

B-Tree – Question and Answer

1. Draw the B-tree of order 3 and order 4 created by inserting the following data arriving in sequence.

92 24 6 7 11 8 22 4 5 16 19 20 78

Order = 3

Maximum number of keys = 2

Minimum number of keys = $\lceil (3/2) - 1 \rceil = (2-1) = 1$

Maximum number of children = 3

Minimum number of children = $\lceil (3/2) \rceil = 2$

B-Tree Question 1.i

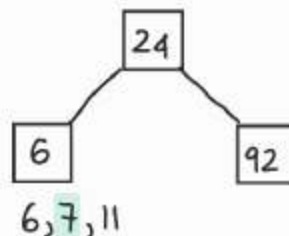
Data: ~~92, 24, 6, 7, 11, 8, 22, 4, 5, 16, 19, 20, 78~~

Order: 3 (Max 2 keys)

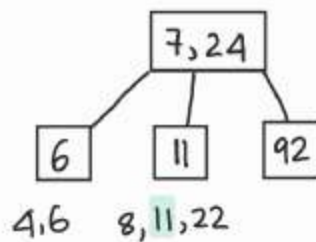
Step 1: Insert first 2 items into root

24, 92 (Max 2 keys)
6, 24, 92

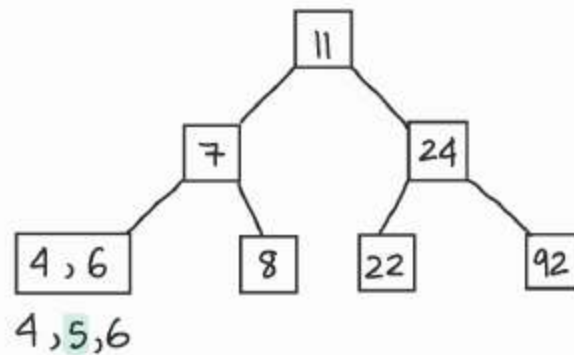
Step 2: Inserting 6 would trespass the rule of order-3 B-tree. Thus, we split and promote 24 to form a new node.



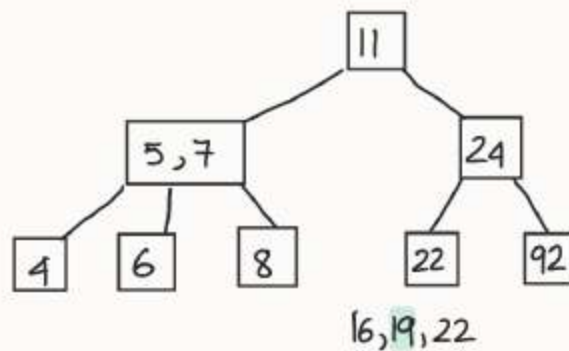
Step 3: Insert 7, 11. Split and promote 7.



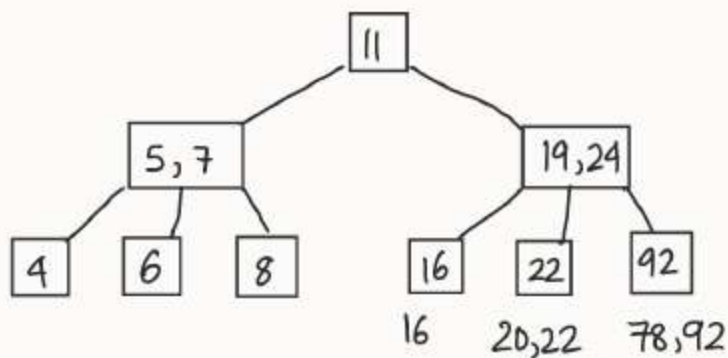
Step 4: Insert 8, 22, 4. Split 8, 11, 22 and promote 11 to the root.
Then, we further split the root and promote 11.



Step 5: After inserting 5, split and promote 5.

