

# **Vector Space Model Implementation**

Presenter

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Slides are obtained from ChengXiang Zhai and Sean  
Massung book

# Outline

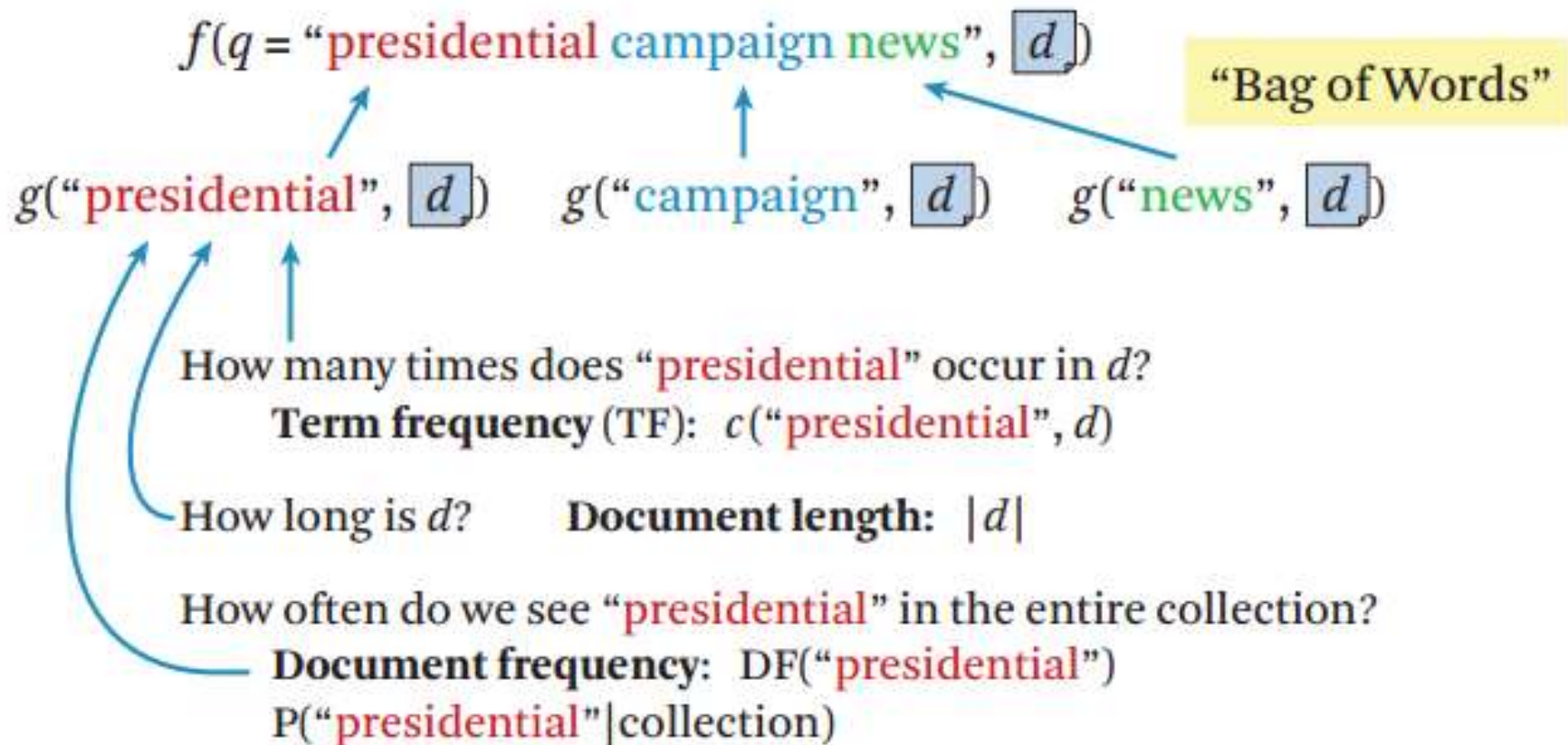
1. Text Retrieval Problem
2. Vector Space Model
3. Vector Space Model Implementation
  - Term At A Time (TAAT) Query Processing
  - Document At A Time (DAAT) Query Processing

# **1. Text Retrieval Problem**

# Text Retrieval Problem

- **Query:**  $q = q_1, \dots, q_m$ , where  $q_i \in V$
- **Document:**  $d = d_1, \dots, d_n$ , where  $d_i \in V$
- **Ranking function:**  $f(q, d) \in \mathcal{R}$
- A good ranking function should rank relevant documents on top of non-relevant ones
- **Key challenge:** how to measure the likelihood that document  $d$  is relevant to query  $q$
- **Retrieval model** = formalization of relevance (give a computational definition of relevance)

