

# Computational Simulation as a Tool of Investigating the Behavior of a Marine Object in Storm Conditions

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Direct computational simulation in naval aerohydrodynamics can serve as an available toolkit for engineering research both in the design of new marine equipment and in storm tests, replacing model and full-scale experiments. The latter is due to the increased power of computing systems and the level of detail of mathematical models possible for computer realization. The paper presents an interactive computational environment for modeling the dynamics and control of ship maneuvering. This virtual testbed can be considered both as a navigator's expert environment and as a simulator for improving skills of effective and safe ship navigation in storm conditions. The paper discusses a reasonable balance between the detail of the applied computational models and computational efficiency. The proposed approach can be implemented in various physical fields, where it is required to develop a virtual testbed to study the behavior of complex technical objects: transport systems, experimental facilities of high-energy physics, etc.

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## INTRODUCTION

The considered problem of modeling the controlled motion of a ship in conditions of intense storm waves is a characteristic problem of practical hydrodynamics in terms of constructing a sequence of direct computational simulation based on significantly different mathematical approaches. To implement this approach, there is a complete understanding of the physics of the phenomenon, dynamics and nature of external forces. This makes it possible to carry out direct simulation of external actions, dynamics of interaction of the object with the external environment. The basis of such modeling is satisfaction of conservation laws. However, the computational model is very resource-intensive in this case. It is even more difficult to connect with it the real-time control of the object. At the same time, absolutely accurate calculations are not required in the set problems. What is required is a result with engineering accuracy, reflecting any situation and physical phenomena. In the first case, it is necessary to choose the most advantageous design of the marine object, to verify safe behavior in difficult storm conditions. In the second case, it is necessary to train competent operation and develop skills of correct reaction to arising critical situations.

Traditionally, these problems have been solved using modeling or full-scale experiments [1]. However, these ways have understandable limitations.

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23 Model experiments in towing tanks: (1) Scaling effect; (2) Inaccurate  
 24 knowledge of object characteristics; (3) Impossibility to provide wide spec-  
 25 trum of experimental conditions; (4) Expensive and longtime of experiments.

26 Full-scale experiments: (1) No way for design and research; (2) Extremely  
 27 costly; (3) Impossible to carry out in complex hazardous environments.

28 Therefore, the task is to build a toolkit based on sufficiently fast, coordi-  
 29 nated models that allow interactive interaction with the object under study.  
 30 These are models of sea waves and wind, models of external forces caused  
 31 by these influences, models of ship dynamics and control models (influence  
 32 of rudder, maneuvering devices, change of speed, etc.). At the same time, in  
 33 order to fully reflect the reality, the ship motion model is built on the basis  
 34 of basic physical conservation laws.

35 An important feature in setting up a computational simulation is the  
 36 possibility of selecting mathematical models, including interactive ones, cor-  
 37 responding to the successive complication of both the hydromechanics of the  
 38 ship and the computer resource intensity.

## 39 1. PECULIARITIES OF DIGITAL CONSTRUCTION OF THE 40 GEOMETRIC MODEL OF THE SHIP

41 Traditional table of heights and additional necessary characteristics of  
 42 the ship's ends are chosen as a digital model of the ship's hull. Integration  
 43 over the contours of the frames sequentially within individual frame-spacing is  
 44 required to construct extremely fast algorithms. Special geometric algorithms  
 45 are used for the surface part of the ship hull, which allow relatively correct  
 46 outlining of the general ship architecture.

47 The surface of the effective waterline is formed from a single center at  
 48 water level and in midship coordinates. Elementary triangles are not grouped  
 49 into separated and unconnected fragments to speed up calculations.

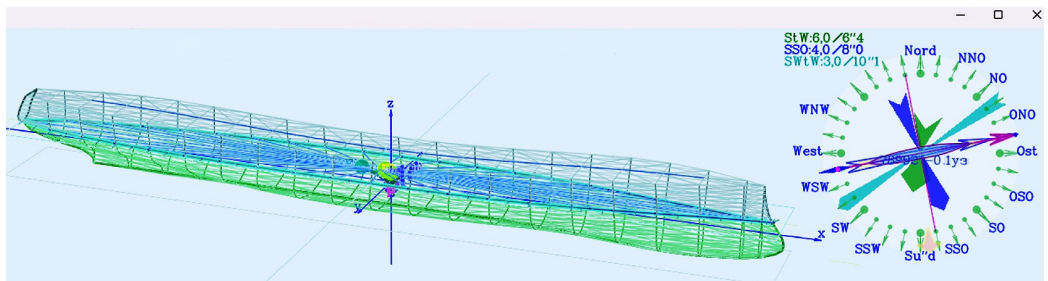


Fig. 1. Ship hull decomposition

50 Figure 1 shows the ship hull with triangulation by the theoretical draw-  
 51 ing's spacing. Triangulation of the waterline surface is made from a single  
 52 center. On the right side is shown compass indicating current and set course;  
 53 rudder position, angle meter and different wind wave systems.

