UNIT 4

Hypotheses Testing:

A hypothesis is a premise or a claim that we want to test or investigate.

∴ Population hypothesis.

⇓

Sample

⇓

Hypothesis test

⇓

Conclude

Null Hypothesis: (H_0)

Default hypothesis or the currently accepted value for a parameter

Alternative Hypothesis: (H_a)

A research or counter hypothesis which is to be tested.

Ex: (1) H_0 : Mean income of Indians > 5000/m

H_a : Mean income of Indians ≤ 5000/m.

Ex (2): It is believed that a candy machine makes chocolate bars on avg of 5g. A worker claims that the machine no longer makes choc. bars of 5g. Write Ho & Ha.

Ho : $\mu = 5g$

Ha : $\mu \neq 5g$

Ho & Ha are mathematical opposites of each other.

If Ho: $\mu \geq c$ then Ha : $\mu < c$

Hypothesis Testing and outcomes:

→ Random sample is taken to test the claim Ho. ~~Bor~~

→ Based on test on the sample

* Reject Ho & accept Ha or

* Fail to reject Ho & reject Ha ($\because$ proving truth of Ho or Ha is difficult).

Statistically significant results

Eg. in Ex (2). Suppose we take 3 samples such that

$\mu_{s1} = 5.12g \neq 5$;

$\mu_{s2} = 5.72g \neq 5$;

$\mu_{s3} = 7.23g \neq 5$

The value $M_{B3} = 7.23$ is statistically significant

Where do we draw the line?

i.e. when can we use the test statistic (like M) to accept or reject H_0?

Level of confidence (c)

$c = 95\%$ or $c = 99\%$ etc.

expresses confidence in our decisions.

Level of significance (α)

$$\boxed{\alpha = 1 - c}$$

$c = 95\% \Rightarrow c = 0.95$

$\therefore \alpha = 0.05$

or $\boxed{\alpha + c = 1}$

Eg: A company makes straws of diameter 4mm. A worker believes it is not so. He draws 100 sample points to perform a hypothesis test with 99% confidence

$\therefore$ ~~H_0~~ $H_0 : \mu = 4\text{mm}$

$H_a : \mu \neq 4\text{mm}$

$c = 0.99$

$\alpha = 1 - 0.99$

$= 0.01$

The more confident alternate hypothesis is, the less significant it will be.

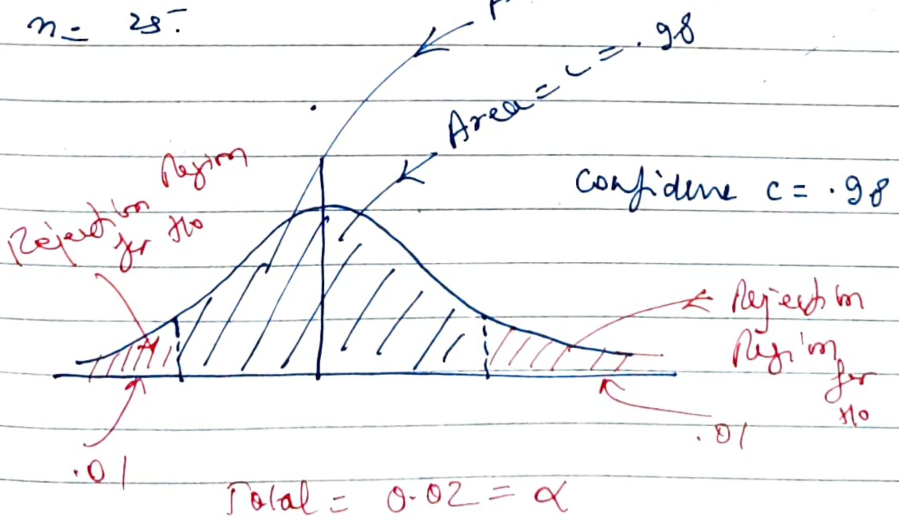
Ex: Doctors believe average teen sleeps on average no longer than 10 hrs/day. A researcher believes that teens on average sleep longer.

$\therefore$ Ho : $\mu \leq 10$ hrs/day
Ha : $\mu \neq 10$ hrs/day ✗

or Ha : $\mu > 10$ hrs/day ✓

Ex: A school board believes "at least 60% students bring phone to school". A teacher claims "This is too high". She samples 25 students at 0.02 significance.

$\therefore$ Ho = $\mu \geq 60\%$
Ha = $\mu < 60\%$
c = 0.98
α = 0.02
n = 25



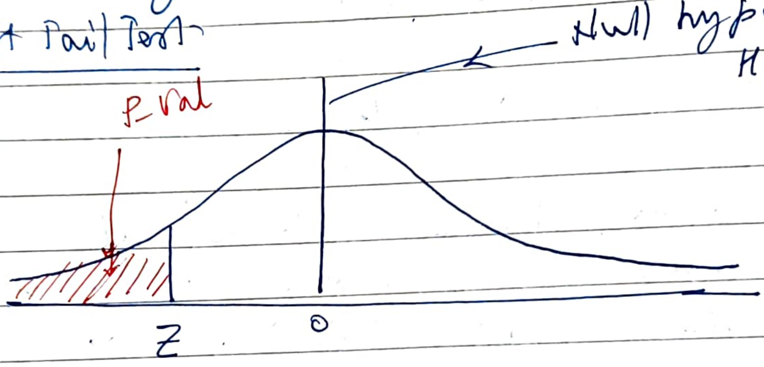
P-values

Qn : Where we reject Ho &
where we fail to reject Ho. ?

