LINEAR AND NON-LINEAR OPTIMISATION LAB

ETCT-258

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Semester : IV

PRACTICAL RECORD

Exp. no	Experiment Name	Date of Performance	Date of Checking	Sign/ Marks
1	*Simplex technique to solve LPP and reading dual solution from the optimal table.*			
2	*Dual Simplex technique to solve L.P.P*			
3	*Illustration of following special cases in LPP using the Simplex method – Unrestricted Variables, Unbounded solution, multiple solutions.*			
4	*To determine local/relative optima of a given unconstrained problem.*			
5	*Test whether the given function is concave/convex.*			
6	*Solution of optimization problems using Karush-Kuhn-Tucker conditions.*			
7	*Solution of Quadratic programming problem by Wolfe's method.*			
8	*Solution of Quadratic programming problem by Beal's method.*			

EXPERIMENT - 1

AIM: Simplex technique to solve LPP and reading dual solution from the optimal table.

Code:

```
clc;

//----------INPUT PARAMETERS

Noofvariables=3;

c=[-1 3 -2]

info = [3 -1 2; -2 4 0; -4 3 8];

b =[ 7; 12 ;10];

n= size(info,"r");

s = eye(n,n);

A =[info s b];

//disp(A);

cost= zeros(1,size(A,"c"));

cost(1:Noofvariables)= c;

 //--------------Constrain BV

BV = Noofvariables+1:size(A,2)-1;

 // --------------Calculate ZjCj

ZjCj = cost(BV)*A - cost;

//disp(ZjCj);

ZCj =[ZjCj;A];

mprintf('\n =========== Simplex Table ============\n')

disp(['x1' 'x2' 'x3' 's1' 's2' 's3' "Sol"], [   ZCj(1:4,1) ,
ZCj(1:4,2),ZCj(1:4,3),ZCj(1:4,4),ZCj(1:4,5),ZCj(1:4,6),ZCj(1:4,7)]);

//simplex table start

// check any negative value
```

```
for i= 1:size(ZjCj,"c")

if (ZjCj(i) <0) then

        mprintf('\n        The current BFS is NOT Optimal \n')

        mprintf('\n ==== The NEXT ITERATION RESULTS==========\n')

        end

end

disp("Old Basic Variable =  ");

disp(BV);

// Finding the Entering Variable

ZC = ZjCj(1:size(ZjCj,"c")-1);

[EnterCol,pvt_col] = min(ZC);

        mprintf('\n The Minimum element in Zj-Cj is %d Corresponding to Column %d
\n',EnterCol,pvt_col);

 //Finding the Leaving Variable

sol= A(:,$);

Column =A(:,pvt_col);

z = size(Column,"r");

 j=0

for i=1:size(Column,"r")

        if Column(i)<0

        j=j+1;

        End

end

// To check UNBOUNDED

disp(j);

if j == z;

        mprintf(' LPP is UNBOUNDED. All entries <= 0 in column  %d \n',pvt_col);

end
```

```
for i=1:size(Column,"r")

    if Column(i)>0

    ratio(i) =sol(i)./Column(i);

  else

    ratio(i)=%inf;

    end

end

[MinRatio,pvt_row] = min(ratio);

mprintf('\n The Minimum Ratio Corresponding to PIVOT Row  %d \n',pvt_row);

//n=pvt_row

disp([' LEAVING variable is'] ,[BV(pvt_row)]);

BV(pvt_row)=pvt_col;

disp(" New Basic Variable (BV)= ");

disp(BV);

 //KEY ELEMENT

pvt_key = A(pvt_row,pvt_col);

//disp(pvt_key);

 //UPDATE THE table for NEXT ITERATION

A(pvt_row,:)=A(pvt_row,:)./pvt_key;

for i= 1:size(A,1)

    if i~=pvt_row

    A(i,:)= A(i,:)-A(i,pvt_col).*A(pvt_row,:);

    end

 ZjCj = ZjCj-ZjCj(pvt_col).*A(pvt_row,:);

ZCj = [ZjCj;A];

mprintf('\n =========== Next Iteration ============\n')

disp(['x1' 'x2' 'x3' 's1' 's2' 's3' "Sol"], [   ZCj(1:4,1) ,
ZCj(1:4,2),ZCj(1:4,3),ZCj(1:4,4),ZCj(1:4,5),ZCj(1:4,6),ZCj(1:4,7)]);
```

```
end

BFS = zeros(1,size(A,2));

BFS(BV) = A(:,$);

BFS($) = sum(BFS.*cost);

mprintf('\n  ============The BFS is ================ \n')

disp(['x1' 'x2' 'x3' 's1' 's2' 's3' "Sol"], [   BFS(1,1) ,
BFS(1,2),BFS(1,3),BFS(1,4),BFS(1,5),BFS(1,6),BFS(1,7)])
```

OUTPUT:

```
=========== Simplex Table ============

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

  1.  -3.   2.   0.   0.   0.   0.
  3.  -1.   2.   1.   0.   0.   7.
 -2.   4.   0.   0.   1.   0.   12.
 -4.   3.   8.   0.   0.   1.   10.

   The current BFS is NOT Optimal

==== The NEXT ITERATION RESULTS==========

 "Old Basic Variable =  "

  4.   5.   6.

The Minimum element in Zj-Cj is -3 Corresponding to Column 2

  1.

The Minimum Ratio Corresponding to PIVOT Row  2

 " LEAVING variable is"

  5.

 " New Basic Variable (BV)= "

  4.   2.   6.

=========== Next Iteration ============

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

 -0.5   0.   2.   0.   0.75   0.   9.
  2.5   0.   2.   1.   0.25   0.   10.
 -0.5   1.   0.   0.   0.25   0.   3.
 -4.    3.   8.   0.   0.     1.   10.
```

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```
 " LEAVING variable is"

  5.

 " New Basic Variable (BV)= "

  4.   2.   6.