## 2. WAVES MODELS

As models of sea waves, the tool uses models that have proven to be computationally efficient and physically adequate to the phenomenon being simulated. First of them is ARMA model [2].

$$\zeta_{x,y,t} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_t} \Phi_{i,j,k} \zeta_{x-i,y-j,t-k} + \varepsilon_{x,y,t}, \quad (1)$$

where  $\Phi_{i,j,k}$  are the generalized coefficients of the AR, and  $\varepsilon_{x,y,t}$  is a field of white noise.

This model is used to simulate spatio-temporal wave fields. The model works well for sloping and weakly nonlinear waves. It simulates well the group structure of waves. The physical adequacy and efficiency of the model are discussed in [3, 4].

A trochoidal wave model is used to consider the effects of substantially nonlinear waves [3]. It accurately describes the three-dimensional distribution of velocities throughout the water column affecting the ship's hull.

## 3. FROUDE-KRYLOV APPROXIMATION

Successive complication of mathematical models of the ship's storm hydrodynamics is provided either by initial conditions setting or interactively, directly in the process of conducting the computational simulation.

The first approximation is the hydrostatic setting of the hull on a wave, as the most dangerous in motion and yawing under the impact of the crests of high waves.

The next approximation is the dynamic Froude-Krylov approximation. It assumes that the ship hull does not make significant changes in the distribution of hydrodynamic pressures under the free surface of waves. In this case we have a potential problem of wave theory. Potential flow represents an inviscid, incompressible fluid flow field via a scalar velocity potential  $\varphi(x, y, z, t)$ , which is described by the system of equations (Laplace equation, dynamic and kinematic boundary conditions)

$$\begin{aligned} \Delta \varphi &= 0, \\ \varphi_t + \frac{1}{2} |\mathbf{v}|^2 + g\zeta &= -\frac{p}{\rho} \quad \text{on } z = \zeta(x, y, t), \\ D\zeta &= \nabla \varphi \cdot \mathbf{n} \quad \text{on } z = \zeta(x, y, t), \end{aligned} \quad (2)$$

where  $\zeta$  is the wave elevation,  $p$  is the wave pressure,  $\rho$  is the water density,  $\mathbf{v} = (\varphi_x, \varphi_y, \varphi_z)$  is the velocity vector,  $g$  is the gravitational acceleration and  $D$  is the substantial derivative. Here coordinate system is  $Oxyz$ : plate  $Oxy$  is coincides with still water surface,  $z$ -axis is upward.

We will take advantage of the fact that it has been shown that the spatio-temporal fields simulated by the wave models are physically adequate. If the

87 wave surface is known initially, then the kinematic boundary condition turns  
 88 out to be linear and the Laplace equation can be solved analytically at any  
 89 given moment of time. The solution to this problem is the realization of  
 90  $\varphi(x, y, z, t)$  and, consequently, all its derivatives with respect to space and  
 91 time [3]. For the two-dimensional case, the solution in computed time  $t$  is as  
 92 follows:

$$\varphi(x, z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{\lambda} e^{\lambda(z+ix)} d\lambda \int_{-\infty}^{\infty} \frac{\zeta_t}{1 - i\zeta_x + i\zeta_x/\sqrt{1 + \zeta_x^2}} e^{-\lambda(\zeta+ix)} dx. \quad (3)$$

93 The three-dimensional problem can be solved with the help of a special  
 94 inversion formula which serves as a modified version of a Fourier transform.  
 95 In polar coordinates, the transform has the following form:

$$F(\rho, \psi) = \int_0^{\infty} \int_0^{2\pi} r f(r, \theta) e^{ir\rho \cos(\psi-\theta) + r\zeta(\rho, \psi)} d\theta dr. \quad (4)$$

96 In this case, the final expression for the velocity potential is [3]

$$\varphi(x, y, z) = \Im^{-1} \left\{ \frac{\Im\{\zeta_t(x, y)\} \Im\{e^{M(iNx+\zeta)}\}}{\Im\{f(x, y)e^{M(iNx+\zeta)}\}} \right\}, \quad (5)$$

