JEE Nexus

Crash Course

Sequence & Series

DPP-1

Questions

1. Which term of the A.P. 2, 5, 8, 11, … is 65?

2. If m^{th} terms of the series 63 + 65 + 67 + 69 + … and 3 + 10 + 17 + 24 + … be equal, then find m.

3. If 9 times the 9th term of an A.P. is equal to 13 times the 13th term, then find the 22nd term of the A.P.

4. In an A.P. if $\frac{a_4}{a_7} = \frac{2}{3}$, then what is the value of $\frac{a_5}{a_{12}}$?

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

5. Find the 13th term from the end of the A.P. 4, 9, 14, 19, …, 124.

6. If the sum of n terms of an A.P. is given by $S_n = 3n + 2n^2$, then the common difference of the A.P. is

(a) 3 (b) 2 (c) 6 (d) 4

7. Find the sum of odd integers from 1 to 101 which are divisible by 3.

(a) 852 (b) 847 (c) 867 (d) 857

8. If the sum of the first 10 terms of an A.P. is 4 times the sum of its 5 terms, then the ratio of first term and common difference is

(a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2

9. 7th term of an A.P. is 40, then the sum of first 13 terms is

(a) 530 (b) 520 (c) 1040 (d) 2080

10. The sum of the first 20 terms common between the series 3 + 7 + 11 + 15 + … and 1 + 6 + 11 + 16 + … is

(a) 4000 (b) 4020 (c) 4200 (d) 4220

11. If the ratio of sum of n terms of two arithmetic progressions is $(3n + 1) : (4n + 15)$, find the ratio of their 9th terms.

12. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then find $S_{3n} : S_n$.

13. If $x^2(y + z), y^2(z + x), z^2(x + y)$ are in A.P., then prove that either x, y, z are in A.P. or $xy + yz + zx = 0$

14. For an A.P. (a_n), where $a_i > 0\ \forall$ i show that: $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_3}+\sqrt{a_4}} = \frac{3}{\sqrt{a_1}+\sqrt{a_4}}$

15. Three number are in A.P. such that their sum is 18 and sum of their squares is 158. The greatest number among them is _______.

16. Divide 32 into four parts which are in A.P. such that the product of extremes is to product of means are 7 : 15.

17. Find the number which should be added to the numbers 2, 14, 62 so that the resulting numbers may be in G.P.

18. Determine the G.P. if its second term is 10 and fifth term is 80.

19. If the *p*th, *q*th and *r*th terms of a G.P. are *x, y, z* respectively, then $x^{q-r}y^{r-p}z^{p-q}$ is equal to

(a) 0 (b) 1 (c) 2 (d) None of these

20. The first term of a G.P. is 1. The sum of the third and fifth term is 90. Find the common ratio of the G.P.

21. The 4th term of a G.P. is square of its second term, and the first term is -3. Find its 7th term.

22. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

23. Let $a_1, a_2, a_3, ...$ be terms of an AP. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

24. Given an AP whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then find its 4th term.

25. If $x_1, x_2, ..., x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, ..., \frac{1}{h_n}$ are two APs such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 h_{10}$ is equal to _______.

26. In a GP consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(a) $\frac{1}{2}\left(1-\sqrt{5}\right)$ (b) $\frac{\sqrt{5}}{2}$ (c) $\sqrt{5}$ (d) $\frac{1}{2}\left(\sqrt{5}-1\right)$

Answer Key

1. 65	2. m = 13	3. 0
4. (b)	5. 64	6. (d)
7. (c)	8. (a)	9. (b)
10. (b)	11. $\frac{52}{83}$	12. 6
15. 11	16. 2, 6, 10, 14	17. 2
18. 5, 10, 20, 40, ...	19. (b)	20. $r = \pm 3$
21. -2187	22. 3072	23. (d)
24. 20	25. 2560	26. (d)

Solutions

1.

Given A.P. = 2, 5, 8, 11, …

Let its n^{th} term is 65

Then $a_n = 65$ and $d = 5 - 2 = 3$

$\Rightarrow a + (n - 1)d = 65$

$\Rightarrow 2 + (n - 1) \times 3 = 65$

$\Rightarrow 2 + 3n - 3 = 65$

$\Rightarrow 3n - 1 = 65$

$$\Rightarrow n = \frac{65+1}{3} = \frac{66}{3} = 22$$

Hence, 22^{nd} term is 65

2.

Given series 63 + 65 + 67 + 69 + … (1)

and 3 + 10 + 17 + 24 + …(2)

Now from (1),

m^{th} term = $a + (m-1)d = 63 + (m-1)2 = (2m+61)$

and from (2),

m^{th} term = $a + (m-1)d = 3 + (m-1)7 = 7m - 4$

Under condition,

$\Rightarrow 7m - 4 = 2m + 61$

$\Rightarrow 5m = 65$

$\Rightarrow m = 13$

3.