# Common Form of a Retrieval Function

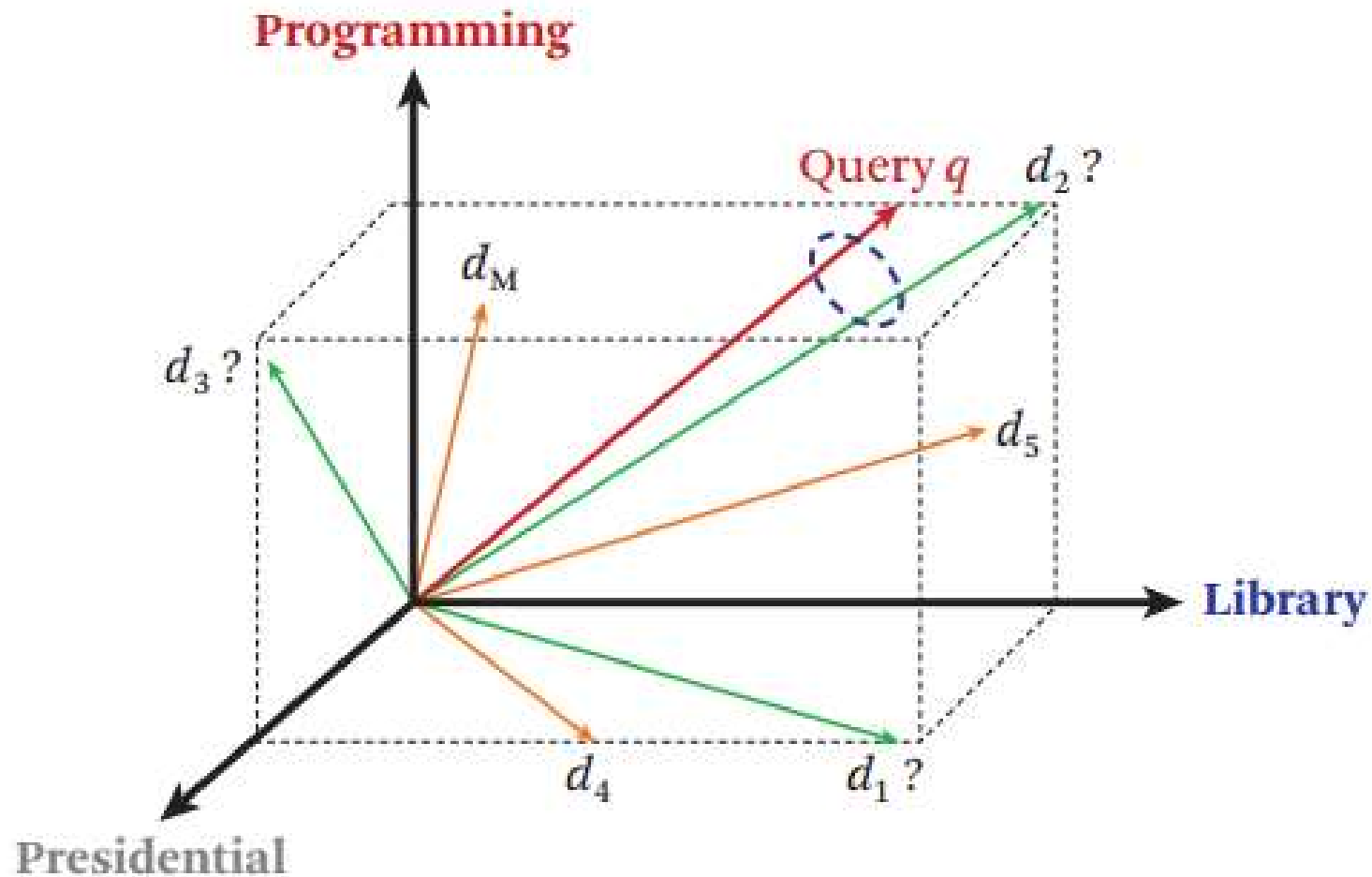


## **2. Vector Space Model**

# Vector Space Model (VSM)

- Represent a document/query by a ***term vector***
  - ***Term***: basic concept, e.g., ***word*** or ***phrase***
  - Each term defines one dimension
  - N terms define a high-dimensional space
  - Element of vector corresponds to term weight
  - E.g.,  $d = (x_1, \dots, x_N)$ ,  $x_i$  is “importance” of term  $i$
- Measure relevance by the distance between the query vector and document vector in the vector space

# Vector Space Model (VSM) Illustration

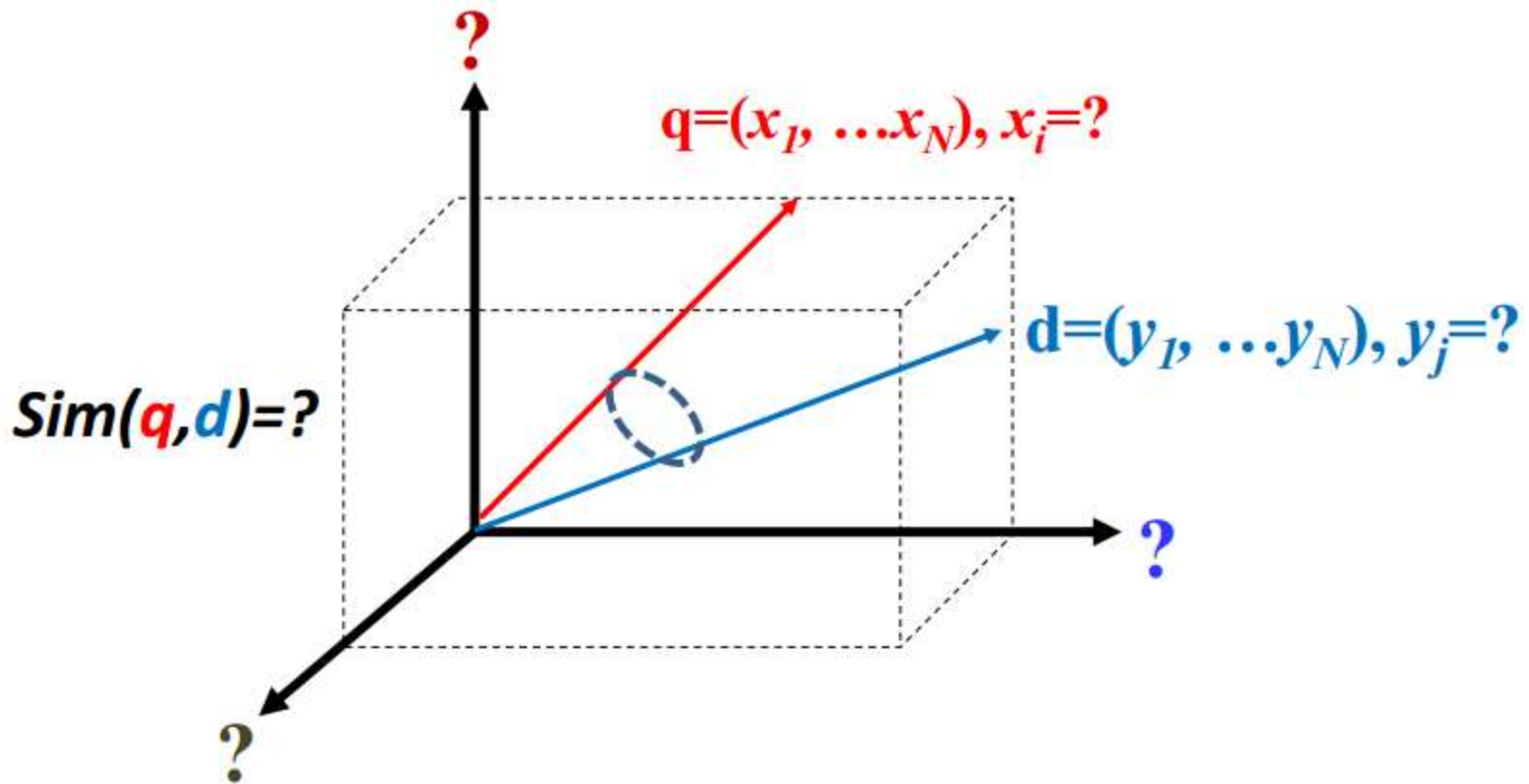




# VSM is a Framework

- How to define/select the terms
  - Terms are assumed to be linearly independent
- How to assign term weights
  - Weight in query indicates importance of term
  - Weight in doc indicates how well the term characterizes the doc
- How to define the similarity/distance measure

# What VSM Doesn't Say



# Term weighting

- Major term weighting heuristics
  - TF weighting and transformation
  - IDF weighting
  - Document length normalization

Term frequency		Document frequency		Normalization	
n (natural)	$tf_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{df_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \dots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times tf_{t,d}}{\max_t(tf_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-df_t}{df_t}\}$	u (pivoted unique)	$1/u$ (Section 6.4.4)
b (boolean)	$\begin{cases} 1 & \text{if } tf_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^\alpha, \alpha < 1$
L (log ave)	$\frac{1 + \log(tf_{t,d})}{1 + \log(\text{ave}_{t \in d}(tf_{t,d}))}$				

<https://nlp.stanford.edu/IR-book/html/htmledition/document-and-query-weighting-schemes-1.html>

# State of the Art VSM Ranking Functions

Pivoted length normalization VSM

$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{\ln(1 + \ln(1 + c(w, d)))}{1 - b + b \frac{|d|}{\text{avdl}}} \log \frac{M + 1}{\text{df}(w)} \quad b \in [0, 1]$$

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$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{(k + 1)c(w, d)}{c(w, d) + k(1 - b + b \frac{|d|}{\text{avdl}})} \log \frac{M + 1}{\text{df}(w)} \quad b \in [0, 1], \quad k \in [0, +\infty)$$

# **3. Vector Space Model Implementation**

# Text Retrieval System Implementation

- Tokenizer
  - This determines how we represent a document
- Indexer
  - This convert documents to data structures that enable fast search
  - Compression when appropriate
- Scorer
  - This use inverted index for fast search

# Inverted Index

- Fast access to all docs containing a given term (along with freq and pos information)
- For each term, we get a list of tuples (docID, freq, pos).
- Given a query, we can fetch the lists for all query terms and work on the involved documents.
  - Boolean query: set operation
  - Natural language query: term weight summing
- More efficient than scanning docs

# Inverted Index Example

**Doc 1**

This is a sample  
document  
with one sample  
sentence

**Doc 2**

This is another  
sample document

**Dictionary**

Term	# docs	Total freq
This	2	2
is	2	2
sample	2	3
another	1	1
...	...	...

