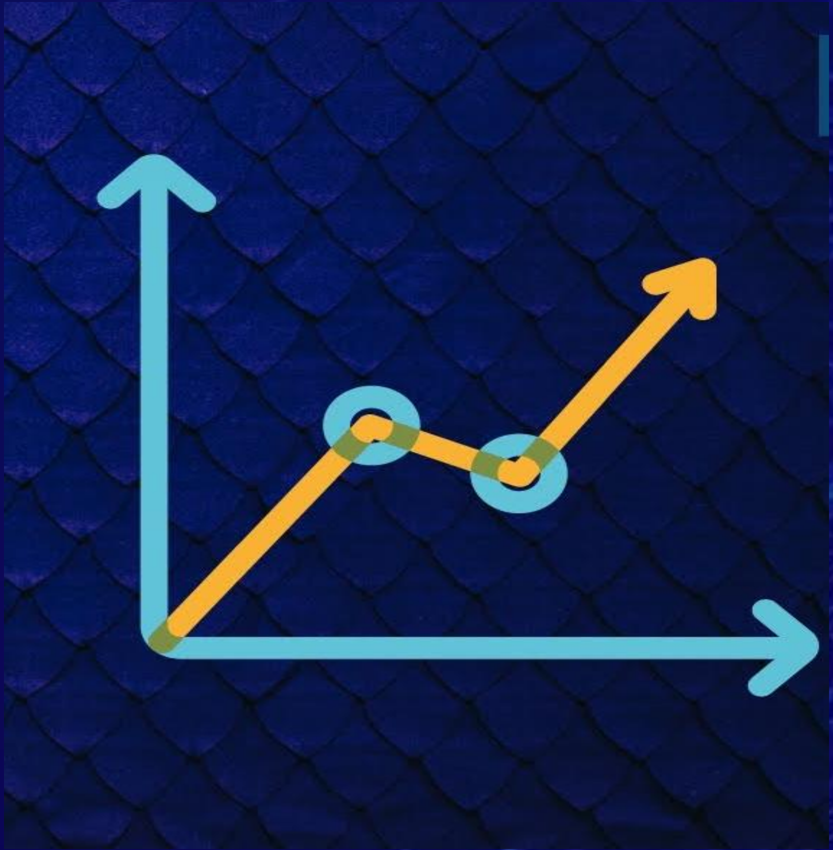
Unit 2.1
Linear Regression



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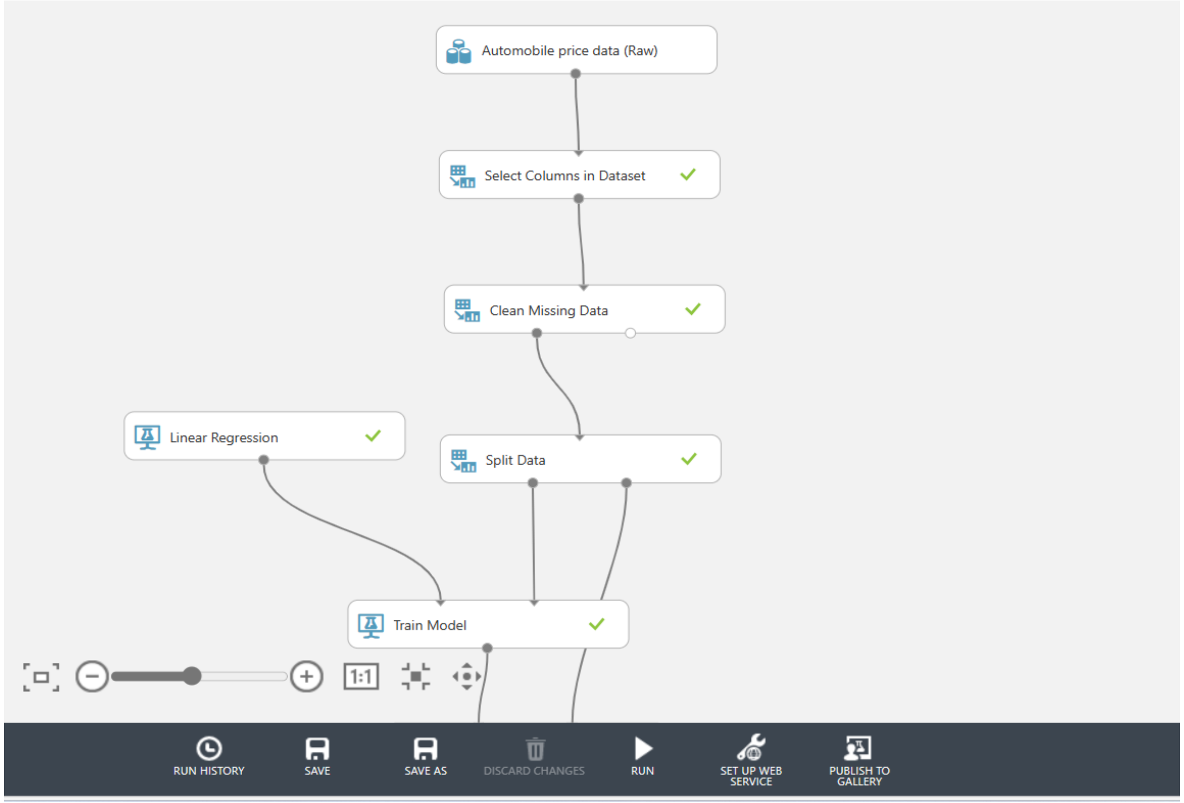
Demo: Let’s Try Linear Regression via GUI



