Foundation Of Computer Science

ASSIGNMENT-3

Khushi
ITE-2
031

Ques 1

Ans For $m = 1$

$m^2 + 2m = 1 + 2 = 3$

3 is divisible by 3. So, that the statement is true for $m = 1$.

For any integer k, $k \geq 1$

$m^3 + 2m = k^3 + 2k$

$= 3m$, where 'm' is an integer

for $m = k+1$

$m^3 + 2m = (k+1)^3 + 2(k+1)$

$= k^3 + 3k^2 + 3k + 2k + 2 + 1$

$= (k^3 + 2k) + 3(k^2 + k + 1)$

$= 3m + 3(k^2 + k + 1)$

$= 3(m + k^2 + k + 1)$

which is divisible by 3.

The statement being true for $n = k$, implies the statement is true for $n = k+1$ and as we have shown it to be true for $n = 1$, the proof of the statement follows by induction.

2. Use mathematical induction to prove that

$1 + 1/4 + 1/9 + \dots + 1/m^2 \leq 2 - 1/m \quad \forall m \in N$

Ans For $m = 2$

$LHS = 1 + \frac{1}{4} = \frac{5}{4}$

RHS $= \frac{3}{2}$

LHS < RHS, so the statement is true for

$m = 2$

For any integer $k \geq 2$

$$S(k) = 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} \leq 2 - \frac{1}{k}$$

For, $m = k+1$

$$S(k+1) = 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$$

LHS: $2 - \frac{1}{k} + \frac{1}{(k+1)^2}$

$$2 - \frac{1}{(k+1)}\left(\frac{k+1}{k} - \frac{1}{k+1}\right)$$

$$2 - \frac{1}{(k+1)}\left(\frac{k^2+k+1}{k(k+1)}\right) < 2 - \frac{1}{k+1}$$

The statement being true for $m = k$ implies the statement is true for $n = k+1$ and as we have shown it to be true for $n = 2$, the proof of the statement follows as induction.

3. Show that if n is an integer greater than 1, then n can be written as the product of primes

Ans If $n = 2$ or $n = 3$, then n falls to be prime and the statement becomes true.

Now assuming that the statement is true for all integers from 2 upto k

This implies that $(k+1)$ is either prime or a product of primes.

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If $(k+1)$ is prime, there are integers C & D such that $1<C<D<(k+1)$ (i.e., m is divisible by numbers other than 1 and itself)

By induction hypothesis, since $2 \le C \le D \le k$, we know that C & D are either prime or are product of primes.

But then $k+1 = CD$, so $k+1$ is a product of primes. Hence, if the statement is true for all integers greater than 1 upto k, then it is also true for $(k+1)$. Hence, it is true for all $n \in Z > 1$.

4. Find an explicit formula for the recurrence relation defined by: $a_n = 5a_{n-1} - 6a_{n-2}$ with the initial conditions $a_1 = 2$ and $a_3 = 1^{n-2}$.

Ans $a_n = 5a_{n-1} - 6a_{n-2}$, $a_1 = 2$, $a_3 = 1$

Characteristic equation: $r^2 - 5r + 6 = 0$

Determinant: $(-5)^2 - 4(1)(6) = 1$

$$r^2 - 5r + 6 = 0$$
$$(r-2)(r-3) = 0$$

$$a_n = \alpha_n 2^n + \alpha_k r^n$$
$$a_1 = 2\alpha_1 + 3\alpha_2 = 2$$
$$a_3 = 8\alpha_1 + 27\alpha_2 = 1$$

So, $\alpha_1 = 1.7$ and $\alpha_2 = -0.467$

$$\therefore a_n = 1.7(2^n) - 0.467(3^n)$$

5. Consider the recurrence relation

$$a_{n+2} = 2a_{n+1} + 4a_n - 8a_{n-1}$$

Find an explicit formula.

Ans $a_{m+2} = 2a_{m+1} + 4a_m - 8a_{m-1}$

$$a_{m+2} - 2a_{m+1} - 4a_m + 8a_{m-1} = 0$$

Given equation is homogenious in nature

Characteristic equation:

$$r^3 - 2r^2 - 4r + 8 = 0$$
$$(r+2)(r^2 - 4r + 4) = 0$$
$$(r+2)(r-2)(r-2) = 0$$

Roots of equation $\Rightarrow$ $r_1 = -2$, $r_2 = r_3 = 2$

Therefore,

$$a_n = A1(-2)^n + A2(2^n) + A3 \cdot n \cdot (2)^n$$

This is the required explicit formula.

