# Signature and beamformer optimization for MIMO-CDMA based on the mean squared error criterion

Article	· April 2003			
CITATIONS	5		READS	
2			20	
2 autho	ors, including:			
All wood	Semih Serbetli NXP Semiconduct	tors		
	25 PUBLICATIONS 8	68 CITATIONS		
	SEE PROFILE			

## Signature and Beamformer Optimization for MIMO-CDMA Based on the Mean Squared Error Criterion

Semih Serbetli and Aylin Yener
Department of Electrical Engineering
The Pennsylvania State University
121 Electrical Engineering East
University Park, PA 16802
e-mail: serbetli@psu.edu, yener@ee.psu.edu

Abstract — We consider the uplink of a CDMA system where the transmitters as well as the receiver are equipped with multiple antennas. Each user transmits a weighted form of its symbol through its transmit antennas. In this work, we adopt the system-wide mean squared error as the performance measure and propose an alternating minimization based iterative algorithm to find the jointly optimum beamformers, signatures and the corresponding optimum receivers for each user. The convergence and the performance of the algorithm are investigated through numerical results.

#### I. Introduction

CDMA is a promising candidate to meet the high data rate demands of next generation wireless systems. However, the performance of a highly loaded CDMA system may suffer from multiaccess interference. While multiuser detection exploits the temporal structure of the multiaccess interference, recently popularized use of multiple antennas can provide additional help to suppress the interference by using spatial signatures of users. It has been shown that the use of multiple antennas increases capacity [1], provides diversity gain and suppresses the interference for narrowband systems [2].

The level of feedback to the transmitter side is an important factor that affects the performance of the transmission schemes. Orthogonal transmit diversity (OTD) [3], space-time spreading (STS) [4] and space-time block coded transmit diversity (STTD) [5] are examples of open loop methods that require no feedback. Performance of the systems can be improved significantly by transmit shaping based on the feedback information from the receiver side. Antenna selection methods [6] are typical examples of limited feedback, whereas beamforming techniques [7] require high level of feedback information. Transmitter diversity schemes for CDMA systems with various (limited) levels of available feedback at the transmitter side have been investigated for downlink scenarios [3, 4, 6]. The downlink performance of multiple antenna CDMA systems is investigated in [8–10].

Performance of a CDMA system is highly dependent on the signature sequence set used. For single antenna systems, information theoretic capacity of synchronous CDMA systems is studied for deterministic sequences in [11], and optimum signature sets are identified for both users with equal received powers [12], and arbitrary received powers [13]. Iterative algorithms that converge to the optimum signature sequence set are proposed in [14–16].

The effect of using multiple receiver antennas in a CDMA system with fixed signatures is well understood [17, 18]. For the multiple antenna CDMA, spectral efficiency under a large system assumption is recently studied [19, 20]. However, the performance of finite size multiple transmitter and receiver antenna CDMA systems deserves further investigation.

Most of the transmitter diversity schemes previously proposed for CDMA systems depend on assigning orthogonal signature sequences to the antennas or symbols and integrating space-time codes to the system [3, 4]. This model is not valid for overloaded systems, i.e., when there are more symbols to send than the processing gain. In the case of a MIMO-CDMA system where users' transmissions interfere with each other, the system objectives should be optimized jointly for all users given the parameters of the users. The relation between the signature set used and multiple antenna transmission scheme remains to be investigated.

The premise of our work is that, similar to the singleuser narrowband case addressed in [7], feedback to the transmitter side is an important factor that can be exploited to improve the performance of a multi-user multiantenna CDMA (MIMO-CDMA) system. We investigate a finite size system where each user is eventually assigned a unique signature sequence and a transmit beamformer in response to system conditions. Our objective in this work is to find the jointly optimum transmitter antenna beamformer weights, signatures and the corresponding linear receivers of all users while considering the systemwide MSE as the performance metric. In the single transmitter antenna case, the problem reduces to finding the optimum signature set for a group of users with given spatial signatures when joint MMSE temporal-spatial receivers are employed at the receiver.