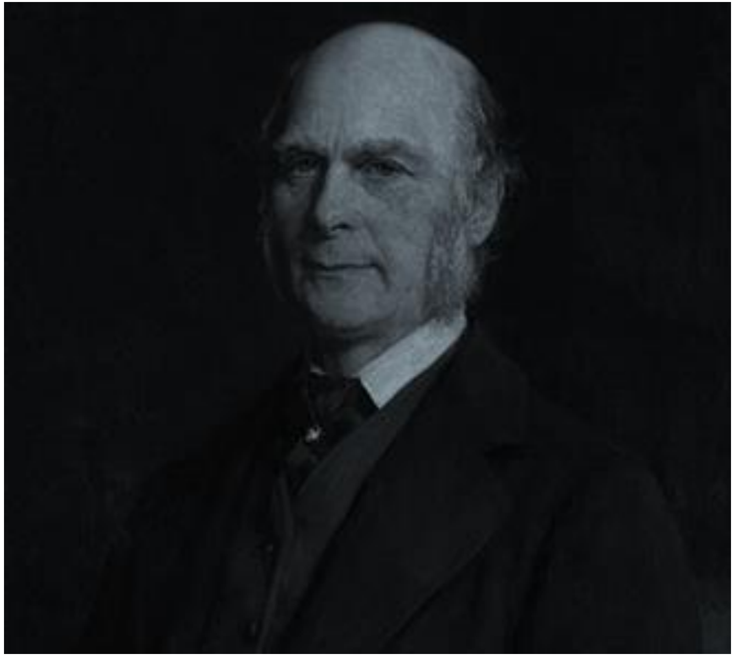
[Reference]

Learning Objectives

- About Linear Regression
- Types of Regression
- Goal
- Regression Modelling
- Performance Metrices
- Bias Variance Tradeoff
- Regularized Regression

Introduction

- In 1800, a person named Francis Galton, was studying the relationship between parents and their children.
- He investigated the relationship between height of fathers and their sons.
- He discovered that a man's son tends to be roughly as tall as his father, **however, tall son's height tended to be closer to the overall average height of all people's sons.**
- **Galton call this phenomenon as "Regression" as "father's son height tends to regress (or drift towards) the mean (average) height of everyone else.**



Linear Regression

- Regression is used to study the relationship between two variables.
- We can use simple regression if both the **dependent variable (DV)** and the **independent variable (IV)** are numerical.
- If the DV is numerical but the IV is categorical, it is best to use Linear Regression.

Example

The following are situations where we can use regression:

1. Testing if IQ affects income (IQ is the IV and income is the DV).
2. Testing if hours of work affects hours of sleep (DV is hours of sleep and the hours of work is the IV).
3. Testing if the number of cigarettes smoked affects blood pressure (number of cigarettes smoked is the IV and blood pressure is the DV).
4. Chances of heart failure due to high body fat

Linear Regression

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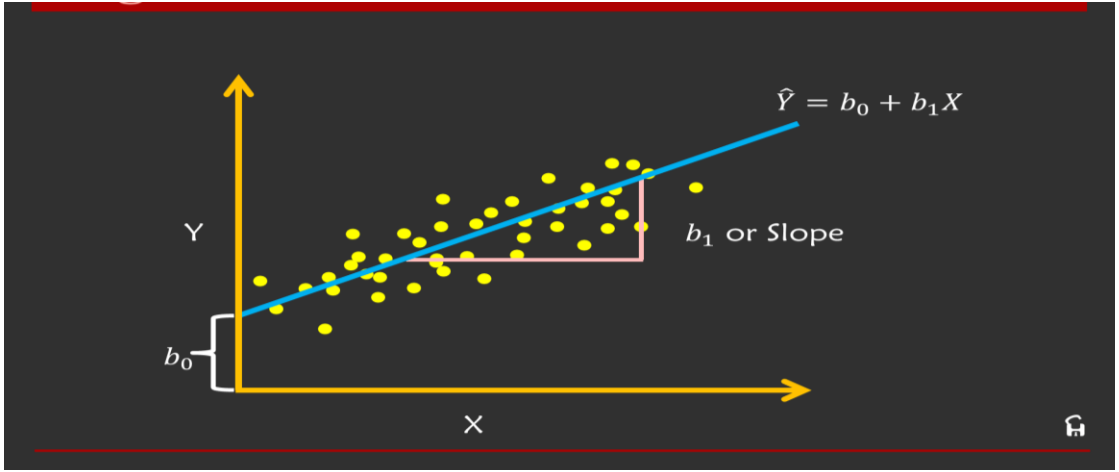
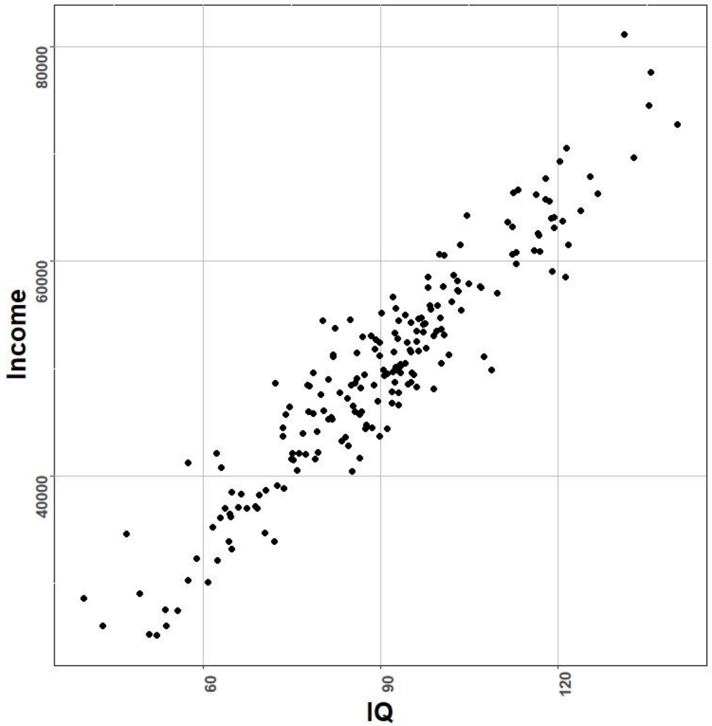


