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Project #1 Report

Introduction

This project is designed to find the shortest solution to solving an 8 piece slide puzzle so all the numbers are in order counting from top-left to bottom-right by counting right. Finding the path requires an AI that uses one of many search algorithms including: Uniform Cost Search (UCS), and Misplaced Tile and Euclidean Distance heuristics applied to A* to calculate all possible moves proceeding a given puzzle. The overall code was implemented as a Tree but no Graph was created and each algorithm uniquely calculates the depth of the Tree and the cost of each transition between two nodes. The code language used was C++, to implement the six default puzzles plus one more and six randomly generated puzzles, and all files work as one to create a code that is fully functional in calculating a multitude of 8 piece puzzles.

Outcomes of Different Algorithms

Uniform Cost Search is simply A* an algorithm where $h(n)$ is hardcoded to 0, and only the cost of $g(n)$ will be calculated by expanding the cheapest node from initial state to the frontier. In the case of this assignment, there are no weights to the expansions, and each expanded node will have a cost of 1.

The Misplaced Tile Heuristic is one of two heuristic search algorithms specified as an A* search. This heuristic looks for any tiles out of place when compared to the goal state. Not counting the blank/missing tile, $g(n)$ is incremented by 1 for each of the tiles not in their current goal state positions. This assigns a number, where lower is better, to node expansion based on how many misplaced tiles there are after a tile has been shifted. When applied to the n-puzzle, queue will expand the node with the cheapest cost, rather than expanding each of the child nodes as Uniform Cost Search would.

The Euclidean Distance Heuristic is similar to the Misplaced Tile Heuristic such that it considers the cost of future expansions and looks at misplaced tiles, but has a different rationale to it. It looks at all the tiles out of place and determines the shortest number of moves to arrive at their goal positions, using the pythagorean theorem and the number of squares moved horizontally and vertically the heuristic determines the total distance used for the edge cost. The resulting $g(n)$ is the sum of all the cost of all the diagonal distances. Again all the distance for all misplaced tiles are calculated except for the blank tile.

Graph/Table Results

Nodes Expanded

In Figure 1, we can see the difference between the number of expanded nodes is much too small when solving the easier puzzles. When we reach the Hard puzzle a gap is present with UCS overtaking both A* heuristic searches and Misplaced is a bit higher than Euclidean. The following puzzle shows the gap widening but after that all three algorithms close in when they should be spread out more. In Figure 3, any puzzles that require more moves have an increased number of nodes for all three algorithms while the Euclidean distance is barely visible in puzzles that are simpler.

Maximum Queue Size

In Figure 2, the flow of each algorithm is similar to those in Figure 1 but the nodes used are different for certain puzzles. The total number of nodes in each queue is fairly close to the expanded nodes in Figure 1 but those for harder puzzles are significantly smaller. In Figure 4, similar concept in Figure 3 but again there are fewer nodes in the queue than those expanded.

Additional Created Puzzles

Here is a list of six puzzles we created ourselves and tested the node expansion and queue size for:

- 1) [{7}, {8}, {2},
 {1}, {3}, {0},
 {4}, {6}, {5}]
- 2) [{5}, {7}, {3},
 {4}, {1}, {6},
 {2}, {0}, {8}]
- 3) [{1}, {5}, {2},
 {0}, {4}, {3},
 {7}, {8}, {6}]
- 4) [{3}, {1}, {6},
 {5}, {0}, {7},
 {4}, {2}, {8}]
- 5) [{7}, {1}, {2},
 {8}, {4}, {3},
 {6}, {5}, {0}]
- 6) [{7}, {4}, {1},
 {8}, {5}, {2},
 {0}, {6}, {3}]

Main Issues

Upon pushing new nodes that were developed after shifting tile up, down, left, or right to the frontier, the error was displayed that was eventually fixed when the push and pop functions in the Tree class were switched. Another problem was a system crash when the total number of expanded nodes reached twenty-thousand nodes in one of the more difficult puzzles. This was partially solved when we fixed the statements for Misplaced Tile Search that was causing a bug. Unfortunately, the node expansion for the two most difficult default puzzles continuously increases without stopping at a specific number of expanded nodes and does not show whether the process has succeeded or failed. So we limited the node expansion for any and all trees created by solving a puzzle.

