Practical No. 1

Limit and continuity

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt{2x}} \right]$

2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \frac{\pi}{6}} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5) Examine the continuity of the following function at given points.

i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \frac{\pi}{2}$ } at $x = \frac{\pi}{2}$

ii) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x < 6$

$= \dfrac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

} at $x=3$ and $x=6$

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6) Find value of k, so that the function $f(x)$ is cts at the indicated Point.

i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $\quad x < 0$

$= k$ $\quad x = 0$ $\quad \Big\}\ x = 0$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$

$= k$ $\quad x = 0$ $\quad \Big\}$ at $x = 0$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \dfrac{\pi}{3}$

$= k$ $\quad x = \dfrac{\pi}{3}$ $\quad \Big\}$ at $x = \dfrac{\pi}{3}$

7) Discuss the Continuity of the following functions which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity.

i) $f(x) = \dfrac{1-\cos 3x}{x \tan x}$ $\quad x \neq 0$

$= a$ $\quad x = 0$ $\quad \Big\}$ at $x = 0$

ii) $f(x) = \dfrac{(e^{3x}-1)\sin x}{x^2}$ $\quad x \neq 0$

$= \dfrac{\pi}{60}$ $\quad x = 0$ $\quad \Big\}$ at $x = 0$

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Q) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x = 0$ find $f(0)$.

Q) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$ is cts at $x = \frac{\pi}{2}$. Find $f\left(\frac{\pi}{2}\right)$.

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→ 1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{(a+2x-3x) \cdot (\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x) \; (\sqrt{a+2x} + \sqrt{3x})} \right]$

$\lim_{x \to a} \frac{(a-x) \cdot (\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x) \cdot (\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x) \cdot (\sqrt{3a+x} + 2\sqrt{x})}{(a-x) \; (\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \cdot \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \; \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$

$\frac{1}{3} = \frac{2}{3\sqrt{3}}$

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→ 2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}.(\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{y}{y\sqrt{a+y}.(\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}.(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}.(\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}.2\sqrt{a}}$$

$$= \frac{1}{2a}$$

→ 3) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$$x = h + \frac{\pi}{6}$$

where $h \to 0$

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$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h.\cos\frac{\pi}{6} - \sin h.\sin\frac{\pi}{6} - \sqrt{3}\sin h\cos\frac{\pi}{6} + \cos h\sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h.\frac{\sqrt{3}}{2}.\sin\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{3}}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{12h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}\times 1$$

$$= \frac{1}{3}$$

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→4) $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

By rationalizing Numerator and Denominator both

$$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$$

$$\lim_{x \to \infty} \left[\frac{(x^2+5 - x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3 - x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

$$\lim_{x \to \infty} \frac{8}{2} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x \to \infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-\frac{3}{x^2})}}$$

After APPlying limit
we get,

$= 4$

→5) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$

at $x = \frac{\pi}{2}$

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$\Rightarrow f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$

$\therefore f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define

ii. $\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting Method

$x - \frac{\pi}{2} = h$

$x = h + \frac{\pi}{2}$

where $h \to 0$

$\lim_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim_{h \to 0} \dfrac{\cos(h + \frac{\pi}{2})}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$

$\lim_{h \to 0} \dfrac{\cos(h + \frac{\pi}{2})}{-2h}$

$\lim_{h \to 0} \dfrac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$

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$$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

6.i.) $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2} \cancel{\sin x}}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \frac{\pi}{2}^-} \cos x$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x = \frac{\pi}{2}$

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ii. $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \dfrac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x = 3$ and $x = 6$

at $x = 3$

i. $f(3) = \dfrac{x^2-9}{x-3} = 6$

f at $x = 3$ define

ii. $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (x+3) = 6$

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \dfrac{x^2-9}{x-3}$

L.H.L = R.H.L

f is continuous at $x = 3$

for $x = 6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = \dfrac{27}{9} = 3$

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2. $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

function is not continuous

6 i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x < 0$

$= k \quad x = 0$ $\Big\}$ at $x = 0$

$\Rightarrow$ f is continuous at $x = 0$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

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$$2 \lim_{x \to 0} \frac{\sin^2 x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x} \right)^2 = k$$

$$2 (2)^2 = k$$

$$\therefore \quad k = 8$$

ii. $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$

$= k \quad x = 0$ } at $x = 0$

sol^n: $f(x) = (\sec^2 x)^{\cot^2 x}$

$$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$$

We know that

$$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$$

$$\therefore \quad = e$$

$$\therefore \quad k = e$$

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iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \frac{\pi}{3}$, $x = \frac{\pi}{3}$ $\Big\}$ at $x = \frac{\pi}{3}$

$= k$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$$f\left(\frac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + h\right)}{\pi - 3\left(\pi/3 + h\right)}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}(1 - \tan\frac{\pi}{3} \cdot \tan h) - \tan\left(\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

