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Assignment No. 1st

Question No. 1st Show that the expected value of a Binomial Random Variable is np i.e.,

$$E(x) = np$$

Where x is binomially distributed. (Use ${}^nC_x\, p^x (1-p)^{n-x}$)

Solution: We know that

$$E(x) = \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!} p^{x-1} (1-p)^{(n-1)-(x-1)}$$

Let $m = n - 1$
$y = x - 1$

Then,

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} {}^{m}C_{y}\, p^y (1-p)^{m-y}$$

We know that,

$$\sum_{y=0}^{m} {}^{m}C_{y}\, p^y (1-p)^{m-y} = (a+b)^m$$

Now, $E(x) = np\,(p + (1-p))^m$

$$= np\,(p+1-p)^m$$

$$= np\,(1)^m$$

$$= np$$

$$\boxed{E(x) = np}$$

Hence proved.

Question no. 2nd Show that the expected value of a normally distributed variable is μ, where $N(\mu, \sigma^2)$

Solution We know that

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Now,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z \Rightarrow x = \mu + \sigma z$

$$dx = \sigma dz$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) e^{-\frac{z^2}{2}} \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \mu e^{-\frac{z^2}{2}} dz + \int_{-\infty}^{\infty} \sigma z e^{-\frac{z^2}{2}} dz \right]$$

Hence, $\int_{-\infty}^{\infty} \sigma z e^{-z^2/2} \, dz$ is an odd function which is equal to zero

Therefore,

$$= \frac{2}{\sqrt{2\pi}} \mu \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz + 0$$

Let $\frac{z^2}{2} = p$

$$\frac{\cancel{2} z \, dz}{\cancel{2}} = dp$$

$$dz = \frac{dp}{\sqrt{2p}}$$

$$E(x) = \frac{2}{\sqrt{2\pi}} \mu \cdot \int_{0}^{\infty} e^{-p} \frac{dp}{\sqrt{2p}}$$

$$= \frac{2}{2\sqrt{\pi}} \mu \int_{0}^{\infty} p^{-1/2} e^{-p} dp$$

$$= \frac{\mu}{\sqrt{\pi}} \Gamma\tfrac{1}{2} \qquad \left[\because \int_{0}^{\infty} x^{n-1} e^{-x} dx = \Gamma n \right.$$

$$= \frac{\mu}{\cancel{\sqrt{\pi}}} \cancel{\sqrt{\pi}}$$

$$\boxed{E(x) = \mu}$$

Hence proved