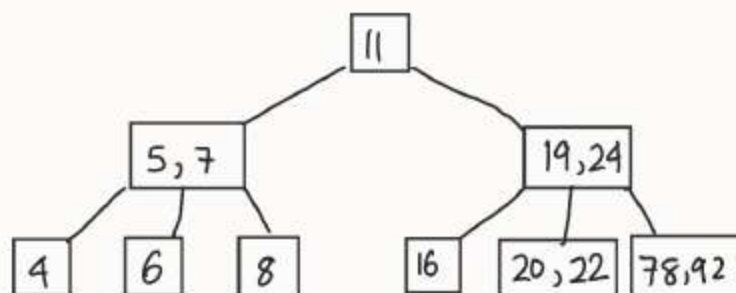


Step 6: Insert 16, 19. Split and promote 19



Step 7: Insert 20, 78

Answer:



Order = 4

Maximum number of keys = 3

Minimum number of keys = $\lceil (4/2) - 1 \rceil = (2-1) = 1$

Maximum number of children = 4

Minimum number of children = $\lceil (4/2) \rceil = 2$

B-Trees Question 1. ii

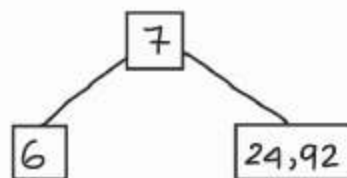
Data: 92, 24, 6, 7, 11, 8, 22, 4, 5, 16, 19, 20, 78

Order: 4 (Max 3 keys)

Step 1: Inserting first 3 keys

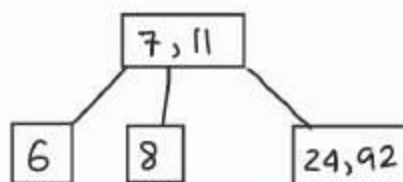
6, 24, 92

Step 2: Inserting 7 to obtain keys 6, 7, 24, 92. Since having 4 keys trespass the rule of an order-4 B-tree, we split the keys and promote 7 in this case (left-biased).



8, 11, 24, 92 (Max 3 keys)

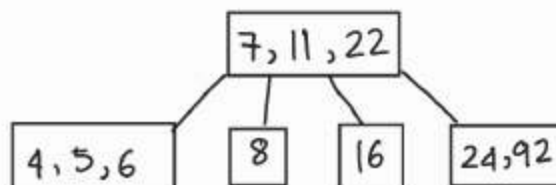
Step 3: Inserting 8, 11. Splitting and promoting 11. (left-biased)



4, 5, 6

16, 22, 24, 92

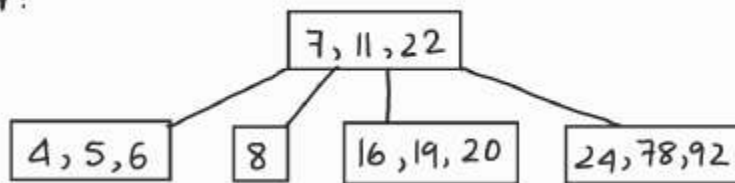
Step 4: Inserting 22, 4, 5, 16. Splitting and promoting 22.



16, 19, 20 24, 78, 92

Step 5: Inserting 19, 20, 78

Answer :



2. Draw the B-tree of order 3 created by inserting data arriving in sequence from the two sets shown below. Compare the two B-trees to determine whether the order of data will create different B-tree.

