Assignment

Unit – 3

1. Prove the following result by the method of Mathematical Induction.
   For all $n \geq 1$, $n^3 + 2n$ is a multiple of 3.

2. Use Mathematical Induction to prove that:

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n} \forall n \in N$$

3. Show that if n is an integer greater than 1, then n can be written as the product of primes.

4. Find an explicit formula for the recurrence relation defined by :

   $a_n = 5a_{n-1} - 6a_{n-2}$ with the initial conditions $a_1 = 2\ and\ a_3 = 1$.

5. Consider the recurrence relation

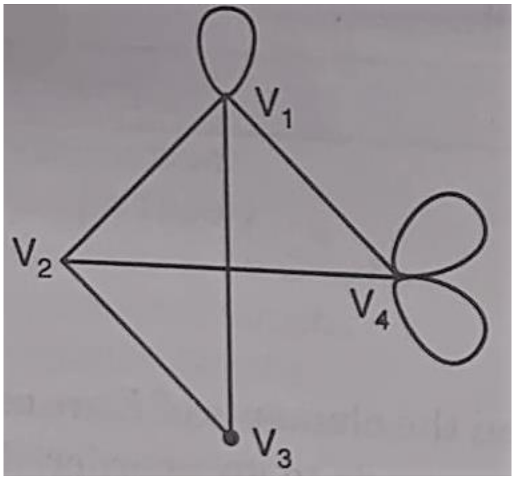
$$a_{n_{+2}} = 2a_{n+1} + 4a_n - 8a_{n-1}$$

   Find an explicit formula.

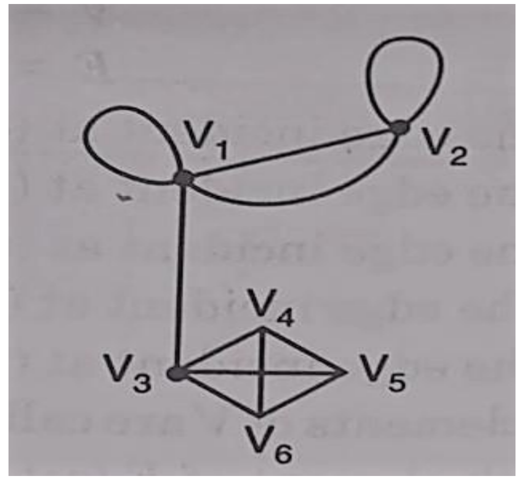
6. Find an explicit formula for the recurrence relation defined by :

   $a_k - 7a_{k-1} + 10a_{k-2} = 2k^2 + 2$, with the initial conditions $a_0 = 0$, $a_1 = 1$.

7. Consider the following graphs given in the figure (a) and (b). Find the degree of each vertex.

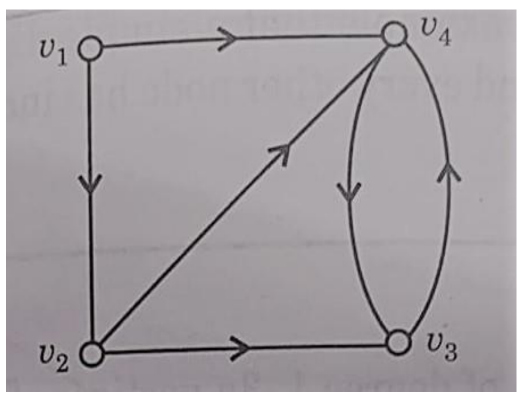


(a)

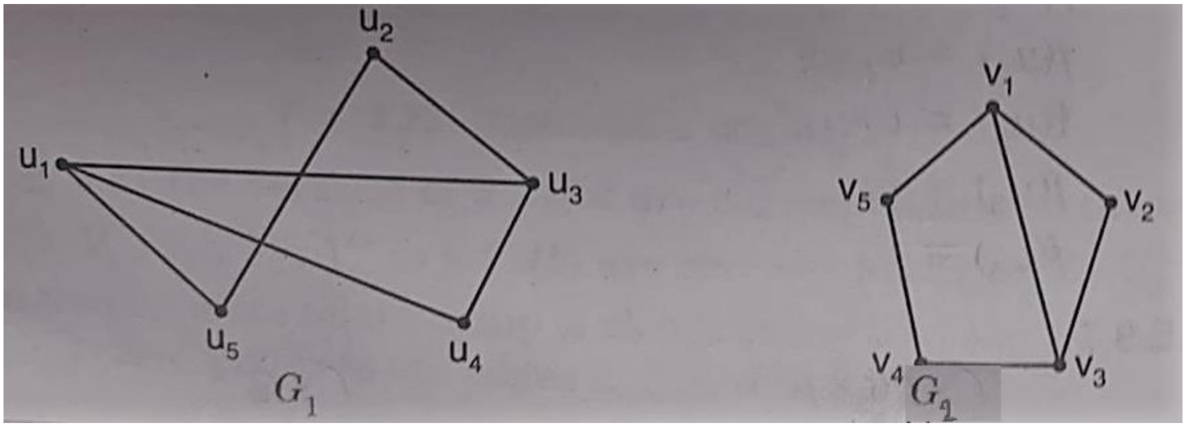


(b)

