

Assignment No: - 5

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- ① Ans: H_0 : There is no significant diff. b/w \bar{x} & μ .
 H_1 : statement is false

$$|z| = \left| \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \right| = \left| \frac{6.75 - 6.8}{1.5/\sqrt{400}} \right|$$
$$= \left| \frac{-0.05}{1.5/20} \right| = \left| \frac{-1}{1.5} \right| \approx \pm 0.67$$

$|z| = 0.67 < 1.96$ at 5% significant level. So H_0 is accepted
i.e. there is no significant difference b/w \bar{x} & μ .

- ② Ans: $\bar{x}_1 = 67.5$, $\bar{x}_2 = 68$ $n_1 = 1000$, $n_2 = 2000$

Step 1:

Null Hypothesis (H_0): $\mu_1 = \mu_2$ Alternative Hypothesis
(H_a) = $\mu_1 \neq \mu_2$

Test statistic: - $\bar{x}_1 - \bar{x}_2 = 67.5 - 68.0 = -0.5$

Since S.D. of the population is known

$$S.E.S = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$= \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = \sqrt{\frac{1}{1000} + \frac{1}{2000}}$$
$$= 0.097$$

$$\therefore z = \frac{\bar{x}_1 - \bar{x}_2}{s} = \frac{-0.5}{0.097} = -5.15 \quad |z| = 5.15$$

level of significance $\alpha = 0.27\%$

critical value

The value of Z_{α} at 0.27% level of significance from table is 3

Since the computed value of $|z| = 5.15$ is greater than critical value $z_{\alpha} = 3$ the hypothesis is rejected.

\therefore The sample cannot be regarded as drawn from same population

- ③ Ans: The following null & alternative hypothesis need to be taken

$$H_0: \mu = 0.700$$

$$H_1: \mu \neq 0.700$$

This corresponds to a two tailed test for which a t-test for one mean, with unknown population standard deviation, using the sample standard deviation will be used

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{0.742 - 0.700}{0.040/\sqrt{15}} \approx 3.3204$$

So yes the work is inferior

4) Ans:-

nine items of a sample - 45, 47, 50, 52, 48, 47, 49, 53, 51

$$\text{Mean} = \frac{(\text{sum of observation})}{\text{Total no. of observation}}$$

$$= \frac{45 + 47 + 50 + 52 + 48 + 47 + 49 + 53 + 51}{9}$$

$$= 442/9 = 49.11$$

Assumed mean = 47.5

The mean difference

$$= \text{Mean} - \text{assumed normal mean}$$

$$= 49.11 - 47.5$$

$$= 1.61$$

Thus, both the mean differ significantly by 1.61

5) Ans: Here $n_1 = 200$, $n_2 = 100$

$$p_1 = \frac{x_1}{n_1} = \frac{46}{200}, \quad n_2 = \frac{19}{100}$$

p = proportion of premium tea brand in population = 0.1

$$q = 1 - p = 0.99$$

Null hypothesis ; H_0 : The manufacturer claim is accepted

Alternative hypothesis H_1 : $p \neq 0.1$

$$\text{Under } H_0, \quad z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.04}{\sqrt{0.1 \times 0.99 \times \frac{3}{200}}} = 26.9$$

conclusion: Since the calculated value of $|z| > 1.645$ & also $|z| > 2.33$.
Hence H_0 is rejected 5% and 1% level of significance

6) Ans:

~~X² test~~

~~H_0 : the experimental results support the theory~~

~~H_a : the experimental results do not support the theory~~

- ⑥ Ans: Null Hypothesis H_0 : The experimental result support the theory i.e. there is no significant difference b/w the observed and theoretical frequency under H_0 , the theoretical frequency

$$E(A) = \frac{1600 \times 9}{16} = 300, E(B) = \frac{1600 \times 3}{16} = 300$$

$$E(C) = \frac{1600 \times 3}{16} = 300, E(D) = \frac{1600 \times 1}{16} = 100$$

Observed frequency O_i	882	313	287	118
Expected frequency E_i	300	300	300	100

$$\frac{(O_i - E_i)^2}{E_i} \quad 0.36 \quad 0.5633 \quad 0.5633 \quad 3.24$$

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 4.7266$$

Conclusion: Table value of χ^2 at 5% level of significance for 3 d.f is 7.815. Since the calculated value of χ^2 is less than the tabulated value. Hence H_0 is accepted i.e. the experimental result support the theory.

- ⑦ Ans: Null Hypothesis (H_0): $\sigma_x^2 = \sigma_y^2$ (sample are drawn from same population).

Alternate Hypothesis (H_1): $\sigma_x^2 \neq \sigma_y^2$ (i.e. samples are not drawn from the same population)

LOS = 5% (two tailed test)

Degree of freedom = $n_1 + n_2 - 2 = 9 + 8 - 2 = 15$

$m = 9, n = 8$

$$\sum (x_i - \bar{x})^2 = 160, \sum (y_i - \bar{y})^2 = 91$$

level of significance

$$\alpha = 0.05$$

Test statistic

$$F = \frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2} = \frac{s_1^2}{s_2^2} \text{ under } H_0$$

$$s_x^2 = \frac{1}{m-1} \sum (x_i - \bar{x})^2 = \frac{160}{8} = 20$$

$$s_y^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2 = \frac{91}{7} = 13$$

$$F_0 = \frac{s_x^2}{s_y^2} = \frac{20}{13} = 1.54$$

Step 6: critical value

Since H_1 is a two sided alternative hypothesis the corresponding critical values are:
critical value for 15 degree of freedom = ± 2.1314

Since $F(8.7)0.95 = -2.314 < F_0 = 1.54 < F(8.7)0.05 = 2.1314$
the null hypothesis is not rejected and we conclude that samples are drawn from same population.

(8) Ans:- Null Hypothesis H_0 : The data are consistent with the hypothesis of equal.

probability for male and female birth i.e. $p = q = 1/2$.

We use binomial distribution to calculate theoretical frequency

$$N(r) = N \times {}^n C_r p^r q^{n-r}$$

where N is total frequency, $N(r)$ is the number of family with r male children, p & q are possibilities of male and female birth respectively, n is the no. of children.

$$N(0) = 800 \times {}^4 C_0 (1/2)^4 = 50, N(1) = 200, N(2) = 300,$$

$$N(3) = 200, N(4) = 50$$

observed frequency O_i	32	178	290	236	64
Expected frequency E_i	50	200	300	200	50
$(O_i - E_i)^2$	324	484	100	1296	196
$\frac{(O_i - E_i)^2}{E_i}$	6.48	2.42	0.333	6.48	3.92

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i} \right] = 19.633$$

Tabulated value of χ^2 at 5% level of significance for $5-1=4$ degrees of freedom = 9.49.

conclusion: since the calculated value of χ^2 is greater than tabulated value, H_0 is rejected i.e. the data are not consistent with the hypothesis that the binomial law holds & that the chance of a male birth is not equal of that of a female birth.