Foundation of Computer Science

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Assignment-2

Q1
Ans (i) Reflexive : A is related to A
(ii) Symmetric : If A is related to B, then B is related to A
(iii) Transitive : If A is related to B and B is related to C, then A is related to C.

Q2
Ans(i) $R = \{(1,1)\ (2,2)\ (3,3)\}$
Ans(ii) $R = \{(1,2), (1,3), (3,1)\}$

Q3 Consider $A = \{a, b, c\}$
$R: \{(a,a), (b,b)\ (c,c)\ (a,b)\ (b,a)\ (a,c)\ (c,a)\ (b,c)\ (c,a)\}$
$R^{-1}: \{(a,a)\ (b,b)\ (c,c)\ (b,a)\ (a,b)\ (c,a)\ (a,c)\ (c,b)\ (a,c)\}$
R^{-1} is reflexive, symmetric and transitive.
So, R^{-1} is an equivalence relation.

Q4 (i) $a \neq b$
For any element $a \in A$, we have $(a,a) \in R$, since $a = a$
$\therefore$ R is not reflexive
Now, let $(a,b) \in R$
$\Rightarrow a \neq b$
$\Rightarrow b \neq a$
$\Rightarrow (b,a) \in R$
$\therefore$ R is symmetric
Now let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow a \neq b$ and $b \neq c$ does not imply that
$\Rightarrow a \neq c$
So, R is not transitive
Since $a \neq b$ is symmetric relation. Hence, it can be asymmetric relation.

(ii) $ab >= 0$

For any element $a \in A$, we have $(a,a) \in R$

Let $a >= 0$

then $a \cdot a >= 0$

$\therefore$ R is reflexive

Now, let $(a,b) \in R$

$\Rightarrow ab >= 0$

$\Rightarrow b \cdot a >= 0$

$\Rightarrow (b,a) \in R$

Therefore, R is symmetric

Since, $ab >= 0$ is symmetric relation.

Hence it can't be asymmetric relation.

Now let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow a \cdot b >= 0$ and $b \cdot c >= 0$

$\Rightarrow a \cdot c \geq 0$

$\therefore$ R is transitive

(iii) $ab \geq 1$

Solv For any element $a \in A$, we have $(a,a) \in R$

Let $a \geq 1$

Then $a \cdot a \geq 1$

Therefore, R is reflexive

Now, Let $(a,b) \in R$

$\Rightarrow a \cdot b \geq 1$

$\Rightarrow b \cdot a \geq 1$

Hence $(b,a) \in R$

Therefore R is symmetric

Now, $ab \geq 1$ is symmetric. Hence it cannot be an asymmetric relation.

Now, let $(a,b) \in R$ and $(b,c) \in R$

$ab \geq 1$ and $bc \geq 1$

$\Rightarrow ac \geq 1$

Hence $(a,c) \in R$

Therefore, R is transitive

Q5

Ans $A = \{a, b, c, d\}$

$R = \{x R y \mid (x,y) \in A \text{ and } x = y\}$

For any element $a \in A$, we have $(a,a) \in R$,

Since, $a = a$

$\therefore$ R is reflexive

Now, let $(a,b) \in R$

$\Rightarrow a = b$

$\Rightarrow b = a$

$\Rightarrow (b,a) \in R$

$\therefore$ R is symmetric

Now, let $(a,b) \in R$ and $(b,c) \in R$.

$\Rightarrow a = b$ and $b = c$

$\Rightarrow a = c$

$\Rightarrow (a,c) \in R$

$\therefore$ R is transitive

Hence, R is an equivalence relation

Q6

Ans A relation R on set S is called a partially ordering if it reflexive, antisymmetric and transitive

i.e (i) $aRa \ \forall \ a \in S$

(ii) aRb and $bRa \Rightarrow a = b$

(iii) aRb and $bRc \Rightarrow aRc$

for $a, b, c \in S$

A set S together with a partial order relation R is called a partially order set or POSET.

It is denoted by (S,R).

Q7

Ans The element y is called the greatest lower bound (glb) of the subset A of a Poset {P, ≤}, if y is a lower bound that is greater than every other lower bound of A.

For example, the greatest lower bound of (5,7) is 5.

The element is called the least upper bound (lub) of the subset A of a POSET {P, ≤} if x is an upper bound that is less than every other upper bound of A.

For example, the least upper bound of the interval (5,7) is 7.

Q8

Ans For subset {a,b,c}

Upper bounds = e, f, h, j

Least upper bound = e

Lower bound = a

Greatest lower bound = a

For subset {i,h}

No upper bound

Lower bounds = f, d, b, e, c, a

Greatest lower bound = g

For subset {a,b,c,d,g}

Upper bound = g, h, i

Least upper bound = g

No lower bound

Q9

Ans Let A be the seat of strings of length eight that start with a 1 bit and B be the seat of strings of length eight that ends with 00 bits. Now, the position of strings of length 8 is _ _ _ _ _ _ _ _

|A| (starts with 1) → 1 _ _ _ _ _ _ _

If 1st place is fixed, then

(8-1) i.e. 7 places can have 2 choices : 0 or 1

Therefore A → 2^7 → 128

|B| (ends with 00) → _ _ _ _ _ _ 0 0

(8-2) i.e. 6 places have 2 choices

∴ B → 2^6 → 64

$|A \cap B|$ (starts with 1 and end with 00)

If the first and last two places are fixed, then

(8-3) i.e. 5 places can have 2 choices

Therefore, A → 2^5 → 32

Putting the values in formula

$$A \cup B = |A| + |B| - |A \cap B|$$
$$= 128 + 64 - 32$$
$$= 160$$

Hence, the number of bit strings of length 8 will either start with 1 or end with 00 is 160.