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$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3(1 - \sqrt{3}\tan h)}$$

$$\lim_{h \to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3} \frac{1}{(1 - \sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right)$$

$$= \frac{4}{3}$$

7.

i. $f(x) = 1 - \cos 3x \quad x \neq 0$

$= a \quad x = 0$ } at $x = 0$

$$f(x) = \frac{1 - \cos 3x}{x \tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

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$$\lim_{x \to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x - \dfrac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$\lim_{x \to 0} f(x) = \frac{9}{2} \qquad g = f(0)$$

$\therefore$ f is not continuous at $x = 0$

Redefine function.

$$f(x) = \begin{cases} \dfrac{1 - \cos 3x}{x \tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\cancel{\lim_{x \to 0}} f(x) = f(0)$

f has removable discontinuity at $x = 0$

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7.ii $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$

$= \dfrac{\pi}{6}$ $\quad x = 0$ $\Big\}$ at $x=0$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim\limits_{x\to 0} \dfrac{e^{3x}-1}{x} \lim\limits_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim\limits_{x\to 0} 3.\dfrac{e^{3x}-1}{3x} \lim\limits_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3\lim\limits_{x\to 0} \dfrac{e^{3x}-1}{3x} \lim\limits_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \log e . \dfrac{\pi}{180} = \dfrac{\pi}{6} = f(0)$

$\therefore$ f is continuous at $x=0$

8. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $\quad x=0$

is continuous at $x=0$

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$\therefore$ Given,

f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 + \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x} \right)^2$$

Multiply with 2 on Num and Denominator

$$= 1 + 2 \times \frac{1}{4}$$

$$= \frac{3}{2}$$

$$= f(0)$$

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9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \frac{\pi}{2}$

$f(x)$ is continuous at $x = \frac{\pi}{2}$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

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Practical 2

Q1. Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

1) $\cot x$

$$f(x) = \cot x$$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x - a) \cdot \tan x \cdot \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(\alpha + h - \alpha) \cdot \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}.$$

formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{(-\alpha - h) - (1 + \tan a \cdot \tan(a+h))}{h \times \tan(a+h) \tan a}.$$

$$= \lim_{h \to 0} \frac{-\tan h \times 1 + \tan a \cdot \tan(a+h)}{\tan(a+h) \tan a}.$$

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$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\tan \sin^2 a}$$

$$= \operatorname{cosec}^2 a.$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)(\sin a \sin(a+h))}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a.$$

ii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cdot \cos a \cdot \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

Formula : $-2\sin\left(\frac{c+D}{2}\right) \cdot \sin\left(\frac{c-D}{2}\right)$

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$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin\frac{-h}{2}}{\cos a \cdot \cos(a+h) \times -\frac{h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= -\frac{1}{2} \times 2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a.$$

Q.2. If $f(x) = 4x+1$, $x \leq 2$
$= x^2+5$ $x > 0$ at $x = 2$, then
Find function is differentiable or not.

Solution:

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

Q.3

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$Df(2^-) = 4$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x = 2$

Q3 If $f(x) = 4x+7$, $x < 3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$, then find f is differentiable or not.

Solution:

RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1 - (3^2+3+3+1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) \quad x^2+6x-3x-18}{x-3}$$

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$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$$= 3+6$$

$$= 9$$

$$Df(3^+) = 9$$

$$LHD = Df(3^-)$$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$Df(3^+) = 4$$

$$RHD \neq LHD$$

f is not differentiable at $x = 3$

Q.4. If $f(x) = 8x - 5$, $x \leq 2$
$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$ Then
Find f is is differentiable or not.

Sol^n:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

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$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 3 \times 2 + 2 = 8$$

$$Df(2^+) = 8$$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 8$$

$$Df(2^-) = 8$$

LHD = RHD

f is differentiable at $x = 3$

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Practical 3

Topic: Application of Derivative.

1) Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

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solutions:

Q1) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$\cancel{3(x^2}$ $3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{\sqrt{3}}}$

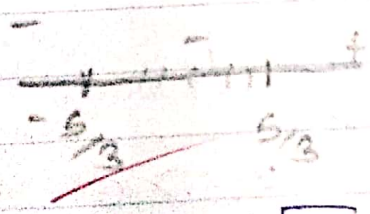
$x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \pm\sqrt{\frac{5}{3}}$



$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

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ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

f is increasing iff $f(x) > 0$

$2x - 9 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x > 2$

$x \in (2, \infty)$

$\therefore f$ is decreasing iff $f(x) < 0$

$2x - 9 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$

$x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

-2 $\quad$ $10/6$

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$\therefore x \in (-\infty, -2) \cup (\frac{10}{6}, \infty)$

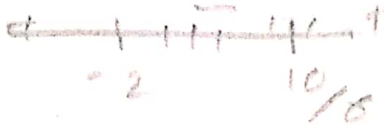
f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x+2) < 0$



$x \in (-2, \frac{10}{6})$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

$f(x)$ is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$x \in (-3, 3)$

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v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

