

MATLAB-based laboratory assignments for Control System Engineering

EXERCISE 1: Basic Operations

Complete the table below in Matlab for the basic arithmetic operators shown: Use values of $a=4$ and $b=-2.1$

| Operation | Operator | Example | Result |
|-----------|----------|---------|--------|
| Plus | + | $a+b$ | |
| Minus | - | $a-b$ | |
| Multiply | * | $a*b$ | |
| Power | ^ | a^b | |
| Divide | / | a/b | |

EXERCISE 2: Vectors

- Enter the first three prime numbers as a row vector.
- Repeat (a), but use column vector.
- The following table shows some measurements a voltage across a resistor as a function of time.

| | | | | | | | | | | |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Time(s) | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 |
| Voltage (V) | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.4 | 1.3 | 1.2 | 1.1 | 1.0 |

- Enter the time as a column vector called *tim*.
- Enter the voltage as ac column vector called *vol*.
- Enter the command *plot (tim,vol)*.

Which variable is on the independent axis?

Which variable is on the dependent axis?

EXERCISE 3: Vector manipulation

Try the following vector operations and write down the results. Ensure that the dimensions of 'x' and 'y' are appropriate for the operations attempted. Note that some of the operators are different from the scalar operators.

| Operation | Operator | Example | Notes |
|-----------|----------|---------|------------------------|
| Plus | + | $x+y$ | Must be same dimension |
| Minus | - | $x-y$ | Must be same dimension |

| | | | |
|--|----|------------------|-------------------------------|
| Multiply | * | $x*y$ | Must be appropriate dimension |
| Multiply element by element | .* | $x.*y$ | Must be same dimension |
| Divide element by element; second vector elements by first vector elements | ./ | $x./y$ | Must be same dimension |
| Divide element by element; first vector elements by second vector elements | .\ | $x.\backslash y$ | Must be same dimension |

Explain in words the three operations: .* , ./ and .\.
Think of an example where we might need this type of operation.

EXERCISE 4: Polynomials

We wish to plot a second-order polynomial $y(t) = 4t^2 + 2t - 3$ as a function of time and check the roots of the equation. We can do this in two ways:

- 1) We can enter the polynomial coefficients as a vector and use the **roots** command.
- 2) We can plot the function 'y' against time and note the points where the function cross the time axis (that is, the point where $y = 0$). To do this, follow these steps.

- (a) Write Matlab code to create a vector 't' which has values evenly spaced from -5 to 5, in steps of 0.5.
- (b) Calculate $y(t) = 4t^2 + 2t - 3$ for all values of the time vector, t. Since t is a vector, think carefully how you will calculate $4t^2 + 2t - 3$.
- (c) Request a plot using **plot(t,y)**. Find the roots on the graph. Are the roots on the graph similar to the answer in item 1 above?

Exercise 5: Matrices

Find the solution of the following set of linear equations:

$$2x_1 + 5x_2 - 3x_3 = 6$$

$$3x_1 - 2x_2 + 4x_3 = -2$$

$$x_1 + 6x_2 - 4x_3 = 3$$

Hint: We write this in the matrix form, that is, $AX = B$, where

A is the matrix of coefficients of x_1 , x_2 and x_3

X is the column vector which will contain the solutions x_1 , x_2 and x_3

B is the column vector on the RHS

Answer: $X = A^{-1}B = ?$

EXERCISE 6: Functions

A sine wave of amplitude **5** and frequency **0.1 Hz** is applied to the input of an analog device which has a constant gain of **1.5** over all frequencies. Write Matlab code to calculate the output signal for the interval 0 to 10 seconds in increments of 0.1 seconds.

Use the command *plot(t,y)* to see a graph of this signal. What is the frequency of the sine wave on the plot? (Use the grid command to help work this out (approximately)).

Does it agree with the frequency of 0.1 Hz given in the question?

EXERCISE 7:

- (a) Enter the complex number $3 + j4$ in Matlab.
- (b) Find the modulus and angle of this complex number.
- (c) Convert the calculated magnitude to dB and the value of angle to degrees.
- (d) Write down the Matlab commands that were entered.

EXERCISE 8: Plotting

The input-output relationship of a n RC circuit can be represented by the following complex function:

$$g(j\omega) = V_o/V_i = K/(j\tau\omega+1)$$

Where K is the gain and τ is the time constant.