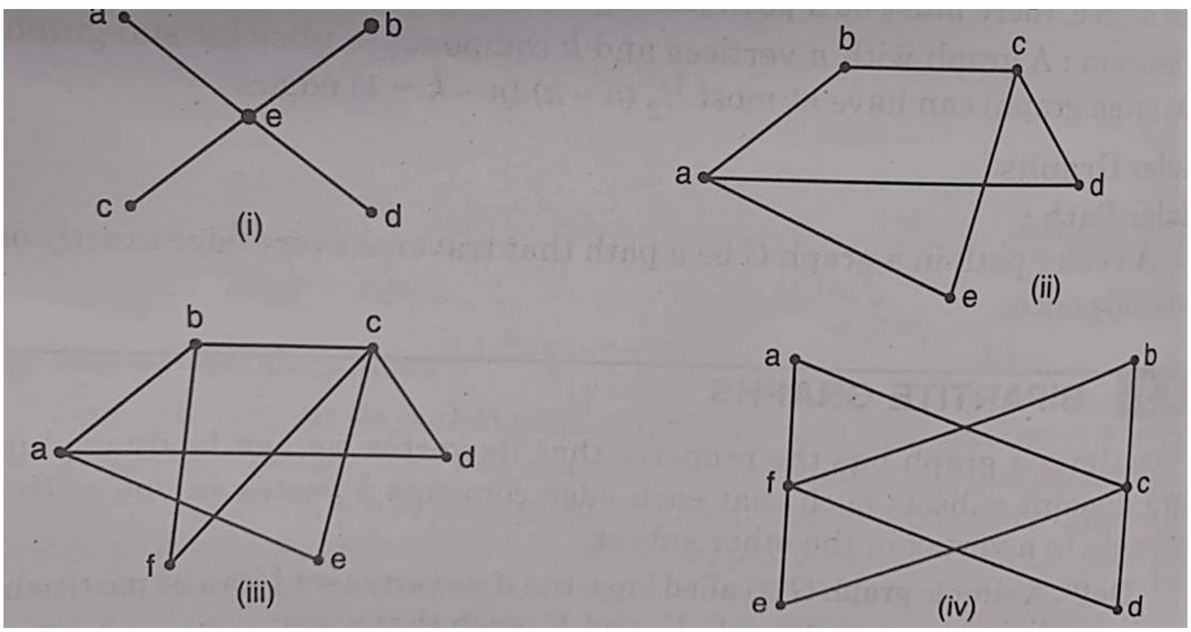
8. Find all the indegrees and outdegrees of the nodes of the graph given in the figure below:



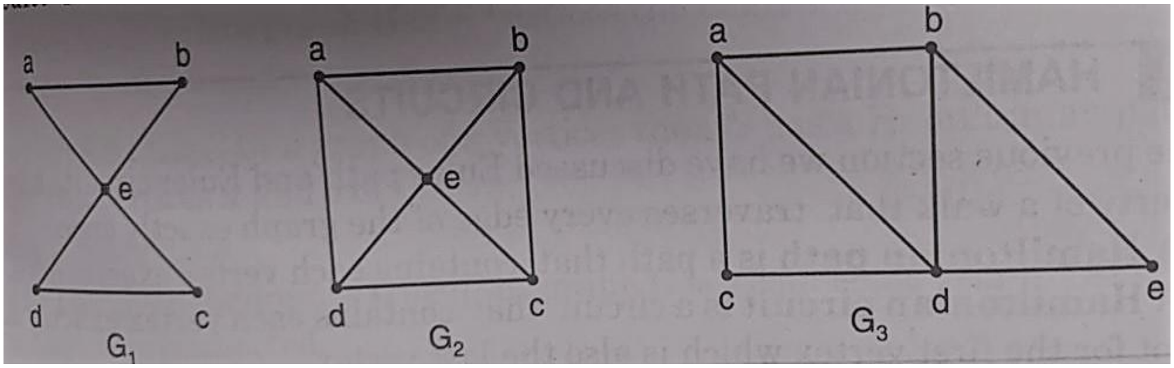
9. Check whether the following graphs are isomorphic or not.



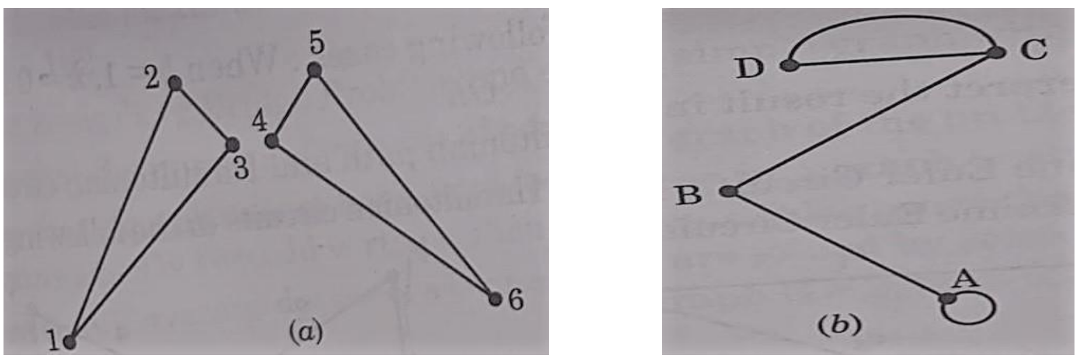
10. Determine whether the following graphs are bipartite :



11. Which of the undirected graphs given below have an Euler circuit? Of those that do not, which have an Euler Path?



12. Verify whether the Hamiltonian path is possible in the following :



13. Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

14. Define the following :
    i. Cut Vertex
    ii. Cut Edge
    iii. Cut Set
    iv. Spanning Tree
    v. Chromatic Number of a Graph

15. Prove that a Planar Graph is 5 – colorable.