Reference

Displaying the Data

- Displaying data for Testing if IQ affects income (IQ is the IV and income is the DV).
- When both the DV and IV are numerical, we can represent data in the form of a scatterplot.

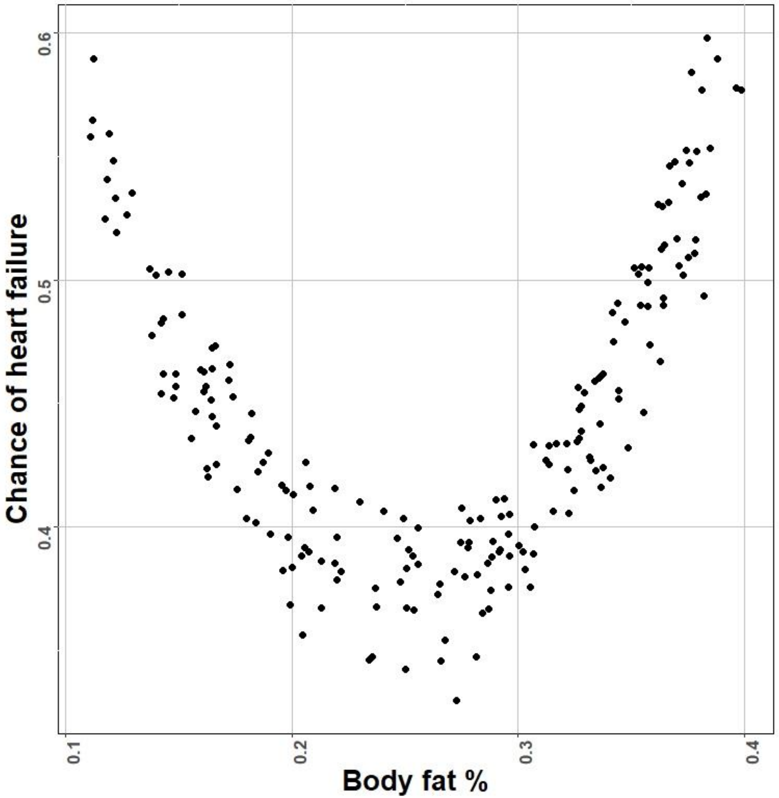


Income V/s IQ

Displaying the Data

- Displaying data of Chances of heart failure due to high body fat

- It is important to perform a scatterplot because it helps us to see if the relationship is linear.



Regression Case

Dataset related to Co2 emissions from different cars.

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSION
0	2.0	4	8.5	196
1	2.4	4	9.6	221
2	1.5	4	5.9	136
3	3.5	6	11.1	255
4	3.5	6	10.6	244
5	3.5	6	10.0	230
6	3.5	6	10.1	232
7	3.7	6	11.1	255
8	3.7	6	11.6	267
9	2.4	4	9.2	?

Regression Case

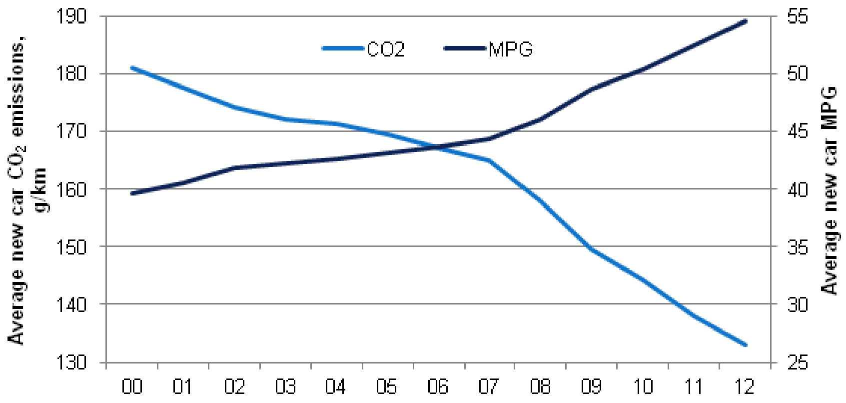
- Looking to the existing data of different cars, can we estimate the approx. CO_2 emission of a car, *which is yet not manufactured*, such as in row 9 ?

- We can use regression methods to predict a continuous value, such as CO_2 Emission, using some other variables.

- In regression there are **two types of variables:**
    - a dependent variable (DV, which we want to predict) and
    - one or more independent variables (IV, existing features).