The system model is introduced and the performance measure is formulated in Section 2. The algorithm to find the signature sequences, the beamformers, as well as the corresponding receivers of the users is given in Section 3. The convergence of the algorithm and the performance improvement achieved by optimizing the transmitters are investigated in Section 4. Section 5 concludes the paper.

#### II. System Model and Performance Metric

We consider the uplink of a single cell synchronous direct-sequence CDMA system with K users and processing gain N. The base station is equipped with  $N_R$  receiver antennas and user i has signature waveform  $s_i(t)$  and  $N_{T_i}$  transmit antennas (Figure 1). A weighted form of the signal,  $f_{ij}s_i(t)b_i$ , is transmitted through its jth transmit antenna where  $f_{ij}$  is the weight of the jth transmit antenna and  $b_i$  is the symbol of user i. We assume that the channels between transmitter antennas and receiver antennas are independent flat fading channels and known perfectly by the receiver. In the presence of additive white Gaussian noise  $n_k(t)$  with zero mean and power spectrum density  $\sigma^2$ , the received signal in one symbol interval is

$$r_k(t) = \sum_{i=1}^{K} b_i s_i(t) \sqrt{P_i} (\mathbf{h}_{ik}^T \mathbf{f}_i) + n_k(t)$$
 (1)

at the kth receiver antenna, where  $\mathbf{h}_{ik} = [h_{ik,1}, h_{ik,2}, \cdots h_{ik,N_{T_i}}]^T$  is the channel gain vector of user i to the kth receiver antenna,  $P_i$  is the transmit power and  $\mathbf{f}_i$  is the unit energy transmit weight vector of user i,  $[f_{i1}, f_{i2}, \cdots, f_{iN_{T_i}}]^T$ . Chip matched filtering the received signal and sampling at the chip rate yields the received signal in vector form

$$\mathbf{r}_k = \sum_{i=1}^K b_i \mathbf{s}_i \sqrt{P_i} (\mathbf{h}_{ik}^T \mathbf{f}_i) + \mathbf{n}_k$$
 (2)

where  $\mathbf{s}_i$  is the chip-sampled version of  $s_i(t)$  and  $\mathbf{n}_k$  is the zero mean Gaussian noise vector with  $E[\mathbf{n}\mathbf{n}^{\dagger}] = \sigma^2 \mathbf{I}$  where  $(\cdot)^{\dagger}$  denotes the complex conjugate. Received signal vectors at all receiver antennas can be represented by a long vector  $\mathbf{r}$  as

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_{N_R} \end{bmatrix} = \sum_{i=1}^K b_i \begin{bmatrix} \mathbf{s}_i \sqrt{P_i} (\mathbf{h}_{i1}^T \mathbf{f}_i) \\ \mathbf{s}_i \sqrt{P_i} (\mathbf{h}_{i2}^T \mathbf{f}_i) \\ \vdots \\ \mathbf{s}_i \sqrt{P_i} (\mathbf{h}_{iN_R}^T \mathbf{f}_i) \end{bmatrix} + \mathbf{n}$$
(3)

Defining the spatial-temporal signature of user i,  $\mathbf{c}_i$ ,

$$\mathbf{c}_{i} = \begin{bmatrix} \mathbf{s}_{i} \sqrt{P_{i}} (\mathbf{h}_{i1}^{T} \mathbf{f}_{i}) \\ \mathbf{s}_{i} \sqrt{P_{i}} (\mathbf{h}_{i2}^{T} \mathbf{f}_{i}) \\ \vdots \\ \mathbf{s}_{i} \sqrt{P_{i}} (\mathbf{h}_{iN_{B}}^{T} \mathbf{f}_{i}) \end{bmatrix}$$

$$(4)$$

the received signal vector becomes

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_{N_R} \end{bmatrix} = \sum_{i=1}^K b_i \mathbf{c}_i + \mathbf{n}$$
 (5)

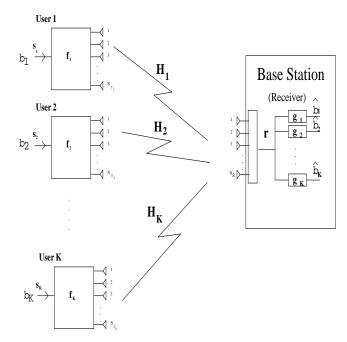


Fig. 1: MIMO CDMA System Model.