============ Next Iteration ============

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

 -0.5   0.   2.   0.   0.75   0.   9.
  2.5   0.   2.   1.   0.25   0.   10.
 -0.5   1.   0.   0.   0.25   0.   3.
 -4.    3.   8.   0.   0.     1.   10.

============ Next Iteration ============

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

 -0.5   0.   2.   0.   0.75   0.   9.
  2.5   0.   2.   1.   0.25   0.   10.
 -0.5   1.   0.   0.   0.25   0.   3.
 -4.    3.   8.   0.   0.     1.   10.

============ Next Iteration ============

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

 -0.5   0.   2.   0.   0.75   0.   9.
  2.5   0.   2.   1.   0.25   0.   10.
 -0.5   1.   0.   0.   0.25   0.   3.
 -2.5   0.   8.   0.  -0.75   1.   1.

 ============The BFS is ================

 "x1"  "x2"  "x3"  "s1"  "s2"  "s3"  "Sol"

  0.   3.   0.   10.   0.   1.   9.
```

EXPERIMENT - 2

AIM: Dual Simplex technique to solve LPP.

Code:

```
clc;

Variables =['x1','x2','x3','s1','s2','Sol'];

cost=[-2 0 -1 0 0 0];

info = [-1 -1 1; -1 2 -4];

b =[-5;-8];

n= size(info,"r");

s = eye(n,n);

A =[info s b];

//----------Finding starting BFS

BV=[];

for j=1:size(s,2)

        for i=1:size(A,2)

        if A(:,i)==s(:,j)

        BV = [BV i];

        end

        end

end

mprintf('\n Basic Variables (BV) = \n');

disp([Variables(BV)]);

ZjCj =cost(BV)*A - cost;

mprintf('\nZjCj = ');

disp(ZjCj);

// for print table
```

```
ZCj =[ZjCj;A];

//mprintf('\n =========== DUAL Simplex Table ============\n')

disp(['x1' 'x2' 'x3' 's1' 's2' "Sol"], [   ZCj(1:3,1) ,
ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5),ZCj(1:3,6)]);

 // DUAL SIMPLEX START

RUN = 1;

while RUN

 Sol = A(:,$);

for i= 1:size(Sol,2)

          if (Sol(i) <0) then

          mprintf('\n=== The current BFS is NOT FEASIBLE ===\n')

//Finding Leaving Variable

          [LeaVal,pvt_row] = min(Sol);

          mprintf('\nLeaving Row = %d \n',pvt_row);

 //Finding Entering Variable

          Row =A(pvt_row,1:$-1);

          ZJ = ZjCj(:,1:$-1)

          for i =1 :size(Row,2)

          if Row(i) <0

          ratio(i) = abs(ZJ(i)./Row(i));

          else

          ratio(i) = %inf;

          end

 end

 [minVAL, pvt_col] = min(ratio);

 mprintf('\nEntering Variable = %d \n',pvt_col);

//Updating the BV
```

```
BV(pvt_row) = pvt_col;

mprintf('\nBasic Variables (BV) = ')

disp([Variables(BV)]);

//Update the table for Next Iteration

 pvt_key=A(pvt_row,pvt_col);

//disp(pvt_key);

A(pvt_row,:) = A(pvt_row,:)./pvt_key;

for i= 1:size(A,1)

        if i~=pvt_row

        A(i,:) = A(i,:)-A(i,pvt_col).*A(pvt_row,:);

        end

ZjCj = cost(BV)*A-cost

mprintf('\nZjCj = ');

disp(ZjCj);

 //for print table

ZCj = [ZjCj;A];

 mprintf('\n =========== Next Iteration ============\n')

disp(['x1' 'x2' 'x3' 's1' 's2' "Sol"], [   ZCj(1:3,1) ,
ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5),ZCj(1:3,6)]);

end

 Else

RUN =0;

        mprintf('\n=== The current BFS is FEASIBLE & OPTIMAL ===\n')

        end

end

End
```

OUTPUT:

```
 Basic Variables (BV) =

  "s1"  "s2"

ZjCj =
   2.   0.   1.   0.   0.   0.

  "x1"  "x2"  "x3"  "s1"  "s2"  "Sol"

   2.   0.   1.   0.   0.   0.
  -1.  -1.   1.   1.   0.  -5.
  -1.   2.  -4.   0.   1.  -8.

=== The current BFS is NOT FEASIBLE ===

Leaving Row = 2

Entering Variable = 3

Basic Variables (BV) =
  "s1"  "x3"

ZjCj =
   1.75   0.5   0.   0.   0.25  -2.

 ============ Next Iteration ============

  "x1"  "x2"  "x3"  "s1"  "s2"  "Sol"

   1.75   0.5   0.   0.   0.25  -2.
  -1.25  -0.5   0.   1.   0.25  -7.
   0.25  -0.5   1.   0.  -0.25   2.

ZjCj =
   1.75   0.5   0.   0.   0.25  -2.
```

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```
 =========== Next Iteration ============

  "x1"  "x2"  "x3"  "s1"  "s2"  "Sol"

   1.75   0.5   0.   0.   0.25  -2.
  -1.25  -0.5   0.   1.   0.25  -7.
   0.25  -0.5   1.   0.  -0.25   2.

=== The current BFS is NOT FEASIBLE ===

Leaving Row = 1

Entering Variable = 2

Basic Variables (BV) =
  "x2"  "x3"

ZjCj =
   1.75   0.5   0.   0.   0.25  -2.

 =========== Next Iteration ============

  "x1"  "x2"  "x3"  "s1"  "s2"  "Sol"

   1.75   0.5   0.   0.   0.25  -2.
   2.5    1.    0.  -2.  -0.5    14.
   0.25  -0.5   1.   0.  -0.25   2.

ZjCj =
   0.5   0.   0.   1.   0.5  -9.

 =========== Next Iteration ============

  "x1"  "x2"  "x3"  "s1"  "s2"  "Sol"

   0.5   0.   0.   1.   0.5  -9.
   2.5   1.   0.  -2.  -0.5   14.
   1.5   0.   1.  -1.  -0.5   9.

=== The current BFS is FEASIBLE & OPTIMAL ===
```

EXPERIMENT - 3

AIM: Simplex technique to solve LPP and reading dual solution from the optimal table.

