

а)

D	F	\bar{D}	$D \vee F$	\bar{F}
0	0	1	0	1
0	1	1	1	0
1	0	0	1	1
1	1	0	1	0

Не имеем согласованных

б)

K	T	$T \rightarrow K$	\bar{K}	$K \vee \bar{T}$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	1
1	1	1	0	1

имеем согласованные

в)

A	B	C	$\neg C \rightarrow B$	$\neg(A \rightarrow (\neg C \rightarrow B))$	$(B \vee C) \wedge A$
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	1

Не имеем

согласованные

г)

B	C	D	$C \rightarrow B$	$D \vee C$	\bar{B}	$A \rightarrow B$
0	0	0	1	0	1	1
0	0	1	1	1	1	0
0	1	0	0	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	0	1	1	1	0	1
1	1	0	1	1	0	1
1	1	1	1	1	0	1

Не имеем согласованные

2

a)	1	$H \rightarrow (R \wedge C)$	Premise
	2	$\neg R \vee \neg C$	Premise
	3	$\neg(R \wedge C)$	DeM 2
	4	$\neg H$	MT 1, 3

b)

1	$K \wedge S$	Premise
2	$\neg K$	Premise
3	K	$\wedge E$ 1
4	\perp	$\neg E$ 2, 3
5	$\neg S$	X 4

c)

1	$A \rightarrow \neg A$	Premise
2	A	Assumption
3	$\neg A$	MP 1, 2
4	$\neg A$	Assumption
5	$\neg A$	R 4
	$\neg A$	LEM 2-3, 4-5

d)

1	$(P \wedge Q) \vee (P \wedge R)$	Premise
2	$P \wedge Q$	Assumption
3	P	$\wedge E$ 2
4	$P \wedge R$	Assumption
5	P	$\wedge E$ 4
	P	$\vee E$ 1, 2-3, 4-5

- 3) a) A - philosophers ponder profound problems
 B - their quandaries quell quotidian quibbles
 C - right reasoning reveals reality

A	B	C	$A \rightarrow B$	$\bar{B} \vee C$	A	\therefore	C
0	0	0	1	1	0	.	0
0	0	1	1	1	0	.	1
0	1	0	1	0	0	.	0
0	1	1	1	1	0	.	1
1	0	0	0	1	1	.	0
1	0	1	0	1	1	.	1
1	1	0	1	0	1	✓	0
1	1	1	1	1	1	✓	1

1	$A \rightarrow B$	Premise
2	$\bar{B} \vee C$	Premise
3	A	Premise
4	B	MP 1, 3
5	\bar{B}	Assumption
6	\perp	$\neg E$ 5, 4
7	$\neg \bar{B}$	$\neg I$ 5-6
8	C	DS 2, 7

- b) A - warthogs are adorable
 B - baby baboons beat bongos
 C - crocodiles can consume cute copy bongs

A	B	C	$A \rightarrow (\bar{B} \vee C)$	B	$\bar{A} \vee C$	\therefore	\bar{A}
0	0	0	1	0	1	.	1
0	0	1	1	0	1	.	1
0	1	0	1	1	1	✓	1
0	1	1	1	1	1	✓	1
1	0	0	1	0	0	.	0
1	0	1	1	0	1	.	0
1	1	0	0	1	0	.	0
1	1	1	0	1	1	X	0

Контрпример: при $A=B=C=1$ $A \rightarrow (\bar{B} \vee C) = 0$ $B=1$ $\bar{A} \vee C = 0$, но $\bar{A} = 0$

- c)
- A - discipline defeat deficiency
 - B - geniuses generally get good grades
 - C - homework has harmed humanity

A	B	C	$\bar{A} \rightarrow B$	$A \rightarrow C \therefore B \vee C$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

1	$\bar{A} \rightarrow B$	premise
2	$A \rightarrow C$	premise
3	$B \vee C$	Assumption
4	$\bar{B} \wedge \bar{C}$	DEM 3
5	\bar{B}	$\wedge E$ 4
6	\bar{C}	$\wedge E$ 4
7	$\neg \bar{A}$	M+ 1, 5
8	A	$\neg \neg E$ 7
9	\bar{A}	M+ 2, 6
10	\perp	$\neg E$ 8, 9
11	$B \vee C$	$\neg I$ 3-10

- d)
- A - crocodiles can consume cute capuchinas
 - B - Incarcerating lguanas is illegal
 - C - Mad monkeys mad mayhem
 - D - dinosaurs do disco dance

A	B	C	D	$A \rightarrow \bar{B}$	$C \wedge D \vee A$	B	$\therefore D \leftrightarrow C$
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	0
0	0	1	0	1	0	0	1
0	0	1	1	1	1	0	1
0	1	0	0	1	0	1	0
0	1	0	1	1	0	1	0
0	1	1	0	1	1	1	1
0	1	1	1	1	1	0	1
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	0
1	0	1	0	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	0	0	1	1	1
1	1	0	1	0	1	1	0
1	1	1	0	0	1	1	0
1	1	1	1	0	1	1	1