Let us denote the linear receiver of user i by  $\mathbf{g}_i$ ; the decision statistic for user i is  $y_i = \mathbf{g}_i^{\dagger} \mathbf{r}$ . Then, the mean squared error incurred by user i,  $MSE_i$ , is

$$MSE_{i} = E[\|y_{i} - b_{i}\|^{2}]$$

$$= \sum_{i=1}^{K} \mathbf{c}_{j}^{\dagger} \mathbf{g}_{i} \mathbf{g}_{i}^{\dagger} \mathbf{c}_{j} - \mathbf{c}_{i}^{\dagger} \mathbf{g}_{i} - \mathbf{g}_{i}^{\dagger} \mathbf{c}_{i} + 1 + \sigma^{2} \mathbf{g}_{i}^{\dagger} \mathbf{g}_{i}$$
(7)

In this work, we aim to find the optimum beamforming vectors  $\{\mathbf{f}_i\}$ , temporal signatures  $\{\mathbf{s}_i\}$  and the corresponding receivers  $\{\mathbf{g}_i\}$  that minimize the system-wide MSE. The system-wide MSE is:

$$MSE = \sum_{i=1}^{K} MSE_{i} = \sum_{i=1}^{K} E[\|y_{i} - b_{i}\|^{2}]$$

$$= \sum_{i=1}^{K} \left( \sum_{j=1}^{K} \mathbf{c}_{j}^{\dagger} \mathbf{g}_{i} \mathbf{g}_{i}^{\dagger} \mathbf{c}_{j} - \mathbf{c}_{i}^{\dagger} \mathbf{g}_{i} - \mathbf{g}_{i}^{\dagger} \mathbf{c}_{i} + 1 + \sigma^{2} \mathbf{g}_{i}^{\dagger} \mathbf{g}_{i} \right)$$
(9)

Total MSE minimization by choosing the transmitters and receivers has recently been studied for synchronous CDMA systems with single antennas in the context of CDMA signature optimization [15]. This performance measure is desirable to work with in transmitter optimization, in contrast with each user minimizing its own MSE as is adapted in receiver optimization [21]. This is because the choice of the transmitter of a user affects the MSE of each user in the system. In the following section, we pose the problem of minimizing the total MSE in the presence of relevant constraints for the MIMO-CDMA system, and devise an iterative algorithm that converges to the solution of the corresponding problem.

#### III. ITERATIVE TRANSMITTER OPTIMIZATION

Our aim in this work is to minimize the system-wide MSE for the MIMO-CDMA system over the signature sequences  $\{\mathbf{s}_i\}$ , beamformers  $\{\mathbf{f}_i\}$ , and the corresponding linear receivers  $\{\mathbf{g}_i\}$ . We assume that the channel of each user is fixed and perfectly known by the receiver. The signature sequences and beamformers are assumed to have unit energy. Formally, the optimization problem is

$$\min \qquad \qquad \underset{\left\{\mathbf{s}_{i}, \mathbf{f}_{i} \ \mathbf{g}_{i}\right\}_{i=1, \dots, K}}{\text{MSE}} \tag{10}$$

s.t. 
$$\mathbf{s}_i^{\dagger} \mathbf{s}_i = 1, \quad \mathbf{f}_i^{\dagger} \mathbf{f}_i = 1 \quad i = 1, \dots, K \quad (11)$$

where we assumed unit energy signatures and beamformers. The Lagrangian for the optimization problem is  $L(\{\mathbf{s}_i\}, \{\mathbf{f}_i\}, \{\mu_{i1}\}, \{\mu_{i2}\}_{k=1}^K) =$ 