- **UNRESTRICTED VARIABLE**

SOLVE LPP Max z = x(0)1 +3x(2) St = x(0) +x(2) <= 2   Here, x(0) = x-x(1)   as x(0) is unrestricted -x(0)+x(2) <=4 x(2) >=0

Code:

```
clc;

//-----------INPUT PARAMETERS

Noofvariables=3;

c=[1 -1 3]

info = [1 -1 1; -1 1 1];

b =[ 2; 4];

n= size(info,"r");

s = eye(n,n);

A =[info s b];

//disp(A);

cost= zeros(1,size(A,"c"));

cost(1:Noofvariables)= c;

 //---------------Constrain BV
```

```
BV = Noofvariables+1:size(A,2)-1;

// ---------------Calculate ZjCj

ZjCj = cost(BV)*A - cost;

//disp(ZjCj);

ZCj =[ZjCj;A];

//disp(ZCj);

mprintf('\n =========== Simplex Table ============\n')

disp(['x' 'x1' 'x2' 's1' 's2' "Sol"], [ ZCj(1:3,1) ,
ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5),ZCj(1:3,6)]);

//simplex table start

 // check any negative value

k=0

for i= 1:size(ZjCj,2)

if (ZjCj(i) <0) then

        k=k+1

end

end

if k>0

        mprintf('\n     The current BFS is NOT Optimal \n')

end

disp("Old Basic Variable =  ");

disp(BV);

// Finding the Entering Variable

ZC = ZjCj(1:size(ZjCj,"c")-1);

[EnterCol,pvt_col] = min(ZC);
```

```
        mprintf('\n The Minimum element in Zj-Cj is %d Corresponding to Column %d
\n',EnterCol,pvt_col);

 //Finding the Leaving Variable

sol= A(:,$);

Column =A(:,pvt_col);

z = size(Column,"r");

 j=0

for i=1:size(Column,"r")

        if Column(i)<0

        j=j+1;

        end

end

// To check UNBOUNDED

//disp(j);

if j == z;

        mprintf(' LPP is UNBOUNDED. All entries <= 0 in column  %d \n',pvt_col);

end

 for i=1:size(Column,"r")

        if Column(i)>0

        ratio(i) =sol(i)./Column(i);

   else

        ratio(i)=%inf;

        end

end

 [MinRatio,pvt_row] = min(ratio);

mprintf('\n The Minimum Ratio Corresponding to PIVOT Row  %d \n',pvt_row);
```

```
//n=pvt_row

disp([' LEAVING variable is'] ,[BV(pvt_row)]);

BV(pvt_row)=pvt_col;

disp(" New Basic Variable (BV)= ");

disp(BV);

 //KEY ELEMENT

pvt_key = A(pvt_row,pvt_col);

//disp(pvt_key);

 //UPDATE THE table for NEXT ITERATION

A(pvt_row,:)=A(pvt_row,:)./pvt_key;

for i= 1:size(A,1)

        if i~=pvt_row

        A(i,:)= A(i,:)-A(i,pvt_col).*A(pvt_row,:);

        end

 ZjCj = ZjCj-ZjCj(pvt_col).*A(pvt_row,:);

ZCj = [ZjCj;A];

mprintf('\n =========== Next Iteration ============\n')

disp(['x' 'x1' 'x2' 's1' 's2' "Sol"], [   ZCj(1:3,1) ,
ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5),ZCj(1:3,6)]);

end

BFS = zeros(1,size(A,2));

BFS(BV) = A(:,$);

BFS($) = sum(BFS.*cost);

mprintf('\n  ============The BFS is ================ \n')
disp(['x' 'x1' 'x2' 's1' 's2' "Sol"], [   BFS(1,1) ,
BFS(1,2),BFS(1,3),BFS(1,4),BFS(1,5),BFS(1,6)]);
```

OUTPUT:

```
============ Simplex Table ============

 "x"  "x1"  "x2"  "s1"  "s2"  "Sol"

 -1.   1.  -3.   0.   0.   0.
  1.  -1.   1.   1.   0.   2.
 -1.   1.   1.   0.   1.   4.

   The current BFS is NOT Optimal

 "Old Basic Variable =  "

  4.   5.

The Minimum element in Zj-Cj is -3 Corresponding to Column 3

The Minimum Ratio Corresponding to PIVOT Row  1

 " LEAVING variable is"

  4.

 " New Basic Variable (BV)= "

  3.   5.

============ Next Iteration ============

 "x"  "x1"  "x2"  "s1"  "s2"  "Sol"

  2.  -2.   0.   3.   0.   6.
  1.  -1.   1.   1.   0.   2.
 -1.   1.   1.   0.   1.   4.
```

```
============ Next Iteration ============

 "x"  "x1"  "x2"  "s1"  "s2"  "Sol"

  2.  -2.   0.   3.   0.   6.
  1.  -1.   1.   1.   0.   2.
 -1.   1.   1.   0.   1.   4.

============ Next Iteration ============

 "x"  "x1"  "x2"  "s1"  "s2"  "Sol"

  2.  -2.   0.   3.   0.   6.
  1.  -1.   1.   1.   0.   2.
 -2.   2.   0.  -1.   1.   2.

 ============The BFS is ================

 "x"  "x1"  "x2"  "s1"  "s2"  "Sol"

  0.   0.   2.   0.   2.   6.
```

- **UNBOUNDED SOLUTIONS**

Consider the linear program:

$$\begin{array}{lrrcll} \text{Maximize} & 2x_1 & +x_2 & & & \\ \text{Subject to:} & & & & & \\ & x_1 & -x_2 & \leq & 10 & (1) \\ & 2x_1 & -x_2 & \leq & 40 & (2) \\ & & x_1,\ x_2 \geq 0\,. & & & \end{array}$$