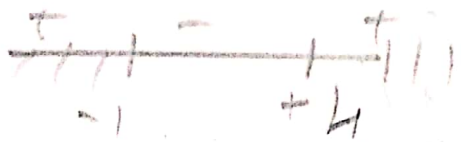
f is increasing iff $f'(x) > 0$

$\therefore \; 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$



$x \in (-\infty, -1) \cup (4, \infty)$

f is increasing iff $f'(x) < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x+1)(x-4) < 0$

$\therefore \; x \in (1, 4)$

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62. i) $y = 3x^2 - 2x^3$

Let,

$f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1-2x)$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1-2x > 0$

$-2x > -1$

$2x < 1$

$x < \frac{1}{2}$

$x \in (-\infty, \frac{1}{2})$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$x > \frac{1}{2}$

$x \in (\frac{1}{2}, \infty)$

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ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\Rightarrow$ Let $f(x) = y$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

f is concave upward iff $f''(x) >$

$12(x^2 - 3x + 2) > 0$

$x^2 - x - 2x + 2 > 0$

$(x-2)(x-1) > 0$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

f is concave downward iff $f''(x) <$

$12(x^2 - 3x + 2) < 0$

$x^2 - x - 2x + 2 < 0$

$(x-2)(x-1) < 0$

$\therefore x \in (1, 2)$

iii) $y = x^3 - 27x + 5$

$\Rightarrow$ Let $f(x) = y$

$\therefore f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave upward iff $f''(x) >$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore \; x \in (-\infty, 0)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

$\Rightarrow$ Let $f(x) = y$.

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$= 12x - 18$

$= 6(2x - 3)$

$f''(x)$ is concave upward iff $f''(x) > 0$

$6(2x-3) > 0$

$2x - 3 > 0$

$x > 3/2$

$x \in (3/2, \infty)$

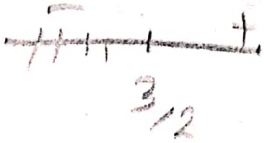
$f''(x)$ is concave downward iff $f''(x) < 0$

$6(2x-3) < 0$

$2x - 3 < 0$

$x < 3/2$

$\therefore \; x \in (-\infty, \frac{3}{2})$



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v) $y = 2x^3 + x^2 - 20x + 4$

$\Rightarrow$ Let $f(x) = y$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x+1)$

$f''(x)$ is concave upward iff $f''(x) > 0$

$2(6x+1) > 0$

$6x+1 > 0$

$x > -1/6$

$-x < 1/6$

$x \in (-1/6, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$2(6x+1) < 0$

$6x+1 < 0$

$x < -1/6$

$x > 1/6$

$\therefore x \in (-\infty, -1/6)$

Q.1.

i)

ii)

iii)

iv)

Q.2

i)

ii)

iii)

iv)

#1
20/4/15

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Practical No: 4

Topic: Application of Derivative of Newton's Method.

Q.1. Find Maximum and Minimum value of following Functions.

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ is $[-\frac{1}{2}, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2. Find the root of following equation by Newton's Method. (Take 4 iteration only). Correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 5.5x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

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Solutions:

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$\therefore \quad 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

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$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$ and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$\therefore f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

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$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

ii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

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$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

ii) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore$ $6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore$ $x = 2$ or $x = -1$

$\therefore$ $f''(x) = 12x - 6$

$\therefore$ $f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore$ $f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

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$\therefore$ f has maximum value at $x = -1$.

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + (12 + 1)$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and
f has minimum value -19 at $x = 2$

Q.2.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0$ .. Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

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$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9461}$$

$$= 0.1712$$

and

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$

$$= 0.0060 - 0.0879 - 9.416 + 9.5$$

$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$

$$= 0.0879 - 1.0272 - 55$$

$$= -55.9393$$

$$\therefore \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 + \frac{0.0011}{55.9393}$$

$$= 0.1712$$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 9$

$$f(2) = 2^3 - 4(2) - 9$$

$$= 8 - 8 - 9$$

$$= -9$$

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$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392$$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.6525 - 10.9568 - 9$

$= 0.696$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.696}{18.5096}$$

$$= 2.7071$$

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$f(x_2) = (2.707)^2 - 4(2.707) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.707)^2 - 4$

$= 21.9861 - 4$

$= 17.9861$

$\therefore \quad x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \dfrac{0.0102}{17.9861}$

$= 2.7071 - 0.0056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + \dfrac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$

$= 2.7065$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17 \qquad [1,2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

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$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2 - \frac{2.2}{5.2}$$

$= 2 - 0.4230$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4769 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6755}{8.2164}$$

$= 1.577 + 0.0822$

$= 1.6592$

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$f(x_2) = (1.6592)^2 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.6677 - 4.853 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.6592 + \frac{0.0204}{7.7143}$$
$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$$x_4 = \cancel{x_3 - \frac{f(x_3)}{f'(x_3}}$$
$$= 1.8618 + \frac{0.0004}{7.6977}$$
$$= 1.6618$$

$\therefore$ The root of equation is 1.6618.