Regression Essentials

- The key point in the regression is that our dependent value should be continuous and cannot be a discreet value.
- However, the independent variable or variables can be either a categorical or continuous.
- We use regression to build such a regression/estimation model which would be used to predict the expected Co2 emission for a new or unknown car.



Reference

Types of Regression Model

1. **Simple regression** is when one independent variable is used to estimate a dependent variable.

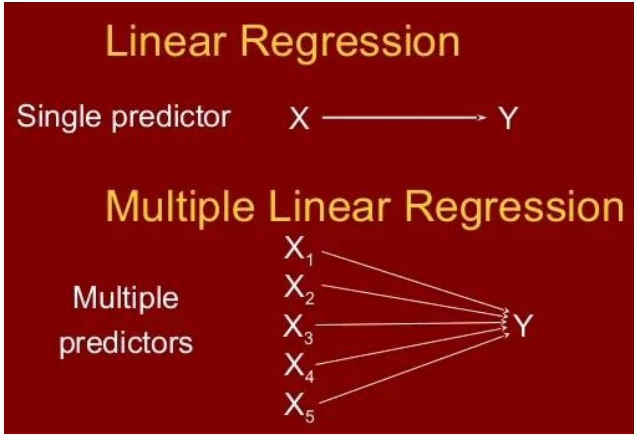
It can be linear or non-linear.

Ex: predicting Co2 emission using the variable of Engine Size.

2. When more than one independent variable is present, the process is called **multiple linear regression**.

Ex: predicting Co2 emission using Engine Size and the number of Cylinders in any given car.

Linearity of regression depends on the relation between dependent and independent variables; it can be either linear or non-linear regression.



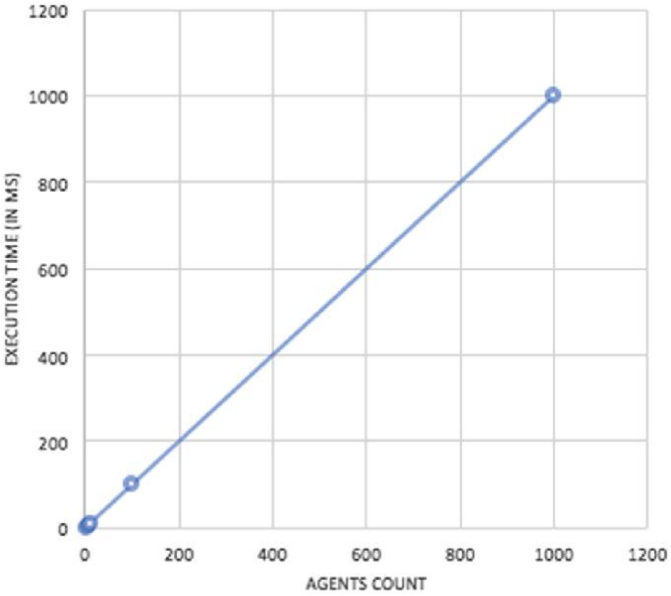
Reference

Regression Application Areas

- Essentially, we use regression when we want to estimate a continuous value.
- You can try to **predict a salesperson's total yearly sales (sales forecast)** from independent variables such as age, education, and years of experience.
- We can use regression analysis to **predict the price of a house** in an area, based on its size, number of bedrooms, and so on.
- We can even use it to **predict employment income** for independent variables, such as hours of work, education, occupation, sex, age, years of experience, and so on.

Simple Linear Regression

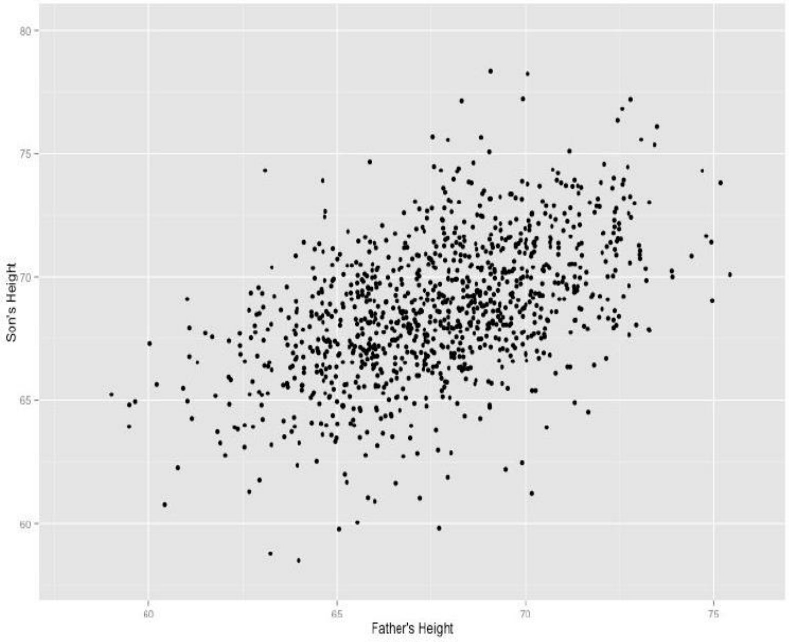
- How to calculate a regression with only 2 data points ?
- In linear regression, we calculate regression line by drawing a connecting line
- For classic linear regression or “Least Square Method”, you only measure the closeness in the “up and down” direction.
- Here we have perfectly fitted line because we have only 2 points.