**Postings**

Doc id	Freq
1	1
2	1
1	1
2	1
1	2
2	1
2	1
...	...

term | #docs | totalfreq | docID1:freq1;docID2:freq2;...



# Positional Inverted Index Example

## Doc 1

web retrieval  
web search  
information

## Doc 2

search engine  
web ranking

## Doc 3

web search  
course  
information  
search

## Dictionary

Term	# docs	Total freq
course	1	1
engine	2	1
information	2	2
ranking	1	1
retrieval	1	1
search	3	4
web	3	4
...	...	...

## Postings

Doc id	Freq	Pos
3	1	2
2	1	1
1	1	4
3	1	3
2	1	3
2	1	1
1	1	3
2	1	0
3	2	1, 4
1	2	0, 2
2	1	2
3	1	1
...	...	...

term | #docs | totalfreq | docID1:freq1,pos1,pos2;docID2:freq2,pos3,pos4,pos5;...

# How to Score Documents Quickly with Inverted Index

## General Form of Scoring Function

The diagram illustrates the general form of a scoring function  $f(q, d)$  with several components and their relationships:

- Final score adjustment**: A box at the top with arrows pointing to  $f_a$ ,  $f_d(d)$ , and  $f_q(q)$ .
- Weight aggregation**: A box with an arrow pointing to  $f_a$ .
- Weight a matched query term in d**: A box with arrows pointing to  $g(t_1, d, q)$  and  $g(t_k, d, q)$ .

The equation is:

$$f(q, d) = f_a(h(\underbrace{g(t_1, d, q)}_{\text{Weight a matched query term in d}}, \dots, \underbrace{g(t_k, d, q)}_{\text{Weight a matched query term in d}}), f_d(d), f_q(q))$$

# TAAT vs DAAT Query Processing

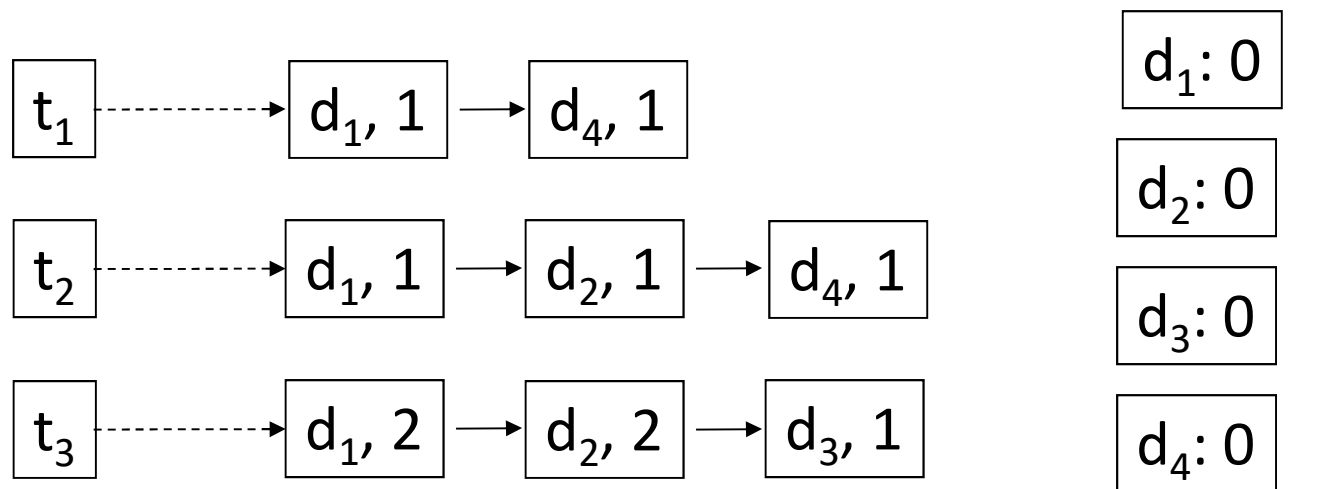
- **TAAT = “Term At A Time”**
  - Scores for all docs computed concurrently, one query term at a time
- **DAAT = “Document At A Time”**
  - Total score for each doc (include all query terms) computed, before proceeding to the next
- Each has implications for how the retrieval index is structured and stored

# Term-at-a-time Ranking

- Read posting lists for query terms  $(t_1, \dots, t_{|q|})$  **successively**
- Maintains an **accumulator** for each result document with value