Conclusion

The lowest total cost to the goal state of an $n \times n$ puzzle can be performed by various algorithms but some are more efficient than others. Based on the data found in each table/graph, it is clear that of the three algorithms A* Euclidean Distance Heuristic performs the best, followed by A* Misplaced Tile Heuristic, and Uniform Cost Search (UCS) as the least efficient. Either of the two heuristics used can greatly improve the efficiency of algorithms as it compares the initial state with the goal state as the process continues on thus creating fewer nodes to expand and a smaller queue size. Although both heuristics improved the run time and space cost of Uniform Cost Search, not all heuristics provide the same results as they process the tree/graph differently. Keep in mind the difficulty of each puzzle and the number of moves it takes to solve the puzzle is determined by the arrangement of each tile but some puzzles may not be solvable at all. The difference between the three graphs is so small when solving easy puzzles but widens when solving more difficult ones. To be precise, the Euclidean Distance is approximately 148.5% more efficient than Misplaced Tiles and 173% more efficient than UCS whereas Misplaced Tile is 116% more efficient than UCS.

Default Puzzles

Number of Nodes Expanded

	UCS	A* MisplacedTiles	A* Euclid Distance
Trivial (1)	0	0	0
Very Easy (2)	12	4	4
Easy (3)	28	8	8
Doable (4)	124	32	28
Hard (5)	14568	3528	2328
Very Hard (6)	300000	225232	53628
Impossible (7)	300000	300000	300000

Nodes Expanded For Each Puzzle

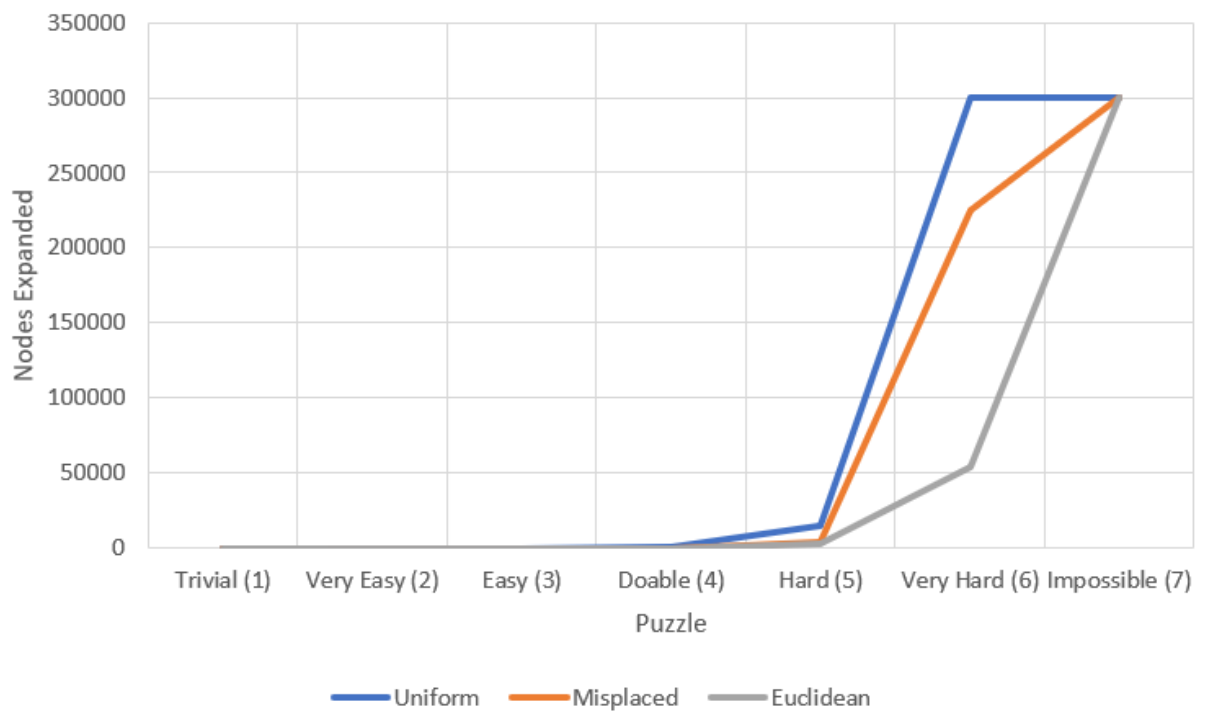


Figure 1. (Y-axis is “Total Number of Nodes Expanded”; X-axis is “Puzzle (Difficulty)”)