Reference

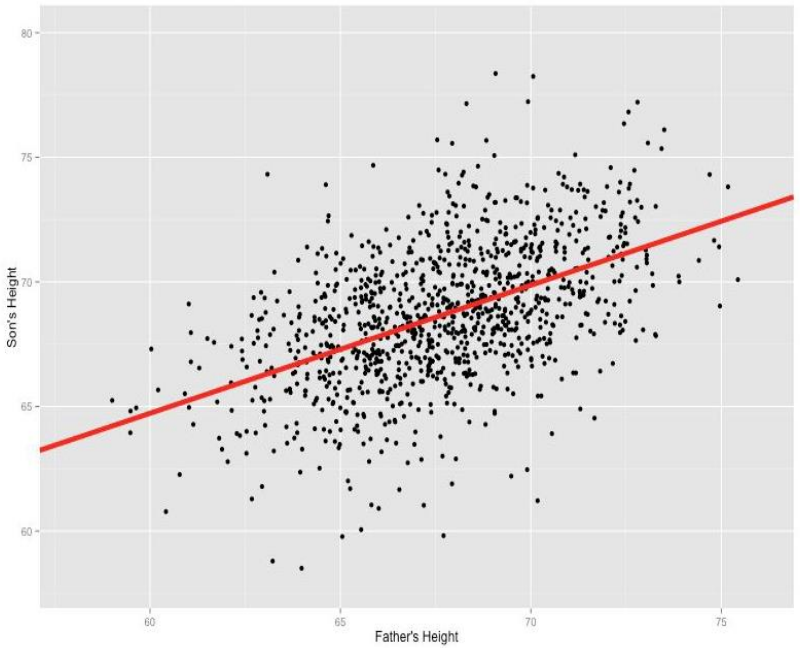
Regression with More Data Points

- Now wouldn't it be great if we could apply this same concept to a graph with more than just two data points?
- By doing this, **we could take multiple men and their son's heights and do things like tell a man how tall we expect his son to be...before he even has a son!**
- This is the idea behind supervised learning!



Regression Goal

- Goal is to determine the best line by minimizing the vertical distance between all the data points and our line.
- Lot of different ways to minimize this, (sum of squared errors, sum of absolute errors, etc).
- All these methods have a general goal of minimizing this distance between your line and rest of data points.



Case Study

- This dataset is related to the Co2 emission of different cars.
- The question is: **Given this dataset, can we predict the Co2 emission of a car, using another field, such as Engine size?**

X: Independent variable

Y: Dependent variable

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSION
0	2.0	4	8.5	196
1	2.4	4	9.6	221
2	1.5	4	5.9	136
3	3.5	6	11.1	255
4	3.5	6	10.6	244
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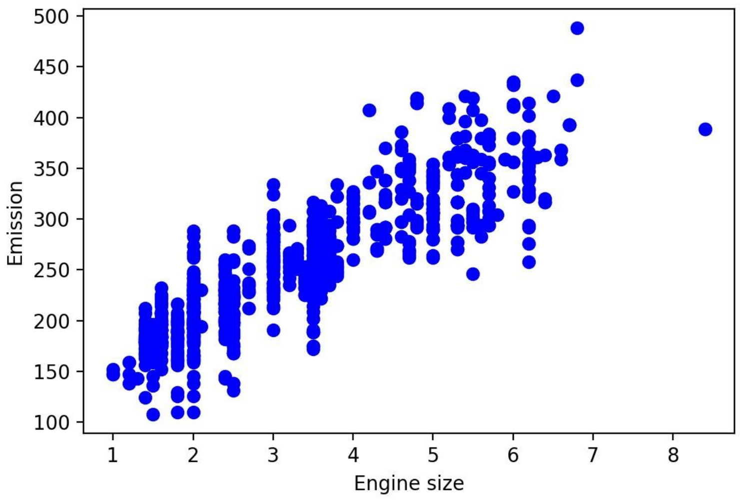
9	2.4	4	9.2	?

Continuous values

Yes!

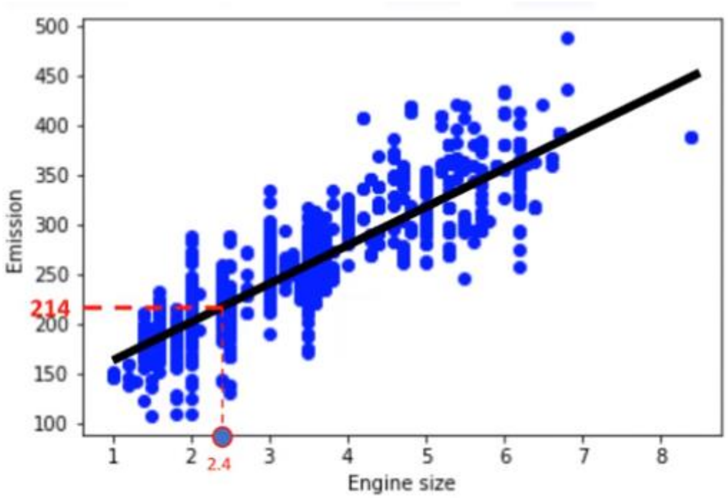
Scatter Plot

- To understand linear regression, we can plot our variables here.
- **Engine size** -- independent variable, **Emission** – dependent/target value that we would like to predict.
- A scatterplot clearly shows the relation between variables where changes in one variable "explain" or possibly "cause" changes in the other variable.
- Also, it indicates that these variables are linearly related.



Inference from Scatter Plot

- **As the Engine Size increases, so do the emissions.**
- How do we use this line for prediction now?
- Let us assume, for a moment, that the line is a good fit of data.
- We can use it to predict the emission of an unknown car.