$$\sum_{i=1}^{K} \left( \sum_{j=1}^{K} \mathbf{c}_{j}^{\dagger} \mathbf{g}_{i} \mathbf{g}_{i}^{\dagger} \mathbf{c}_{j} - \mathbf{c}_{i}^{\dagger} \mathbf{g}_{i} - \mathbf{g}_{i}^{\dagger} \mathbf{c}_{i} + 1 + \sigma^{2} \mathbf{g}_{i}^{\dagger} \mathbf{g}_{i} \right)$$

$$+ \sum_{i=1}^{K} \mu_{i1} (\mathbf{s}_{i}^{\dagger} \mathbf{s}_{i} - 1) + \sum_{i=1}^{K} \mu_{i2} (\mathbf{f}_{i}^{\dagger} \mathbf{f}_{i} - 1) \quad (12)$$

where  $\{\mu_{i1}\}$  and  $\{\mu_{i2}\}$  are the Lagrange multipliers associated with the constraints of the signature sequences and beamformers, respectively. The first order condition with respect to the receivers results in the well-known MMSE receiver for user i

$$\mathbf{g}_{i} = \left(\sum_{i=1}^{K} \mathbf{c}_{i} \mathbf{c}_{i}^{\dagger} + \sigma^{2} \mathbf{I}\right)^{-1} \mathbf{c}_{i}$$
 (13)

When MMSE receivers are used by all users, the total MSE function can be reformulated as

$$MSE = K - \sum_{i=1}^{K} \mathbf{c}_i^{\dagger} \mathbf{T}^{-1} \mathbf{c}_i = K - N_R N + \sigma^2 \operatorname{tr} \{ \mathbf{T}^{-1} \}$$
 (14)

where  $\mathbf{T} = \sigma^2 \mathbf{I} + \sum_{i=1}^K \mathbf{c}_i \mathbf{c}_i^{\dagger}$ . Using matrix inversion lemma,  $\mathbf{T}^{-1}$  can be expressed as

$$\mathbf{T}^{-1} = \mathbf{E}_k^{-1} - \frac{\mathbf{E}_k^{-1} \mathbf{c}_k \mathbf{c}_k^{\dagger} \mathbf{E}_k^{-1}}{1 + \mathbf{c}_k^{\dagger} \mathbf{E}_k^{-1} \mathbf{c}_k}$$
(15)

with  $\mathbf{E}_k = \sum_{i \neq k} \mathbf{c}_i \mathbf{c}_i^{\dagger} + \sigma^2 \mathbf{I}$ . In this form, the MSE can be expressed as a function of parameters of one plus the contributions of the remaining users as follows:

MSE = 
$$K - N_R N + \sigma^2 \operatorname{tr} \{ \mathbf{E}_k^{-1} - \frac{\mathbf{E}_k^{-1} \mathbf{c}_k \mathbf{c}_k^{\dagger} \mathbf{E}_k^{-1}}{1 + \mathbf{c}_k^{\dagger} \mathbf{E}_k^{-1} \mathbf{c}_k} \}$$
  
=  $C_k - \sigma^2 \frac{\mathbf{c}_k^{\dagger} \mathbf{E}_k^{-2} \mathbf{c}_k}{1 + \mathbf{c}_k^{\dagger} \mathbf{E}_k^{-1} \mathbf{c}_k}$  (16)

where  $C_k$  represents terms independent of user k. The above construction suggests an iterative way of decreasing the MSE. Specifically, an iterative algorithm that minimizes the MSE at each step can be devised by optimizing

