Khushi

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ASSIGNMENT-3

Ans 1 Probability that bomb will hit the target = 0.2

bomb will not hit the target = 0.8

$n = 6$

(i) Probability that exactly two will hit the target $= {}^6C_2 (0.2)^2 (0.8)^4$

$= 0.2457$

(ii) Probability that atleast two will hit target = 1 − (probabity of one or none hit the target)

$= 1 - [{}^6C_0 (0.2)^0 (0.8)^6 + {}^6C_1 (0.2)^1 (0.8)^5]$

$= 1 - 0.655$

$= 0.345$

Ans-2 1. $n \to \infty$

2. $p \to 0$

3. $np = \lambda$

Proof

$$= \lim_{n\to\infty} P(x)$$

$$= \lim_{n\to\infty} {}^nC_x p^x q^{n-x}$$

$$= \lim_{n\to\infty} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \lim_{n\to\infty} \frac{n(n-1)\dots(n-(x-1))\cancel{(n-x)!}}{x!\cancel{(n-x)!}} p^x (1-p)^{n-x}$$

Put $p = \lambda/n$

$$= \lim_{n\to\infty} \frac{n^x \left[(1-1/n)\dots(1-(x-1))\right]}{x!} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \lim_{n\to\infty} \frac{1}{x!} \lambda^x \left(1-\frac{\lambda}{n}\right)^n \underbrace{\left(1-\frac{\lambda}{n}\right)^{-x}}_{=1}$$

$$= \lim_{n\to\infty} \frac{1}{x!} \lambda^x \left(1-\frac{\lambda}{n}\right)^x$$

$$= \frac{\lambda^x e^{-\lambda}}{x!}$$

Hence Proved

(b) Mean $= np = 2000(0.001) = 2$

Prob that more than 2 will get a bad reaction = 1 − (prob. that no one gets a bad reaction + prob that two gets + prob that one gets bad reaction)

$$= 1 - \left[e^{-\lambda} + \frac{\lambda e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!}\right]$$

$$= 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2}\right]$$

$$= 1 - \frac{5}{e^2}$$

$$= 0.32$$

Ans 4 Let the equation of the line to be fitted is

$$y = a + bx$$

	x	y	x^2	xy
	1	14	1	14
	2	13	4	26
	3	9	9	27
	4	5	16	20
	5	2	25	10
Σ	15	43	55	97

Normal Form

$$\Sigma y = \Sigma(a + bx)$$

$$\Sigma y = na + b\Sigma x \quad \ldots (1)$$

$$xy = ax + bx^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \ldots (2)$$

Putting values in equation (1) from table

$$43 = 5a + 15b \quad \ldots (III)$$

equation (II)

$$97 = 15a + 55b \quad \ldots (IV)$$

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$43 = 5a + 15b \quad \times 3$

$97 = 15a + 55b \quad \times 1$

$129 = 15a + 75b$

$\underline{97 = \cancel{15a} + 55b}$

$32 = 20b$

$\boxed{b = 1.6}$

Putting value of b in (i)

$43 = 5a + 15 \times 1.6$

$\frac{43 - 24}{5} = a$

$\boxed{a = 3.8}$

Putting value of a and b

$y = 3.8 + 1.6x$

Ans 5

Let the equation of the parabola to be fitted

$y = a + bx + cx^2$

Normal equation

$\Sigma y = \Sigma a + b\Sigma x + c\Sigma x^2 \quad \ldots (i)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \ldots (ii)$

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \ldots (iii)$

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$b = 5.415$, $c = 0.036$

$$19 = 5a + 15 \times 5.415 + 55 \times 0.036$$
$$19 - 81.225 - 1.98 = 5a$$
$$a = -\frac{64.205}{5} = -12.841$$

So equation parabola

$$y = -12.841 + 5.415x + 0.036x^2$$

Ans-5 We know that line of regression, intersect at means

$$3\bar{x} + 2\bar{y} = 26 \quad \ldots (1)$$
$$6\bar{x} + \bar{y} = 31 \quad \ldots (11)$$
$$\bar{y} = 31 - 6\bar{x}$$

Putting value of y in (1)

$$3\bar{x} + 2\bar{y}(31 - 6\bar{x}) = 26$$
$$3\bar{x} + 62 - 12\bar{x} = 26$$
$$32 = 9\bar{x}$$
$$\bar{x} = \frac{32}{9}$$

$$\bar{y} = 31 - \frac{\cancel{6}^{2} \times 32}{\cancel{9}3} = 31 - \frac{64}{3} = \frac{93 - 64}{3} = \frac{29}{3}$$