(a) For $K=10$ and $\tau = 5$, write Matlab code to calculate the gain and phase of V_o/V_i for $\omega = 0.01, 0.05, 0.1, 2, 5, 10$ rad/s.

(a) Plot the gain and phase using the *semilogx* command.

Hints:

- 1. Enter the values of K and τ as K and tau. Define a frequency vector, w.
- 2. Calculate $g = K ./ (j*\tau*w+1)$. Note the operation ./.
- 3. Calculate the gain in dB and the phase in degrees.
- 4. Plot the response using *semilogx(w, gain)* and *semilogx(w, phase)*.
- 5. Label the axes and insert a title for the plot.

EXERCISE 9: Given the following transfer functions:

$$g_1(s) = \frac{3(s+1)}{s^2+3s+1}, \quad g_2(s) = \frac{2s+3}{s^3+1}$$

- (a) Enter $g_1(s)$ using row vectors
- (b) Enter $g_2(s)$ using the *s=tf('s')* notation. Record from the screen the MATLAB commands that were entered.

- (c) Find the roots of the numerator and denominator polynomials using the `roots` command.
- (d) Compare with the results using the `zero` and `pole` commands.
- (e) Examine where the roots are using the `pzmap` function.

Exercise 10:

1) Create a time vector t with points spaced at 0.1 seconds between 0 and 10 seconds. Plot the function $y = 2 \sin t + 3 \cos t$ and use the `max` function to find the maximum value of the function. Verify using the cursor.

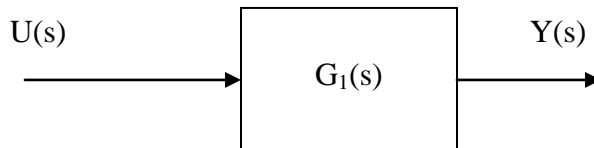
2) write Matlab code to find the roots of the following polynomial equations:

- a) $s^3 + 2s^2 + 1 = 0$
- b) $p^3 + 1 = 0$
- c) $-3s^2 - 5s + 2 = 0$

3) For the following transfer function descriptions, enter the Matlab commands to provide the appropriate transfer function expression.

- a) System G_A has a unity gain at $\omega = 0$ rad/sec, a pole at -3 and a zero at -4 .
- b) System G_B has a gain of 10 at $\omega = 0$ rad/sec, a zero at -1 and two poles at $-1+j0.5$ and $-1-j0.5$.
- c) System G_C has an infinite gain at $\omega = 0$ rad/sec, a zero at -6 , two poles at the origin and a pole at -4 .

4) For the following system, where $G_1(s) = 1/(\tau s + 1)$ and $U(s)$ is a unit step input, write an M-file to plot the output $y(t)$ for the three cases $\tau = 2$, $\tau = 4$ and $\tau = 6$ seconds. Plot the three responses on the same graph.



5) Given the following transfer functions:

$$g_1(s) = \frac{s^2 + 2s + 1}{s^3 + 3s^2 + 1}, \quad g_2(s) = \frac{2s^2 - 3}{(s^3 + 1)(s^2 + s + 1)}$$

- a) Enter $g_1(s)$ using row vector.
- b) Enter $g_2(s)$ using the `s = tf('s')` notation. Record from the screen the Matlab commands that were entered.
- c) Write Matlab code to find the zeros of the numerator and denominator of $g_1(s)$ using the `roots` command.

- d) Write Matlab code to compare with the results using the **zero** and **pole** commands. Examine where the roots are using the **pzmap** function.
- e) Write Matlab code to find the zeros and poles of $g_2(s)$ using appropriate commands. Use the **pzmap** function to ascertain if the system is stable.

6) Write an M-file to solve the following set of equations:

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

7) The Laplace transform of a time delay is e^{-Ts} . The following transfer function has a time delay of 2 seconds.

$$G(s) = (10 e^{-2s}) / (5s+1)$$

- a) Enter a transfer function g_1 which represents $g(s)$ without the time delay.
- b) Enter **get(g_1)**; we see in the list of attributes of g_1 when input delay is zero.
- c) Use the following commands to enter $g(s)$:

g = g1;
set (g, 'inputdelay',2)

d) Plot the step response of the system with and without the time delay and compare the results.

