Uniform distribution

Uniform distribution with values between a and b: $x \in [a, b]$

Probability distribution function: $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$

Mean: $\mu = \frac{a+b}{2}$

Standard deviation: $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

The probability that x is between c and d:

$$P(c < x \leq d) = \int_c^d f(x)dx = \left[\frac{x}{b-a} + C\right]_c^d = \frac{d-c}{b-a} \text{ for } a \leq c \leq b \text{ and } a \leq d \leq b$$

All other probabilities can be derived using this equation:

$$P(x < g) \to c = a, d = g$$

$$P(x \geq g) \to c = g, d = b$$

Examples of calculating probabilities for uniform distributions:

https://courses.lumenlearning.com/odessa-introstats1-1/chapter/the-uniform-distribution/

Binomial Distribution

Binomial distribution with n trials, x successes (the random variable) and a chance p of success in any one trial

Probability mass function: $f(x, n, p) = \binom{n}{x} p^x (1-p)^{n-x}$

Mean: $\mu = np$

Standard deviation: $\sigma = \sqrt{np(1-p)}$

The probability of achieving k successes (with k<n) is:

$$f(x = k, n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

The probability of obtaining a number of successes k between c and d:

$$P(c < x \leq d) = \sum_{x=c}^{x=d} f(x, n, p)$$

All other probabilities can be derived using this equation:

$$P(x \leq g) \to c = 0, d = g$$

$$P(x \geq g) \to c = g, d = n$$

Examples of calculating probabilities for binomial distributions:

https://www.intmath.com/counting-probability/12-binomial-probability-distributions.php

Normal distribution

Probability distribution function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Mean: $\mu = \mu$

Standard deviation: $\sigma = \sigma$

The probability that x is between c and d:

$$P(c < x \leq d) = \int_c^d f(x)dx = \int_c^d \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

This function cannot be integrated symbolically and will generally be solved numerically (through approximations of the integral). In python one can use the scipy.stats module to obtain the surface under the probability distribution function (i.e. the cumulative distribution function):

$$P(c < x \leq d) = scipy.stats.norm(mean, std).cdf(d) - scipy.stats.norm(mean, std).cdf(c)$$

https://courses.lumenlearning.com/odessa-introstats1-1/chapter/using-the-normal-distribution/