Maximum Queue Size

	UCS	A* MisplacedTiles	A* Euclid Distance
Trivial (1)	1	1	1
Very Easy (2)	7	3	3
Easy (3)	13	4	4
Doable (4)	27	11	11
Hard (5)	2185	535	393
Very Hard (6)	33540	27294	7670
Impossible (7)	33540	33540	33540

Queue Size For Each Puzzle

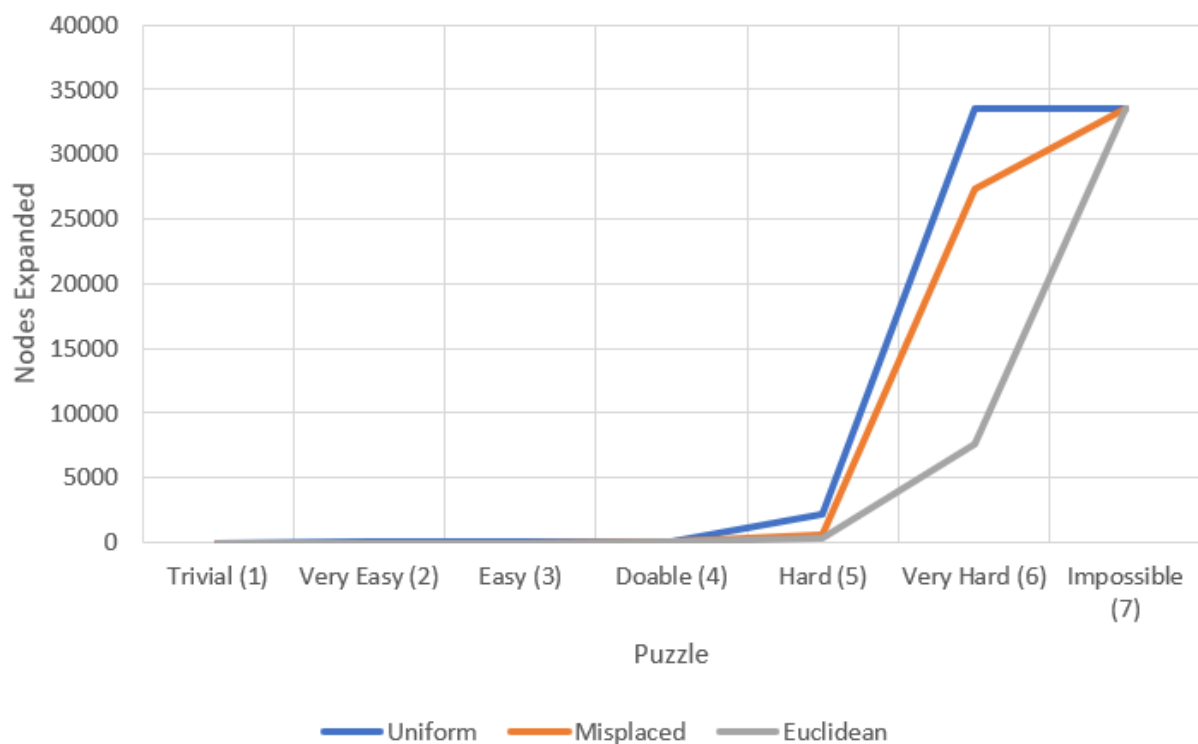


Figure 2. (Y-axis is "Maximum Queue Size"; X-axis is "Random Puzzles")

Created Puzzles

Number of Nodes Expanded

	UCS	A* MisplacedTiles	A* Euclid Distance
Puzzle 1	111932	19716	9232
Puzzle 2	121140	21740	11580
Puzzle 3	316	84	84
Puzzle 4	477692	143160	68240
Puzzle 5	27036	4556	1904
Puzzle 6	27760	4148	1800