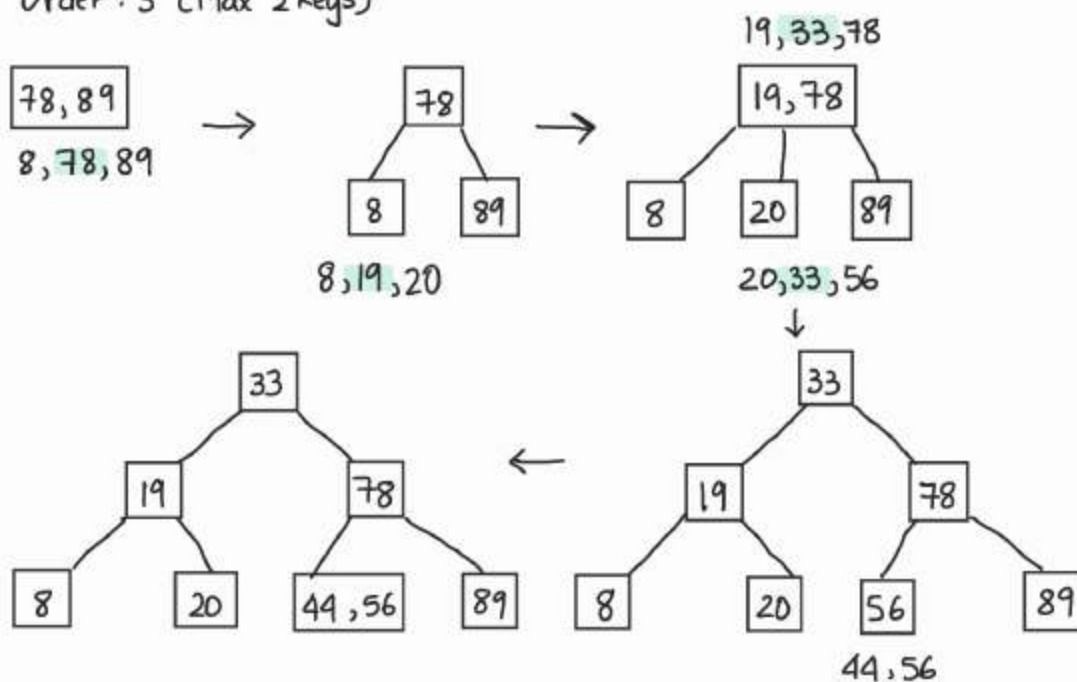
Data 1: 89 78 8 19 20 33 56 44

Data 2: 44 56 33 20 19 8 78 89

B-Tree Question 2

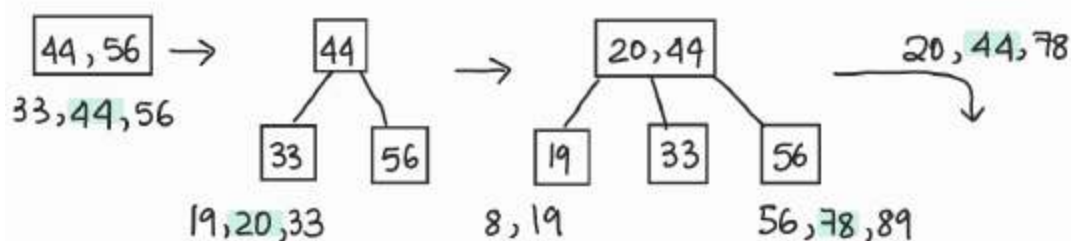
Data 1: ~~89, 78, 8, 19, 20, 33, 56, 44~~

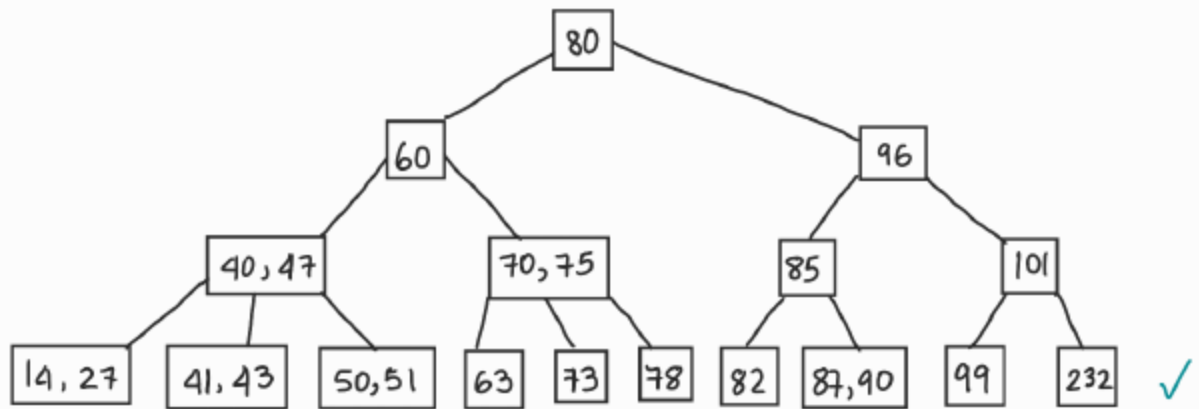
Order : 3 (Max 2 keys)