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Practical 5

Integration.

Q.1. Solve the following integration

i) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^{7} \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \dfrac{1}{x^3} \sin\left(\dfrac{1}{x^2}\right) dx$

viii) $\int \dfrac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx$

x) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right) dx$

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i) $\int \frac{1}{x^2+2x-3} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

substitute

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln\left(|x + \sqrt{x^2-a^2}|\right)$

$= \ln\left(|t + \sqrt{t^2-4}|\right)$

$t = x+1$

$= \ln\left(|x+1 + \sqrt{(x+1)^2-4}|\right)$

$= \ln\left(|x+1 + \sqrt{x^2+2x-3}|\right)$

$= \ln\left(|x+1 + \sqrt{x^2+2x-3}|\right) + c$

2) $\int (4e^{3x}+1) \, dx$

$= \int 4e^{3x} \, dx + \int 1 \, dx$

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F: $\int e^{ax}\,dx = \frac{1}{x} \times e^{ax}$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

5)

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

F: $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10 \times \sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\,dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$

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$$= \frac{x^{5/2}+1}{5/2+1}$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$$

5) $\int t^7 \times \sin(2t^4)\, dt$

Put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\, du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\, du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du = \frac{t^4 \times \sin(2t^4)}{8}\, du$$

substitute t^4 with $\frac{u}{2}$

$$= \int \frac{u/2 \times \sin(u)}{8}\, du$$

$$= \int \frac{u \times \sin(u)}{2} \Big/ 8\, du$$

$$= \int \frac{u \times \sin(u)}{16}\, du$$

$$= \frac{1}{16} \int u \times \sin(u)\, du$$

$\int u\,dv = uv - \int v\, du$

Where $u = u$

$dv = \sin(u) \times du$

$du = 1\, du$

$v = -\cos(u)$

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$$= \frac{1}{16} \quad (u \times (-\cos(u)) - \int -\cos(u)\, du$$

$$\int \cos x \, dx = \sin(x)$$

$$= \frac{1}{16} \times \left(u \times (-\cos u) + \sin(u) \right)$$

Return the substitution $u = 2t^4$

$$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4) \right)$$

$$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$$

vi) $\sqrt{x}\,(x^2 - 1)\, dx$

$$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$$

$$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$$

$$= \int x^{5/2} - x^{1/2}\, dx$$

$$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$$

$$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7}$$

$$= \frac{2 \cdot x^3 \sqrt{x}}{7}$$

$$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$$

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vii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} \, dx$

$= \int \frac{\cos x}{\sin x^{2/3}} \, dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)} \, dt$

$= \frac{1}{\sin x^{3/2}} \, dt$

$= \frac{1}{t^{2/3}} \, dt$

$I = \int \frac{1}{t^{2/3}} \, dt = \frac{-1}{(2/3 - 1) t^{2/3 - 1}} = \frac{-1}{(2/3 - 1) t^{2/3 - 1}}$

$= \frac{-1}{-1/3 \, t^{2/3 - 1}} = \frac{1}{1/3 \, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3 t^{1/3}$

$= 3 \sqrt[3]{t}$

Return substition $t = \sin x$

$= 3 \sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \, dx$

Put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} \, dt$

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$$= \int \frac{3x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x} \, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3} \, dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)} \, dt = \int \frac{1}{3t} \, dt$$

$$= \frac{1}{3} \int \frac{1}{t} \, dt \qquad \int \frac{1}{x} \, dx = \ln|x|$$

$$= \frac{1}{3} \times \ln|t| + c$$

$$= \frac{1}{3} \times \ln(x^3 - 3x^2 + 1) + c$$

AK
03/01/2020

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Practical No. 6

Application of Integration and Numerical Integration.

Q.1. Find the length of the following curve.

i) $x = t\sin t$, $y = 1 - \cos t$ $\quad t \in [0, 2\pi]$

→ $x = t\sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dx} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2(1-\cos t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{4\sin^2 t/2}\, dt$$

$$= \int_0^{2\pi} \sin t/2\, dt$$

$$= 2\left[\cos \frac{t}{2}\right]_0^{2\pi}$$

$$= 2\left[-\cos \pi + \cos 0\right]$$

$$= 4$$

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2) $y = \sqrt{4-x^2} \qquad x \in [-2, 2]$

$\Rightarrow$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$y = \sqrt{4-x^2}$$

$$\therefore \frac{dy}{dx} = 2\int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}(x/2)\right]_0^2$$

$$= 2\pi$$

3) $y = x^{3/2}$ in $[0, 4]$

$\Rightarrow$ $f'(x) = 3/2\, x^{1/2}$

$$[f'(x)]^2 = 9/4\, x$$

$$L = \int_0^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9x}{4}}\, dx$$

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$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\int_0^4 \frac{(4+9x)^{1/2}}{1/2 + 1.9}\, dx$$

$$= \frac{1}{27}\left[[4+0]^{3/2} - [4+36]^{3/2}\right]$$

$$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$$

4) $x = 3\sin t \qquad y = 3\cos t$

$\Rightarrow \frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9}\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[t]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

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5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $\qquad y \in [1,2]$

$$\Rightarrow \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12}$$

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Q.2. Using Simpson's Rule solve the following.