Code:

```
clc;

//-----------INPUT PARAMETERS

Noofvariables=2;

c=[2 1 ]

info = [1 -1; 2 -1];

b =[ 10; 40];

n= size(info,"r");

s = eye(n,n);

A =[info s b];

//disp(A);

cost= zeros(1,size(A,"c"));

cost(1:Noofvariables)= c;


//---------------Constrain BV

BV = Noofvariables+1:size(A,2)-1;


// ---------------Calculate ZjCj

ZjCj = cost(BV)*A - cost;

//disp(ZjCj);
```

```
ZCj =[ZjCj;A];

//disp(ZCj);

mprintf('\n =========== Simplex Table ============\n')

disp(['x1' 'x2' 's1' 's2' "Sol"], [ ZCj(1:3,1) , ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5)]);

//simplex table start

k=0

 // check any negative value

for i= 1:size(ZjCj,2)

 if (ZjCj(i) <0) then

            k=k+1

 end

if k>0

            mprintf('\n       The current BFS is NOT Optimal \n')

 disp("Old Basic Variable =  ");

disp(BV);

// Finding the Entering Variable

ZC = ZjCj(1:size(ZjCj,"c")-1);

[EnterCol,pvt_col] = min(ZC);

            mprintf('\n The Minimum element in Zj-Cj is %d Corresponding to Column %d
\n',EnterCol,pvt_col);


//Finding the Leaving Variable

sol= A(:,$);

Column =A(:,pvt_col);

z = size(Column,"r");

 j=0

for i=1:size(Column,"r")
```

```
if Column(i)<0
            j=j+1;
            end
end
// To check UNBOUNDED
  if j == z;
            mprintf(' LPP is UNBOUNDED. All entries <= 0 in column  %d \n',pvt_col);
 break
   end;
for i=1:size(Column,"r")
            if Column(i)>0
            ratio(i) =sol(i)./Column(i);
   else
            ratio(i)=%inf;
            end
end
 [MinRatio,pvt_row] = min(ratio);
mprintf('\n The Minimum Ratio Corresponding to PIVOT Row  %d \n',pvt_row);
//n=pvt_row
disp([' LEAVING variable is'] ,[BV(pvt_row)]);
BV(pvt_row)=pvt_col;
disp(" New Basic Variable (BV)= ");
disp(BV);
 //KEY ELEMENT
pvt_key = A(pvt_row,pvt_col);
//disp(pvt_key);
```

```
//UPDATE THE table for NEXT ITERATION
A(pvt_row,:)=A(pvt_row,:)./pvt_key;
for i= 1:size(A,1)
        if i~=pvt_row
        A(i,:)= A(i,:)-A(i,pvt_col).*A(pvt_row,:);
        end
 ZjCj = ZjCj-ZjCj(pvt_col).*A(pvt_row,:);
ZCj = [ZjCj;A];
mprintf('\n =========== Next Iteration ============\n')
disp(['x1' 'x2' 's1' 's2' "Sol"], [   ZCj(1:3,1) , ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5)]);
end
BFS = zeros(1,size(A,2));
BFS(BV) = A(:,$);
BFS($) = sum(BFS.*cost);
mprintf('\n  ============The BFS is ================ \n')
disp(['x1' 'x2' 's1' 's2' "Sol"], [   BFS(1,1) , BFS(1,2),BFS(1,3),BFS(1,4),BFS(1,5)]);
 end
End
```

OUTPUT:

```
=========== Simplex Table ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

 -2.  -1.   0.   0.   0.
  1.  -1.   1.   0.   10.
  2.  -1.   0.   1.   40.

   The current BFS is NOT Optimal

 "Old Basic Variable =  "

  3.   4.

The Minimum element in Zj-Cj is -2 Corresponding to Column 1

The Minimum Ratio Corresponding to PIVOT Row  1

 " LEAVING variable is"

  3.

 " New Basic Variable (BV)= "

  1.   4.

=========== Next Iteration ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

  0.  -3.   2.   0.   20.
  1.  -1.   1.   0.   10.
  2.  -1.   0.   1.   40.
```

```
=========== Next Iteration ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

  0.  -3.   2.   0.   20.
  1.  -1.   1.   0.   10.
  0.   1.  -2.   1.   20.

 ============The BFS is ================

 "x1"  "x2"  "s1"  "s2"  "Sol"

  10.   0.   0.   20.   20.

   The current BFS is NOT Optimal

 "Old Basic Variable =  "

  1.   4.

The Minimum element in Zj-Cj is -3 Corresponding to Column 2

The Minimum Ratio Corresponding to PIVOT Row  2

 " LEAVING variable is"

  4.

 " New Basic Variable (BV)= "

  1.   2.
```

```
=========== Next Iteration ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

  0.   0.  -4.   3.   80.
  1.   0.  -1.   1.   30.
  0.   1.  -2.   1.   20.

 ============The BFS is ================

 "x1"  "x2"  "s1"  "s2"  "Sol"

  30.   20.   0.   0.   80.

   The current BFS is NOT Optimal

 "Old Basic Variable =  "

  1.   2.

The Minimum element in Zj-Cj is -4 Corresponding to Column 3
LPP is UNBOUNDED. All entries <= 0 in column  3
```

