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FOUNDATION of COMPUTER SCIENCE

ASSIGNMENT-1

Q1

p	q	r	$(p \wedge q)$	$\sim r$	$(p \wedge q) \vee (\sim r)$	$((p \wedge q) \vee (\sim r)) \leftrightarrow p$
T	T	T	T	F	T	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	F	T	T	T
F	T	T	F	F	F	T
F	T	F	F	T	T	F
F	F	T	F	F	F	T
F	F	F	F	T	T	F

Q2

$(p \vee \sim q) \rightarrow (p \wedge q)$

$(\sim(p \vee \sim q)) \vee (p \wedge q)$ (∵ Logical Equivalence)

$(\sim p \wedge q) \vee (p \wedge q)$ (∵ De Morgan law)

$q \wedge (p \vee \sim p)$ (∵ De Morgan negation law)

$q \wedge T = q$ (∵ De Morgan law)

∴ Contengency

Q3

Johan is poor if John is a poet.

Converse: If John is poor the Jhon is a poet.

Contrapositive: If John is not poor then he is not a poet.

Inverse: If Joh is not poor the he is not a poet.

Q4

p	q	$p \vee q$
T	F	T
T	T	T

∴ If p is true, then $p \vee q$ is also true.

Q5, i) $Q \leftrightarrow (R \wedge \sim P)$

$\sim P$ = It is not snowing.
$R \wedge \sim P$ = I have time and it is not snowing.

$Q \leftrightarrow (R \wedge \sim P)$ = I will go to town if and only if I have time and it is not snowing.

ii) $\sim(R \vee Q)$
= I have no time and I will not go to town.

Q6
Sol (i) Let P be the preposition "Alice is a Math major"
and q be the preposition "Alice is a CSI major".
This argument is of the form $\therefore p \vee q$.

This is an argument that uses addition rule.
Jerry is a Math major and CSI major. Therefore Jerry is a math major.

(ii) Let P be the preposition "Jerry is a Math major".
and q be the preposition "Jerry is a CSI major".
This argument is of the form $p \wedge q$
$\therefore p$
This is an argument that uses the simplification rule.

Sol (iii) Let P be the preposition "It is rainy" and q be the proposition.
This argument is of the of the form $p \to q$
p
$\therefore q$
This is an argument that uses the Modus bonens Inference rule

Q7

Sol (i) Jan is either not rich or not happy.

Sol (ii) Carlos will neither bicycle nor run tomorrow.

Sol (iii) Mei neither walks nor takes the bus to class.

Sol (iv) Ibrahim is either not smart or not harworking.

Q8

$(p \cap (\sim q)) \cup q$

$(p \cup q) \cap (\sim q \cup q)$

$(p \cup q) \cap (T)$

$p \cup q$

Hence proved

Q9

(i) True

(ii) False

Q10

(a) Here, the argument is not valid by Modus Ponen Rule or any rule:

P : Sides of triangle are equal

q : opposite angles are equal

$p \rightarrow q$

$\sim p$

$\therefore \neg q$, Hence argument is not valid.

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Q11

$P(n)$ = n is greater than 5
$Q(n)$ = n^2 is greater than 25

$$\sim(\forall x\,[P(n) \rightarrow Q(n)])$$

Q12

(a) $\exists n\ K(n)$
(b) $\exists n\ (K(n) M(o))$
(c) $\exists n\ (K(n) \rightarrow N(n))$

Q13

Solu Let $\sqrt{5}$ be a rational number
Then it must be in form of p/q where, $q \neq 0$ (p and q are coprime)

$$\sqrt{5} = \frac{p}{q}$$

$$\sqrt{5} \times q = p$$

Squaring on both sides

$$5q^2 = p^2 \quad \text{---} \quad (1)$$

p^2 is divisible by 5
So, p is divisibly by 5

$$p = 5c$$

Squaring on both sides,

$$p^2 = 25c^2 \quad \text{---} \quad (2)$$

Substituting p^2 in eqn (1)

$$5q^2 = 25c^2$$

$$q^2 = 5c^2$$

So q is divisible by 5
P and q have a common factor of 5
So there is contradiction, as p e our assumption.

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We have assumed p and q are co-prime but here they have a common factor of 5.
∴ The above statement contradicts our assumption.
Therefore $\sqrt{5}$ is an irrational number.

Q14

NAND: The negation of AND operator give output result for NAND and it is indicated by $(\sim\wedge)$.

p	q	$p \wedge q$	$\sim(p \wedge q)$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

Q15

$(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

$(p \wedge q) = (p \wedge q) \wedge (r \vee \sim r)$
$= (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r)$

$(\sim p \wedge r) = (\sim p \wedge r) \wedge (q \vee \sim q)$
$= (\sim p \wedge r \wedge q) \vee (\sim p \wedge \sim q \wedge r)$

$(q \wedge r) = (p \wedge \sim p) \wedge (q \wedge r)$
$= (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r)$

$\Rightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge r \wedge q) \vee (\sim p \wedge \sim q \wedge r)$
$\vee (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r)$

$\Rightarrow (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$

$\therefore$ This is required PDNF of $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

Q17

Solⁿ $(c \vee d), (c \vee d) \longrightarrow \sim h, \sim h \longrightarrow (a \wedge \sim b), (a \wedge \sim b) \rightarrow (r \vee s)$ are primises

1.	$c \vee d$	RP
2.	$(c \vee d) \rightarrow \sim h$	RP
3.	$\sim h$	RT from 1 & 2
4.	$\sim h \rightarrow (a \wedge \sim b)$	RP
5.	$a \wedge \sim b$	RT from 3 & 4
6.	$(a \wedge \sim b) \rightarrow (r \vee s)$	RP
7.	$r \vee s$	RT from 5 & 6

Q18

Solⁿ 1. Nested quantifives :-

$\forall x \forall y P(x,y)$ Domain $\rightarrow$ Integer and $P(x,y)$ is $xy = yx$. For all real numbers x, for all real numbers y $\boxed{xy = yx}$.

2. Infusion :-

$f(x) = 3x - 1$, x is a free variable because there is no limitation set on it.

3. For all x, $(x+1)^2 = x^2 + 2x + 1$, here x becomes a Bound variable.

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Q19.

Solution $\forall x \, \forall y \; ((x>0) \wedge (y<0) \rightarrow (xy<0))$

→ For all x, for all y, if x is greater than zero and y is smaller than zero then it implies that product of x and y is always smaller than zero.

Q20

1. False
2. True
3. False
4. True
5. True