ETMA-202 **ASSIGNMENT :-1**

Q1. Solve the PDE

(i) $r = a^2 t$

(ii) $(D^4 - D'^4)z = 0$

(iii) $(D^4 + D'^4)z = 0$

(iv) $(D^3 - 2D^2 D')z = 0$

Q2. Solve

(i) $(D^2 + 3DD' + 2D'^2)z = x + y$

(ii) $(D^2 + D'^2)z = \sin mx \sin ny$

(iii) $(D^2 + DD' - 6D'^2)z = y\cos x$

(iv) $(D_x^3 - 7D_x D_y^2 - 6D_y^3)z = \sin(x+2y) + e^{3x+y}$

(v) $(r + 5s + 6t) = \frac{1}{(y-2x)}$

Q3(i) Solve $(DD' + D - D' - 1)z = xy$

(ii) $(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^y$

(iii) $(D - 3D' - 2)^2 z = 2e^{2x}\sin(y+3x)$

Q4 (i) $(D^2 - D')z = 2y - x^2$

(ii) $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$

Q5. Using the method of separation of Variables, Solve

(i) $3u_x + 2u_y = 0$, $u(x,0) = 4e^{-x}$

(ii) $4u_x + u_y = 3u$, $u(0,y) = 3e^{-y} - e^{-5y}$

(iii) $u_t = u_{xx}$ Given $u = 0$ when $t \to \infty$ and $u = 0$ at $x = 0$ and $x = l$

(iv) $u_{tt} = 4u_{xx}$ Subject to Condition $u = \sin t$ at $x = 0$ and $\frac{\partial u}{\partial x} = \sin t$ at $x = 0$

Q6 (i) The Vibrations of an elastic string is governed by the p.d.e $u_{tt} = u_{xx}$ the length of the String is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the Vibrating String for $t > 0$.

(ii) The points of trisection of a string of length 'l' are pulled aside through the same distance "d" on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the String at Subsequent time and Show that the mid point of the string always remains at rest.

Q7 (i) A rod of length "l" with insulated Sides is initially at a uniform temperature u. Its ends are suddenly cooled to $0^\circ C$ and are kept at that temperature. Prove that the temperature function $u(x,t)$ is given by $u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{-\frac{c^2 n^2 \pi^2}{l^2} t}$

Where b_n is determined from the equation $u(x,0) = u_0$, Find the Value of b_n.

(ii) A bar AB of length 20 Cm has its ends A and B kept at 30° and 80° temperature respectively until steady state Condition is reached. Then the temperature at A is lowered to $40^\circ C$ and at B to $60^\circ C$ these temperatures are maintained. Find the Subsequent temperature distribution in the bar.

ASSIGNMENT 1

1. Solve the partial differential equation

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

2. Solve the partial differential equation

$$(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{2x+y}$$

3. Solve the partial differential equation

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$$

4. Solve the partial differential equation

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy$$

5. Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ where } u(x,0) = 6e^{-3x} \ ; x > 0, t > 0$$

6. Solve the equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given $u = 3e^{-y} - e^{-5y}$ when $x = 0$

7. A thin uniform tightly stretched vibrating string fixed at the points $x = 0$ and $x = l$ satisfies the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$; $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ and released from rest from this position. Find the displacement $y(x,t)$ at any x and any time t.

8. Solve the differential equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u = \sin t$ at $x = 0$ and $\frac{\partial u}{\partial x} = \sin t$ at $x = 0$.

9. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to $0^\circ c$ and are kept at that temperature. Prove that the temperature function $u(x,t)$ is given by

$$u(x,t) = \frac{4u_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin\frac{(2n-1)\pi x}{l} e^{-\frac{c^2(2n-1)^2\pi^2 t}{l^2}}.$$

10. Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions

(i) $u \to 0$ as $y \to \infty$ for all x (ii) $u = 0$ at $x = 0$ for all y

(iii) $u = 0$ at $x = l$ for all y (iv) $u = lx - x^2$ if $y = 0$ for all $x \in (0, l)$