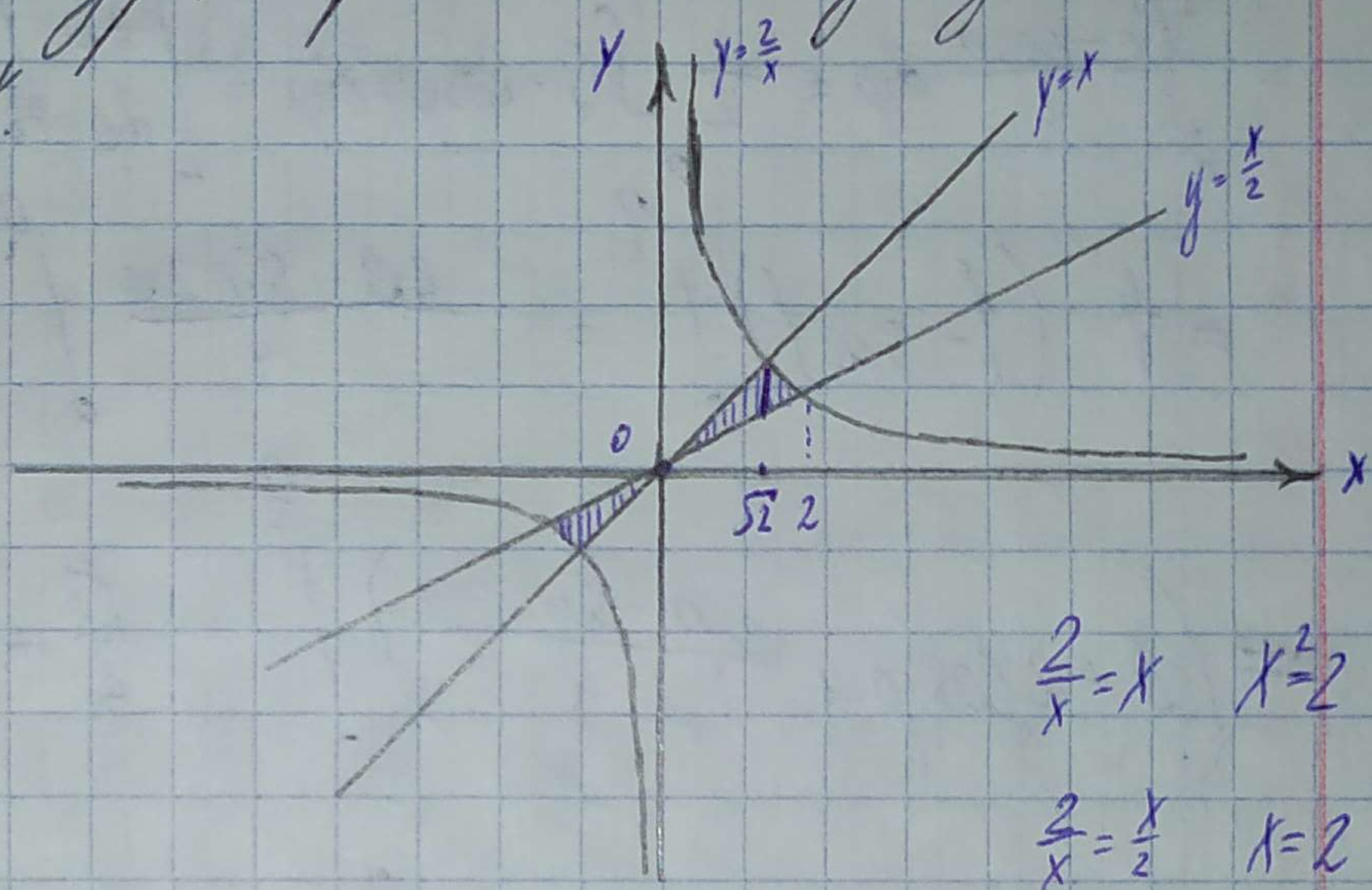
Хроменко Данил ИСб-20-2-0 РГЗ определённый интеграл

Вариант-20 №№ 3, 4, 5

3. Вычислить площади фигур, ограниченных указанными линиями. Выполнить чертёж.



