Probability And Statistics

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ASSIGNMENT-4

Ques Determine the estimated value of the parameter λ in the poission distribution using the maximum likelihood estimation.

Soln $$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Now, the likelihood function,

$$L(\lambda; x_1, \dots x_n) = \prod_{j=1}^{n} \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}$$

Taking natural log

$$l(\lambda; x_1, \dots x_n) = \ln\left(\prod_{j=1}^{n} \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}\right)$$

$$= \sum_{j=1}^{n} \ln\left(\frac{\lambda^{x_j} e^{-\lambda}}{x_j!}\right)$$

$$= \sum_{j=1}^{n} \left[\ln(\lambda^{x_j}) + \ln(e^{-\lambda}) - \ln(x_j!)\right]$$

$$= \sum_{j=1}^{n} \left[x_j \ln(\lambda) - \lambda - \ln(x_j!)\right]$$

$$= -n\lambda + \ln(\lambda) \sum_{j=1}^{n} x_j - \sum_{j=1}^{n} \ln(x_j!)$$

Calculating derivative w.r.t λ,

$$\frac{\partial}{\partial\lambda} l(\lambda; x_1, \dots x_n) = \frac{\partial}{\partial\lambda}\left(-n\lambda + \ln\lambda \sum_{j=1}^{n} x_j - \sum_{j=1}^{n} \ln(x_j!)\right)$$

$$= -n + \frac{1}{\lambda}\sum x_j$$

Putting it equal to 0.

$$-n + \frac{1}{\lambda}\sum x_j = 0$$

$$\Rightarrow \boxed{\lambda = \frac{1}{n}\sum_{j=1}^{n} x_j}$$

This is equivalent to the sample mean of n observations in the sample.

Que What is p value in hypothesis testing? Explain with suitable Example.

Ans The p-value is a number, calculated from a statistical test, that describes how likely you are to have found a particular set of observation if the null hypothesis were true.

p-values are used in hypothesis testing to help decide whether to reject the null hypothesis. The smaller the p value, the more likely you are to reject the null hypothesis.

Example

Q The average weight of all residents in town XYZ is 168 lbs. A nutritionist belives the true mean to be different. She measured the weight of 36 individuals and found the mean to be 169.5 lbs with a standard deviation of 3.9.

At a 95% confidence level, is there enough evidence to discard the null hypothesis? (use p value method)

Solu it's a two tailed test

$H_0 : \mu = 168$, $n = 36$

$H_a : \mu \neq 168$, $C = 0.95$

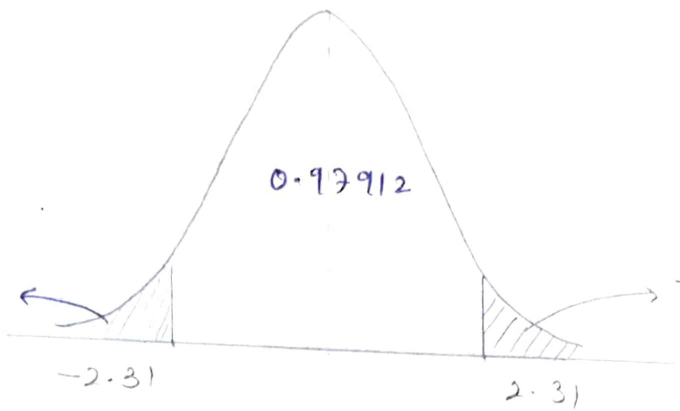
$\bar{x} = 169.5$, $S = 3.9$

$\alpha = 1 - C = 0.05$

$$Z_c = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

$$= \frac{169.5 - 168}{3.9/\sqrt{36}}$$

$$= \frac{1.5}{0.65} = 2.31$$



Similary

0.01044

This area

$= 1 - 0.98956$

$= 0.01044$

pvalue = sum of area on both sides

$= 0.01044 + 0.01044$

$= 0.02088$

Now

$\alpha = 0.05$

$\boxed{\text{pvalue} < \alpha}$

Hence, the null hypothesis is rejected