- **MULTIPLE SOLUTIONS**

Maximize $2000x_1 + 3000x_2$

subject to

$6x_1 + 9x_2 \leq 100$
$2x_1 + x_2 \leq 20$

$x_1, x_2 \geq 0$

CODE:

```
clc;

//-----------INPUT PARAMETERS

Noofvariables=2;

c=[4 14]

info = [2  7; 7 2];

b =[ 21; 21];

n= size(info,"r");

s = eye(n,n);

A =[info s b];

//disp(A);

cost= zeros(1,size(A,"c"));

cost(1:Noofvariables)= c;

 //---------------Constrain BV

BV = Noofvariables+1:size(A,2)-1;

 // ---------------Calculate ZjCj

ZjCj = cost(BV)*A - cost;

//disp(ZjCj);

ZCj =[ZjCj;A];
```

```
//disp(ZCj);

mprintf('\n =========== Simplex Table ============\n')

disp(['x1' 'x2' 's1' 's2' "Sol"], [ ZCj(1:3,1) , ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5)]);

//simplex table start

 // check any negative value

k=0

for i= 1:size(ZjCj,2)

if (ZjCj(i) <0) then

        k=k+1

end

end

if k>0

        mprintf('\n       The current BFS is NOT Optimal \n')

end

disp("Old Basic Variable =  ");

disp(BV);

// Finding the Entering Variable

ZC = ZjCj(1:size(ZjCj,"c")-1);

[EnterCol,pvt_col] = min(ZC);

        mprintf('\n The Minimum element in Zj-Cj is %d Corresponding to Column %d
\n',EnterCol,pvt_col);

 //Finding the Leaving Variable

sol= A(:,$);

Column =A(:,pvt_col);

z = size(Column,"r");

 j=0

for i=1:size(Column,"r")
```

```
            if Column(i)<0
            j=j+1;
            end
end
// To check UNBOUNDED
//disp(j);
if j == z;
            mprintf(' LPP is UNBOUNDED. All entries <= 0 in column  %d \n',pvt_col);
end
 for i=1:size(Column,"r")
            if Column(i)>0
            ratio(i) =sol(i)./Column(i);
   else
            ratio(i)=%inf;
            end
end
 [MinRatio,pvt_row] = min(ratio);
mprintf('\n The Minimum Ratio Corresponding to PIVOT Row  %d \n',pvt_row);
//n=pvt_row
disp([' LEAVING variable is'] ,[BV(pvt_row)]);
BV(pvt_row)=pvt_col;
disp(" New Basic Variable (BV)= ");
disp(BV);
 //KEY ELEMENT
pvt_key = A(pvt_row,pvt_col);
//disp(pvt_key);
```

```
//UPDATE THE table for NEXT ITERATION
A(pvt_row,:)=A(pvt_row,:)./pvt_key;
for i= 1:size(A,1)
        if i~=pvt_row
 A(i,:)= A(i,:)-A(i,pvt_col).*A(pvt_row,:);
        end

ZjCj = ZjCj-ZjCj(pvt_col).*A(pvt_row,:);
ZCj = [ZjCj;A];
mprintf('\n =========== Next Iteration ============\n')
disp(['x1' 'x2' 's1' 's2' "Sol"], [   ZCj(1:3,1) , ZCj(1:3,2),ZCj(1:3,3),ZCj(1:3,4),ZCj(1:3,5)]);
end
BFS = zeros(1,size(A,2));
BFS(BV) = A(:,$);
BFS($) = sum(BFS.*cost);
//disp(BFS);
mprintf('\n  ============The BFS is ================ \n')
disp(['x1' 'x2' 's1' 's2' ], [   BFS(1,1) , BFS(1,2),BFS(1,3),BFS(1,4)]);
for i= 1:2
 if  BFS(1,i)== 0 then
        disp("Multiple Solutions as Zj-Cj value corresponding to non basic variable is zero")
        else
 end
end
```

OUTPUT:

```
=========== Simplex Table ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

 -4.  -14.   0.   0.   0.
  2.   7.    1.   0.   21.
  7.   2.    0.   1.   21.

   The current BFS is NOT Optimal

 "Old Basic Variable =  "

  3.   4.

The Minimum element in Zj-Cj is -14 Corresponding to Column 2

The Minimum Ratio Corresponding to PIVOT Row  1

 " LEAVING variable is"

  3.

 " New Basic Variable (BV)= "

  2.   4.

=========== Next Iteration ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

  0.      0.   2.      0.   42.
  0.286   1.   0.143   0.   3.
  7.      2.   0.      1.   21.

=========== Next Iteration ============

 "x1"  "x2"  "s1"  "s2"  "Sol"

  0.      0.   2.      0.   42.
  0.286   1.   0.143   0.   3.
  6.429   0.  -0.286   1.   15.

 ============The BFS is ================

 "x1"  "x2"  "s1"  "s2"

  0.   3.   0.   15.

 "Multiple Solutions as Zj-Cj value corresponding to non basic variable is zero"
```

EXPERIMENT - 5

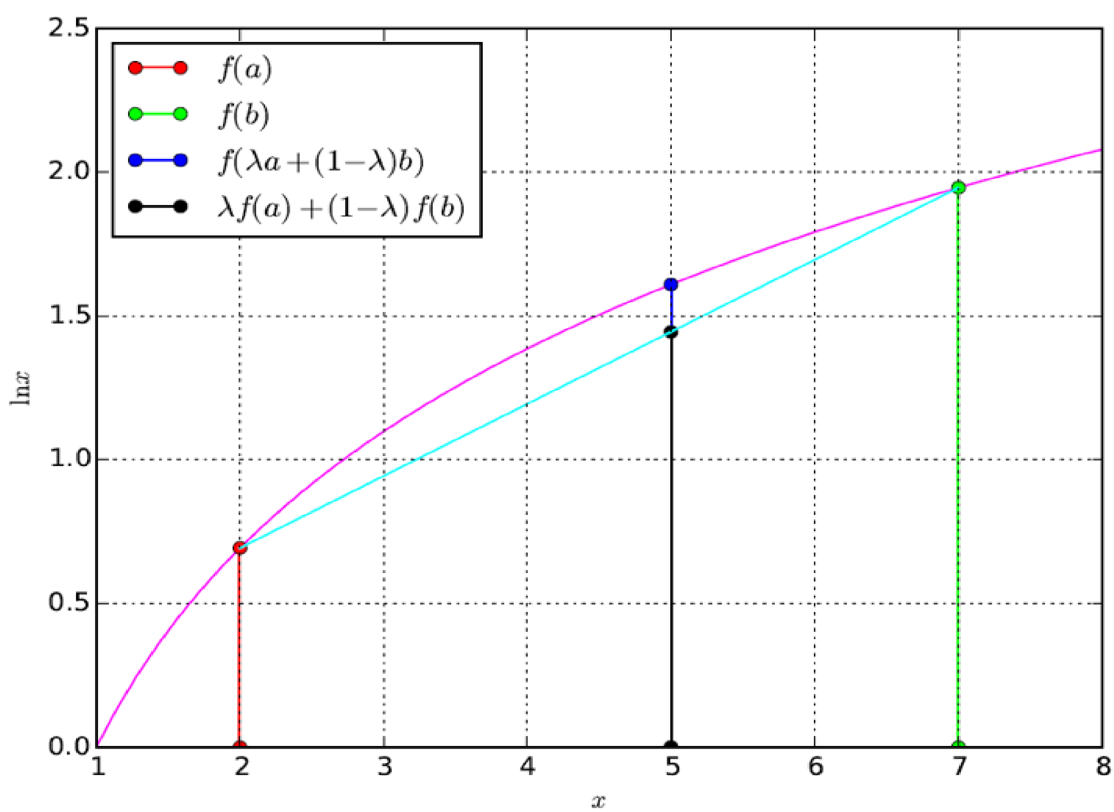
AIM: Simplex technique to solve LPP and reading dual solution from the optimal table.