Data 2: ~~44, 56, 33, 20, 19, 8, 78, 89~~

Order : 3 (Max 2 keys)



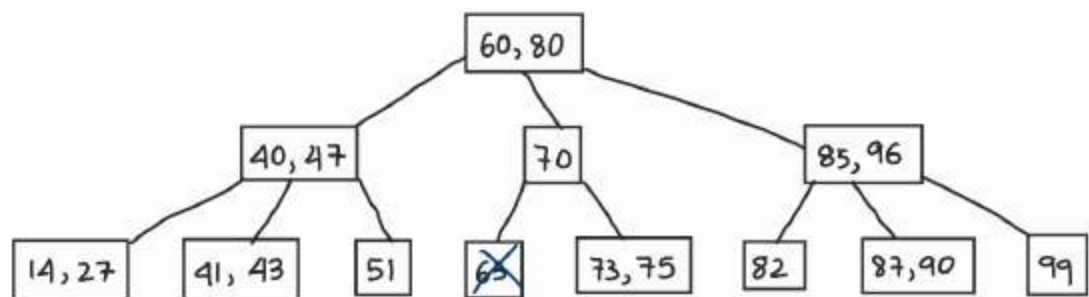


4. Using the same B-tree order 3 above, delete 63, 90, 41 and 60.

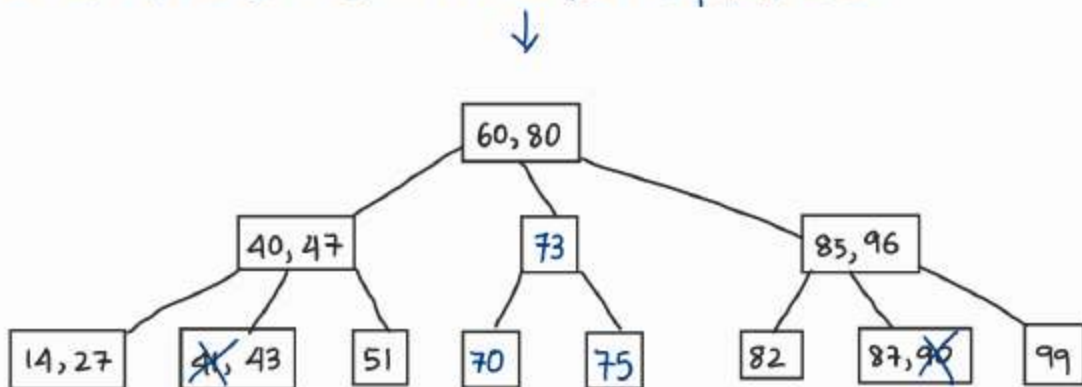
B-Tree Question 4

Order: 3 (Max 2 keys)