Tab. 1: Alternating Minimization Algorithm

the parameters of one user, say user k,  $\mathbf{c}_k$ , at each step. Note that  $\mathbf{c}_k$  depends on the temporal signature,  $\mathbf{s}_k$ , and the beamformer weights,  $\mathbf{f}_k$ . The second term, dependent on user k in the MSE expression can be expressed in terms of only signature sequence,  $\mathbf{s}_k$  or beamformer  $\mathbf{f}_k$  when all other variables are fixed. Defining  $N \times N$  block matrices  $(\mathbf{E}_k^{-1})_{ij}$ , we have

$$\mathbf{E}_{k}^{-1} = \begin{bmatrix} (\mathbf{E}_{k}^{-1})_{11} & (\mathbf{E}_{k}^{-1})_{12} & \cdots & (\mathbf{E}_{k}^{-1})_{1N_{R}} \\ (\mathbf{E}_{k}^{-1})_{21} & (\mathbf{E}_{k}^{-1})_{22} & \cdots & (\mathbf{E}_{k}^{-1})_{2N_{R}} \\ \vdots & \vdots & \ddots & \vdots \\ (\mathbf{E}_{k}^{-1})_{N_{R}1} & (\mathbf{E}_{k}^{-1})_{N_{R}2} & \cdots & (\mathbf{E}_{k}^{-1})_{N_{R}N_{R}} \end{bmatrix}$$

$$(17)$$

We can define  $(\mathbf{E}_k^{-2})_{ij}$  in the same manner as an  $N \times N$  block of  $\mathbf{E}_k^{-2}$ . Using these two definitions, we can easily express the total MSE in terms of the beamformer of user k,  $\mathbf{f}_k$  when all other variables are fixed as

$$MSE = C_k - \sigma^2 \left( \frac{\mathbf{f}_k^{\dagger} \mathbf{A}_k \mathbf{f}_k}{\mathbf{f}_k^{\dagger} \mathbf{B}_k \mathbf{f}_k} \right)$$
 (18)

where

$$\mathbf{A}_{k} = \sum_{i=1}^{N_{R}} \sum_{j=1}^{N_{R}} P_{k}(\mathbf{h}_{k,i}^{*} \mathbf{h}_{k,j}^{T}) \mathbf{s}_{k}^{\dagger} (\mathbf{E}_{k}^{-2})_{ij} \mathbf{s}_{k}$$
(19)

$$\mathbf{B}_{k} = \mathbf{I} + \sum_{i=1}^{N_R} \sum_{i=1}^{N_R} P_k(\mathbf{h}_{k,i}^* \mathbf{h}_{k,j}^T) \mathbf{s}_k^{\dagger} (\mathbf{E}_k^{-1})_{ij} \mathbf{s}_k \quad (20)$$

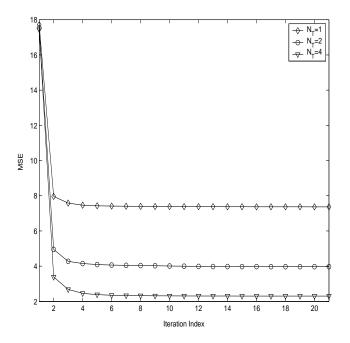


Fig. 2: K=30 user MIMO CDMA system with  $N=16,\,N_R=2$  and  $N_T=1,2,4$ 

In the same manner, we can express the total MSE in terms of the signature sequence of user k,  $\mathbf{s}_k$  when all other variables are fixed as

$$MSE = C_k - \sigma^2 \left( \frac{\mathbf{s}_k^{\dagger} \mathbf{C}_k \mathbf{s}_k}{\mathbf{s}_k^{\dagger} \mathbf{D}_k \mathbf{s}_k} \right)$$
 (21)

where

$$\mathbf{C}_{k} = \sum_{i=1}^{N_{R}} \sum_{j=1}^{N_{R}} P_{k} (\mathbf{h}_{k,i}^{T} \mathbf{f}_{k})^{*} (\mathbf{h}_{k,j}^{T} \mathbf{f}_{k}) (\mathbf{E}_{k}^{-2})_{ij}$$
 (22)

$$\mathbf{D}_{k} = \mathbf{I} + \sum_{i=1}^{N_{R}} \sum_{j=1}^{N_{R}} P_{k} (\mathbf{h}_{k,i}^{T} \mathbf{f}_{k})^{*} (\mathbf{h}_{k,j}^{T} \mathbf{f}_{k}) (\mathbf{E}_{k}^{-1})_{ij} (23)$$