$$acc(d) = \sum_{i \leq j} score(t_i, d)$$

after the first  $j$  posting lists have been read

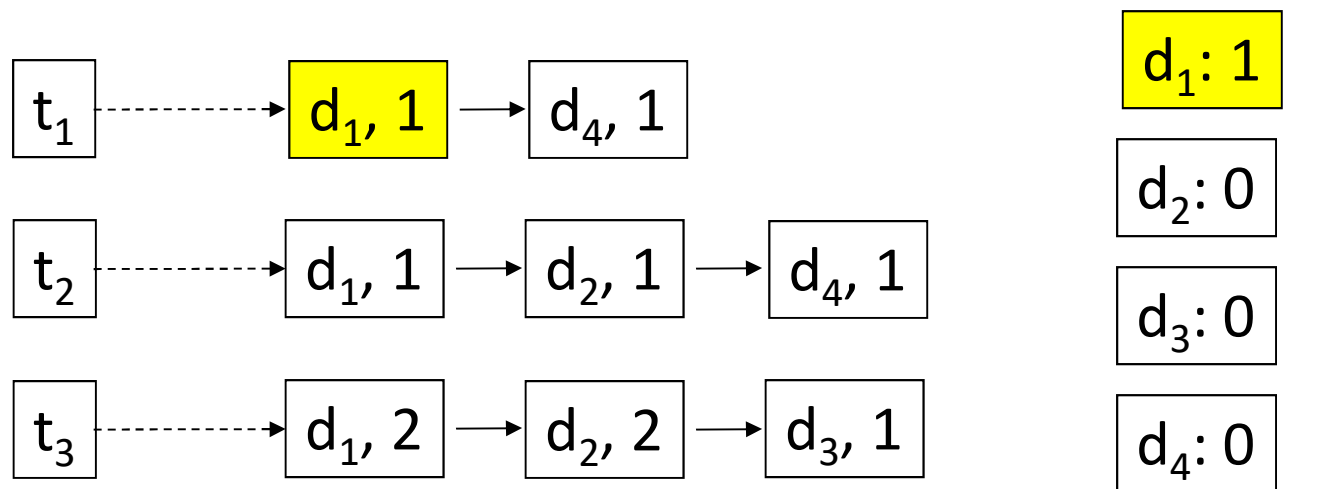


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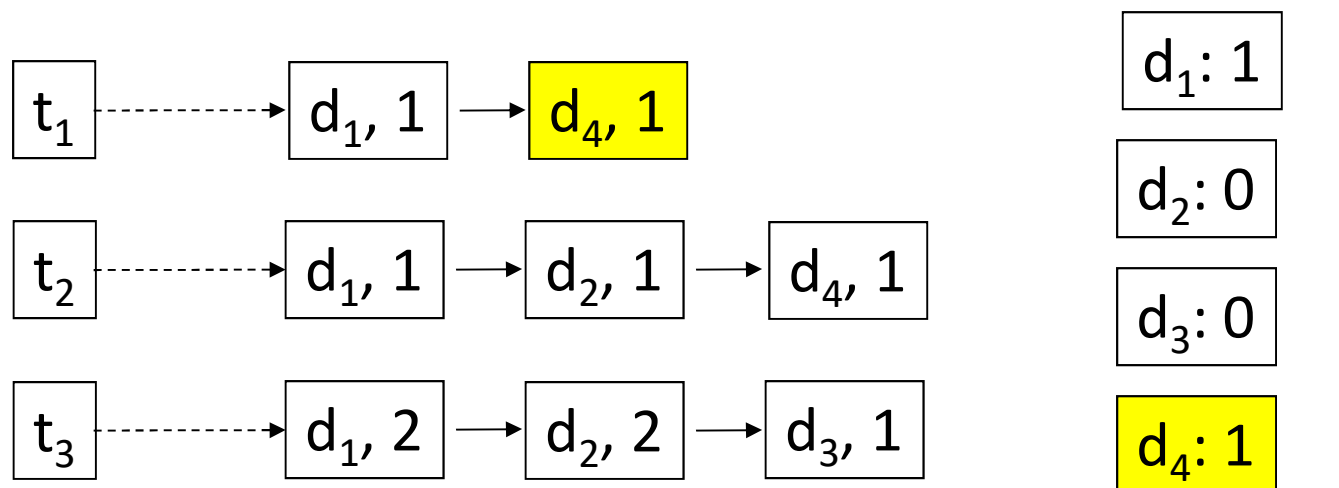


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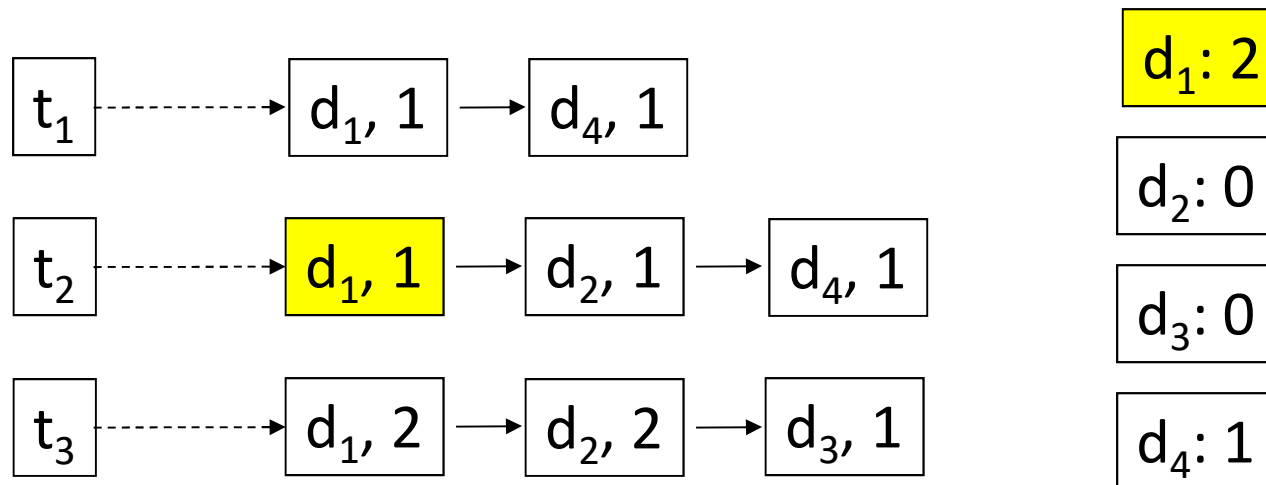
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Accumulators

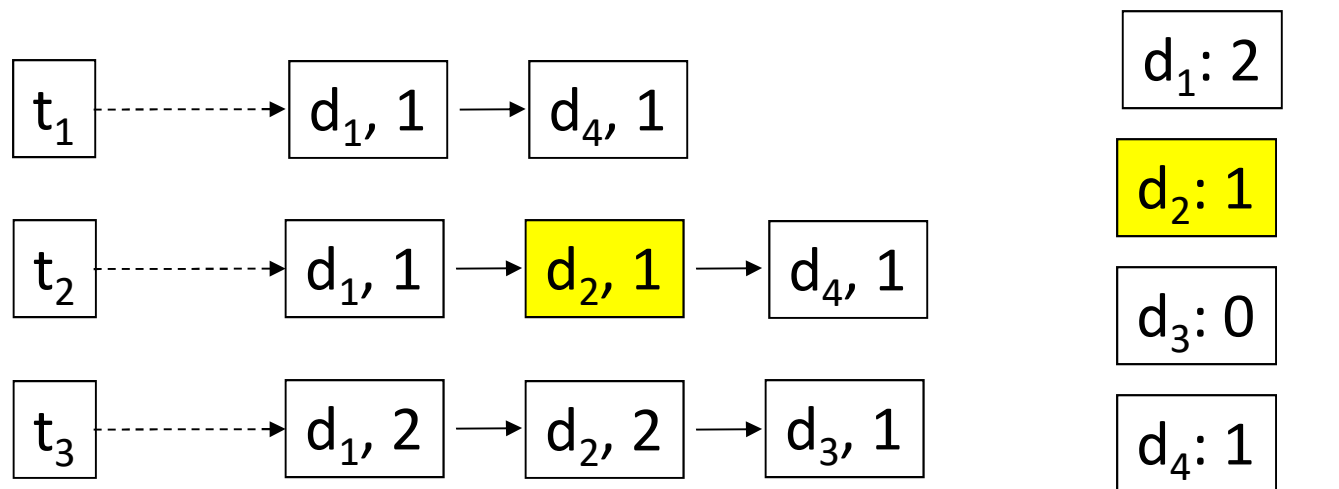


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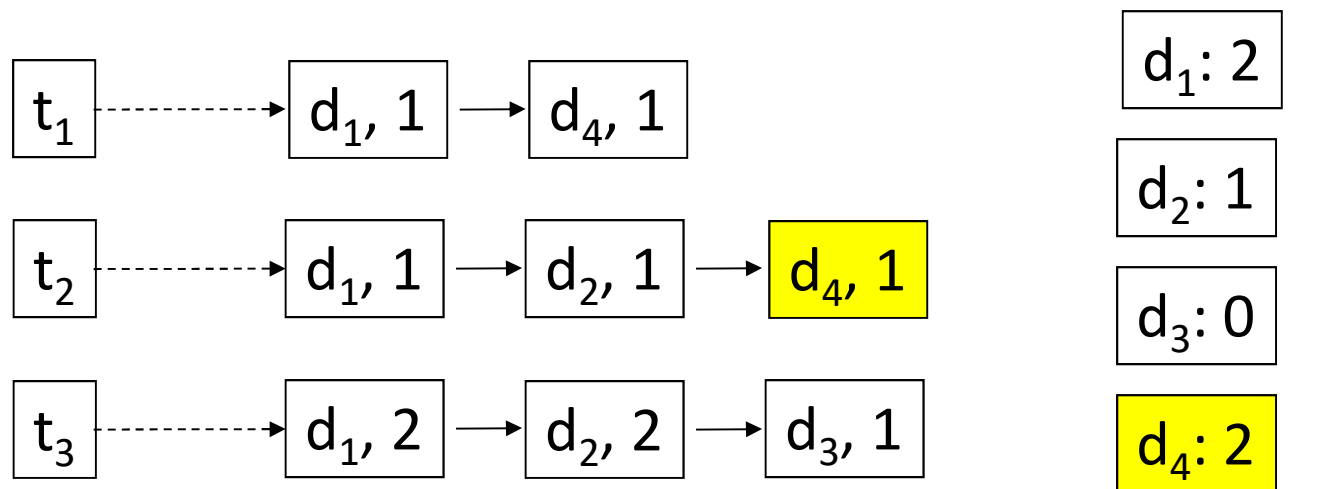