CODE:

```
import numpy as np

import matplotlib.pyplot as  plt

# Plotting log( x )

x = np.linspace( 1 , 8 , 5 0 )

# p o i n t s on t h e x a x i s

f=np.log( x ) # Ob j e c t i v e f u n c t i o n

plt.plot(x,f,color =( 1 , 0 , 1 ))

plt.grid( )

plt.xlabel('$x$ '')

plt.ylabel('$\lnx$' )

# Co n v e x i t y / Co n c a v i t y

a = 2

b = 7

lamda =0.4

c =lamda∗a+(1−lamda)∗b

f_a=np.log(a)

f_b=np.log(b)

f_c=np.log(c)

f_c_hat=lamda∗f_a+(1−lamda)∗f_b
```

```
# Plot commands

plt.plot([a,a],[0,f_a],color=(1,0,0),marker='o',label="$f(a)$")

plt.plot([b,b],[ 0 , f_b],color=(0,1,0),marker ='o',label= "$f( b )$")

plt.plot([c,c],[0,f_c],color=(0,0,1),marker ='o',label= " $f(\lambda a+(1−\lambda)b) $")

plt.plot ([c,c] , [0,f_c_hat ] , color=( 1 / 2 , 2 / 3 , 3 / 4), marker= 'o' ,label="
$\lambdaf(a)+(1−\lambda)f(b)$")

plt.plot([a,b], [f_a,f_b],color=(0 , 1 , 1))

plt.legend(loc=2)

# plt. save fig (  '. . / f i g s / 1 . 1 . eps ' )

plt.show( )
```

OUTPUT:



EXPERIMENT - 6

AIM: Solution of optimization problems using Karush-Kuhn-Tucker conditions.

> Solve
>
> $$\min_{\mathbf{x}} \quad x_1 + x_2 \qquad (2.53)$$
>
> with the constraints
>
> $$x_1^2 - x_1 + x_2^2 \leq 0 \qquad (2.54)$$
>
> where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

THEORY :

In general for solving a convex optimization problem like

using Lagrange Multipliers. These are called Karush-Kuhn-Tucker(KKT) conditions.

Using the method of Lagrange multipliers,

$\nabla \{ f(x) + \mu g(x)\} = 0, \mu \geq 0$

resulting in the equations

$2x_1 \mu - \mu + 1 = 0$, $2x^2\mu + 1 = 0$, $x^2{}_1 - x_1 + x_2 = 0$

so, $\mu = \sqrt{2}$.