From the perspective of user k, MSE can be minimized by choosing  $\mathbf{f}_k$  and  $\mathbf{s}_k$  to maximize the second terms in equations (18) and (21). This is accomplished by choosing  $\mathbf{f}_k$  to be the maximum generalized eigenvalued eigenvector of  $\mathbf{A}_k$  and  $\mathbf{B}_k$  and choosing  $\mathbf{s}_k$  to be the maximum generalized eigenvalued eigenvector of  $\mathbf{C}_k$  and  $\mathbf{D}_k$ . Thus, an iterative algorithm that minimizes the MSE can be devised as follows: each user takes turns in optimizing the MSE function from its perspective as explained above, monotonically decreasing the MSE function at each iteration of signature sequence and beamformer update. The algorithm is shown in Table 1.

The algorithm iterates over the users decreasing the system-wide MSE at each step. Clearly the MSE function is bounded below. This implies that the algorithm, which produces a decreasing sequence that is lower bounded, is convergent.

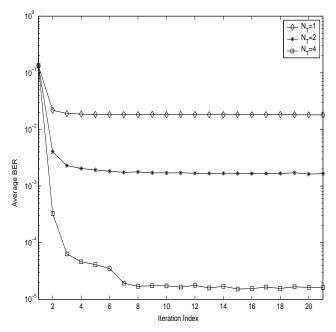


Fig. 3: K=30 user MIMO CDMA system with  $N=16,\,N_R=2$  and  $N_T=1,2,4.$  BER analysis at each iteration

#### IV. Numerical Results

In this section, we present numerical results related to the performance of the proposed algorithm. We also compare the performance of the resulting transmission scheme with Alamouti's scheme to investigate the benefit gained by exploiting the channel state information.

The simulations are performed for a MIMO-CDMA system with a processing gain of N=16. Each user is equipped with multiple antennas as stated for different cases and the receiver has  $N_R=2$  antennas. The channel realizations used are a sample realization of a flat fading channel model where all links are assumed to be independent. Variance of the AWGN noise used in the simulations is 0.4. Each plot shows the MSE, BER or SINR values versus the algorithm iteration index, where an iteration signifies updating all users' transmit weight vector and signature sequence, i.e., 2K updates.

We consider a K = 30 user MIMO-CDMA system for three different number of transmit antennas: 1, 2 and 4. It is observed that the total MSE monotonically decreases and converges to its minimum value for each case. Figure 2 shows the evolution of the algorithm as it converges to the optimum MSE value for different number of transmit antenna cases. We observe the effect of the number of transmit antennas on the minimum value of MSE. As number of transmit antennas is increased, lower MSE values can be achieved as expected since each added transmit antenna increases the flexibility of the user to find a better transmit weight vector defined in the subspace of the channel matrix. For the  $N_T = 1$  case, users do not have any flexibility to beamform at the transmitter side. As the number of transmit antennas increases, the flexibility in forming the spatial signature and diversity

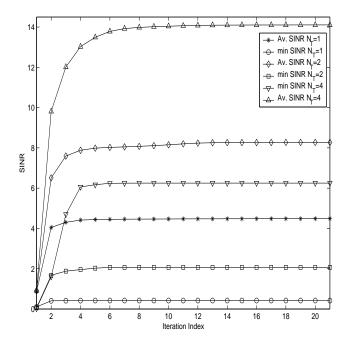


Fig. 4: K=30 user MIMO CDMA system with  $N=16,\ N_R=2$  and  $N_T=1,2,4.$  SINR analysis at each iteration

increases. For example, when  $N_{T_i} = 2$ , each user has a 2 dimensional subspace to form its spatial signature.

The system is analyzed in terms of MSE, average SINR, minimum SINR and average BER of the users. The updates show that the overall performance of the system is improved by updating the signatures and transmit weight vectors. Figure 3 and Figure 4 show the evolution of the average BER, the minimum SINR and the average SINR values as the MSE converges to the optimum.