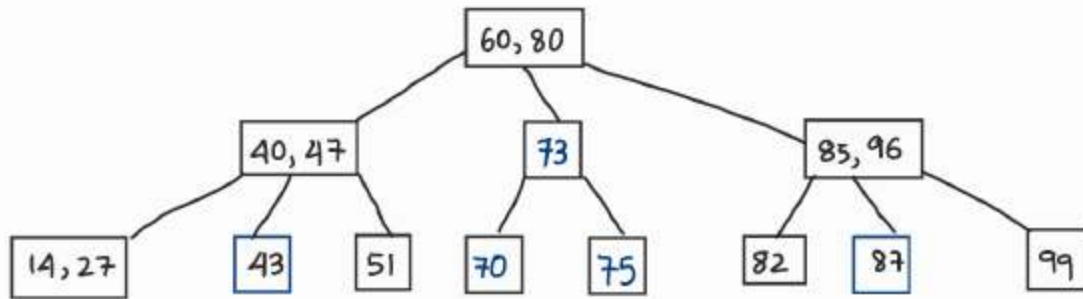
Delete: 63, 90, 41, 60



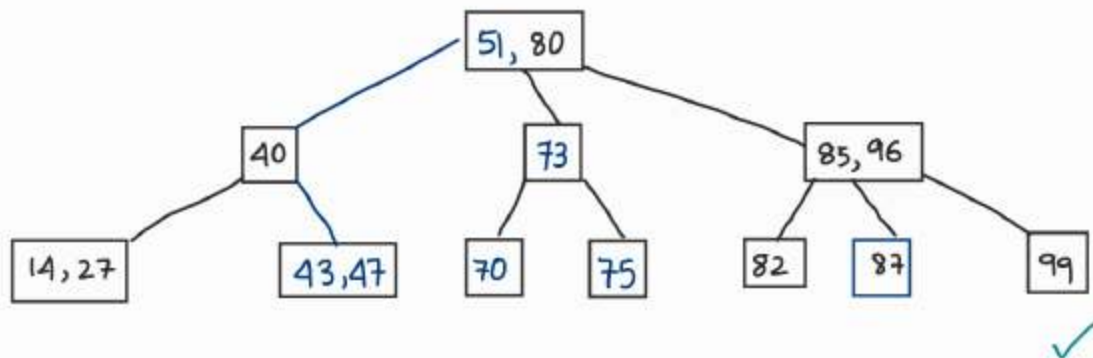
In order to delete 63, we borrow from the parent node.



Leaf node 90 and 41 can be deleted easily since the number of keys was greater than minimum.



Using inorder predecessor 51 to delete 60.



5. Which of the following are legal 2-3-4 trees? If it is not, justify your answer.

Order = 4

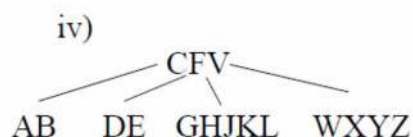
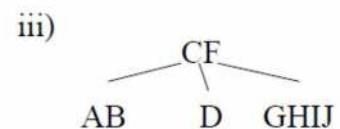
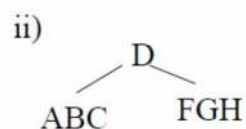
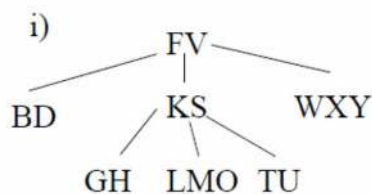
Maximum number of keys = 3

Minimum number of keys = $\lceil 4/2 \rceil - 1 = (2-1) = 1$

Maximum number of children = 4

Minimum number of children = $\lceil 4/2 \rceil = 2$

2-3-4 trees are B-trees of order 4 (also known as 2-4 trees). This indicated that the maximum number of keys should be 3 and the minimum number of keys should be $\lceil 4/2 \rceil - 1 = 1$. The maximum number of children is 4 and the minimum number of children is $\lceil 4/2 \rceil = 2$.