6. Explicit formula for the recurrence relation defined by:

$a_k - 7a_{k-1} + 10a_{k-2} = 2k^2 + 2$, with the Initial conditions $a_0 = 0, a_1 = 1$.

A) $a_k - 7a_{k-1} + 10a_{k-2} = 2k^2 + 2$

Given equation is non-homogenous in nature

(i) Characteristic equation:-

$$r^2 - 7r + 10 = 0$$
$$(r-5)(r-2) = 0$$

Roots of equation: $r_1 = 5$, $r_2 = 2$

Therefore, $a_n = A_1(5)^n + A_2(2)^n$

(ii) Let $a_k = d_0 + d_1 k + d_2 k^2$

$$a_{k-1} = d_0 + d_1(k-1) + d_2(k-1)^2$$
$$a_{k-2} = d_0 + d_1(k-2) + d_2(k-2)^2$$

$\therefore (d_0 + d_1 k + d_2 k^2) - 7[d_0 + d_1(k-1) + d_2(k-1)^2] + 10(d_0 + d_1(k-2) + d_2(k-2)^2]$

$= 2k^2 + 2$

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$$d_0 + d_1 k + d_2 k^2 - 7d_0 - 7d_1 k + 7d_1 + (-7d_2 k^2 - 7d_2 + 14d_2 k)$$
$$+ 10d_0 + 10d_1 k - 20d_1 + 10d_2 k^2 + 40d_2 - 40kd_2 = 2k^2 + 2$$

$$(4d_0 - 13d_1 - 33d_2) + (4d_1 - 26d_2)k + (4(d_2))k^2 = 2k^2 + 2$$

Comparing

$4d_2 = 2$, $4d_1 - 26d_2 = 0$

$d_2 = \frac{1}{2}$, $4d_1 - 26\left(\frac{1}{2}\right) = 0$

$$4d_1 = 13$$
$$d_1 = 13/4$$

and $4d_0 - 13d_1 - 33d_2 = 2$

$$4d_0 - 13\left(\frac{13}{4}\right) - 33\left(\frac{1}{2}\right) = 2$$

$$16d_0 - 169 - 66 = 8$$
$$d_0 = \frac{243}{16}$$

Thus, $a_k = \frac{243}{16} + \left(\frac{13}{4}\right)k + \left(\frac{1}{2}\right)k^2$

We know

$$a_k = a_k^m + a_k^p$$

$$a_k = A_1(5)^m + A_2(2)^m + \frac{243}{16} + \left(\frac{13}{4}\right)k + \left(\frac{1}{2}\right)k^2$$

7. Considering the following graphs in the given figure (a) and (b). Find the degree of each vertex.

(a) $V_1 = 5$

$V_2 = 3$

$V_3 = 2$

$V_4 = 6$

(b) $V_1 = 5$, $V_5 = 3$

$V_2 = 4$, $V_6 = 3$

$V_3 = 4$

$V_4 = 3$

8. Find all the indegrees and outdegrees of the nodes of the graph given in the figure.

Vector	Indegree	Outdegree
V_1	0	2
V_2	1	2
V_3	2	1
V_4	3	1

9. Check whether the following graphs are isomorphic or not.

Ans

Condition	G_1	G_2
Vertices	5	5
Edge	6	6
Degree seq.	3,2,3,2,2	3,2,3,2,2

10. Determine whether the following graphs are bipartite.

Ans (i) Bipartite
(ii) Bipartite
(iii) Bipartite
(iv) Bipartite

11. Which of the undirected graphs given below have a euler circuit? Of those that do not, which have an Euler path?

Ans G_1 has Euler circuit: a b e d c e a
G_3 has Euler path: a b c d e a d f

12. Verify whether the Hamiltonian Path is possible in the following.

Ans (a) Hamiltonian path is not possible
(b) Hamiltonian path is possible : A B C D

Q13

$V = 20$

Using hand shaking, $E = 30$

$R + V = E + 2$

$\therefore R = 12$

Q14

(i) Cut Vertex: A cut vertex is a single vertex whose removal disconnects a graph.

(ii) Cut Edge: A cut edge or ~~vertex~~ bridge is a single edge whose removal disconnects a graph.

(iii) Cut set: A cut set of a connected graph G is a set S of edges with following properties:

⇒ removal of all edges in S disconnects G.

⇒ removal of some edges in S doesn't disconnect G.

(iv) Spanning Tree: It is a tree that connects all the vertices of a graph with the minimum possible member of edges.

(V) Chromatic Number of Graph: The chromatic number of a graph is the minimum number of colours needed to produce a proper colouring of a groups.