97 where  $\Im$  is an ordinary forward Fourier transform, with  $M, N$  – scales of  
 98 wave number and space (dimensionless transformation  $(x, y) \rightarrow (xN, yN)$ ),

$$f(x, y) = M \frac{N^2 + i\zeta_x(\sqrt{N^2 + \zeta_x^2 + \zeta_y^2} - N)}{N\sqrt{N^2 + \zeta_x^2 + \zeta_y^2}}.$$

99 In this case, Bernoulli's equation can be used to directly calculate the hydro-  
 100 dynamic pressures at any point.

$$p(x_0, y_0, z_0) = -\rho\varphi_t - \frac{\rho}{2}(\varphi_x^2 + \varphi_y^2 + \varphi_z^2) - \rho g z_0. \quad (6)$$

101 There is no easy way to derive an analogous formula for the derivatives  
 102 of the velocity potential  $(\varphi_x, \varphi_y, \varphi_z)$ . However, numerical experiments have  
 103 shown that there is no need to do this. These derivatives can be obtained  
 104 numerically via finite difference formulae. Fewer integral transforms means  
 105 less numerical error and faster computation.

106 Perturbing force and moment can be found by integrating the pressures  
 107 over the surface of the ship's wetted surface:

$$\mathbf{F} = - \int_S p \cdot \mathbf{n} dS \quad \mathbf{M} = - \int_S p(\mathbf{r} \times \mathbf{n}) dS, \quad (7)$$

108 where  $\mathbf{r}$  is the radius-vector of the element  $dS$  relative to the center of gravity.

109 The vector-tensor representation of the ship's hull makes it easy to cal-  
 110 culate the integration nodes on the ship's hull and the direction of the local  
 111 normal. This makes it possible to create a computationally efficient proce-  
 112 dure.

#### 4. INFLUENCE OF VISCOSITY AND REFLECTED WAVE-MAKING

The described modeling variant takes into account only the potential nature of the phenomenon. It does not take into account the effect of water viscosity, as well as the reflection of waves from the hull, which leads to changes in the pressure field near the ship. The solution of the full problem taking into account viscosity is feasible, but does not have a deep meaning. The influence of damping, firstly, affects exclusively near the hull (boundary layer), and its effect is significant only at strong oscillations. In this case, the computational complexity of the model increases very high. In the applied computational models the influence of damping is reduced to the weakening of natural velocities relative to the external environment. Therefore, the induced velocity damping parameters are given by the velocity damping coefficients. In the initial data, the damping parameter is given as a positive value with some optimal values  $\nu \in [0; \infty)$ :

$$\nu_{\xi, \eta, \zeta} = 0.05, 0.4, 0.3 - \text{surge } \xi, \text{ sway } \eta, \text{ heaving } \zeta.$$

$$\nu_{\theta, \psi, \chi} = 0.1, 0.3, 0.4 - \text{rolling } \theta, \text{ pitching } \psi, \text{ yawing } \chi.$$

At the same time, artificial component-by-component constraints can be used to speed up calculations, to satisfy various special cases (hull lines, ship architecture, etc.) and the specifics of the digital model of the ship hull. Practice shows that the use of such approximate approaches is justified within the framework of the problem formulation.

In the first approximation, the transformation of the wave field surrounding the ship can be performed without taking into account interference, refraction, and other effects of mutual influence of wave structures. Practical hydromechanics of the force interaction of the ship hull with sea waves can be based on the methods of solving problems of ship wave formation, when vortex sources are distributed along the wetted hull plating. Their distribution is performed to satisfy the condition of no-flow-loss and balance of the integral flow along the hull.

A simple fiber with a discontinuity of the normal velocity component on the wetted hull plate formally defines the width (thickness) of the hull. It is modeled by a layer of sources and sinks, and compensates for oncoming running and wave currents:  $\delta \hat{\mathbf{f}}_n / \rho = (\Delta p_n / \rho) \cdot \delta \hat{\mathbf{s}} = 0.5 \cdot (\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_n) \cdot |\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_n| \cdot \delta \hat{\mathbf{s}}$ , where  $\Delta p$  is change of water pressure on the onboard plate element;  $\delta \mathbf{f}$  is the computational vector of force;  $\mathbf{n}$  is the unit normal vector on a board plate element,  $\mathbf{s}$  is the unit tangent vector on a board plate element in local flow direction,  $\mathbf{v}_n$  is normal component of particles velocity near hull. Here  $\hat{\phantom{x}}$  over for vector means values projected on the local basis. Notation (using left arrow instead of  $\hat{\phantom{x}}$ ) of finite-difference representations of vectors and tensors are defined in [5].

