**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANSWER**

**Since the work started after 10 mins so the time left is 50 min.**

**Probability that the service manager cannot meet his commitment**

**= P(x>50) = 1 – P(x<=50) (x is the time taken to complete the work.**

**Standard normal variable Z = (x- µ)/σ=(x-45)/8**

**= P(x<=50)**

**= P (Z <= (50-45)/8)**

**= P(Z<=0.625)**

**= 0.73237 = 73.27% (Z-Score)**

**100 - 73.27 = 26.73% = 0.2676**

**So, option B is correct.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**ANSWER**

**P(X>44) = 1 – P(X<=44)**

**Z = (X- µ) / σ=(X-38)/6**

**The question can be answered by using normal table to find**

**P(X<=44) = P (Z <= (44-38)/6) = P(Z<=1) = 84.1345%**

**Probability that the employee will be greater than age of 44 = 100-84.1345 = 15.86%**

**So, the probability of number of employees between 38-44 years of age = P(X<44) – 0.5 = 84.1345 – 0.5 = 34.1345%**

**Therefore, the above statement (A) is FALSE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANSWER P(X<30)**

**Z = (X- µ) / σ= (30-38)/6**

**The question can be answered by using normal table to find**

**P(X<=30) = P (Z<= (30-38)/6) = P (Z<= -1.333) = 9.12%**

**So, the number of employees with probability 0.912 of them being under 30 = 0.0912\*400 = 36.48 or 36 employees.**

**Therefore, the above statement (B) is TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANSWER**

**As we know that if *X* ∼ *N*(μ1, σ1^2) and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ *N*(μ1 + µ2, , σ1^2 + σ2^2 ) and X – Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ).**

**Similarly if Z = aX + bY, the linear combination of X and Y, then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).**

**Therefore, in the above question**

**2X1∼ N(2 u,4 σ^2) and X1+X2 ∼ N(µ + µ, σ^2 + σ^2 ) ∼ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

**The mean of  and  is same but the var() of   is 2 times more than the variance of .**

**The difference between the two says that the two given variables are identically and independently distributed.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANSWER**

**We have to work in the reverse order.**

**The probability of getting value of a & b should be 0.99**

**The Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99)**

**The Probability towards left from a = -0.005 (ie. 0.01/2).**

**The Probability towards right from b = +0.005 (ie. 0.01/2).**

**Since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.**

**By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.**

**Z = (X- μ) / σ**

**For Probability 0.005 the Z Value is -2.57 (from Z Table).**

**Z \* σ + μ = X**

**Z (-0.005) \* 20+100 = - (-2.57) \* 20+100 = 151.4**

**Z (+0.005) \* 20+100 = (-2.57) \* 20+100 = 48.6**

**So, option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**ANSWER**

**(A) Range is 99.00 – 980.99**

**(B) 5th percentile of profit is Rs.170**

**(C) stats.norm.cdf(0,5,3) > stats.norm.cdf(0,7,4)**

**i.e, 0.477 > 0.400**