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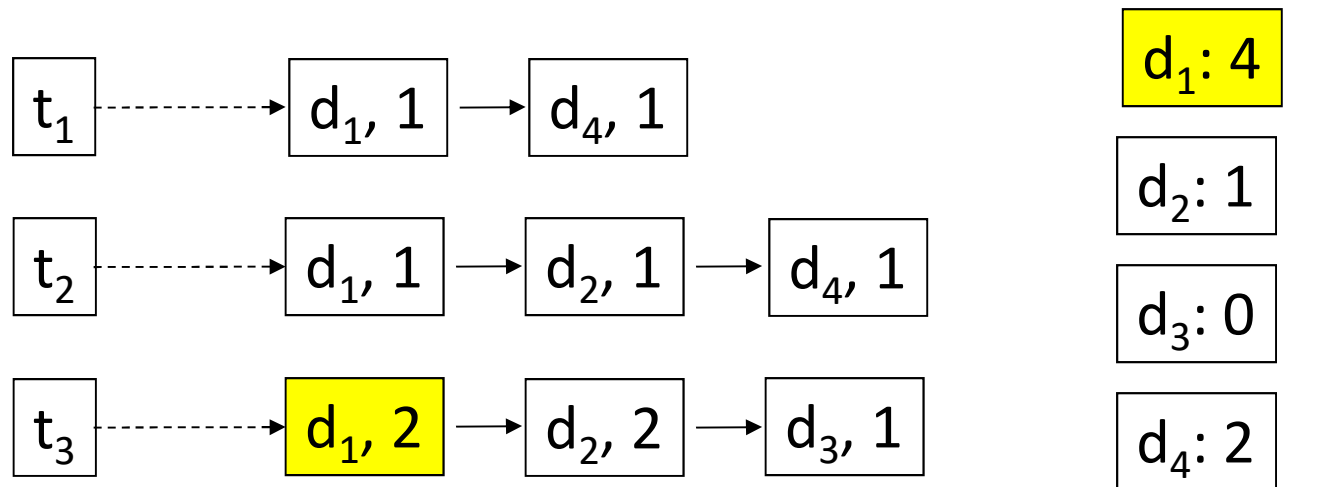


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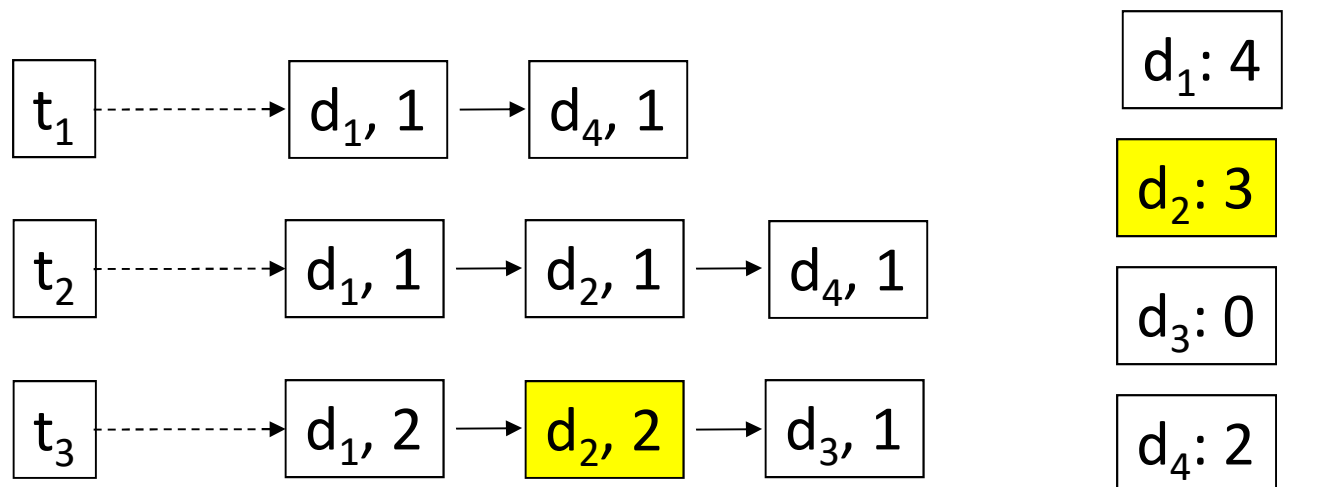


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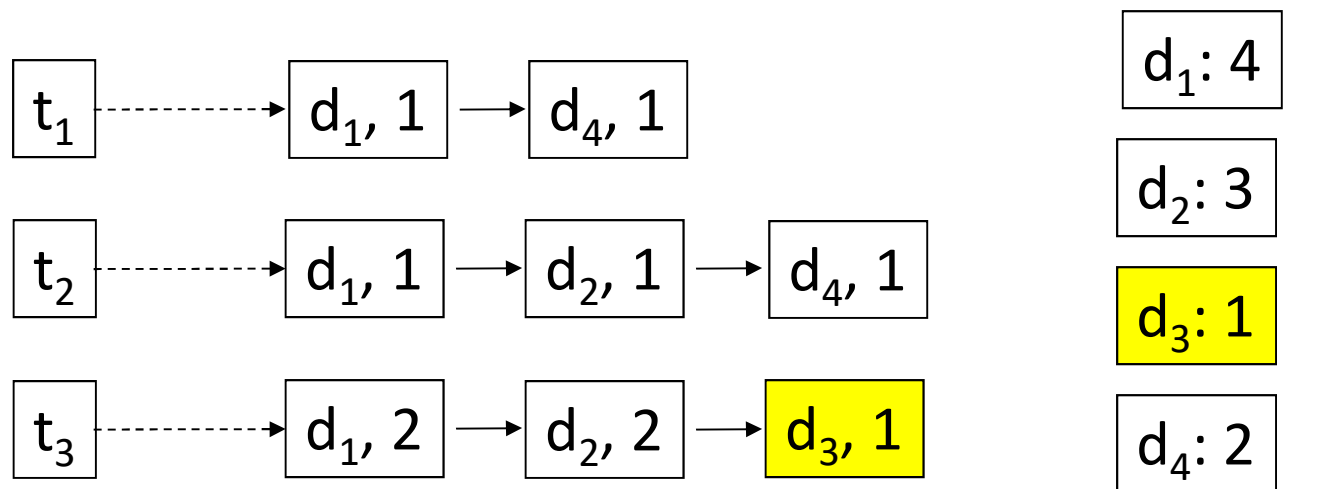