а). $y=\frac{2}{x}$ $y=x$ $y=\frac{x}{2}$

$\frac{2}{x}=x$ $x^2=2$ $x=\pm\sqrt{2}$

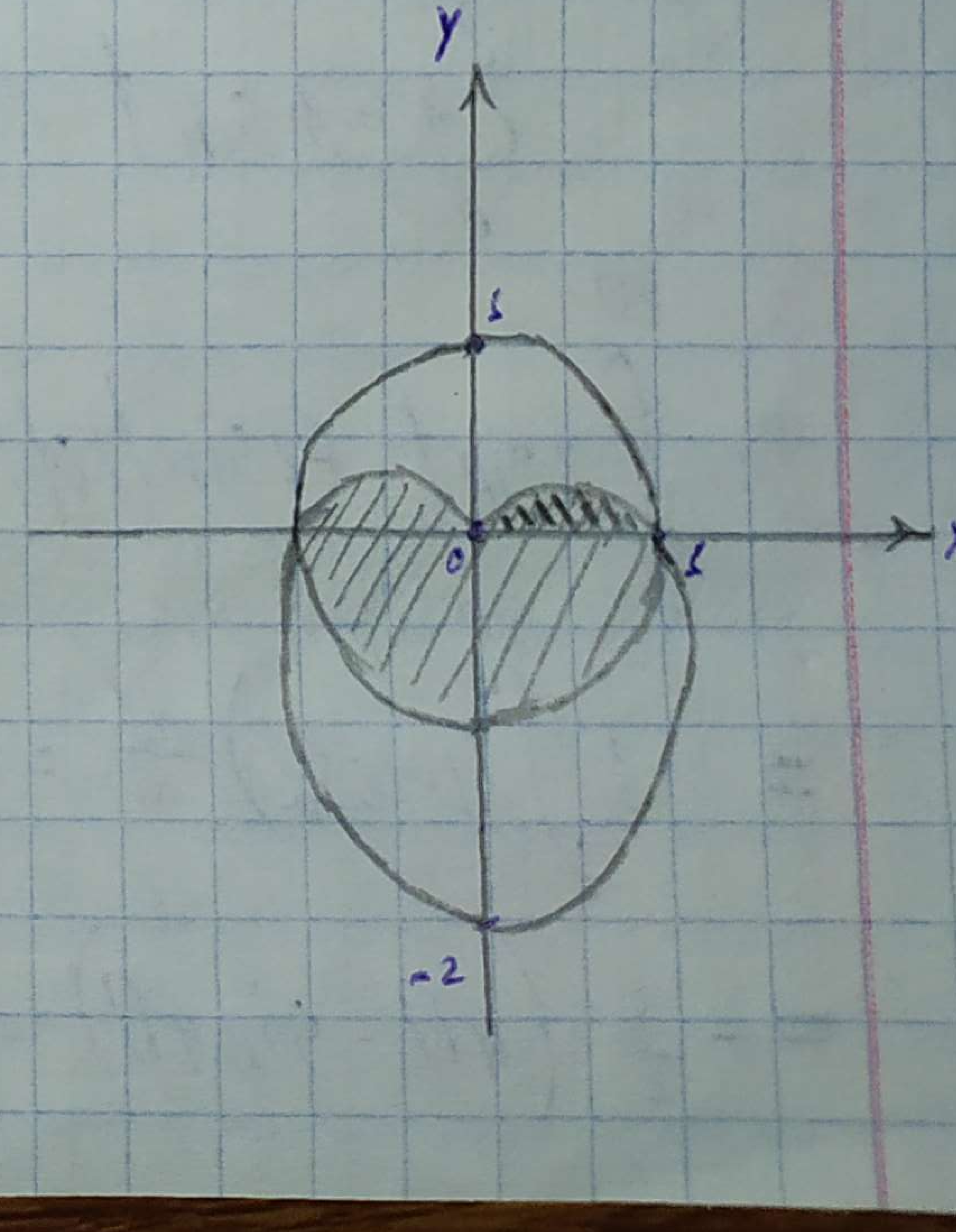
$\frac{2}{x}=\frac{x}{2}$ $x=2$

$$S=2\cdot\left(\int_0^{\sqrt{2}}\left(x-\frac{x}{2}\right)dx+\int_{\sqrt{2}}^{2}\left(\frac{2}{x}-\frac{x}{2}\right)dx\right)=$$