Figure 5 shows the evolution of the algorithm for the case of 2 transmit and 2 receiver antennas with five random starting points. Total MSE monotonically decreases and converges to its minimum value for each starting transmit weight vector and signature sequence set. Throughout the simulations, for each channel realization, our algorithm always converged to the same MSE value for different starting points.

Figure 6 shows the MSE evolution when additional 10 users become active in the previous MIMO-CDMA system where each user is equipped with 2 transmit antennas. The minimum achievable MSE of the system increases as expected due to the higher number of users and channel constraints. When the system is overloaded, the number of symbols transmitted is larger, and the MSE minimization does not constrain the MSE achieved by each user. There are many possibilities that achieve same total MSE value with resulting different BER values. Our simulations showed that the BER at the fixed point is always lower than starting transmit weight vectors and signature sequences.

For performance comparison, Alamouti's transmit diversity scheme is investigated in terms of MSE, average BER, minimum SINR and average SINR for a 30 user

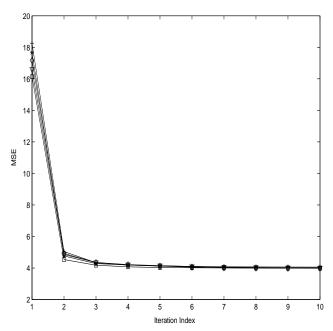


Fig. 5: K=30 user MIMO CDMA system with  $N=16,\ N_R=2$  and  $N_T=2.$  Performance of the algorithm with 5 different starting points

Scheme	MSE	Av.BER	Min SINR	Av.SINR
Alamouti	13.9211	0.0114	0.9361	4.1252
Iter. Alg.	7.9410	0.0017	2.0727	8.2717

Tab. 2: Comparison of Transmit Diversity Schemes

MIMO-CDMA system with a processing gain of 16 where each transmitter as well as the receiver is equipped with 2 antennas. The proposed algorithm achieves much better performance than Alamouti's transmit diversity technique as presented in Table IV. Performance enhancement with respect to Alamouti's transmit diversity technique is due to the multiuser structure of the proposed algorithm. The algorithm uses all system parameters to optimize the system objective jointly whereas Alamouti's technique is based on single-user transmission case. When the system is underloaded where each user can be assigned orthogonal codes the performance of transmit diversity techniques designed for single MIMO systems can be directly applied. However, for overloaded multiuser systems, the interfering structure of multiuser systems should be considered for performance enhancement.

#### V. Conclusion

In this paper, we proposed a transmit beamformer and signature sequence update algorithm that is geared towards enhancing the system performance by minimizing the total MSE in a MIMO-CDMA system. We investigated the relationship between the number of transmit antennas each user has and the performance of the system. Specifically, we observed that the performance of the system can be enhanced by minimizing system-wide MSE and in case of flat fading channel in closed loop

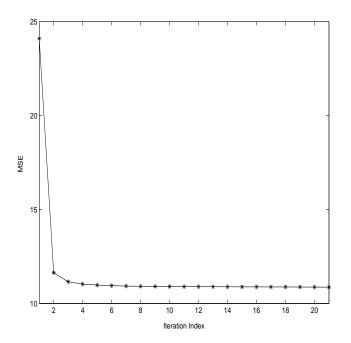


Fig. 6: K=40 user MIMO CDMA system with  $N=16,\,N_{T_i}=2$  and  $N_R=2$ 

systems performance of the proposed algorithm is better than applying Alamouti's scheme on each user.

It is important to note that the algorithm we propose here relies on the updates through error-free, low-delay feedback channels and highlights the performance versus complexity tradeoff between open and closed loop transmit schemes.