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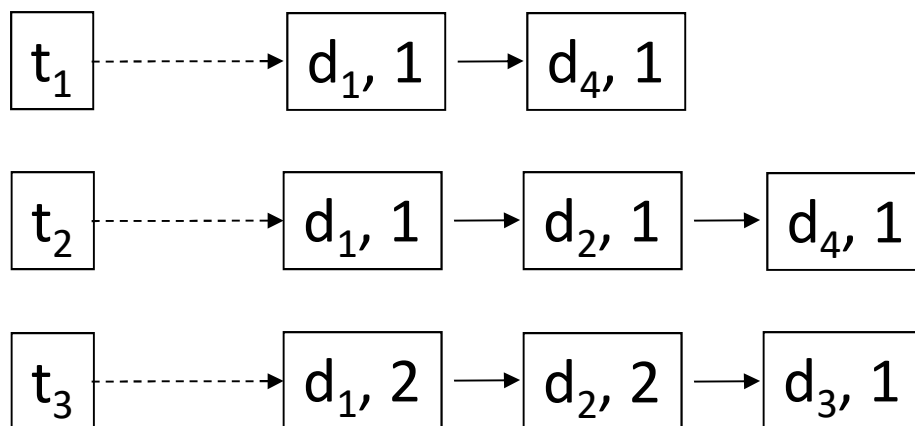
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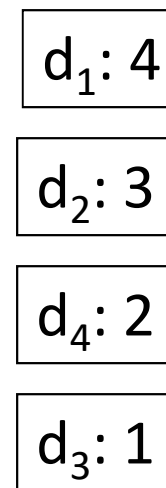
$$acc(d) = \sum_{i \leq j} score(t_i, d)$$

after the first  $j$  posting lists have been read

- **Top- $k$  results** can be determined by **sorting accumulators** at the end



Sorted Accumulators



# Term-at-a-time Ranking

```
scores = {}      // score accumulator maps doc IDs to scores
for  $w \in q$  do
    for  $d, count \in \text{Idx.fetch\_docs}(w)$  do
         $scores[d] = scores[d] + score\_term(count)$ 
    end for
end for
return top  $k$  documents from scores
```

# Disadvantage of term-at-a-time ranking

- The size of the score accumulators *scores* will be the size of the number of documents matching at least one term.
- This set is still large.

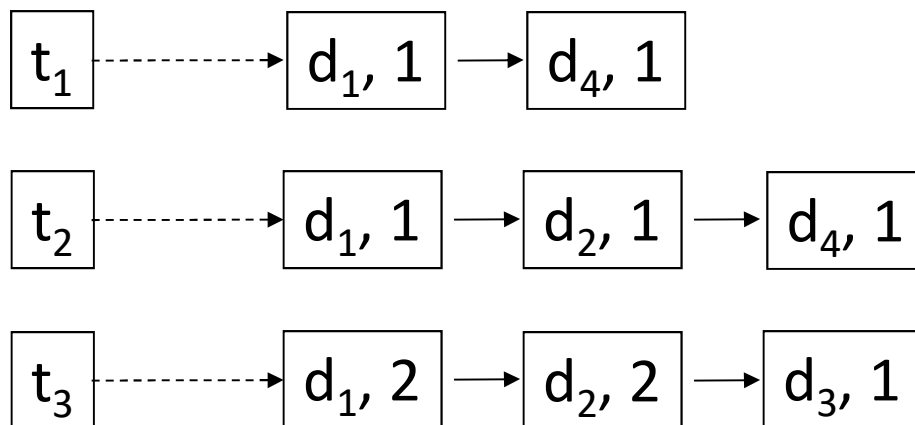
# Document-at-a-time Ranking

- Since most searches are top- $k$  searches, we can only keep the top- $k$  documents at any one time.
- We can hold the  $k$  best completely scored documents with a priority queue.



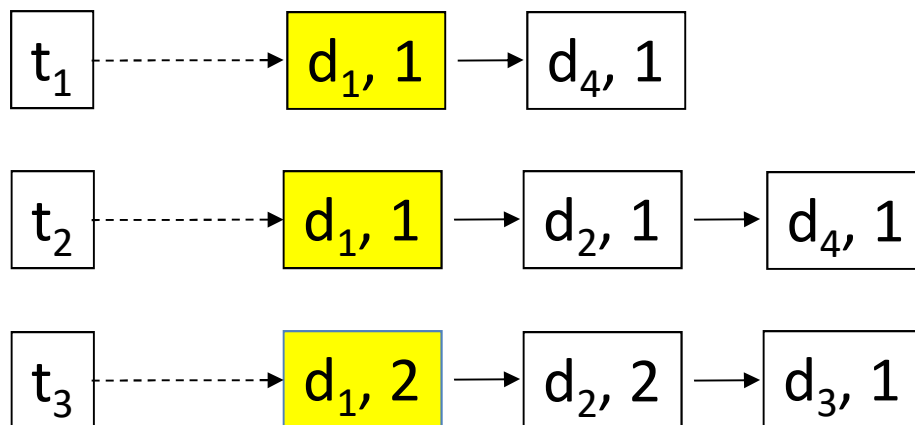
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- Read posting lists for query terms  $(t_1, \dots, t_{|q|})$  **concurrently**
- Computes score when **same document** is seen in one or more posting lists
- Always advances posting list with **lowest current document id**
- **Top-k results** can be determined by keeping results in **priority queue**