$$=2\cdot\left(\left(\frac{x^2}{2}-\frac{x^2}{4}\right)\Big|_0^{\sqrt{2}}+\left(2\ln|x|-\frac{x^2}{4}\right)\Big|_{\sqrt{2}}^{2}\right)=2\cdot\left(\frac{\sqrt{2}^2}{2}-\frac{\sqrt{2}^2}{4}+2\ln 2-\frac{2^2}{4}-2\ln\sqrt{2}+\frac{\sqrt{2}^2}{4}\right)=$$

$$=2-1+4\ln 2-2-4\ln\sqrt{2}+1=4(\ln 2-\ln\sqrt{2})=4\ln\frac{2}{\sqrt{2}}$$

Ответ: $S=4\ln\frac{2}{\sqrt{2}}$



б). $r=1$ $r=1-\sin\varphi$

$S=\frac{1}{2}$ окружности + 2 части кардиоиды в I и II четверти

$$S_{\frac{1}{2}\text{окр.}}=\frac{1}{2}\cdot\frac{1}{2}\int_0^{2\pi}1^2\,d\varphi=\frac{1}{4}(2\pi-0)=\frac{\pi}{2}$$

$$S_{\text{частей кардиоиды}} = 2\cdot\frac{1}{2}\int_0^{\pi/2}(1-\sin\varphi)^2\,d\varphi = \int_0^{\pi/2}(1-2\sin\varphi+\sin^2\varphi)\,d\varphi =$$

$$= \int_0^{\frac{\pi}{2}}\left(1-2\sin\varphi+\frac{1-\cos2\varphi}{2}\right)d\varphi = \varphi+2\cos\varphi\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}}\frac{1-\cos2\varphi}{2}\,d\varphi =$$

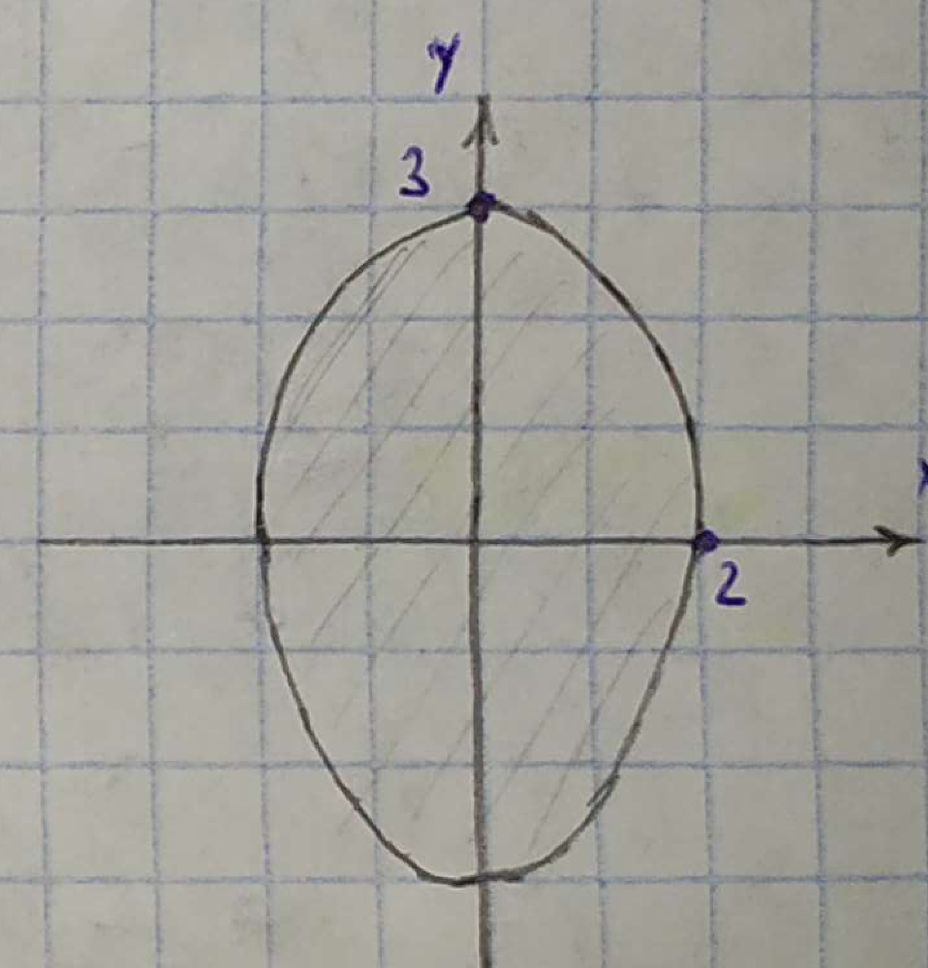
$$\int_0^{\frac{\pi}{2}}\frac{1-\cos\varphi}{2}\,d\varphi = \frac{1}{2}\int_0^{\frac{\pi}{2}}(1-\cos\varphi)\,d\varphi = \begin{bmatrix}2\varphi=t\\ d\varphi=\frac{dt}{2}\end{bmatrix} = \frac{1}{2}\int_0^{\frac{\pi}{2}}(1-\cos t)\frac{dt}{2} =$$