Graphical solution: The constraint can be expressed

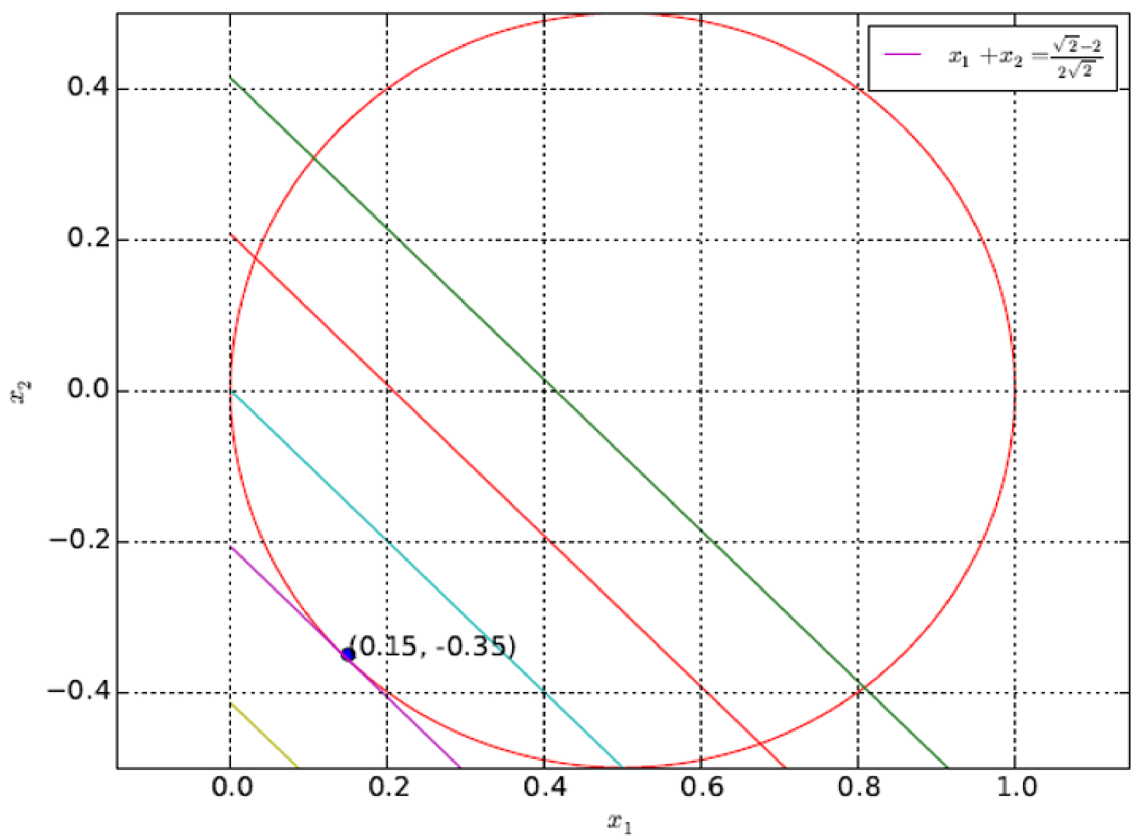
$x^21 - x_1 + x^2{}_1 <= 0$

CODE:

```
import numpy a s np

import matplotlib.pyplotasplt

sol = np.zeros ((2,1) )

# Printing minimum

sol[0] =(np.s qrt(2)−1) / (2 ∗ np.sqrt( 2 ) )

sol[ 1] = −1/ (2∗ np.sqr t (2 ) )

# Plotting the circle

circle = plt.Circle( (0.5 ,0) , 0.5 ,

color=’r’,fill =False)

fig , ax = plt.sub plots( )

ax .addartist(cir cle)

 A = np.around ( sol[ 0 ] , decimals =2)

B = np.around ( sol [ 1 ] , decimals =2)

plt.plot (A,B, ’ o ‘)

for xy in zip (A,B) :

ax . annotate ( ‘(%s , %s )’ %xy , xy=xy , textcoords=’data ’ )

print (sol)

# Plotting the line

p =(sol[0]+ sol[1]) ∗np . arange( −2 ,3)

x = np.linspace ( 0 , 1 ,100 )

na=np.newaxi s

x line = x [ : ,na ]

y line = p [ na , : ] − x[ : ,na ]

bx= plt . plot ( x line , y line , ’− ’ )
```

```
plt . axis ( ' equal ' )
plt . grid( )
plt . xlabel ( ' $x 1$ ' )
plt .ylabel ( '$x 2$ ' )
plt .ylim ( −0 .5 , 0. 5 )
plt . legend ( [ bx [ 3 ] ] , [ '$x 1+x 2=\\ frac { \ sqr t { 2 } −2 } { 2 \ sqr t { 2 } } $ ' ] ,
loc= ' best ' , prop ={ 'size ' : 11 } )
plt.show ( )
```

<u>OUTPUT:</u>



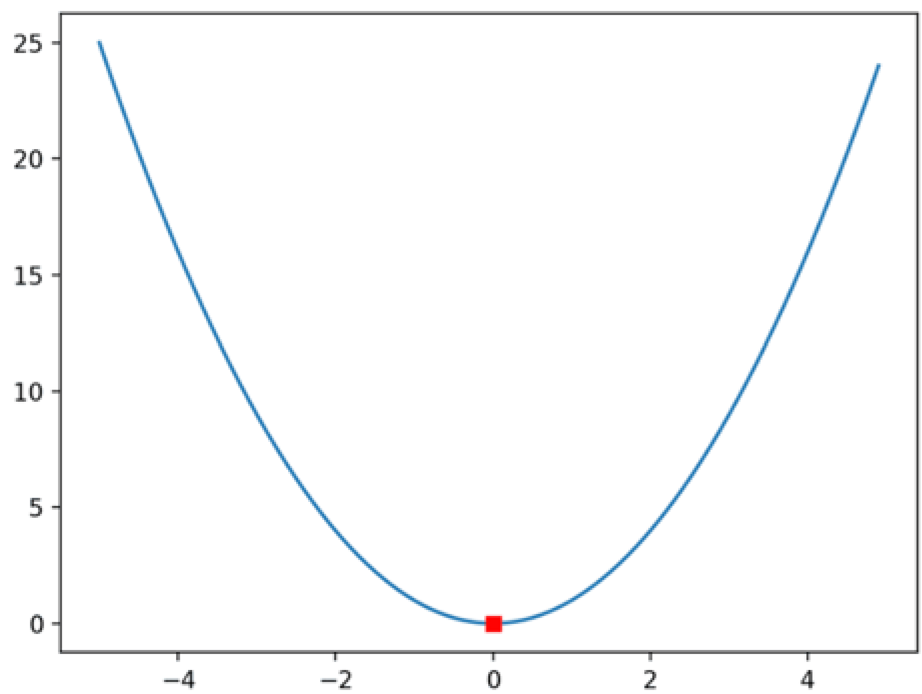
EXPERIMENT - 4

AIM: To determine a given unconstrained problem's local/relative optima.

Code:

```
from numpy import arange

from matplotlib import pyplot

  # objective function

def objective(x):

        return x**2.0

  # define range for input

r_min, r_max = -5.0, 5.0

# sample input range uniformly at 0.1 increments

inputs = arange(r_min, r_max, 0.1)

# compute targets

results = objective(inputs)

# create a line plot of input vs result

pyplot.plot(inputs, results)

# define the known function optima

optima_x = 0.0

optima_y = objective(optima_x)

# draw the function optima as a red square

pyplot.plot([optima_x], [optima_y], 's', color='r')

# show the plot

pyplot.show()
```

OUTPUT:



EXPERIMENT - 7

AIM: Solution of Quadratic programming problem by Wolfe's method.