1) $\int_0^2 e^{x^2} dx$ with $n=4$

$\Rightarrow$ $a=0$, $b=2$, $n=4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.6877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule.

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1+54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$$

$$= 1.779$$

2) $\int_0^4 x^2 \, dx$

$\Rightarrow$ $L = \frac{4-0}{4} = 1$

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x	0	1	2	3	4
y	0	1	4	9	16

$$= \int_0^4 x^2\,dx = \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= 64/3$$

$$= \int_0^4 x^2\,dx = 21.533$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \qquad n = 6$

$\Rightarrow$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$\frac{4\pi}{18}$	$6\pi/18$	$\frac{8\pi}{18}$
y	0	0.4166	0.58	0.70	0.8087	0.8722	0.9908

$$\Rightarrow \int_0^{\pi/3} \sqrt{\sin x} = \frac{\pi}{54} \times 12.1163$$

$$= 6.7049$$

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Practical no. 7

Differential Equation. 61

Q1. solve the following differential equation.

1) $x \frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x \, dx}$

$= x$

$y(IF) = \int Q(x)(IF) \, dx + c$

$= \int \frac{e^x}{x} \cdot x \, dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\rightarrow \frac{dy}{dx} + \frac{2e^x y}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

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$P(x) = 2$ $\qquad Q(x) = e^{-x}$

$$\int P(x)\,dx$$

$$IF = e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int e^{-x} \cdot e^{2x}\,dx + c$$

$$= \int e^{x}\,dx + c$$

$$y \cdot e^{2x} = e^{x} + c$$

3) $x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$$\Rightarrow x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$P(x) = 2x$ $\qquad Q(x) = \dfrac{\cos x}{x^2}$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2x\,dx}$$

$$= x^2$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int \frac{\cos x}{x^2} - x^2\,dx + c$$

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$$= -\int \cos x + c$$

$$x^2 y = \sin x + c$$

4) $x \dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{3y}{x} = \dfrac{\sin x}{x^3}$

$P(x) = 3/x \qquad Q(x) = \sin x / x^3$

$P(x) = \int 3/x \, dx$

$= x^3$

$IF = e^{\int P(x) dx}$

$= x^3$

$Y(IF) = \int Q(x)(IF) \, dx + c$

$= \int \dfrac{\sin x}{x^3} \cdot x^3 \, dx + c$

$= \int \sin x + c$

$x^3 y = -\cos x + C$

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5) $e^{2x}\dfrac{dy}{dx} + 2e^{2x}y = 2x$

$\Rightarrow \dfrac{dy}{dx} + 2y = \dfrac{2x}{e^{2x}}$

$P(x) = 2$ $\qquad Q(x) = 2x/e^{2x}$
$= 2xe^{-2x}$

$If = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx} = e^{2x}$

$y(If) = \int Q(x)(If)\,dx + c$

$= \int 2xe^{-2x}\,e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

(vi) $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\dfrac{\sec^2 x}{\tan x}\,dx = \dfrac{-\sec^2 y}{\tan y}\,dy$

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$$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$$

$$\log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x - \tan y| + c$$

$$\tan x \cdot \tan y = e^c$$

(Ex 11) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1=u$

Differentiating BTS.

$$x-y+1 = u$$

$$1 - \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{1-du}{dx} = dy/dx$$

$$1 - \frac{du}{dx} = \sin^2 u$$

$$\frac{du}{dx} = 1 - \sin^2 u$$

$$\frac{du}{dx} = \cos^2 u$$

$$\frac{du}{\cos^2 u} = dx$$

$$\int \sec^2 u\, du = \int dx$$

$$\tan u = x + c$$

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$\tan(x+y-1) = x+c$

(viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+c}$$

Put $2x+3y = u$

$$2 + \frac{3dy}{dx} = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{du}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{du}{dx} - 2\right) = \frac{1}{3}\frac{(u-1)}{(u+2)}$$

$$\frac{du}{dx} = \frac{u-1}{u+2} + 2$$

$$\frac{du}{dx} = \frac{u-1+2u+4}{u+2}$$

$$= \frac{2u+3}{u+2}$$

$$= \frac{3(u+1)}{u+2}$$

$$= \int\left(\frac{u+2}{u+1}\right) du = 3dx$$

$$\int \frac{u+1}{u}\, dx = \int \frac{1}{u+1}\, du = 3x$$

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$$u + \log|u| = 3x + c$$

$$2x + 3y + \log|2x + 3y + 1| = 3x + c$$

$$3y = x - \log|2x + 3y + 1| + c$$

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10/9/2020

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Practical No. 8

Topic : Euler's Method.