$$= \frac{1}{4}(t-\sin t)\Big|_0^{\frac{\pi}{2}} = \frac{2\varphi-\sin2\varphi}{4}\Big|_0^{\frac{\pi}{2}}$$

$$= \left(\varphi+2\cos\varphi+\frac{2\varphi-\sin2\varphi}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}+2\cos\frac{\pi}{2}+\frac{\pi-\sin\pi}{4}-2$$

$$S_{\text{общ.}} = \frac{\pi}{2}+\frac{\pi}{2}+2\overset{0}{\cos\frac{\pi}{2}}+\frac{\pi}{4}-\frac{\overset{0}{\sin\pi}}{4}-2 = \frac{5\pi}{4}-2 \approx 1.926$$

Ответ: $S \approx 1.926 = \frac{5\pi}{4}-2$



б) $\begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases}$ $\qquad x' = (2\cos t)' = -2\sin t$

$$\int_0^{2\pi}3\sin t\cdot(-2\sin t)\,dt = -6\int_0^{2\pi}\sin^2 t\,dt = -3\int_0^{2\pi}(1-\cos2t)\,dt = \begin{bmatrix}2t=\alpha\\ dt=\frac{d\alpha}{2}\end{bmatrix} =$$

$$= -3\int_0^{2\pi}(1-\cos\alpha)\frac{d\alpha}{2} = -1{,}5\,(\alpha-\sin\alpha)\Big|_0^{2\pi} = -\frac{3}{2}(2t-\sin2t)\Big|_0^{2\pi} =$$

$$= -\frac{3}{2}(4\pi-\overset{0}{\sin4\pi}) = -\frac{3\cdot4\pi}{2} = -6\pi \approx -18{,}85$$

Ответ: $S = -6\Pi \approx -18.85$

4. Вычислить длину дуг кривых

$$y = \frac{x^2}{2} - \frac{\ln x}{4}, \quad 1 \le x \le 3 \Rightarrow x \in [1; 3]$$

$$l = \int_1^3 \sqrt{1 + \left(\left(\frac{x^2}{2} - \frac{\ln x}{4}\right)'\right)^2}\, dx$$

$$\left(\frac{x^2}{2} - \frac{\ln x}{4}\right)' = x - \frac{1}{4x} \qquad \left(x - \frac{1}{4x}\right)^2 = x^2 - \frac{2x}{4x} + \frac{1}{16x^2} = x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$l = \int_1^3 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\, dx = \int_1^3 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\, dx = \int_1^3 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx =$$

$$= \int_1^3 \left(x + \frac{1}{4x}\right) dx = \left(\frac{x^2}{2} + \frac{\ln|x|}{4}\right)\Bigg|_1^3 = \frac{3^2}{2} + \frac{\ln 3}{4} - \frac{1^2}{2} - \frac{\ln 1}{4} =$$