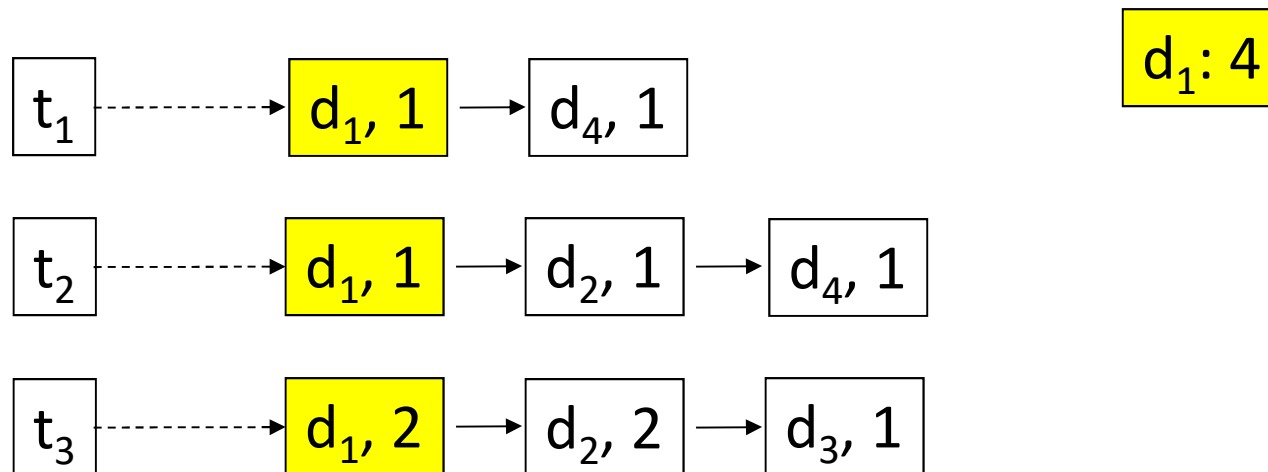
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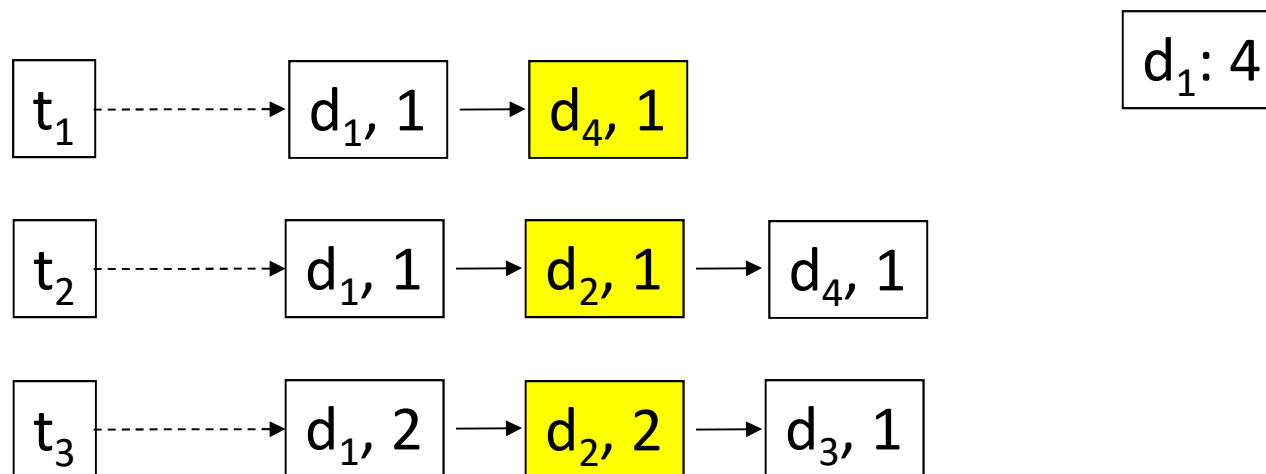
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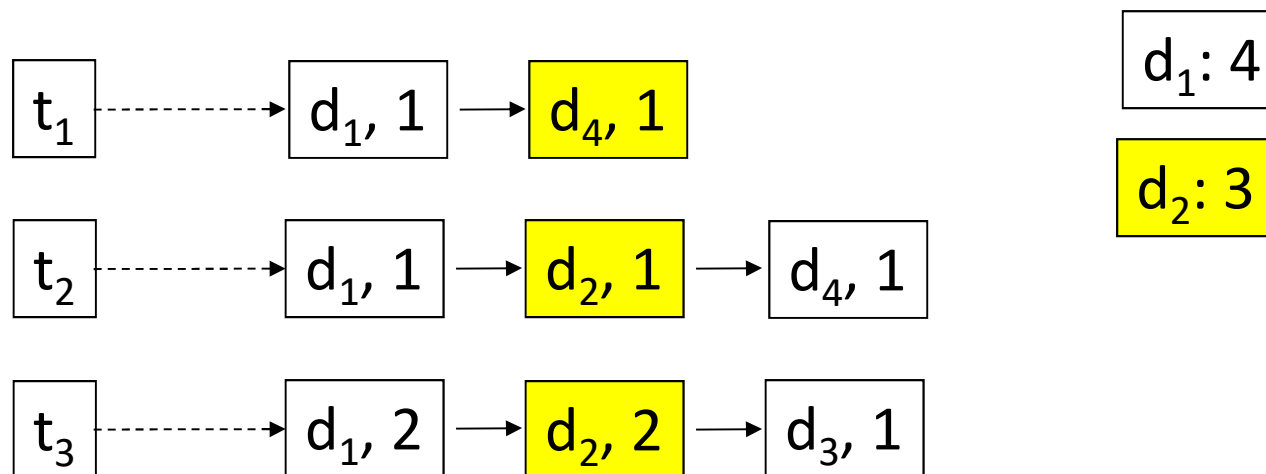
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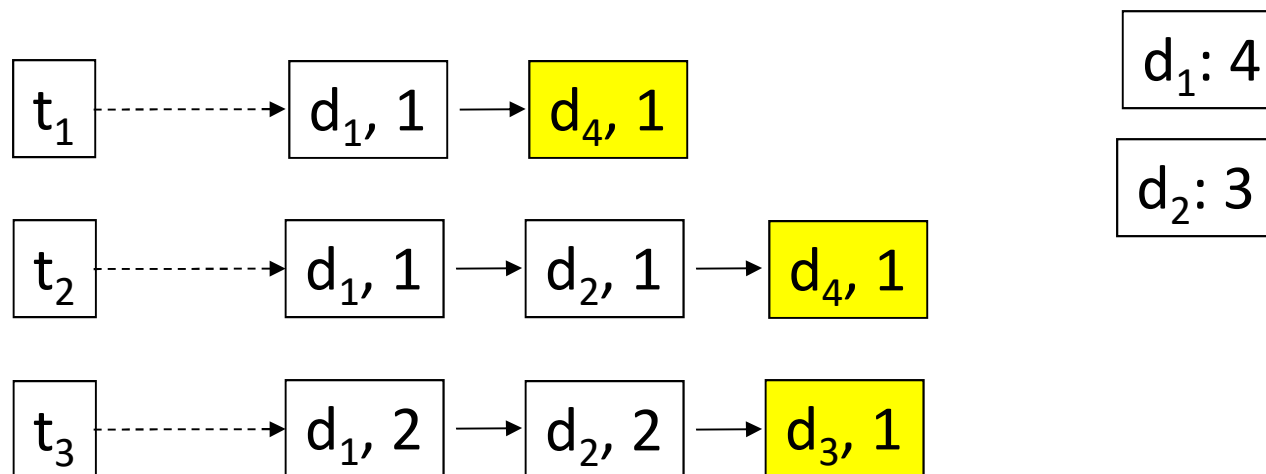
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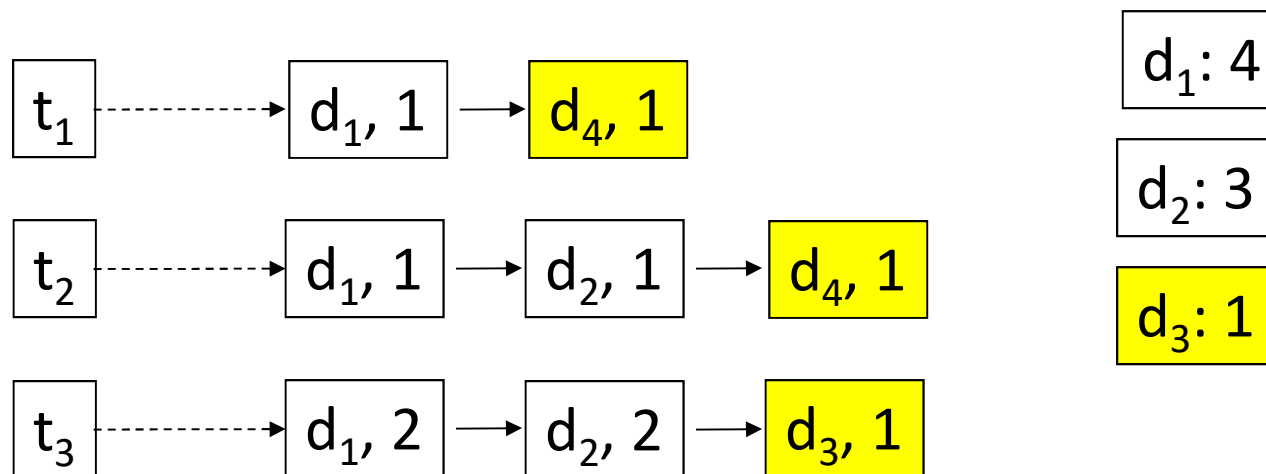
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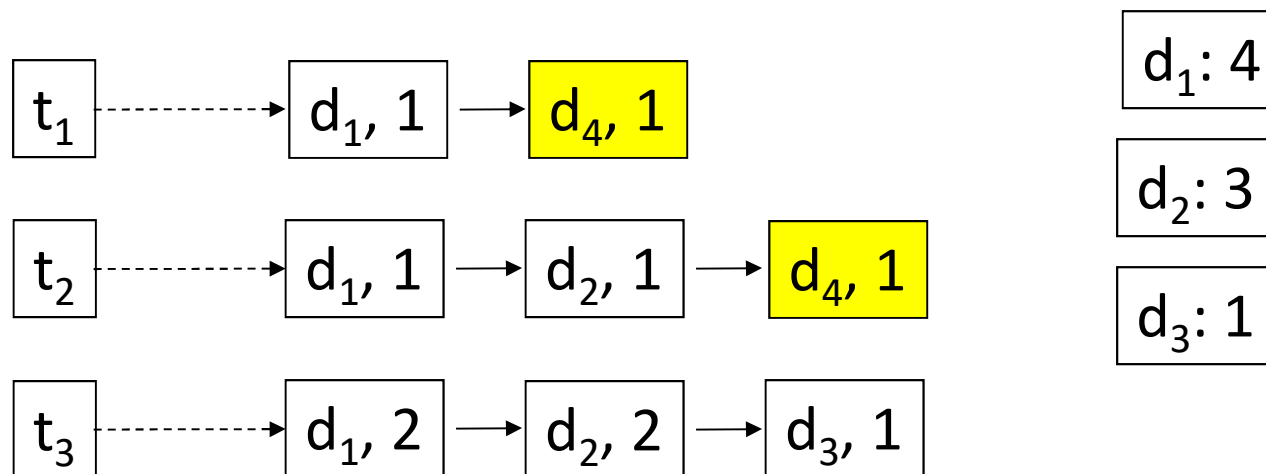
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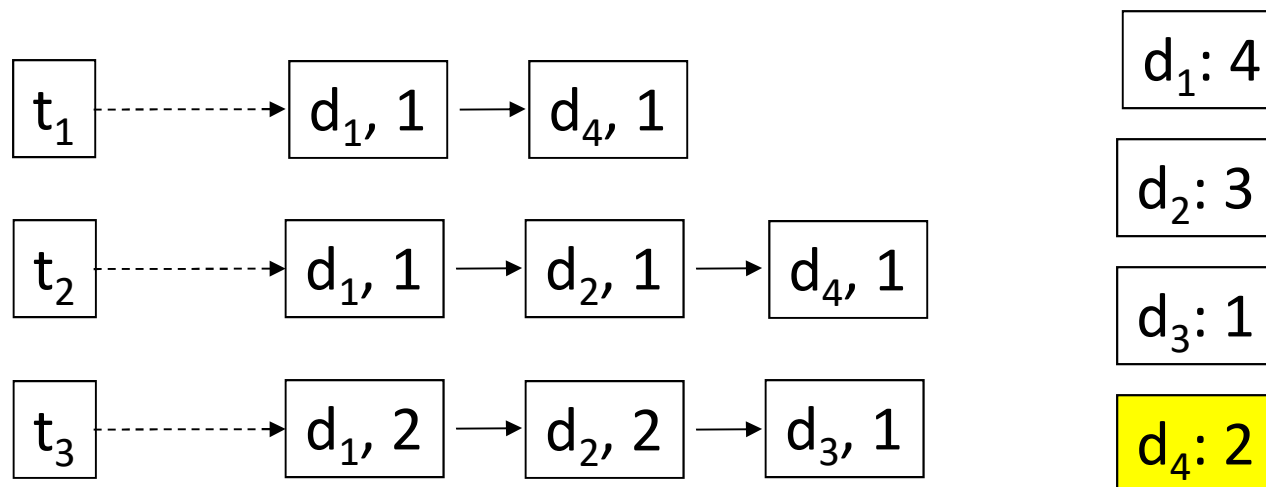
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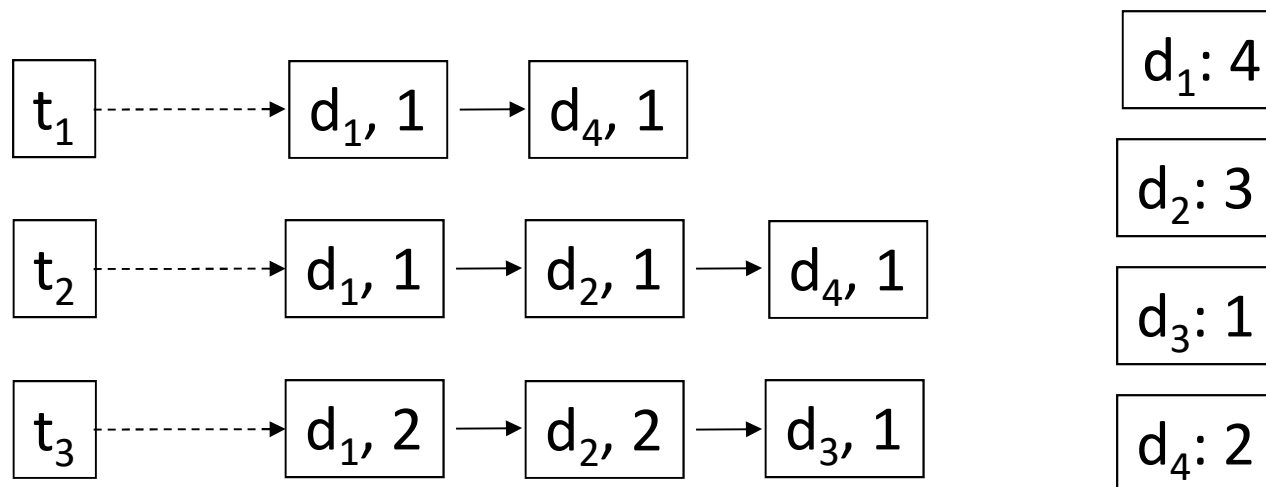
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# Document-at-a-time Ranking

```
context = {}    // maps a document to a list of matching terms
for  $w \in q$  do
    for  $d, count \in Idx.fetch\_docs(w)$  do
        context[ $d$ ].append(count)
    end for
end for
priority_queue = {}    // low score is treated as high priority
for  $d, term\_counts \in context$  do
    score = 0
    for count  $\in term\_counts$  do
        score = score + score_term(count)
    end for
    priority_queue.push( $d, score$ )
    if priority_queue.size() >  $k$  then
        priority_queue.pop()    // removes lowest score so far
    end if
end for
Return sorted documents from priority_queue
```

# References

- ChengXiang Zhai and Sean Massung, *Text Data Management and Analysis: A Practical Introduction to Information Retrieval and Text Mining*, ACM Books, 2016.
  - Chapter 6, Section 6.1-6.3 (Vector space model)
  - Chapter 8, Section 8.1-8.3 (Vector space model implementation)

# Questions