[11] Write a Matlab function to check whether the number N is prime. (Hint: investigate the **mod** function)

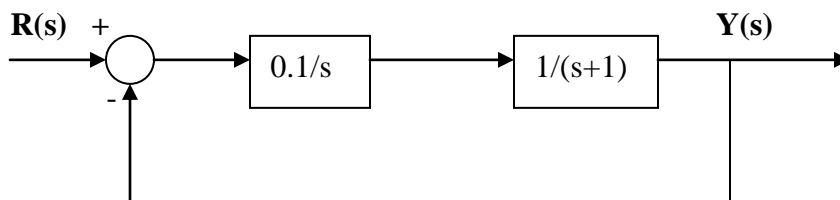
[12] The input-output relationship of a servo system can be represented by the following complex function:

$$G(j\omega) = \frac{\theta}{V_i} = \frac{K}{j\omega(\tau j\omega + 1)}$$

Where K is the gain and τ is the time constant.

- a) For $K=10$ and $\tau = 2.5$, calculate the gain and phase of $\frac{\theta}{V_i}$ for $\omega = 0.01, 0.05, 0.1, 2, 5, 10$ (rad/sec).
- b) Plot the gain and phase using the **semilogx** command.

[13] Write an M-file to find and plot the response of the following system to an input signal of $R(s) = 10/s$.



[14] Plot the frequency response of the following system for $K=1$.

$$G(s) = \frac{10K}{s(s+5)}$$

- a) Determine the frequency at which the magnitude plots crosses the 0 dB line.
- b) What value of K would ensure that the magnitude plot crosses 0 dB at 5 rad/s?

[15] Write an M-file which enters the following transfer function and plots the two step responses corresponding to $k=1$ and $k=2$ on the same plot.

$$g1(s) = \frac{k}{(s+1)}.$$

[16] For a unity feedback system with $G(s) = \frac{7}{(s^2 + 4s - 2)}$. Write Matlab code to determine transient and steady-state specifications. Also implement the same using unity-feedback system in SIMULINK.

[17] Consider a system described by the following differential equations:

$$Y1(s) = \frac{2}{3s+1}U1(s)$$
$$Y2(s) = 6Y1(s) + \frac{5}{6s+1}U2(s)$$

- (a) Implement the model in simulink.
- (b) Plot $y2(t)$ for $u1(t)=u2(t)=1$.

[18] Write an M-file to find the transient response specifications to a step input of magnitude 3 and also plot the response for the following systems:

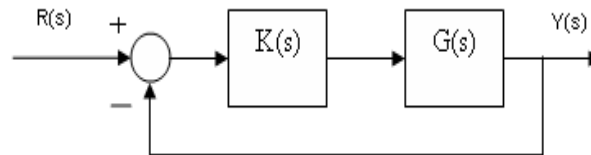
$$(a) G(s) = \frac{10}{2s+1} \quad (b) G(s) = \frac{3}{s+4}$$

[19] An open-loop system is given by

$$G(s) = \frac{(2s+1)}{(0.5s+1)(12s+1)}$$

The system is placed in a unity-feedback loop with a controller $\mathbf{K(S)} = \frac{(10s + 1)}{s}$ in the forward path. Using MATLAB, determine the closed-loop poles and zeros. Comment on the stability of the system.

[20] The figure below shows a typical closed-loop system with the system transfer function $G(S)$ and the controller transfer function $K(s)$.



$$G(s) = \frac{6}{(s + 1)}.$$

Write Matlab code to determine the close-loop transfer function and the steady-state error for the controller $K(s)$ being:

- (a) A P controller
- (b) PD controller
- (c) PI controller
- (d) PID controller. Comment on the error produced for each controller.

[21] For the following system write MATLAB code to plot the root locus.

$$(a) \quad G(s) = \frac{(s - 4)}{(s + 1)(s + 2)(s + 3)}. \quad (b) \quad G(s) = \frac{K}{s(s + 2)}, K = 1.$$

[22] For the following systems

$$(a) \quad G(s) = \frac{1}{(s^2 + 0.6s + 1)} \quad (b) \quad G(s) = \frac{400}{(s + 10)(s^2 + 0.4s + 4)}$$

Write MATLAB code to plot the frequency response and determine stability margins using:

- (a) Bode plot
- (b) Nichols plot
- (c) Nyquist plot.

Comment on stability of the systems.