Q10

Ans (i) Pigeonhole Principle:

It states that if n items are put into ~~n~~ items are put into m containers with n>m, then atleast one container must contain more than one item.

(ii) Generalisation of Pigeonhole Principle

This illustrates a general principle called the pigeonhole principle, which states that if there are more pigeons than pigeonhole then there must be atleast one pigeonhole with atleast two pigeons in it.

Q11

Ans A partially ordered set $(L, \le)$ is called a lattice in which every pair of elements $a, b \in L$ has a GLB and LUB

We denote

GLB of a subset $\{a, b\} \le L$ by $a \wedge b$

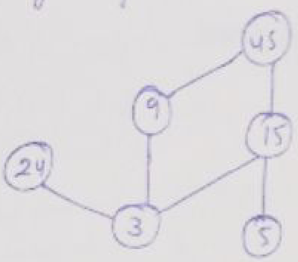
LUB of a subset $\{a, b\} \le L$ by $a \vee b$

For examples: Let $A = \{1, 2, 3, 6\}$

where a is related to b

by divisibility, meaning, "a divides b".

Q12 Ans(i) Hasse diagram for Poset $(\{3, 5, 9, 15, 24, 15, 45\}, |)$ is :



There are two maximal elements in this Hasse diagram: 24 and 45.

Also there are two minimal elements: 3 and 5

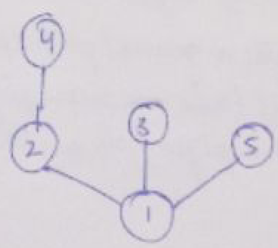
(ii) Both the maximal elements are at the same level. They are not dividing each other. So, there is no maximum or greatest element here.

Also, both the minimal elements are at the same level. There is no minimum element.

Q13 Operation table

(i) LUB

$\vee$	1	2	3	4	5
1	1	2	3	4	5
2	2	2	X	4	X
3	3	X	3	X	X
4	4	4	X	4	5
5	5	X	X	X	5



(ii) GLB

∧	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Since $(\{1,2,3,4,5\}, 1\})$ is a POSET and every pair of element of this POSET don't have LUB. So, it is not a Lattice

Ans-14 A lattice L is said to be distributive if for any element $a, b, c \in L$, we have the following.

$\rightarrow a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$\rightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Q15

(i) Supremum

Ans The element x is called supremum of the subset A of a posel $\{P, \le\}$ if x is an upper bound that is less than every other upper bound of A.

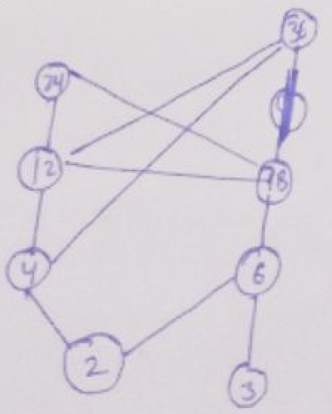
(ii) Infimum

The element y is infimum of the subset A of a POSET $\{p, \le\}$, if y is a lower bound that is a greater than every other lower of A.

(iii) Maximal elements

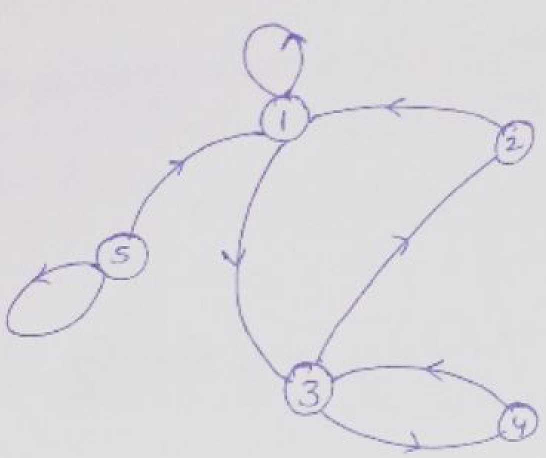
A maximal element of a subset S of some preordered set is an element of S that is not smaller than any other element of S.

Q16 Ans



Q17

Ans



Q19

Ans (i) R_2 is reflexive

(ii) R_1 is symmetric and R_3 is also symmetric

(iii) R_3 is transitive

Q20

Ans
$$f(x) = 2x+3$$
$$g(x) = 3x+2$$
$$f \circ g = f[g(x)]$$
$$= 2[3x+2]+3$$
$$= 6x+4+3 = 6x+7$$
$$g \circ f = g[f(x)]$$
$$= 3[2x+3]+3$$
$$= 6x+9+3 = 6x+12$$

Q21

Ans (i) For $f \circ g$.