1	$A \rightarrow \bar{B}$	premise
2	$C \wedge D \vee A$	premise
3	B	premise
4	\bar{B}	assumption
5	\perp	$\neg E$ 1, 3
6	$\neg \bar{B}$	$\neg I$ 4-5
7	\bar{A}	MT 1, 6
8	$C \wedge D$	PS 2, 7
9	C	$\wedge E$ 8
10	D	$\wedge E$ 8
11	C	Assumption
12	\perp	R 10
13	\bar{D}	Assumption
14	\perp	R 9
15	$C \leftrightarrow D$	$\leftrightarrow I$ 11-12, 13-14

4) a) $\neg \neg A \therefore A$

1	$\neg \neg A$	premise
2	$\neg A$	assumption
3	\perp	$\neg E$ 1, 2
	A	IP 2-3

c) $(\neg B \rightarrow \neg A) \therefore (A \rightarrow B)$

1	$\neg B \rightarrow \neg A$	premise
2	A	Assumption
3	$\neg B$	Assumption
4	$\neg A$	MP 1, 3
5	\perp	$\neg E$ 2, 4
6	B	IP 3-5
7	$A \rightarrow B$	$\rightarrow I$ 2-6

d) $\neg(A \vee B) \therefore (\neg A \wedge \neg B)$

1	$\neg(A \vee B)$	premise
2	A	Assumption
3	$A \vee B$	$\vee I$ 2
4	\perp	$\neg E$ 1, 3
5	$\neg A$	$\neg I$ 2-4
6	B	Assumption
7	$A \vee B$	$\vee I$ 6
8	\perp	\negE 1, 7
9	$\neg B$	$\neg I$ 6-8
10	$\neg A \wedge \neg B$	$\wedge I$ 5, 9

b) $(A \rightarrow B) \rightarrow A \therefore A$

1	$(A \rightarrow B) \rightarrow A$	premise
2	$\neg A$	Assumption
3	A	Assumption
4	\perp	$\neg E$ 2, 3
5	B	X 4
6	$A \rightarrow B$	$\rightarrow I$ 3, 5
7	A	MP 1, 6
8	\perp	\negE 2, 7
9	$\neg \neg A$	\negI 2-8
10	A	Assumption
11	\perp	\negE 9, 10
12	A	IP 10-11

$$2) (\neg A \wedge \neg B) \therefore \neg(A \vee B)$$

1	$\neg A \wedge \neg B$	premise
2	$\neg A$	$\wedge E$ 1
3	$\neg B$	$\wedge E$ 1
4	$A \vee B$	Assumption
5	A	Assumption
6	\perp	$\neg E$ 2, 5
7	B	Assumption
8	\perp	\negE 3, 7
9	\perp	$\vee E$ 4, 5-6, 7-8
10	$\neg(A \vee B)$	$\neg I$ 4-9

$$3) (A \rightarrow B) \wedge (\neg A \rightarrow B) \therefore B$$

1	$A \rightarrow B \wedge (\neg A \rightarrow B)$	premise
2	$A \rightarrow B$	$\wedge I$ 1
3	$\neg A \rightarrow B$	$\wedge I$ 1
4	A	Assumption
5	B	MP 2, 4
6	$\neg A$	Assumption
7	B	MP 3, 6
8	B	

1	$(A \rightarrow B) \wedge (\neg A \rightarrow B)$	premise
2	$\neg B$	Assumption
3	A	Assumption
4	B	MP 1, 3
5	\perp	\negE 2, 4
6	$\neg A$	\negI 3-5
7	B	MP 1, 6

$$4) (A \rightarrow B) \wedge (\neg A \rightarrow B) \therefore B$$

1	$(A \rightarrow B) \wedge (\neg A \rightarrow B)$	premise
2	$A \rightarrow B$	$\wedge E$ 1
3	$\neg A \rightarrow B$	$\wedge E$ 1
4	$\neg B$	Assumption
5	A	Assumption
6	B	MP 2, 5
7	\perp	\negE 4, 6
8	$\neg A$	\negI 5-7
9	B	MP 3, 8
10	\perp	\negE 4, 9
11	B	\negI 4-10

b) Докажем, что $\overline{A \rightarrow B} \therefore A \wedge \overline{B}$ и далее сгруппируем это в C, \perp