Q.1. $\frac{dy}{dx} = y + e^{x} = e$

$f(x, y) = y + e^{x} - 2$, $y_0 = 2$, $x_0 = 0$ $h = 0.5$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.6	2.487	3.57435
2	1	3.5743	4.2927	5.3615

$y_{n+1} = y_n + h f(x_n, y_n)$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	8.3615	7.8431	9.28305
4	2	9.8431		

$\therefore$ By Euler's formyla.

$y(2) = 9.2831$

Q.2. $\frac{dy}{dx} = 1 + y^2$

$f(x, y) = 1 + y^2$, $y_0 = 0$ $x_0 = 0$ $h = 0.2$

Using Euler's iteration formyla

$y_{n+1} = y_n + h f(x_n, y_n)$

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n.	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	161.64	0.408
2	0.4	0.408	1.1664	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula

$y(1) = 1.2942$

Q.3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1 \quad x_0 = 0, \quad h = 2$

Using Euler's iteration formula.

$$y_{n+1} = y_n + h f(x_n, y_n)$$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

$y(1) = ?$

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Q.4. $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$, $x_0=1$ $h=0.25$

for $h=0.5$

Using Euler's iteration formula.

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

$\therefore$ By Euler's formula.

$y(2) = 28.5$

for $h=0.25$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.35
3	1.75	6.3359	10.1815	8.50
4	2	8.9048		
50				

By Euler's formula

$y(2) = 8.9048$

Q.5. $\frac{dy}{dx} = \sqrt{xy} + 2$ $y_0 = 1$ $x_0 = 1$ $h = 0.2$

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Using Euler's formula iteration

$$y_{n+1} = y_n + h f(x_n, y_n)$$

h	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula.

$y(1.2) = 1.6$

17/01/2020

Practical No. 9

Limits and Partial order derivatives.

Q.1.

i) $\lim_{(x,y) \to (4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{4(-1) + 5}$

$= \dfrac{-64 + 3 + 1 - 1}{4 + 5}$

$= \dfrac{-61}{9}$

ii) $\lim_{(x,y) \to (2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{(x + 3y)}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \dfrac{(0+1)((2)^2 + 0 - 4(2))}{2+0}$

$= \dfrac{1(4 + 0 - 8)}{2}$

$= \dfrac{-4}{2}$

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ii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2-z^2}{x^3-x^2yz}$

At (1, 1, 1), Denominator = 0

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{(x+yz)\cancel{(x-yz)}}{(x-yz)\,x^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On Applying limit

$= \dfrac{1+1(1)}{(1)^2}$

$= 2$

Q.2.

i) $f(x,y) = xye^{x^2}+y^2$

$\therefore f_x = \dfrac{d}{dx}(f(x,y))$

$= \dfrac{d}{dx}(xye^{x^2}+y^2)$

$= ye^{x^2}+y^2(2x)$

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$\therefore f_x = 2xye^{x^3+y^2}$

$f_y = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{d}{dx}(f(x,y))$

$\frac{d}{dx}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{d}{dy}(f(x,y))$

$\therefore \frac{d}{dy}(e^x \cos y)$

$f_y = -e^x \sin y$

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ii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q.3.

i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x\frac{d}{dx}(1+y^2)}{(1+y^2)}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

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$$\frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{(1+y^2)(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)(2)}$$

$$= 0$$

Q.4.

1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

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$$f_x = \frac{x^2 \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{d}{dx}\left(-x^2y - 2x(xy^2 - xy)\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{d}{dx}x^4}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6} \quad \ldots (1)$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad \ldots (2)$$

$$f_{xy} = \frac{d}{dy}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x}\right) \quad \ldots (3)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{d}{dx}\frac{(2y - x)}{x^2}$$

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$$= \frac{x^{4}\frac{d}{dx}(2y-x) - (2y-x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{-x^4 - 4xy + 2x^2}{x^4} \quad \ldots \quad (4)$$

from (3) and (4);

$f_{xy} - f_{yx}$

i) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{d}{dx}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{d}{dy}(x^2 + 3x^2y^2 - \log(x^2+1))$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\frac{d}{dx}(2x) - 2x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}\right)$$

$$6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \ldots \quad (1)$$

$f_{yy} = \frac{d}{dy}(6x^2y)$

$6x^2 \quad \ldots \quad (2)$

$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy \quad \ldots \quad (3)$

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$$f_{yx} = \frac{d}{dx}(6x^2y)$$

$$= 12xy$$

From (3) and (4) .... (4)