Let the first term and common difference of given A.P. be a and d, respectively.

It is given that $9.\ a_9 = 13.\ a_{13}$

$\Rightarrow 9(a + 8d) = 13(a + 12d)$

$\Rightarrow 9a + 72d = 13a + 156d$

$\Rightarrow 4a + 84d = 0$

$\Rightarrow (a + 21d) = 0$

$\Rightarrow a_{22} = 0$

4.

Given $\frac{a_4}{a_7}=\frac{2}{3}$

$\Rightarrow \frac{a+3d}{a+6d}=\frac{2}{3}$

$\Rightarrow 3a+9d=2a+12d$

$\Rightarrow a=3d$

$\therefore \frac{a_5}{a_{12}}=\frac{a+4d}{a+11d}=\frac{3d+4d}{3d+11d}=\frac{7d}{14d}=\frac{1}{2}$

5.

Given AP = 4, 9, 14, 19, …,124

a = 4, d = 9 – 4 = 5, l = 124

13th term from last $=l-(n-1)d$

= 124 – (13 – 1) × 5

= 124 – 12 × 5

= 124 – 60

= 64

Hence, 13th term from last is 64.

6.

Given, $S_n = 3n + 2n^2$

$S_1 = 3(1) + 2(1)^2 = 5 = a_1$

$S_2 = 3(2) + 2(2)^2 = 14 = a_1 + a_2$

Now, $S_2 – S_1 = 9 = a_2$

$d = a_2 - a_1 = 9 – 5 = 4$

7.

Odd integers from 1 to 101 are 1, 3, 5, 7, 9 ,…, 101.

Integers which are divisible by 3 are 3, 9, 15, 21 ,…, 99.

First term of this series a = 3

Common difference d = 9 – 3 = 6

Last term = 99

Let nth term be the last term, then

last term = 99 = 3 + (n – 1) × 6 [From formula, l = a + (n – 1)d]

$\Rightarrow 6(n-1) = 99 - 3$

$\Rightarrow (n-1) = \frac{96}{6} = 16$

$\Rightarrow n = 16 + 1 = 17$

Hence, sum of 17 terms.

$$S_{17} = \frac{17}{2}\left[2\times3+(17-1)\times6\right]$$

$$= \frac{17}{2}[6+96] = \frac{17}{2}\times102$$

$= 17 \times 51 = 867$

Hence, sum of odd integers from 1 to 101, which is divisible by 3 is 867.

8.

Under conditions, we get

$$\frac{10}{2}\{2a+(10-1)d\} = 4\left[\frac{5}{2}\left[2a+(5-1)d\right]\right]$$

$\Rightarrow 2a+9d = 4a+8d$ or $\frac{a}{d} = \frac{1}{2}$

Hence $a:d = 1:2$

9.

7^{th} term of an A.P. = 40

$a + 6d = 40$

$$S_{13} = \frac{13}{2}\left[2a+(13-1)d\right] = \frac{13}{2}\left[2(a+6d)\right]$$

$$= \frac{13}{2}.2.40 = 520$$

10.

Two A.P. are

3,7,11,15,19,23,27,31… and

1,6,11,16,21,26,31 …

Common terms are 11,31,51 …

Its an A.P. with first term 11 and common difference 20.

Thus, sum to 20 common terms is

$$S_{20} = \frac{20}{2}\left[2(11)+(20-1)(20)\right] = 4020$$

11.

Let for 1st arithmetic progression, first term = a_1 and common difference = d_1

for 2nd arithmetic progression, first term = a_2 and common difference = d_2

We are given that

$$\frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}=\frac{3n+1}{4n+15}$$

$$\Rightarrow \frac{\left[2a_1+(n-1)d_1\right]}{\left[2a_2+(n-1)d_2\right]}=\frac{3n+1}{4n+15}$$

$$\Rightarrow \frac{\left[a_1+\left(\frac{n-1}{2}\right)d_1\right]}{\left[a_2+\left(\frac{n-1}{2}\right)d_2\right]}=\frac{3n+1}{4n+15} \qquad \text{...(i)}$$

For 9th term, $\left(\frac{n-1}{2}\right)=8$

$\Rightarrow n=17$

Substituting n = 17 in (i), we get

$$\frac{\left[a_1+8d_1\right]}{\left[a_2+8d_2\right]}=\frac{3(17)+1}{4(17)+15}=\frac{52}{83}$$

$$\Rightarrow \frac{9^{th}\text{ term of first A.P.}}{9^{th}\text{ term of second A.P.}}=\frac{52}{83}$$

12.

Let first term be a and common difference be d.