- i. No, the first tree is not a B-tree. This is because all the leaves nodes are not at the same level.
- ii. Yes, the second tree is also a B-tree of order 4. This is because it has a minimum number of 1 key in the root. Moreover, the leaf nodes have three keys each.
- iii. No, the third tree is not a B-tree of order 4 because it violates the condition of B-tree by having more than 3 keys on its leaf node.
- iv. No, the fourth tree is not a 2-3-4 tree because it trespasses the rule of B-tree and has more than the maximum number of keys on its leaf nodes.
- v. No, the last tree is not a 2-3-4 tree because the root node has four keys which is unacceptable in B-trees of order 4.

6. Show the B-tree of order 5 when inserting R,Y,F,X,A,M,C,D,E,T,H,V,L,W,G in this order.

Data: R, Y, F, X, A, M, C, D, E, T, H, V, L, W, G

Order: 5

Maximum number of keys = 4

Minimum number of keys = $\lceil 5/2 \rceil - 1 = (3-1) = 2$

Maximum number of children = 5

Minimum number of children = $\lceil 5/2 \rceil = 3$

B-Tree Question 6

Data: R, Y, F, X, A, M, C, D, E, T, H, V, L, W, G

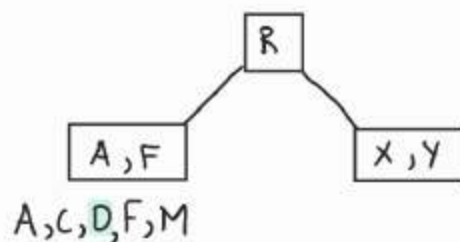
Order: 5 (Max 4 keys)

Step 1: First four keys are inserted into root.

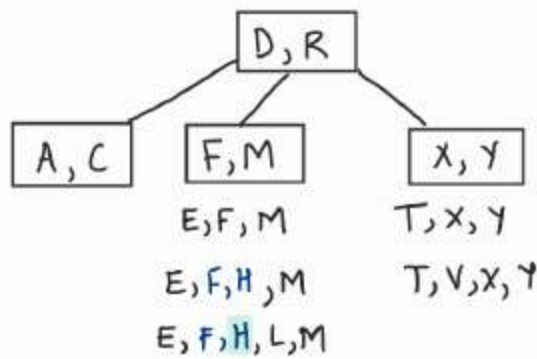
F, R, X, Y - Max 4 keys

Step 2: A, F, R, X, Y

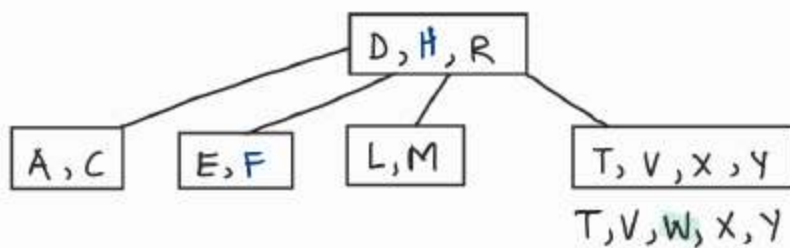
Inserting A. Splitting and promoting R.



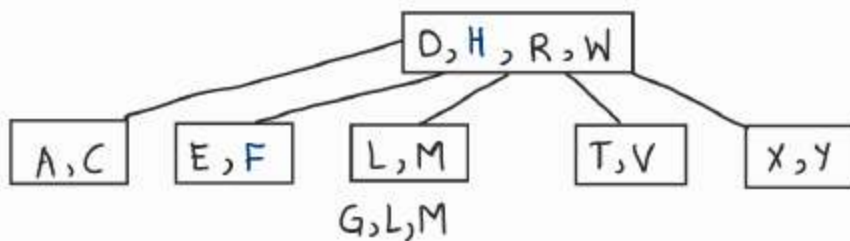
Step 4: Inserting M, C, D, E. Splitting A, C, D, F, M and promote D to root.