$$\therefore f_{xy} = f_{yx}$$

ii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{(x+y)}$

$$\therefore f_{xx} = \frac{d}{dx}\left[y\cos(xy) + e^{x+y}\right]$$

$$= -y\sin(xy).(y) + e^{x+y}(1)$$

$$= -y^2\sin(xy) + e^{x+y} \quad \ldots \quad (1)$$

$$f_{yy} = \frac{d}{dy}\left[x\cos(xy) + e^{x+y}\right]$$

$$= -x\sin(xy)(x) + e^{x+y}(1)$$

$$= -x^2\sin(xy) + e^{x+y} \quad \ldots \quad (2)$$

$$f_{xy} = \frac{d}{dy}\left(y\cos(xy) + e^{x+y}\right)$$

$$= -y^2\sin(xy) + \cos(xy) + e^{x+y}, \ldots \quad (3)$$

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$$f_{yx} = \frac{d}{dx}\left[x\cos(xy) + e^{x+y}\right]$$

$$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \quad \ldots (4)$$

From (3) and (4)

$$f_{xy} \neq f_{yx}$$

Q.5.

i) $f(x, y) = \sqrt{x^2 y^2}$ at $(1, 1)$

$\rightarrow$ $f(1, 1) = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x) \qquad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{x}{\sqrt{x^2+y^2}} \qquad = \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1, 1) = \frac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1, 1) = \frac{1}{\sqrt{2}}$

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$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 + 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0$

$= -1$

$f_y = 0 - 0 + \sin x$

f_y at $(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2}$

$= 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= 1 - x + y$

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iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

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Practical 10

Q.1. Find the directional derivative of the following function of given Points and in the direction of given vector.

i) $f(x,y) = x+2y-3$, $a = (1,-1)$ $u = 3i - j$

-> Here,

$u = 3i - j$ is not a unit vector

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\bar{i} - \bar{j})$

$= \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now,

$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= 1+2(-1)-3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) + \left(1+\frac{3h}{\sqrt{10}}, -1-\frac{h}{\sqrt{10}}\right)$

$\cancel{= 4 + \frac{3h}{\sqrt{10}} - 4}$ $\quad 1+\frac{3h}{\sqrt{0}} + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1-2-3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

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$$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$$

$$= \lim_{h \to 0} \frac{h}{h\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}}$$

ii) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

→ Here,

$u = 3i + 5j$ is not a unit vector.

$\bar{u} = 3\bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i} + \bar{5j})$

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \frac{1}{\sqrt{26}}, \frac{5}{\sqrt{36}}$$

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Now,

$$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 - \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

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iii) $f(x,y) = 2x + 3y$ $\quad a = (1,2), \quad u = 3i + 4j$

→ Here,

$u = 3i + 4j$ is not a unit vector.

$\bar{u} = \bar{3}i + \bar{4}j$

$|\bar{u}| = \sqrt{25} = 5$

$\therefore$ Unit vector along $u = \dfrac{\bar{u}}{|\bar{u}|} = \dfrac{1}{5}(\bar{3}i + \bar{4}j)$

$= \dfrac{1}{5}(3,4)$

$= \left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

$= f\left(1 + \dfrac{3h}{5}, 2 + \dfrac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \dfrac{3h}{5}\right) + 3\left(2 + \dfrac{4h}{5}\right)$

$= 2 + \dfrac{6h}{5} + 6 + \dfrac{12h}{5}$

$= \dfrac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \dfrac{\frac{18h}{5} + 8 - 8}{h}$

$= \dfrac{18}{5}$

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Q2. Find gradient vector for following function at given point.

i) $f(x, y) = x^y + y^x$ $\quad a = (1, 1)$

$f(x) = y x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x, y) = (f_x, f_y)$

$= (y x^{y-1} + y^2 \log y, x^y \log x + x y^{x-1})$

$f(1, 1) = (1 + 0, 1 + 0)$

$= (1, 1)$

ii) $f(x, y) = (\tan^{-1} x) \cdot y^2$ $\quad a = (1, 1)$

$\Rightarrow$ $f_x = \dfrac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x, y) = (f_x, f_y)$

$= \left(\dfrac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1, -1) = \left(\dfrac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\dfrac{1}{2}, \dfrac{\pi}{4}(-2)\right)$

$= \left(\dfrac{1}{2}, \dfrac{-\pi}{2}\right)$

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iii) $F(x,y,z) = xyz - e^{x+y+z}$ $\quad a = (1,-1,0)$

$F_x = yz - e^{x+y+z}$
$F_y = xz - e^{x+y+z}$
$F_z = xy - e^{x+y+z}$

$\nabla F(x,y,z) = F_x, F_y, F_z$
$= yz - e^{x+y+z}, \; xz - e^{x+y+z}, \; xy - e^{x+y+z}$

$F(1,-1,0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$
$= (0 - e^0, 0 - e^0, -1 - e^0)$
$= (-1, -1, -2)$

Q.3. Find the equation of tangent and normal to the following using curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$F_x = \cos y \, 2x + e^{xy} \, y.$
$F_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1, 0)$ $\quad \therefore x_0 = 1, \; y_0 = 0$

Eq^n of tangent.