Here $f(g(x)) = f \circ g$

$= f \circ g(1) = f[g(1)] = f(3) = 4$

Similarly

$$f(g(x)) = f \circ g$$
$$= f \circ g(2)$$
$$= f[g(2)]$$
$$= f[5] = 3$$

$$f(g(x)) = f \circ g$$
$$= f \circ g(3)$$
$$= f \circ g[g(3)]$$
$$= f[1] = 2$$

$$f(g(x)) = f \circ g = f \circ g(4)$$
$$= f[g(4)]$$
$$= f[2] = 1$$

$$f(g(x)) = f \circ g = f \circ g(5)$$
$$= f[g(5)]$$
$$= f[4]$$
$$= 5$$

$$\therefore f \circ g = \{(1,4)\ (2,3)\ (3,2)\ (4,1)\ (5,5)\}$$

For $g \circ f$

Here $g[f(x)] = g \circ f$
$$= g \circ f(1)$$
$$= g[f(1)]$$
$$= g[2] = 5$$

Similarly, $g[f(x)] = g \circ f$
$$= g \circ f(2)$$
$$= g[f(2)]$$
$$= g[1]$$
$$= 3$$

$$g[f(x)] = g \circ f$$
$$= g \circ f(4)$$
$$= g[f(4)]$$
$$= g[5] = 4$$

$$g[f(x)] = g \circ f$$
$$= g \circ f(5]$$
$$= g[f(3)]$$
$$= g[4] = 2$$

$$g[f(x)] = g \circ f$$
$$= g \circ f(5)$$
$$= g[f(5)]$$
$$= g[3] = 1$$

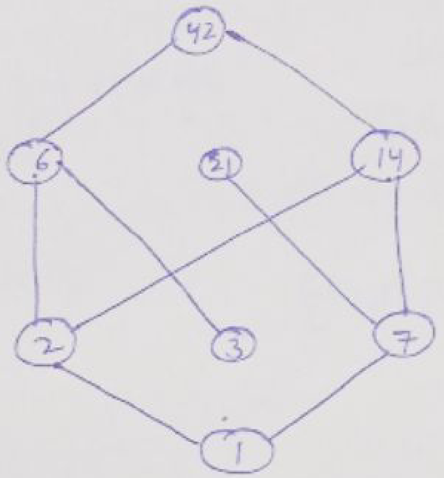
$g \circ f = \{(1,5)(2,3)(3,2)(4,4)(5,1)\}$

Hence, $f \circ g \neq g \circ f$

Q22

Ans $D_{42} = \{1, 2, 3, 6, 7, 14, 21, 42\}$

The Hasse diagram is shown by



The hasse Diagram has the greatest element $I = 42$ and least element $0 = 1$ considle $1 \in D_{42}$. let its complement be t. Then by defination $lub(1, b) = I = 42$, This is tone when $b = 42$.

Similory, by defination $glb(I, b) = 0 = 1$, which is again tone when $b = 42$

Thus complement of 1 is 42, that is $1' = 42$, By symmetry complement of 42 is 1,

that is $42' = 1$

Similarly,

$2' = 21$ Since $glb(2, 21) = 1 = 0$, $21' = 2$

$3' = 14$ Since $glb(3, 14) = 1 = 0$, $14' 3$

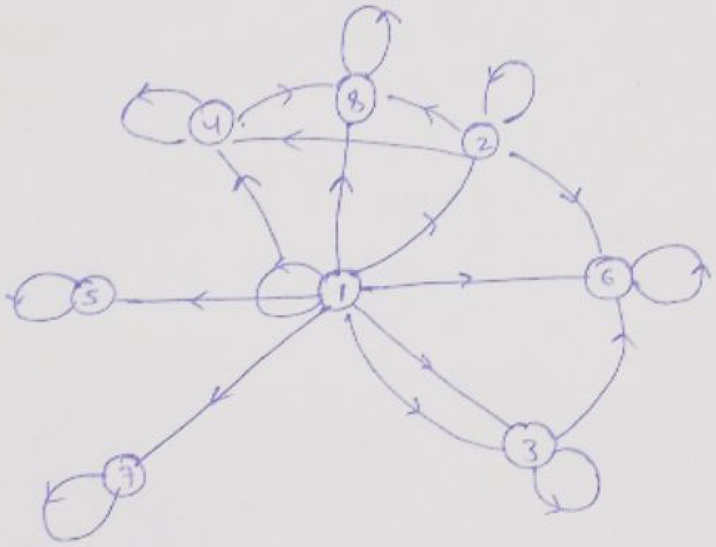
$7' = 6$ Since $glb(7, 6) = 1 = 0 = 6' 7$

Q23

Ans partical ording on set
$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ is defined as $\{(a,b)\}$ such that a divides b, so, the Relation R can be defined as

$R = \{ (1,1) (1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)$
$(2,2)(2,4)(2,6)(2,8)$
$(3,3),(4,8),(3,6),(4,4),(5,5)(6,6)(7,7)(8,8) \}$

Digraph:



Hasse diagram