1	$A \rightarrow B$	premise
2	$\overline{A \vee B}$	Assumption
3	A	Assumption
4	\overline{A}	Assumption
5	\perp	$\neg E$ 4, 3
6	$\neg \overline{A}$	$\neg I$ 4-5
7	B	OS 2, 6
8	$A \rightarrow B$	$\rightarrow I$ 3-7
9	\perp	$\neg E$ 1, 8
10	$\overline{A \vee B}$	$\neg I$ 2-9
11	$\neg \overline{A} \wedge \overline{B}$	DEM 10
12	$\neg \neg A$	$\wedge E$ 11
13	\overline{B}	$\wedge E$ 11
14	A	$\neg \neg E$ 12
15	$A \wedge \overline{B}$	$\wedge I$ 14, 13

i	$A \rightarrow B$
\therefore	$A \wedge \overline{B}$ $\neg I$ $\neg I$

c) $(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$

1	$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$	premise
2	$(\neg B \rightarrow \neg A) \wedge ((\neg B \rightarrow A) \rightarrow B)$	$\neg I$ 1
3	$\neg B \rightarrow \neg A$	$\wedge E$ 2
4	$((\neg B \rightarrow A) \rightarrow B)$	$\wedge E$ 2
5	$(\neg B \rightarrow A) \wedge \overline{B}$	$\neg I$ 4
6	$\neg B \rightarrow A$	$\wedge E$ 5
7	\overline{B}	$\wedge E$ 5
8	$\neg A$	MP 3, 7
9	A	MP 6, 7
10	\perp	$\neg E$ 8, 9

$$d) (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$$

1	$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$	premise
2	$(A \rightarrow (B \rightarrow C)) \wedge ((A \rightarrow B) \rightarrow (A \rightarrow C))$	$\wedge I$ 1
3	$A \rightarrow (B \rightarrow C)$	$\wedge E$ 2
4	$(A \rightarrow B) \rightarrow (A \rightarrow C)$	$\wedge E$ 2
5	$(A \rightarrow B) \wedge A \rightarrow C$	$\wedge I$ 4
6	$A \rightarrow B$	$\wedge E$ 5
7	$A \rightarrow C$	$\wedge E$ 5
8	$A \wedge \bar{C}$	$\wedge I$ 7
9	A	$\wedge E$ 8
10	\bar{C}	$\wedge E$ 8
11	$B \rightarrow C$	MP 3, 9
12	B	MP 6, 9
13	C	MP 11, 12
14	\perp	$\neg E$ 10, 13

5 a) $(A \rightarrow B) \vee (B \rightarrow A)$

1 $(A \rightarrow B) \vee (B \rightarrow A)$ premise 2

2 $A \rightarrow B \wedge B \rightarrow A$ DEM 1

3 $A \rightarrow B$ 1E 2

4 $A \vee B$ Assumption

5 A Assumption

6 $\neg A$ Assumption

7 \perp 7E 6, 5

8 $\neg A$ 7I 6-7

9 B DS 4, 8

10 $A \rightarrow B \rightarrow I$ 5-9

11 \perp 7E 3, 10

12 $A \vee B$ 7I 4-11

13 $\neg A \wedge B$ DEM 12

14 B 7E 13

15 $B \rightarrow A$ 1E 2

16 $B \vee A$ Assumption

17 B Assumption

18 $\neg B$ Assumption

19 \perp 7E 18, 17

20 $\neg B$ 7I 18-19

21 A DS 16, 20

22 $B \rightarrow A \rightarrow I$ 17-21

23 \perp 7E 15, 22

24 $B \vee A$ 7I 16-23

25 $\neg B \wedge A$ DEM 24

26 $\neg B$ 1E 25

27 B 7E 26

28 \perp 7E 14, 27

6) $A \rightarrow (B \rightarrow A)$

1 $A \rightarrow (B \rightarrow A)$ premise

2 $\neg A \vee (B \rightarrow A)$ Assumption

3 A Assumption

4 $\neg A$ Assumption

5 \perp 7E 4, 3

6 $\neg A$ 7I 4-5

7 $B \rightarrow A$ DS 2, 6

8 $A \rightarrow (B \rightarrow A) \rightarrow I$ 3-7

9 \perp 7E 1, 8

10 $\neg A \vee (B \rightarrow A) \rightarrow I$ 2-9

11 $\neg A \wedge B \rightarrow A$ DEM 10

12 $\neg A$ 1E 11

13 A 7E 12

14 $B \rightarrow A$ 1E 11

15 $B \vee A$ Assumption

16 B Assumption

17 $\neg B$ Assumption

18 \perp 7E 17, 16

19 $\neg B$ 7I 17-18

20 A DS 15, 19

21 $B \rightarrow A \rightarrow I$ 16-20

22 \perp 7E 14, 21

23 $B \vee A$ 7I 15-22

24 $\neg B \wedge A$ DEM 23

25 $\neg B$ 1E 24

26 \perp 7E 25, 13