$F_x(x - x_0) + F_y(y - y_0) = 0$

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$F_x(x_0, y_0) = \cos 0 \cdot 2(1) + e^0 \cdot 0$

$= 1(2) + 0$

$= 2$

$F_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

$\therefore$ The eq^n of tangent is $2x + y - 2$

Eq^n of normal : $ax + by + c = 0$

$\therefore bx + ay + d = 0$

$\therefore 1(1) + 2(y) + d = 0$

$\therefore 1 + 2y + d = 0$ at $(1, 0)$

$0 = 1 + 2(0) + d$

$\therefore d + 1 = 0$

$\therefore d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$F_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$F_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$\therefore x_0 = 2, \; y_0 = -2$

$(x_0, y_0) = (2, -2)$

$\therefore F_x(x_0, y_0) = 2(2) - 2 = 2$

$F_y(x_0, y_0) = 2(-2) + 3 = -1$

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Eq^n of tangent.

$$F_x(x-x_0) + F_y(y-y_0) = 0$$

$$2(x-2) + (-1(y+2)) = 0$$

$$2x - 2 - y - 2 = 0$$

$$2x - y - 4 = 0$$

$\therefore$ The required equation of tangent is $2x-y-4=0$

Eq^n of Normal:

$$ax + by + c = 0$$

$$bx + ay + d = 0$$

$$\therefore -1(x) + 2(y) + d = 0$$

$$-x + 2y + d = 0 \qquad \text{at } (2,-2)$$

$$-2 + 2(-2) + d = 0$$

$$-2 - 4 + d = 0$$

$$-6 + d = 0$$

$$d = 6$$

Q.4. Find the eq^n of tangent and Normal line to each of the following surface.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$$F_x = 2x - 0 + 0 + z$$

$$F_x = 2x + z$$

$$F_y = 0 - 2z + 3 + 0$$

$$= 2z + 3$$

$$F_z = 0 - 2y + 0 + x$$

$$= -2y + x$$

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$(x_0, y_0, z_0) = (2, 1, 0)$ $\quad \therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$F_x(x_0, y_0, z_0) = 2(2) + 0 = 4$
$F_y(x_0, y_0, z_0) = 2(0) + 3 = 3$
$F_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

$\therefore$ Equation of tangent.

$f_x(x - x_0) + F_y(y - y_0) + F_z(z - z_0) = 0$

$\therefore 4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x - 8 + 3y - 3 + 0 = 0$

$4x + 3y - 11 = 0$

Equation of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

$3xy^2 - x - y + z = -4$ $\quad$ at $(1, -1, 2)$

$3xyz - y + z + 4 = 0$ $\quad$ at $(1, -1, 2)$

$F_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$F_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$F_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

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$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1,\ y_0 = -1,\ z_0 =$

$\therefore f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$\therefore f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$\therefore f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Eq^n of tangent:

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

Eq^n of normal:

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5. Find the local maxima and minima for the following functions

i) $F(x, y) = 3x^2 + 3y^2 - 3xy + 6x - 4y$

$\therefore F_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$\therefore F_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$F_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

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$2x - y + 2 = 0$

$2x - y = -2 \quad \ldots \quad (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad \ldots \quad (2)$

Multiply eqn 1 with 2

$\therefore \; 4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = -2 \qquad \therefore \; y = 2$

$\therefore$ critical Points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore$ F has maximum at $(0, 2)$

$\therefore \; 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

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ii) $F(x,y) = 2x^4 + 3x^2y^2 - y^2$

$F_x = 8x^3 + 6xy$

$F_y = 3x^2 - 2y$

$F_x = 0$

$= 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \quad \ldots \quad (1)$

$F_y = 0$

$3x^2 - 2y = 0 \quad \ldots \quad (2)$

Multiply eq^n (1) with 3

(2) with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$17y = 0$

$\therefore y = 0$

Substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical Point is $(0,0)$

$r = F_{xx} = 24x^2 + 6x$

$t = F_{yy} = 0 - 2 = -2$

$s = F_{xy} = 6x - 0 = 6x = 6(0) = 0$

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r at $(0,0)$

$= 24(0) + 6(0)$

$= 0$

$\therefore r = 0$

$F(x,y)$ at $0,0)$

$= 2(0)^4 + 3(0)^2(0) - 0$

$= 0 + 0 - 0$

$= 0$

$\therefore rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0$

$= 0$

$\therefore r = 0$ and $rt - s^2 = 0$

(nothing to say)

$F(x,y) = x^2 - y^2 + 2x + 8y - 70$

$F_x = 2x + 2$

$F_y = -2y + 8$

$F_x = 0$ $\quad \therefore 2x + 2 = 0$

$\therefore x = -1$

$F_y = 0$ $\quad -2y + 8 = 0$

$y = \frac{-8}{-2}$

$\therefore y = 4$

Critical Point is $(-1, 4)$

$r = F_{xx} = 2$

$t = F_{yy} = -2$

$s = F_{xy} = 0$

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$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$F(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

AB

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