

BNF :- (Bakus - Naur form)

- Developed by John - Bakus and peter Naur.
- Used for specifying the syntax of the language.
- This notation can be expressed as,

$\langle \text{symbol} \rangle ::= \text{Exp1} \mid \text{Exp2} \mid \text{Exp3} \dots$

Sequence of
symbols
(can be T as well as NT)

$\langle \text{Symbol} \rangle \rightarrow$ Nonterminal
(written in angle brackets)

Exp \rightarrow Consisting one or more symbol
separated by "|"

reduced by
definition
comb

$\therefore \rightarrow$ left side must be replaced
with the exp of right side.

eg:- $\langle \text{Number} \rangle ::= \langle \text{digit} \rangle \mid \langle \text{Number} \rangle \langle \text{digit} \rangle$

$\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

OR
Terminal

The Language of a Grammar :-

If $G(V, T, P, S)$ is a CFG, the Language of G , denoted $L(G)$, is the set of terminal strings that have derivations from the start symbol. That is,

$$L(G) = \{ w \in T^* \mid S \xRightarrow[G]{*} w \}$$

- If a Language L is the Language of some CFG, the L is said to be Context free language (CFL).

Closure properties of (CFL)

Content free language:-

- The context free Language is closed under,

① Union

② Concatenation

③ Kleene closure.

① Union:-

- let $G_1 = (V_1, T_1, P_1, S_1)$ and $G_2 = (V_2, T_2, P_2, S_2)$ be the two context free ~~CFG~~ Grammars and L_1 and L_2 be two context free languages.

- We have to show that the union of G_1 and G_2 i.e. $L_1 + L_2$ is also context free
- for that we have to show there exists a CFG G such that G can generate either the strings generated by G_1 or strings generated by G_2

let $G = (V, T, P, S)$ be such CFG

Where,

$$V = V_1 \overset{\text{NT of } G_1}{\cup} V_2 \overset{\text{NT of } G_2}{\cup} S$$

$$T = T_1 \cup T_2$$

P consists of

$$P_1 \cup P_2 \cup \{S \rightarrow S_1 | S_2\}$$

S = start symbol

- Here, S is the start symbol of G . The production rule $S \rightarrow S_1$ tells that G can transit from its start symbol to start symbol of G_1 . i.e. It can use the production rule P_1 of G_1 and hence generate the strings generated by G_1 and
- Similarly, when it uses the production rule $S \rightarrow S_2$ tells that G transit from its start symbol ~~of~~ to start symbol of G_2 i.e. it can use the production rules P_2 of G_2 and generates the strings generated by G_2 .
- As a result of which, we can say that G can produce either G_1 or G_2 . Hence the class of language is closed under union.

② Concatenation:-

- let $G_1 = (V_1, T_1, P_1, S_1)$ and $G_2 = (V_2, T_2, P_2, S_2)$ be two context free grammars and L_1 & L_2 be ~~the~~ their respective context free language.

We have to show that the concatenation of G_1 and G_2 i.e. $L_1 \cdot L_2$ is also context free. For that, we have to show there exists a CFG G such that G can generate the strings generated by G_1 and (followed) strings generated by G_2 .

Let $G = (V, T, P, S)$ be such CFG,

where

$V = V_1 \cup V_2 \cup S$

$T = T_1 \cup T_2$

P consists of

$P_1 \cup P_2 \cup \{S \rightarrow s_1 s_2\}$

$S = \text{start symbol}$.

Here, S is the start symbol of G . The production rule $S \rightarrow s_1 s_2$ tells that G can transition from its start symbol to start symbol of G_1 and hence generates the string generated by G_1 .

After that G transit from G_1 to G_2 to generate the string generated by G_2 .

The class of language is closed under concatenation.

③ Closure :-

⇒ Let $G_1 = (V_1, T_1, P_1, S_1)$ be the context free grammar & L_1 be the respective context free language.

- We have to show that the closure of G_1 i.e. L_1^* is also context free.

- for that we have to show there exists a grammar G such that G can generate the string generated by G_1 any number of times including empty.

- let $G = (V, T, P, S)$ be such CFG where,

$$V = V_1 \cup S$$

$$T = T_1$$

P consists of -

$$P, U \{ S \rightarrow SS \mid \epsilon \}$$

S = start symbol

- Here, S is a start symbol of G , the production rule $S \rightarrow SS$ tells that G transits from start symbol of G to start symbol of G_1 & hence generating the

string generated by G_1 .

- \emptyset taking ϵ empty symbol. G_1 repeats any number of times to generate the closure of G_1 .
- G_1 can transit to G by taking the empty symbol \emptyset as well.
- As a result G can produce the closure of G_1 . Hence the class of language is closed under closure.