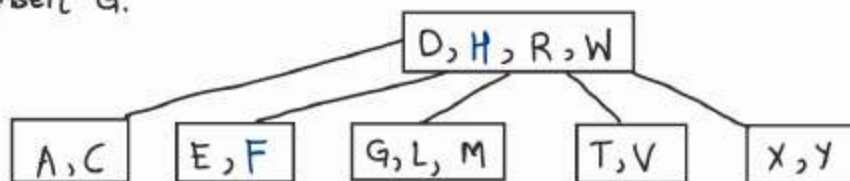
Step 5: Inserting E, T, H, V, L and splitting E, F, H, L, M and promoting H to root node to obtain D, F, R.



Step 6: Inserting W. Splitting T, V, W, X, Y and promoting W to root node to obtain D, H, R, W.



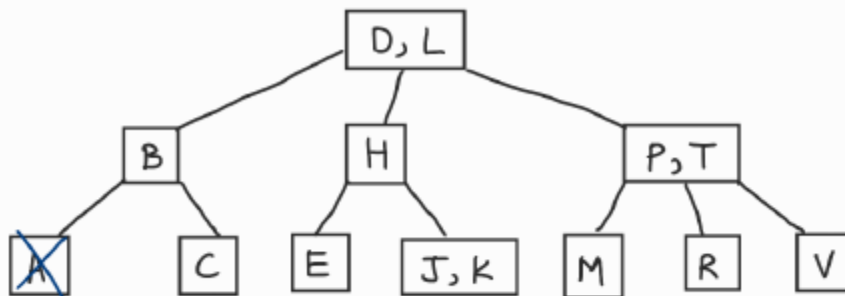
Step 7. Insert G.



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7. Show the B-tree of order 3, when deleting A, then deleting V and then deleting P from the following original B-Tree.

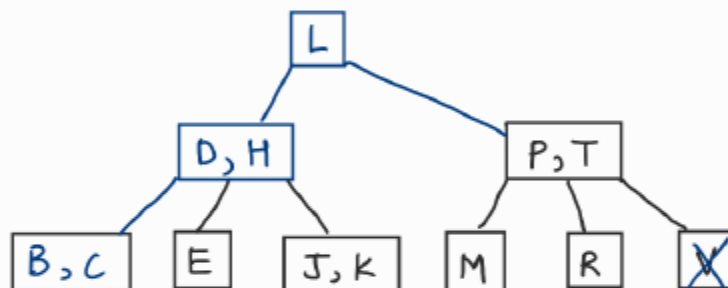
B-Tree Question 7



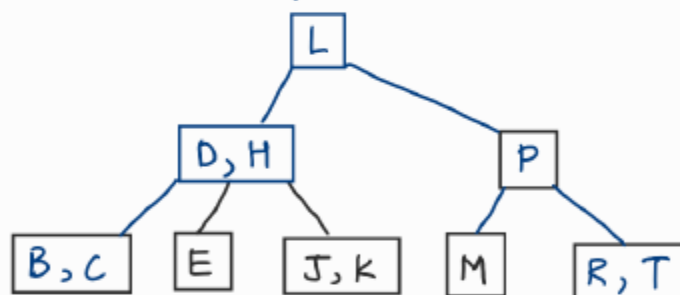
Order : 3

Delete A, V, P

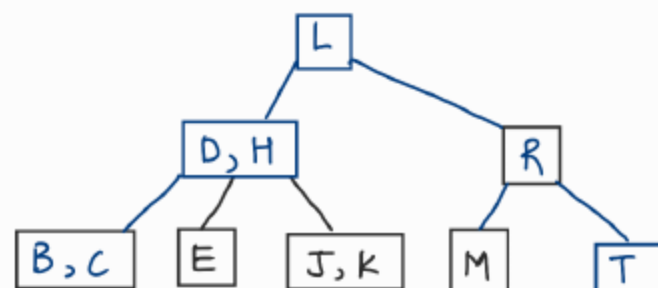
↓ Delete A (Borrow and Merge)



↓ Delete V (Borrow and Merge)



↓ Delete P (Inorder Successor: R)



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