The flow near the wetted ship skin is modeled by a vortex layer. It partially affects the water pressure near the ship skin in accordance with Bernoulli's law and generates a small force vector in the direction normal to

the elementary computational plates:

$$\delta \hat{\mathbf{f}}_s / \rho = (\Delta p_s / \rho) \cdot \delta \hat{\mathbf{s}} = 0.5 \cdot (V^2 - |\hat{\mathbf{n}} \times \hat{\mathbf{v}}_s|) \cdot \delta \hat{\mathbf{s}},$$

where  $V$  is ship speed and wave inflow,  $\hat{\mathbf{v}}_s$  is tangential component of particles velocity near hull.

By analogy with the wing theory methods, the intensity of the distributed vorticity layer can be redistributed so that an continuous flow is formed under the after rake and at the rudder. This formally forms the wing effect with a significant shift of the hydrodynamic center of lateral drag to the forward end. In general

$$\delta \hat{\mathbf{f}}_\Delta / \rho = (\Delta p / \rho) \cdot \delta \hat{\mathbf{s}} = g \cdot h \cdot \delta \hat{\mathbf{s}} = 0.5 \cdot (V^2 - |\hat{\mathbf{n}} \times \hat{\mathbf{v}}_s|^2 \pm (\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_n)^2) \cdot \delta \hat{\mathbf{s}}, \quad (8)$$

where  $h$  is water level over wave slope near hull skin.

In a computational simulation, the oncoming underway flow affects the dynamic attitude (draft, heel, trim) of the ship; the running trim or the design balance of the ship's wave formation; the hydroplaning conditions or the tendency to negative running trim. A similar interpretation of flows in the water column under storm waves, for example, can serve to verify the hypothesis of increased danger for characteristic hull lines at hydrostatic staging of the ship on a storm wave. In this case, it is possible to refine the hull lines optimization based on a series of computational simulations on an intense trochoidal wave.

## 5. TROCHOIDAL WAVES IN SIMULATION OF SHIP MOTION

As mentioned above numerical model of trochoidal waves is used for simulation of nonlinear waves in storm conditions. The mathematical model of trochoidal waves  $\zeta(s, z, t)$  [m] is written in vector form on the wave profile plane  $w \in s, z$  in the direction of wave front propagation  $s$  [m] with  $z$  [m] vertical. The specifics of numerical modeling of trochoidal waves and their impact on the ship are given in [6].

In such case fluid particles move along their own trochoidal trajectories, where the wave pressure is equalized by the relations of depth and local velocity of these particles under the wave slopes. To increase the computational efficiency of the applied mathematical model without reducing the modeling accuracy, it is possible to introduce a symmetric correction with respect to the wave phase  $C_r$  to the local depth  $r_z$ . It agrees the averaged or equal-volume water level for ridges with pointed tops and ridges of cnoidal waves.

Based on observations of wave processes modeled in computational simulation, the mid-level correction will serve as the wave steepness coefficient:  $C_r \approx H_w$ . Here  $H_w = 8/3\pi h/\lambda$  is coefficient of wave height relative to the

ultimate high (breaking) crest of progressive trochoidal wave ( $h = 2r_w$  is the ultimate wave height as twice the conditional radius of the surface trochoid).

$$r_z \times = \exp(2\pi \cdot r_z \cdot C_r \cdot \cos \varphi_w / \lambda),$$

If necessary, an additional asymmetric correction to the wave profile is introduced to account for wind stresses  $W_d$ :

$$r_z \times = \exp(2\pi \cdot r_z \cdot W_d \cdot (\sin \varphi_w - 1) / \lambda),$$

185 which can be applied exclusively to wind waves with an averaged significant  
186 shear, as:  $W_d \approx \sqrt{0.5} \approx 0.7071$ . Here  $r_z \times$  is expressed in the form of  
187 C programming language notation (operations  $x+ =, x- =$ , etc.). Here  
188  $\varphi_w = 2\pi(t \cdot C_w - s - L_b/2)/\lambda$  is phase angle for trochoidal waves, where  $\lambda$  is  
189 wave length with speed  $C_w$  in plate  $w \in \{s, z\}$  at the time moment  $t$  in initial  
190 position  $s$ .  $L_b$  is the length of model water area. More detailed computations  
191 are given in [3, 6].