Regression Modeling - Fitting Line

- Fitting line help us to predict the target value, Y, using the independent variable 'Engine Size' represented on X axis
- The fit line is shown traditionally as a polynomial.
- In Simple regression Problem (single x), the form of the model would be

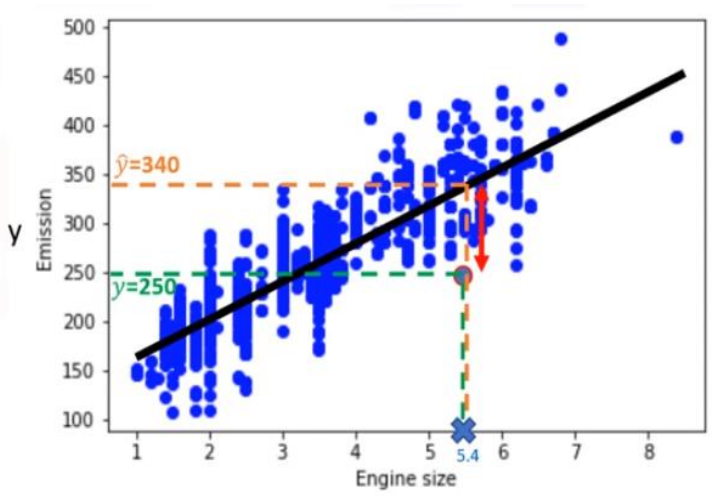
$$\hat{y} = \theta_1 + \theta_2 x_1$$

θ_1 = intercept θ_2 = slope of the line

- Where Y is the dependent variable, or the predicted value and X is the independent variable.
- θ_1 and θ_2 are coefficient of linear equation

Model Error

- If we have, for instance, a car with engine size x_1 =5.4, and actual Co2=250,
- Its Co2 should be predicted very close to the actual value, which is y=250, **based on historical data**.
- But, if we use the fit line it will return $\hat{y}$ =340.
- **Compare the actual value with we predicted using our model, you will find out that we have a 90-unit error.**
- Prediction line is not accurate. This error is also called the **residual error.**



Error = $\hat{y} - y$ = 340-250 = 90

Reference

Linear Regression

Mean Absolute Error

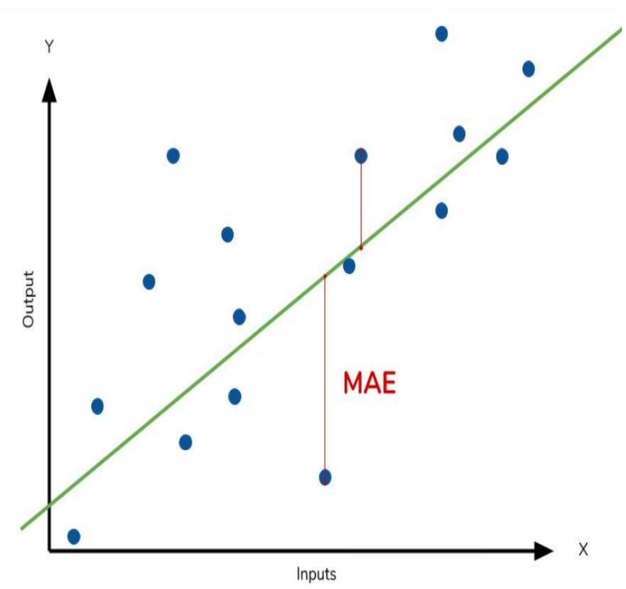
Divide by the total number of data points

Predicted output value

Actual output value

$$MAE = \boxed{\frac{1}{n}} \underset{\text{Sum of}}{\sum} \underbrace{\left| \boxed{y} - \boxed{\hat{y}} \right|}_{\text{The absolute value of the residual}}$$

In this, the residual for every data point, taking only the absolute value of each so that negative and positive residuals do not cancel out. Then take the average of all these residuals.



Mean Squared Error

- The mean square error (MSE) is just like the MAE.
- But squares the difference before summing them all instead of using the absolute value. We can see this difference in the equation below.

$$MSE = \frac{1}{n} \Sigma \underbrace{\left(y - \widehat{y} \right)^2}_{\text{The square of the difference between actual and predicted}}$$

R2 Score

- Statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model.
- If the R^2 of a model is 0.50, then approximately half of the observed variation can be explained by the model's inputs.

- **Formula for R-Squared**

$$R^2 = 1 - \frac{\text{Unexplained Variation}}{\text{Total Variation}}$$

Adjusted R2 Score

- “R-squared will always increase when a new predictor variable is added to the regression model.”
- Regression model with a large number of predictor variables.
- Has a high R-squared value, even if the model doesn’t fit the data well.

Adjusted R^2 = 1 – [(1-R^2)*(n-1)/(n-k-1)]

where:

R^2: The R^2 of the model

n: The number of observations

k: The number of predictor variables

Formula for Adj. R-Squared

Parameter Estimation

- **The objective of linear regression is to minimize this MSE equation, and to minimize it, we should find the best parameters, θ_0 and θ_1 .**
- How to find θ_0 and θ_1 in such a way that it minimizes this error?

We have two options here:

Option 1 - We can use a mathematic approach Or,

Option 2 - We can use an optimization approach.