Hence Means are $\bar{x} = \frac{32}{9}$ and $\bar{y} = \frac{29}{3}$

Let line

X on Y

$3x+2y=26$

$x=-\frac{2y}{3}+\frac{26}{3}$

$\rightarrow b_{xy}=-\frac{2}{3}$

Y on X

$6x+y=31$

$y=-6x+31$

$b_{yx}=-6$

$r=\sqrt{b_{xy}\times b_{yx}}=\sqrt{\left(\frac{-2}{3}\right)(-6)}>1$

Hence not possible

It contradicts our assumption

Hence

line X on Y

$6x+y=3$

$x=-\frac{y}{6}+\frac{3}{6}$

$\rightarrow b_{xy}=-\frac{1}{6}$

line Y on X

$3x+2y=26$

$y=-\frac{3x}{2}+\frac{26}{2}$

$b_{yx}=-\frac{3}{2}$

$r=\sqrt{b_{xy}\times b_{yx}}$

$=-\sqrt{\left(\frac{-1}{6}\right)\left(\frac{-3}{2}\right)}=-\sqrt{\frac{1}{4}}=-0.5$

Hence, correlation coefficient between x and y is -0.5.

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Ans-7 Let the Equation of Regression line Y on X

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

b_{yx} ↳ slope

where $\bar{x} = \frac{\Sigma x}{n}$ and $\bar{y} = \frac{\Sigma y}{n}$

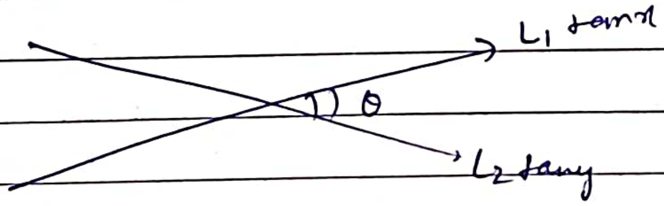
$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

X on Y

$$(x-\bar{x}) = b_{xy}(y-\bar{y})$$

Slope $= \frac{1}{b_{xy}}$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$



Let θ is the acute angle between the two lines of regression.

We know that

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$m_1 = b_{yx} = \frac{r\sigma_y}{\sigma_x}$ $m_2 = b_{xy} = \frac{r\sigma_x}{\sigma_y}$

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$$\tan\theta = \left| \frac{r\,\sigma_y/\sigma_x - \sigma_y/r\sigma_x}{1 + \frac{r\sigma_y}{\sigma_x} \times \frac{\sigma_y}{r\times\sigma_x}} \right|$$

$$= \left| \frac{\left(\frac{r^2-1}{r}\right)\left(\frac{\sigma_y}{\sigma_x}\right) \times \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right|$$

$$= \left| \left(\frac{r^2-1}{r}\right) \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \right|$$

For acute, $\tan\theta = -1 \quad |$

$$\tan\theta = \left| \left(\frac{r^2-1}{r}\right) \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \right|$$

Hence proved

Ans 9 $n=10,\ \Sigma x = 20,\ \Sigma y = 40,\ \Sigma x^2 = 240,\ \Sigma y^2 = 410,\ \Sigma xy = 200.$

$$r = \frac{Cov(x,y)}{\sigma_x\sigma_y}$$

$$Cov(x,y) = n\Sigma xy - \Sigma x\Sigma y = 10(200) - (20)(40)$$
$$= 1200$$

$$\sigma_x = \sqrt{n\Sigma x^2 - (\Sigma x)^2} = \sqrt{10(240) - (20)^2}$$

$$= \sqrt{2400-400} = \sqrt{2000} = 20\sqrt{5}$$

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$$\sigma_y = \sqrt{10(410) - 1600}$$
$$= \sqrt{4100 - 1600}$$
$$= \sqrt{2500} = 50$$

$$\gamma = \frac{\cancel{1200}^{6}}{\cancel{20}\sqrt{5} \times 5\cancel{0}}$$
$$= \frac{1.2}{\sqrt{5}} = \frac{1.2}{2.23} = 0.538$$

$$\longrightarrow \boxed{\gamma = 0.538}$$

Ans 10

Maths(m)	Chemistry (c)	R_m	R_c	$d^2 = (R_m - R_c)^2$
3	5	3	1	4
2	4	4	2	4
4	3	2	3	1
1	2	5	4	1
5	1	1	5	16
				26

Rank Coorelation, $\rho = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$
$$= 1 - \frac{\cancel{6} \times \cancel{26}\,13}{5 \times \cancel{24}\,\cancel{4}\,2}$$
$$= 1 - 1.3$$
$$= -0.3$$