Then, $S_{2n} = 3S_n$

$$\Rightarrow \quad \frac{2n}{2}\left[2a+(2n-1)d\right]=\frac{3n}{2}\left[2a+(n-1)d\right]$$

$\Rightarrow \quad 2[2a + (2n – 1)d] = 3[2a + (n – 1)d]$

$\Rightarrow \quad 4a + (4n – 2)d = 6a + (3n – 3)d$

$\Rightarrow \quad 2a = (n + 1)d \qquad$ …(i)

$$\text{Now, } \frac{S_{3n}}{S_n}=\frac{\frac{3n}{2}\left[2a+(3n-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{3\left[2a+(3n-1)d\right]}{\left[2a+(n-1)d\right]}$$

$$=\frac{3\left[(n+1)d+(3n-1)d\right]}{\left[(n+1)d+(n-1)d\right]}=\frac{3\left[4nd\right]}{2nd}=6$$

13.

$\because x^2(y+z), y^2(z+x), z^2(x+y)$ are in A.P.

$\therefore$ On adding xyz in each terms

$x^2(y+z)+xyz, y^2(z+x)+xyz, z^2(x+y)+xyz$ also will be in A.P.

or $x(xy+yz+zx), y(xy+yz+zx), z(xy+yz+zx)$ also will be in A.P.

$\therefore 2y(xy+yz+zx) = x(xy+yz+zx)+z(xy+yz+zx)$

$\Rightarrow 2y(xy+yz+zx) = (xy+yz+zx)(x+z)$

$\Rightarrow 2y(xy+yz+zx) - (xy+yz+zx)(x+z) = 0$

$\Rightarrow (xy+yz+zx)(2y-x-z) = 0$

If $2y-x-z=0$

Then $2y = x+z$

$\Rightarrow x, y, z$ are in A.P.

or $xy+yz+zx=0$

14.

Let terms in A.P. be $a_1, a_2, a_3, \ldots..a_n$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots. = a_n - a_{n-1} = d$

Now, $\dfrac{1}{\sqrt{a_1}+\sqrt{a_2}} + \dfrac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dfrac{1}{\sqrt{a_3}+\sqrt{a_4}}$

Rationalize

$$\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \frac{\sqrt{a_3}-\sqrt{a_4}}{a_3-a_4}$$

$$\Rightarrow \frac{1}{-d}[\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+\sqrt{a_3}-\sqrt{a_4}]$$

$$\Rightarrow -\frac{1}{d}[\sqrt{a_1}-\sqrt{a_4}]$$

$$\Rightarrow -\frac{1}{d}\left[\frac{a_1-a_4}{\sqrt{a_1}+\sqrt{a_4}}\right]$$

$$\Rightarrow -\frac{1}{d}\left[\frac{a_1-(a_1+3d)}{\sqrt{a_1}+\sqrt{a_4}}\right]$$

$$\Rightarrow -\frac{1}{d}\left[\frac{-3d}{\sqrt{a_1}+\sqrt{a_4}}\right] = \frac{3}{\sqrt{a_1}+\sqrt{a_4}}$$

15.

Let three number of A.P. $a-d, a, a+d$

Sum = 18, and $(a-d)^2 + a^2 + (a+d)^2 = 58$

$a - d + a + a + d = 18$

$a=6$ and $(6-d)^2+36+(6+d)^2=158$

$\Rightarrow 36+d^2+36+d^2=122$

$\Rightarrow 2d^2+72=122$

$2d^2=50$

$d=\pm 5$

When $d=5$, the numbers are 1, 6, 11

and when d = -5, the numbers are 11,6,1

Greatest number is 11

16.

Let the required parts be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

Then $(a-3d)+(a-d)+(a+d)+(a+3d)=32$

$\Rightarrow 4a=32$

$\Rightarrow a=8$

Also $\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2}=\frac{7}{15}$

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=\frac{512}{128}$

$\Rightarrow d=\pm 2$

Thus $a=8$ and $d=2$ or $a=8$ and $d=-2$

Hence, required numbers are 2, 6, 10, 14 or 14, 10, 6, 2.

17.

Suppose that the added number be x, then $x+2, x+14, x+62$ be in G.P.

Therefore $(x+14)^2=(x+2)(x+62)$

$\Rightarrow x^2+196+28x=x^2+64x+124$

$\Rightarrow 36x=72$

$\Rightarrow x=2$

18.

Second term $a_2 = ar = 10$ ...(i)

Fifth term $a_5 = ar^4 = 80$ ...(ii)

Dividing (ii) by (i)

$\frac{ar^4}{ar} = \frac{80}{10}$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$

Put the value of r in (i)

$2a = 10$

Required G.P. is 5, 10, 20, 40, ...

19.

Let the first term be a and ratio be R then

$a_p = aR^{p-1} = x,$

$a_q = aR^{q-1} = y,$

$a_r = aR^{r-1} = z$

$\therefore\ x^{q-r} y^{r-p} z^{p-q} = \left(aR^{p-1}\right)^{q-r} \left(aR^{q-1}\right)^{r-p} \left(aR^{r-1}\right)^{p-q}$

$= a^{(q-r)+(r-p)+(p-q)} R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= a^0 R^0 = 1$

20.

