

Foundation of Computer Science

Assignment - 1

Unit - 1

1. Construct the Truth Table for the following:

a) $((p \rightarrow q) \rightarrow r) \rightarrow s$

b) $((p \wedge q) \vee \neg r) \leftrightarrow p$

2. Show whether the statement $(p \vee \neg q) \rightarrow (p \wedge q)$

Is a tautology, a contradiction or a contingency.

3. Determine the converse, inverse and contrapositive statement for the statement :

“John is poor if John is a poet”.

4. Show that the following rule is valid using Truth Table Method:

$$p \therefore p \vee q$$

5. Let P be “It is snowing”.

Q be “I will go to town”. &

R be “I have time”.

Write the following statement in English:

i) $Q \leftrightarrow (R \wedge \neg P)$

ii) $\neg(R \vee Q)$

6. Which rule of inference is used in each argument below?

☐ Alice is a Math major. Therefore, Alice is either a Math major or a CSI major.

☐ Jerry is a Math major and a CSI major. Therefore, Jerry is a Math major.

☐ If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

7. Use De Morgan's laws to find the negation of each of the following statements.

☐ Jan is rich and happy.

☐ Carlos will bicycle or run tomorrow.

☐ Mei walks or takes the bus to class.

☐ Ibrahim is smart and hard working.

8. Prove: $(p \wedge \neg q) \vee q \Leftrightarrow p \vee q$ using logical equivalences.

9. If $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of the following:

a) $\exists x \in A(x+5=10)$

b) $\forall x \in A(x+1 < 5)$

10. Test the validity of the following arguments:

a) If two sides of a triangle are equal, then the opposite angles are equal.

Two sides of a triangle are not equal.

Therefore, the opposite angles are not equal.

b) If Today is Tuesday, then I have a test in Mathematics or Economics.

If my Economics Professor is sick, then I will not have a test in Economics.

Today is Tuesday and my Economics Professor is sick.

Therefore, I have a test in Mathematics.

11. Negate the statement:

For all real numbers x , if $x > 5$ then $x^2 > 25$ [Hint: use predicates and quantifier]

12. Let $K(x)$: x is a student.

$M(x)$: x is clever.

$N(x)$: x is successful.

Express using quantifier :

a) There exists a student.

b) Some students are clever.

c) Some students are not successful

13. Use the method of contradiction to show that $\sqrt{5}$ is irrational.

14. Define NAND (\uparrow) and NOR (\downarrow) connectives. Construct their corresponding Truth Tables.

15. Express in PDNF:

☐ $(p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r)$

☐ If John attends classes, then John passes in university exams.

16. Express in PCNF:

☐ $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$

☐ Ken is playing football or Ken is sleeping.

17. Show that $(r \vee s)$ follows logically from the premises $(c \vee d)$, $(c \vee d) \rightarrow \neg h$, $\neg h \rightarrow (a \wedge \neg b)$, $(a \wedge \neg b) \rightarrow (r \vee s)$

18. Give an example of Nested Quantifiers, Bounded and Free Variable. [Hint: Two quantifiers are nested if one is within the scope of the other.]

19. Translate into English:

$$\forall x \forall y ((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$$

20. Consider Universe of Discourse, $P = \{1, 2, 4, 5, \text{cat}, \text{table}, \text{fan}\}$

Let $N(x) \equiv x$ is a number, $E(x) \equiv x$ is even

Let $A(x) \equiv N(x) \rightarrow E(x)$

- ☐ What is the Truth value of $A(x)$? i.e. $N(x) \rightarrow E(x)$?
- ☐ What is the Truth value of $A(2)$? i.e. $N(2) \rightarrow E(2)$?
- ☐ What is the Truth value of $A(1)$?, i.e. $N(1) \rightarrow E(1)$
- ☐ What is the Truth value of $N(1) \rightarrow E(2)$?
- ☐ What is the Truth value of $A(\text{cat})$ i.e. $(\text{cat}) \rightarrow E(\text{cat})$?