P-values tell where to reject Ho.

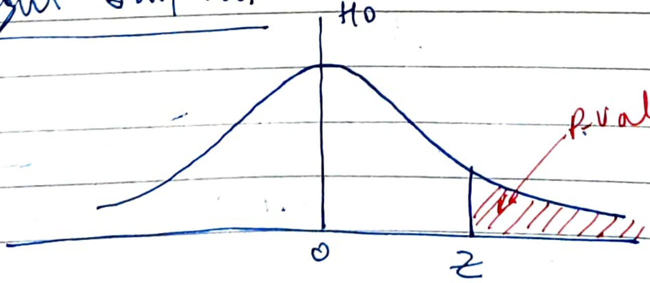
P-val: Probability of obtaining a sample "more extreme" than the ones observed in the sample assuming Ho is true.

Left Tail Test



Test statistic : $\boxed{z = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$

Right Tail Test



more extreme than data.

Ex: Left Tail Test

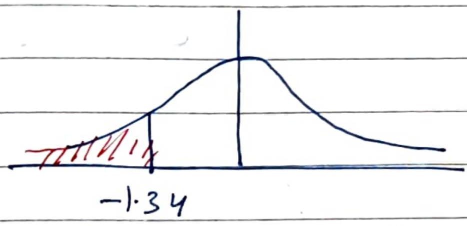
$H_0 : \mu \geq 0.15$

$H_a : \mu < 0.15$

& from the data (sample)

$z = -1.34$

$\therefore$



From the 'N' chart find $P(z < -1.34)$

$=$ shaded area

look in the $N(0,1)$ chart for ~~1.34~~ $P(Z \leq -1.34)$

$= 0.0901$

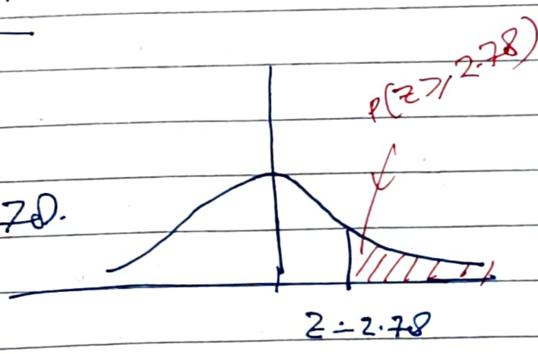
$\therefore$ P-val $= 0.0901$

Ex: Right tail Test

$H_0 : \mu \leq 0.43$

$H_a : \mu > 0.43$

From data $z = 2.78$.



$\therefore$ From $N(0,1)$

Find $P(Z \geq 2.78)$

$\therefore$ $N(0,1)$ is CDF $\therefore$ it is evaluated for $P(Z \leq 3)$.

$P(Z > 2.78) = 1 - P(Z < 2.78)$

$= 1 - 0.9973$

$= 0.0027$

$\therefore$ $\boxed{\text{P-val} = 0.0027}$

T-Test & Z-Test

T-Test (Sample size < 30)

- Compares the means of exactly two groups.
- Assumes ~~normal~~ student's t-distribution
- population variance not known
-

$$\text{T-test} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$\bar{x}$ ≡ sample mean
s ≡ sample s.d.
n ≡ sample size
μ ≡ population mean

Z-Test (Sample size > 30)

$$\text{Z-test} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$\bar{x}$ ≡ sample mean
σ ≡ population s.d.
n ≡ sample size
μ ≡ population mean

Z is normal $N(0,1)$

- Adopted when population variance is known

Ex: A factory has a machine that dispenses 80 ml of fluid in a bottle. An employee believes the average amount of fluid ≠ 80 ml. Using 40 observations as sample he measures the average amount dispensed = 78 ml. with a s.d. of 2.5.

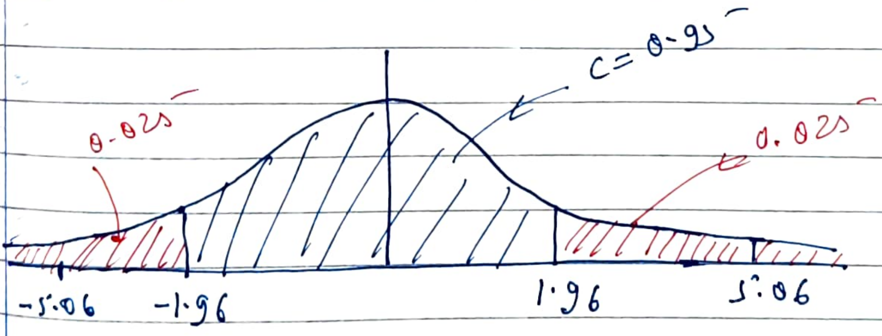
(a) Write H_0 & H_a

(b) At a 95% confidence level, is there enough evidence to support the idea that the machine is not working properly?

Sol (a) $H_0 : \mu = 80$

$H_a : \mu \neq 80$

(b). Plot



From $N(0,1)$ table $P(z \leq z) = -0.025$

$\Rightarrow z = -1.96$

Now calculate z-score from sample

$\bar{x} = 78 \quad \sigma = 2.5 \quad n = 40 \quad \mu = 80$

$\therefore z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{78 - 80}{2.5/\sqrt{40}}$

$= -2/0.39528$

$\therefore H_0$ rejected

Rejection region → $\boxed{z = -5.06}$