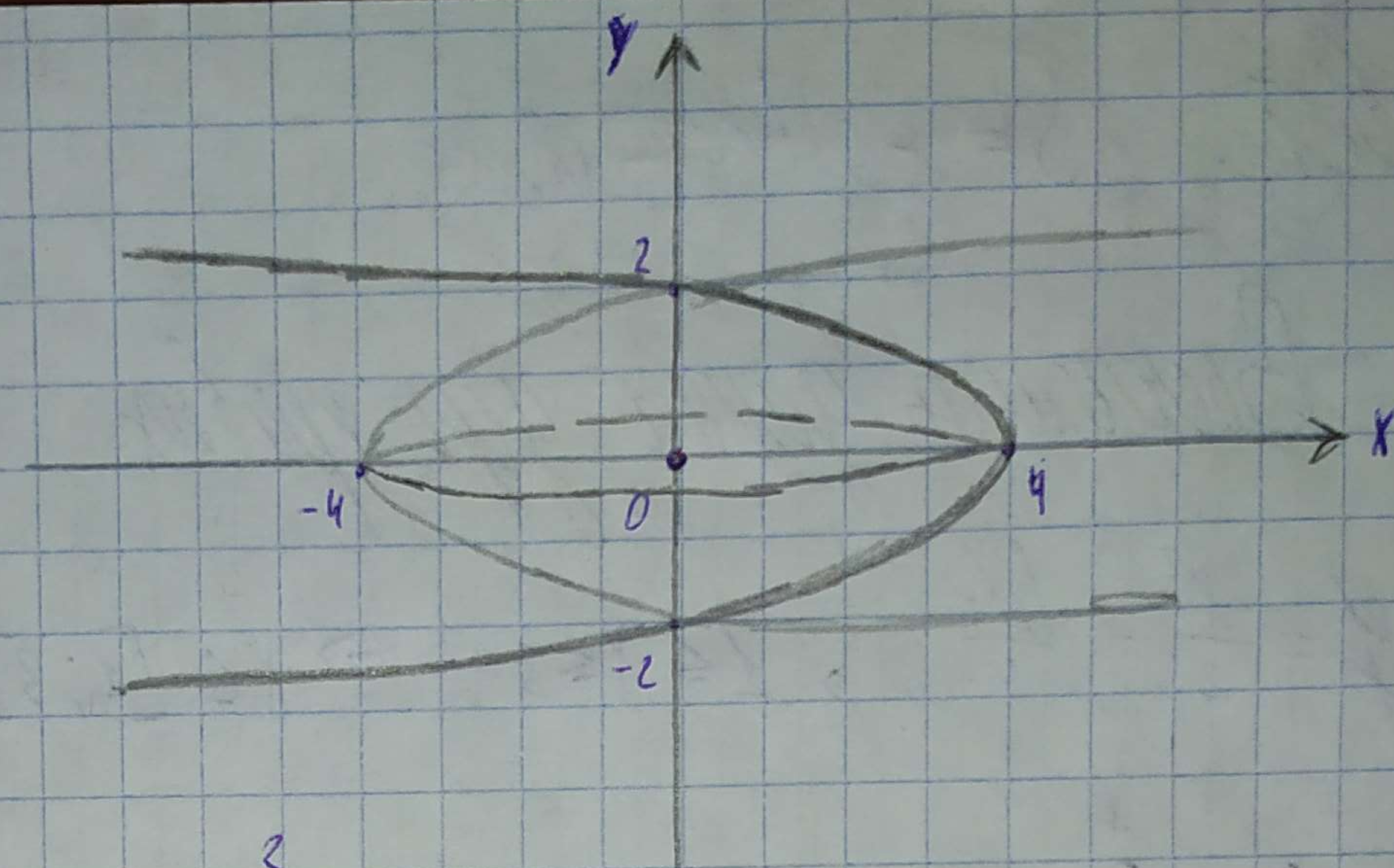
$$= 4{,}5 + \frac{\ln 3}{4} - 0{,}5 = \frac{\ln 3 + 16}{4} \approx 4{,}27$$

Ответ: $l = \frac{\ln 3 + 16}{4} \approx 4{,}27$

5. Найти объём тела, полученного вращением указанных линий. Выполнить чертёж

$$y^2 = 4 - x, \quad x = 0 \quad (Oy)$$

x	0	2	4
y	2/-2	$\sqrt{2}$/$-\sqrt{2}$	0



$x = 4 - y^2$

$$V = \pi \int_{-2}^{2} (4-y^2)^2\,dy = \pi \int_{-2}^{2} (16 - 8y^2 + y^4)\,dy =$$

$$= \pi \left(16y - \frac{8y^3}{3} + \frac{y^5}{5}\right)\Big|_{-2}^{2} = \pi\left(16\cdot 2 - \frac{8\cdot 2^3}{3} + \frac{2^5}{5} - 16\cdot(-2) + \frac{8\cdot(-2)^3}{3} - \frac{(-2)^5}{5}\right) =$$

$$= \pi\left(32 - \frac{64}{3} + \frac{32}{5} + 32 - \frac{64}{3} + \frac{32}{5}\right) = \pi\left(64 - \frac{128}{3} + \frac{64}{5}\right) =$$

$$= \pi\left(\frac{960 - 640 + 192}{15}\right) = \frac{\pi \cdot 512}{15} \approx 107.23$$

Ответ: $V = \frac{512\pi}{15} \approx 107.23$