### References

- E. Telatar. Capacity of multi-antenna gaussian channels. 1995. AT&T Bell Labs Memorandum.
- [2] J. Winters, S. Salz, and R. D. Gitlin. Impact of antenna diversity on the capacity of wireless communication systems. *IEEE Transactions on Communications*, 42:1740– 1751, Jan./Feb./Mar. 1994.
- [3] J. Rohani and L. Jalloul. Orthogonal transmit diversity for direct-sequence spread CDMA. In ETSI Special Mobile Group Plenary Meeting, SMG2 97', Sept. 1997.
- [4] B. Hochwald, T.L. Marzetta, and C.B. Papadias. A transmitter diversity scheme for wideband CDMA systems based on space-time spreading. *IEEE Journal on Selected Areas in Communications*, 19(1):48–60, Jan. 2001.
- [5] A. Dabak, S. Hosur, and R. Negi. Space time block coded transmit antenna diversity scheme for WCDMA. In *IEEE* Wireless Communications and Networking Conference, WCNC 99', pages 1466–1469, 1999.
- [6] A. Hottinen and R. Wichman. Transmit diversity by antenna selection in CDMA downlink. In *IEEE 5th International Symposium on Spread Spectrum Techniques and Applications* 98', pages 767–770, Sept. 1998.
- [7] H. Sampath, P. Stoica, and A. Paulraj. Generalized linear precoder and decoder design for MIMO channels using

- the weighted MMSE criterion. *IEEE Transactions on Communications*, 49(12):2198–2206, December 2001.
- [8] R. L. Choi, K. B. Letaief, and R. D. Murch. MIMO CDMA antenna systems. In *IEEE International Confer*ence on Communications, ICC 01', pages 888–898, 2001.
- [9] M. Lenardi, A. Medles, and D. T. M. Slock. Comparison of downlink transmit diversity schemes for RAKE and SINR maximizing receivers. In *IEEE International Con*ference on Communications, ICC 01', pages 1679 –1683, 2001.
- [10] M. Ahmed, J. Pautler, and K. Rohani. CDMA receiver performance for multiple-input multiple-output antenna systems. In *IEEE Vehicular Technology Conference*, VTC 01', pages 1309 –1313, 2001.
- [11] S. Verdú. Capacity region of gaussian CDMA channels: The symbol–synchronous case. In 24th Annual Allerton Conference on Communication, Control and Computing, pages 1025–1034, October 1986.
- [12] M. Rupf and J. L. Massey. Optimum sequence multisets for synchronous code-division multiple-access channels. *IEEE Transactions on Information Theory*, 40(4):1261– 1266, July 1994.
- [13] P. Viswanath and V. Anantharam. Optimal sequences and sum capacity of synchronous CDMA systems. *IEEE Transactions on Information Theory*, 45(6):1984–1991, September 1999.
- [14] S. Ulukus and R. D. Yates. Iterative construction of optimum signature sequence sets in synchronous CDMA systems. *IEEE Transactions on Information Theory*, 47(5):1989–1998, July 2001.
- [15] S. Ulukus and A. Yener. Iterative transmitter and receiver optimization for synchronous CDMA systems. In IEEE International Symposium on Information Theory, ISIT'02, page 48, June 2002.
- [16] C. Rose, S. Ulukus, and R. D. Yates. Wireless systems and interference avoidance. *IEEE Transactions on Wireless Communications*, 1(3):415–428, July 2002.
- [17] A. F. Naguib, A. J. Paulraj, and T. Kailath. Capacity improvement with base-station antenna arrays in cellular CDMA. *IEEE Transactions on Vehicular Technology*, 43(3):691–698, August 1994.
- [18] A. Yener, R. D. Yates, and S. Ulukus. Interference management for CDMA systems through power control, multiuser detection, and beamforming. *IEEE Transactions on Communications*, 49(7), July 2001.
- [19] B.L. Hughes and P. Sudarshan. On the spectral efficiency of CDMA with multiple antennas. In *IEEE Information Theory Workshop 01*', pages 101–103, 2001.
- [20] A. Mantravadi and V.V. Veeravalli. Sum capacity of CDMA systems with multiple transmit antennas. In IEEE International Symposium on Information Theory, ISIT 02', page 280, 2002.
- [21] S. Verdú. Multiuser Detection. Cambridge University Press, 1998.