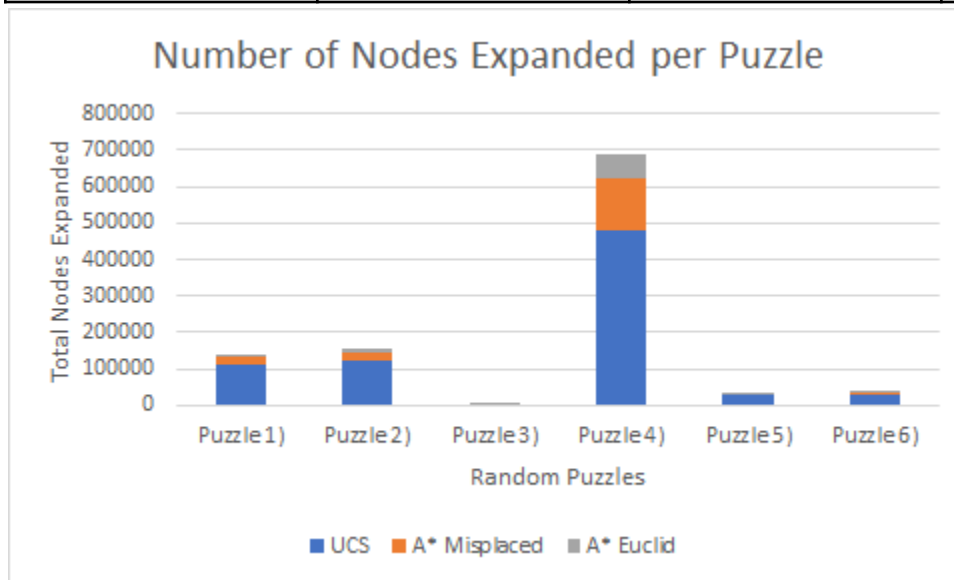


Figure 3.

Maximum Queue Size

	UCS	A* MisplacedTiles	A* Euclid Distance
Puzzle 1	15080	3035	1386
Puzzle 2	16090	3304	1756
Puzzle 3	57	19	19
Puzzle 4	33258	18294	9516
Puzzle 5	4072	712	335
Puzzle 6	4108	669	313

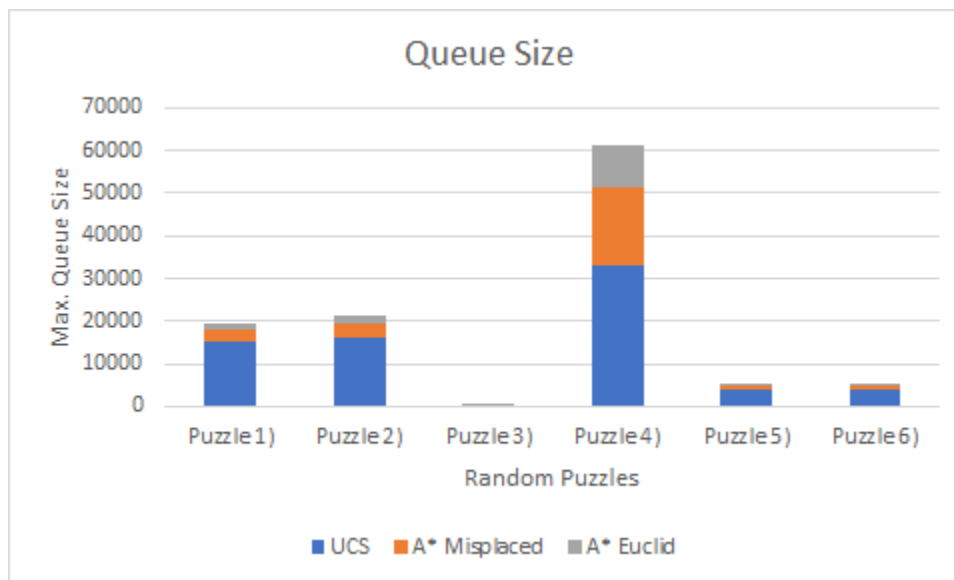


Figure 4.