Code:

```
function [x, fval]=wolf(D, l, b, Mat, inq, minimize)

n = length(l);

m = length(b);

 if ~isequal(size(Mat,1),m) || ~isequal(length(inq),m) || ~isequal(size(D,1),size(D,2)) ||
~isequal(size(D,1),n) || ~isequal(size(Mat,2),n)

fprintf('\nError: Dimension mismatch!\n');

          return

end

 if nargin < 4 || nargin > 6

          mprintf('\nError:Number of input arguments are inappropriate!\n');

          return

end

 if nargin < 5

          minimize = 0;

          inq = -ones(m,1);

elseif nargin < 6

          minimize = 0;

end

 if minimize == 1

          l = -l;

          D = -D;

end
```

```
if min(spec(-D)) < 0 % Checking convexity of Hessian
    mprintf('\nError: Wolf method may not converge to global optimum!\n');
    return
elseif (min(spec(-D)) == 0) && ~isempty(find(l,1))
    mprintf('\nError: Wolf method may not converge to global optimum!\n');
    return
end
 count = n;
 for i = 1 : m
    if (inq(i) > 0)
    Mat(i,:) = -Mat(i,:);
    b(i) = -b(i);
    elseif (inq(i) == 0)
    count = count + 1;
    Mat(i,count) = -1;
    l(count) = 0;
    D(count,count) = 0;
    end
end
 a = [-2*D Mat' -eye(count,count) zeros(count,m);Mat zeros(m,m + count) eye(m,m)];
d = [l;b];
 for i = 1 : count + m
    if(d(i) < 0)
    d(i) = -d(i);
    a(i,:) = -a(i,:);
    end
end
```

```
 cb = zeros(1,count + m);
bv = zeros(1,count + m);
nbv = (1 : 2 * (count + m));
c = zeros(1,2 * (count + m));
rem = zeros(1,count + m);
 for i = 1 : count + m
        if(a(i,count + m + i) == -1)
        bv(i) = 2 * (count + m) + i;
        cb(i) = -1;
        elseif(a(i,count + m + i) == 1)
        rem(i)=count + m + i;
        bv(i) = count + m + i;
        cb(i) = 0;
        end
end
 [h,j,k] = find(rem);
a(:,k) = [];
c(k) = [];
nbv(k) = [];
r = cb * a - c;
exitflg = 0;
iter = 0;
z = cb * d;
[w,y] = size(a);
opt = 0;
 while(exitflg == 0)
iter = iter + 1;
```

```
    mprintf('\n\n %d th tableau:\n',iter );

    mprintf('\n\t\t\tBV\t');disp(nbv);

    disp([bv' d a ;0 z r]);

    r_new = r;

    found = 0;

    while found == 0

    [u,v] = min(r_new);

    leave = 0;

    if ~(u < 0)

    if abs(z) > 10^-6

    mprintf('\nError: Wolf method fails to find optimum!\n');

    exitflg = 1;

    found = 1;

    else

    mprintf('\nThe optimum has achieved!\n');

    exitflg = 1; opt = 1;

    found = 1;

    end

    else

ratio = 15;

    check = 0;

for i = 1 : w

    if bv(i) <= 2 * (count +  m) && abs(bv(i) - nbv(v)) == count + m

            check = 1;

    end

    end

    if check == 0
```

```
for i = 1 : w
        if a(i,v) > 0 && (d(i) / a(i,v)) < ratio
        ratio = d(i) / a(i,v);
        leave = i;
        end
    end
    mprintf('\nEntering Variable:'); disp(nbv(v));
    mprintf('\nLeaving Variable:'); disp(bv(leave));
    for i = 1 : w
        for j = 1 : y
        if i ~= leave && j ~= v
        a(i,j) = a(i,j) - a(i,v) * a(leave,j) / a(leave,v);
        end
        end
    end
    z = z - d(leave) * r(v) / a(leave,v);
    for j = 1 : y
        if j ~= v
        r(j) = r(j) - r(v) * a(leave,j) / a(leave,v);
        a(leave,j) = a(leave,j) / a(leave,v);
        end
    end
    for i = 1 : w
        if i ~= leave
        d(i) = d(i) - a(i,v) * d(leave) / a(leave,v);
        a(i,v) = -a(i,v) / a(leave,v);
        end
```

```
end
        d(leave) = d(leave) / a(leave,v);
        a(leave,v) = 1 / a(leave,v);
        r(v) = -r(v) / a(leave,v);
        temp = nbv(v);
        nbv(v) = bv(leave);
        bv(leave) = temp;
        found = 1;
        elseif check == 1
        r_new(v) = 1;
        end
        end
        end
end
if opt == 1
        x = zeros(n,1);
        for i = 1 : w
        if bv(i) <= n
        x(bv(i)) = d(i);
        end
        end
        fval = x'*l+x'*D*x;
        if minimize == 1
        fval = -fval;
        end
end
end
```

OUTPUT:

Scilab 6.1.1 Console

```
--> exec('C:\Users\DELL\Desktop\simplex11.sce', -1)

--> D=[-3 1;1 -2];l=[2;3];b=[1;12];Mat=[1 1;3 4];inq=[1 -1];

-->

--> wolf(D,l,b,Mat,inq);


 1 th tableau:

                        BV
   2.   3.   4.   5.   6.   7.   8.

   9.    2.   -2.  -1.   3.  -1.   0.   0.   0.
   10.   3.    4.  -1.   4.   0.  -1.   0.   0.
   11.   1.    1.   0.   0.   0.   0.  -1.   0.
   8.    12.   4.   0.   0.   0.   0.   0.   1.
   0.   -6.   -3.   2.  -7.   1.   1.   1.   0.

Entering Variable:
   2.

Leaving Variable:
   10.


 2 th tableau:

                        BV
   10.   3.   4.   5.   6.   7.   8.

   9.    3.5    0.5   -1.5    5.  -1.  -0.5    0.   0.
   2.    0.75   0.25  -0.25   1.   0.  -0.25   0.   0.
   11.   0.25  -0.25   0.25  -1.   0.   0.25  -1.   0.
   8.    9.    -1.     1.    -4.   0.   1.     0.   1.
   0.   -3.75   12.    1.25  -4.   1.   0.25   1.   0.

Error: Wolf method fails to find optimum!
```