Mathematical Approach

- θ_0 and θ_1 (intercept and slope of the line, termed Beta parameter) are the coefficients of the fit line.
- Need to calculate the mean of the independent and dependent or target columns, from the dataset.
- **Notice:** All of the data must be available.
- It can be shown that the intercept and slope can be calculated using these variables.
- We can start off by estimating the value for θ_1.

Parameter Estimation

	ENGINESIZE	CYLINDERS	FUELCONSUMPTION_COMB	CO2EMISSION
0	2.0	4	8.5	196
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$$\hat{y} = \theta_0 + \theta_1 x_1$$

$$\boldsymbol{\theta_1} = \frac{\sum_{i=1}^{s}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{s}(x_i - \bar{x})^2}$$

$\bar{x} = 3.34$

$\bar{y} = 256$

$$\Theta_1 = \frac{(2-3.34)(196-256)+(2.4-3.34)(221-256)+ \ldots\ldots\ldots}{(2.0-3.34)^2+(2.4-3.34)^2+ \ \ldots\ldots}$$

$$\boxed{\Theta_1 = 39}$$

$$\boxed{\Theta_0 = \bar{y} - \Theta_1 \bar{x} = 125.74}$$

Making Predictions

- We can write down the polynomial of the line.

$$\hat{y} = \mathbf{125.74 + 39x_1}$$

- Making predictions is as simple as solving the equation for a specific set of inputs.
- Imagine we are predicting Co2 Emission(y) from EngineSize(x) for the Automobile in record number 9. So, looking to the dataset, $\mathbf{x_1 = 2.4}$
- Implementing x_1 in above equation, **we can predict the CO2 emission of this specific car (row 9) with engine size 2.4 :**

$$\hat{y} = \mathbf{218.6}$$

Lab 1: [Linear Regression on Car Emission data]

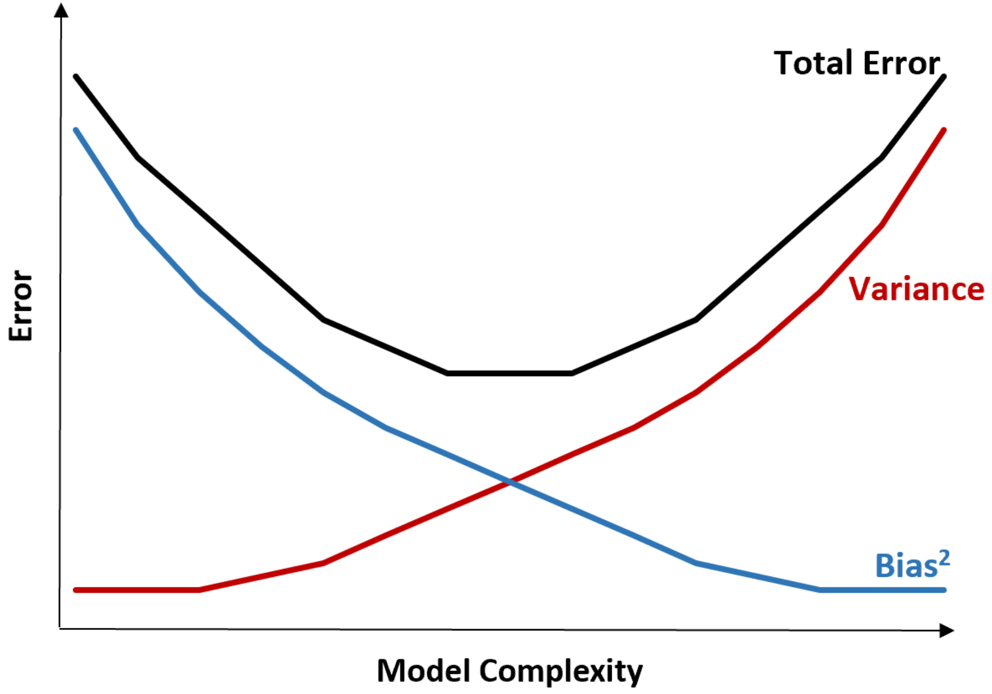
Bias-Variance Tradeoff

- **Variance:** Defines model complexity

  eg:- Non Linear models

- **Bias:** Defines model imperfection

  eg: Linear model for complex cases



Reference

Lasso & Ridge Regression

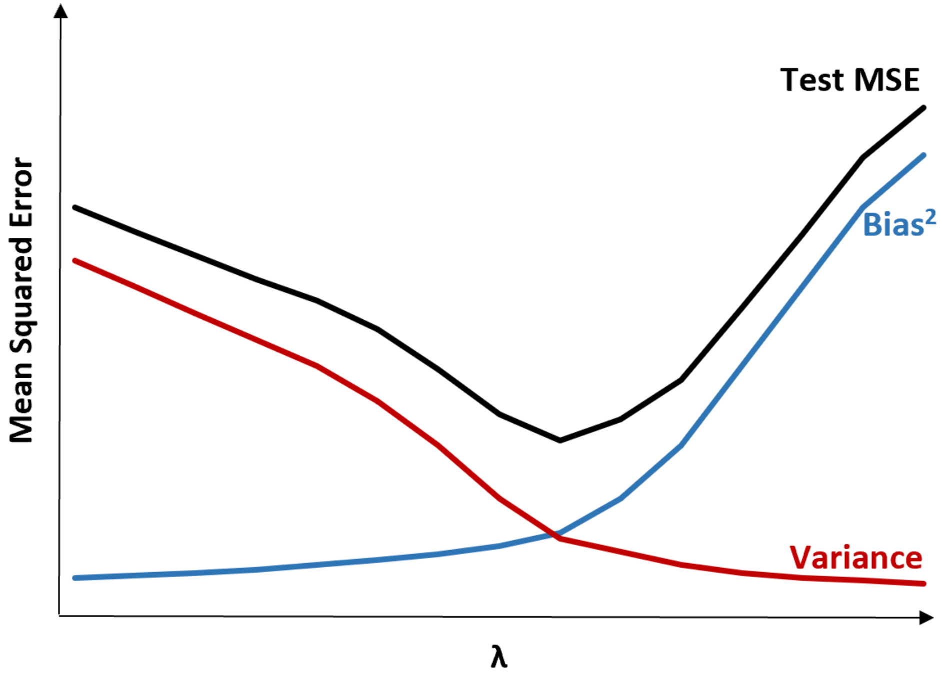
- To handle bias-variance tradeoff
- Ridge regression seeks to minimize the following:

$$\mathbf{MSE + \lambda\Sigma\beta j^2}$$

- Lasso regression seeks to minimize the following:

$$\mathbf{MSE + \lambda\Sigma|\beta j|}$$

- Second term is known as a **shrinkage penalty**.



Reference

Regularization: An Overview

- The idea of regularization revolves around modifying the loss function L; in particular, we add a regularization term that penalizes some specified properties of the model parameters.

$$L_{reg}(\beta) = L(\beta) + \lambda R(\beta),$$

- where λ is a scalar that gives the weight (or importance) of the regularization term.
- Fitting the model using the modified loss function L_{reg} would result in model parameters with desirable properties (specified by R).

LASSO Regression

- Since we wish to discourage extreme values in model parameter.
- Need to choose a regularization term that penalizes parameter magnitudes.
- For our loss function, we will again use MSE.
- Together our regularized loss function is:

Note that $\sum_{j=1}^{J} |\beta_j|$ is the ***l*$_1$** norm of the vector ***b***

$$L_{LASSO}(\beta) = \frac{1}{n}\sum_{i=1}^{n} |y_i - \boldsymbol{\beta}^\top \boldsymbol{x}_i|^2 + \lambda \sum_{j=1}^{J} |\beta_j|. \qquad \sum_{j=1}^{J} |\beta_j| = \|\boldsymbol{\beta}\|_1$$

Ridge Regression

- Can choose a regularization term that penalizes the squares of the parameter magnitudes. Then, our regularized loss function is:

$$L_{Ridge}(\beta) = \frac{1}{n}\sum_{i=1}^{n} |y_i - \boldsymbol{\beta}^\top \boldsymbol{x}_i|^2 + \lambda \sum_{j=1}^{J} \beta_j^2.$$

- Note that $\sum_{j=1}^{J} |\beta_j|^2$ is the square of the ***l*$_2$** norm of the vector ***b***

$$\sum_{j=1}^{J} \beta_j^2 = \|\boldsymbol{\beta}\|_2^2$$

Choosing Lambda ?

In both ridge and LASSO regression, larger choice of the **regularization parameter** l, the more heavily penalize large values in ***b,***

- If l is close to zero, we **recover the MSE**, i.e. ridge and LASSO regression is just ordinary regression.
- If l is sufficiently large, the **MSE term will be insignificant,** and the regularization term will force $\boldsymbol{b}_{\text{ridge}}$ and $\boldsymbol{b}_{\text{LASSO}}$ to be close to zero.

To avoid ad-hoc choices, we should select K using cross-validation.

Lab 2: Lasso & Ridge Regression for House Price Estimation

Summary

In this session we have learned:

- What does it mean by term regression.
- Understood the dependent and independent variable roles.
- Framework to formulae linear regression model.
- Approach to attain solution using a mathematical approach
- How model complexity adds cost to the model
- How to avoid overfitting using regularization
- Realization of Lasso & Ridge Models

Quiz

Q1. What is the primary objective of linear regression analysis?

a) To classify data points into different categories

b) To predict a continuous outcome variable based on one or more predictor variables

c) To find the median value of a dataset

d) To calculate the mode of a dataset

Correct Answer: b) To predict a continuous outcome variable based on one or more predictor variables

Quiz

Q2. In simple linear regression, how many predictor variables are used to predict the outcome variable?

a) One

b) Two or more

c) None

d) It varies depending on the dataset

Correct Answer: a) One

Quiz

Q3. What is the goal of minimizing the sum of squared errors (SSE) in linear regression?

a) To maximize the R-squared value

b) To minimize multicollinearity among predictor variables

c) To find the best-fitting line that minimizes the difference between predicted and observed values

d) To maximize the p-value of the regression coefficients

Correct Answer: c) To find the best-fitting line that minimizes the difference between predicted and observed values

Quiz

Q4. Ridge and Lasso Regression are techniques primarily used for:

a) Feature extraction

b) Dimensionality reduction

c) Regularization in linear regression

d) Classification

Correct Answer: c) Regularization in linear regression

References

- https://en.wikipedia.org/wiki/Linear_regression
- https://web.stanford.edu/~hastie/ElemStatLearn//printings/ESLII_print12.pdf
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- https://scikit-learn.org/stable/modules/linear_model.html#ordinary-least-squares
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- https://www.r-bloggers.com/2020/09/linear-regression-in-r/
- "Introduction to Linear Regression Analysis" by Douglas C. Montgomery, Elizabeth A. Peck, and G. Geoffrey Vining
- "Applied Linear Regression" by Sanford Weisberg

Thank you...!