)$$

$$l'(p) = \frac{x}{p} + \frac{n-x}{1-p} = 0$$

$$\hat{p} = \frac{X}{n}$$

b) Fisher information:

$$I(\hat{p}) = -E\left[\frac{\partial^2}{\partial p^2} lnf(x|p)\right]$$

$$I(\hat{p}) = -E\left[-\frac{x}{p^2} - \frac{n-x}{(1-p)^2}\right] = -\left[-\frac{E(x)}{p^2} - \frac{n-E(x)}{(1-p)^2}\right]$$

$$I(\hat{p}) = -\left[-\frac{np}{p^2} - \frac{n-np}{(1-p)^2}\right] = \frac{n}{p} + \frac{n}{1-p} = \frac{n}{p(1-p)}$$

Asymptotic variance of this MLE:

$$V(\hat{p}) \geq \frac{1}{nI(\hat{p})} = \frac{p(1-p)}{n^2}$$

The variance of the MLE $\hat{p}$:

$$Var(\hat{p}) = Var\left(\frac{1}{n}X\right) = \frac{Var(X)}{n^2} = \frac{p(1-p)}{n^2}$$

→ The MLE has attained Cramer-Rao Lower Bound