Let r be the common ratio of the G.P. It is given that the first term is $a = 1$. Now,

$a_3 + a_5 = 90$

$\Rightarrow ar^2 + ar^4 = 90$

$\Rightarrow r^2 + r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$

$\Rightarrow \left(r^2 + 10\right)\left(r^2 - 9\right) = 0$

$\Rightarrow r^2 - 9 = 0$

$\Rightarrow r = \pm 3$

21.

Let a be the first term and r be the common ratio of the G.P.

$\therefore a = -3$

It is known that, $a_n = ar^{n-1}$

$\therefore a_4 = ar^3 = (-3)r^3$

$a_2 = ar = (-3)r$

According to the given condition,

$\Rightarrow -3r^3 = 9r^2$

$r = -3\,(r \neq 0)$

$\therefore a_7 = ar^{7-1}$

$a_7 = ar^6$

$a_7 = (-3)(-3)^6 = -(3^7) = -2187$

Thus, the seventh term of the G.P. is -2187

22.

Common ratio, r = 2

Let a be the first term of the G.P.

$\therefore a_n = ar^{n-1}$

$a_8 = ar^7$

$192 = ar^7$

$192 = a(2)^7$

$a = \frac{3}{2}$

$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$

23.

Given $\frac{S_p}{S_q} = \frac{p^2}{q^2},\ p \neq q$

$$\Rightarrow \frac{\frac{p}{2}[2a+(p-1)d]}{\frac{q}{2}[2a+(q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{a+\left(\frac{p-1}{2}\right)d}{a+\left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

Put $\frac{p-1}{2} = 5$ and $\frac{q-1}{2} = 20$

$$\Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

24.

Let $200 < a_1 + a_2 + a_3 + \cdots a_9 < 220$

or $\quad 20 < \frac{9}{2}[2a_1 + 8d] < 220$

Therefore, $200 < 9a_1 + 36d < 220$ ..... (1)

Since $a_2 = 12 \Rightarrow a_1 + d = 12$. Therefore, $d = 12 - a_1$

Now from Eq. (1), we have

$200 < 9\,a_1 + 36\,(12 - a_1) < 220$

$\Rightarrow 200 < 9\,a_1 + 432 - 36\,a_1 < 220$

or $200 < 432 - 27\,a_1 < 220$

$\Rightarrow -232 > 27\,a_1 > 212$

Therefore, $a_1 = 8 \Rightarrow d = 12 - 8 = 4,\ a_3 = 16$ and $a_4 = 20$

25.

Given two APs: $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \cdots, \frac{1}{h_n}$

Now, $x_3 = x_1 + 2d_1$, Where, d_1 is common difference of first AP.

And, $\frac{1}{h_2} = \frac{1}{h_1} + d_2$, Where, d_2 is common difference of second AP.

We know that $x_3 = h_2$

So, $8 = x_1 + 2d_1$ $\cdots$ (1)

$\frac{1}{8} = \frac{1}{h_3} + d_2$ $\cdots$ (2)

And $x_8 = h_7 = 20$

$x_1 + 7d_1 = 20$ $\cdots$ (3)

$\frac{1}{20} = \frac{1}{h_1} + 6d_2$ $\cdots$ (4)

From Eq. (1) and Eq. (3), we get

$d_1 = \frac{12}{5}$ and $x_1 = \frac{16}{5}$

From Eq. (2) and Eq. (4), we get

$$\frac{1}{20} - \frac{1}{8} = 5d \Rightarrow \frac{-12}{20 \times 8} = 5d_2 \Rightarrow d_2 = \frac{-3}{200}$$

And $$\frac{1}{h_1} = \frac{1}{20} + \frac{3 \times 6}{200} \Rightarrow \frac{1}{h_1} = \frac{10 + 18}{200} \Rightarrow \frac{1}{h_1} = \frac{7}{50}$$

So, $$x_5 \cdot h_{10} = (x_1 + 4d_1)\frac{1}{h_1 + 9d_2}$$

$$= \left(\frac{16}{5} + 4 \cdot \frac{12}{5}\right) \cdot \frac{1}{\frac{28}{200} - \frac{27}{200}}$$

$$= \frac{200}{5}(48 + 16) = \frac{200 \times 64}{5} = 40 \times 64 = 2560$$

26.

Given that $ar^{n-1} = ar^{n} + ar^{n+1}$

$\Rightarrow 1 = r + r^{2}$

$\Rightarrow r^{2} + r - 1 = 0$

$\Rightarrow r = \frac{\sqrt{5}-1}{2} \left(r \neq \frac{-\sqrt{5}-1}{2} \right)$