## 192 6. INTERACTIVE CONTROL OF COURSE AND SHIP SPEED

193 An important function of any computational simulation is its online con-  
194 trol [7]. For described experiments it is the smooth control of the running  
195 speed and circulation speed while maintaining the specified storm running  
196 mode, which does not limit the free running dynamics and yawing on the  
197 course under the influence of storm waves.

198 The above-described linear damping operation is quite adequately ap-  
199 plied for small deviations from the set running speed  $\dot{\xi}_i$  (i-th time moment)  
200 in storm navigation conditions. Accelerated reduction to the set speed is for-  
201 mally predetermined by the conditional power of the engines, and does not  
202 particularly affect its small pulsations with a small increase in the damping  
203 factor:  ${}^+\dot{\xi}/ = 1 - \tanh^2(\dot{\xi}_i - \dot{\xi})/16$  (the meaning of notation  ${}^+X$  is described  
204 in [5]).

205 Course (heading) control is performed by small increments of the heading  
206  $\delta\chi$ , which in calm water provides full turn in one of three modes: "Rudder  
207 hard over" – in 1 minute; "Rudder half over" – in 2 minutes; and "Slower"  
208 – in 4 minutes. In rough seas, the "Slower" mode is activated automatically  
209 when the ship deviates from the set course by 1 rhomb ( $11^\circ 15'$ ), but this may  
210 not be sufficient to return the ship to the commanded heading. To speed up  
211 the turn and get the ship back on course, manual control can be used by  
212 selecting "Rudder hard over" or "Rudder half over" modes. After the ship  
213 has reached the specified course, the rudder is automatically set straight,  
214 with resumption of automatic control with rudder shifting to the "Slower"  
215 mode.

216 On a large wave, the circulation speed may be insufficient and the model  
217 may turn broadside on the wave, or go into dangerous broaching (the current  
218 version of tool does not yet evaluate the reverse action of the rudder under  
219 the overtaking flow in the crest of a storm wave).

Stroke control with the help of engine thrust and smooth course control allow to simulate the occurrence of heel on the circulation with adequate running trim during acceleration and braking; with the effect of stroke loss on storm waves and other effects of ship motion and propulsion on all six degrees of free maneuvering of the ship.

## CONCLUSION

Direct computational experiments for a ship navigating in extreme seas can serve as a justification for the design of ships and vessels with increased seaworthiness in heavy seas. The experimental computing environment includes a full-fledged three-dimensional visualization of the structure of ocean waves, with the trajectory and instantaneous position of a ship. The new toolkit opens a novel direction of research in the field of ship hydromechanics. Significant advantages of such a “virtual testbed” are specification of the optimal amount of computer memory and computing performance.

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## CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## REFERENCES

1. *Girs I. V. R.A.A.N.Y.A.* Ship seaworthiness tests. — Leningrad: Shipbuilding, 1977. — P. 190.
2. *Rozhkov V. A. T.Y.A.* Probability Models of Oceanological Processes. — Leningrad: Hydrometeoizdat, 1990. — P. 270.
3. *Degtyarev A.B., Khrumushin V.N.* First-principles algorithms for ship motion simulation based on ARMA and trochoidal wave models // *Ocean Engineering*. — 2024. — V. 310. — P. 118395. — <https://doi.org/10.1016/j.oceaneng.2024.118395>.
4. *Degtyarev A.B., Reed A.M.* Synoptic and short-term modeling of ocean waves // *International Shipbuilding Progress*. — 2013. — V. 60(1–4). — P. 523–553. — <https://doi.org/10.3233/ISP-130091>.



- 253 5. *Degtyarev A.B., Khramushin V.N.* Coordinate systems, numerical objects  
254 and algorithmic operations of computational experiment in fluid mechan-  
255 ics // EPJ Web of Conferences. — 2016. — V. 108, no. 02018. — P. 6. —  
256 <https://doi.org/10.1051/epjconf/201610802018>.
- 257 6. *Degtyarev A.B., et al.* Computational model of unsteady hydromechanics  
258 of large amplitude Gerstner waves // EPJ Web Conf. — 2020. — V. 226,  
259 no. 02009. — P. 4. — <https://doi.org/10.1051/epjconf/202022602009>.
- 260 7. *Degtyarev A.B., Khramushin V.N.* Real-time simulation, decomposition  
261 of mathematical models and algorithms of interactive control and visu-  
262 alization // Physics of Particles and Nuclei. — 2024. — V. 55, no. 3. —  
263 P. 313–316. — <https://doi.org/10.1134/S1063779624030304>.