An Internship Report

on

Graph Theory and Analysis of Algorithms

submitted in the fulfillment of

Summer Research Internship

submitted by

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under the supervision of

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I am also thankful for having this opportunity to uplift my knowledge and remarkable learning during the tenure. Their approachable nature and readiness to help created a positive and engaging learning environment.My sincere appreciation extends to my peers, whose valuable discussions and collaborative spirit added greatly to the learning experience. A special note of thanks to the authors of the graph theory textbook and the researchers whose papers and resources I have studied during this internship. Their rigorous and insightful work formed the backbone of this report and enriched my understanding of the subject.

INTRODUCTION

Throughout the internship, I engaged in tasks and activities that encouraged hands-on learning. I didn't just read or watch explanation. I applied theories, solved problems, and discussed real-life scenarios where graphs and algorithmic analysis are used (such as social networks, scheduling, and resource allocation). This blend of theoretical knowledge and practical experience made the learning more impactful and memorable. There are multiple learning output during the interval and that is :

1. **Strong Emphasis on Basics**

- Revisited core concepts of graph theory (nodes, edges, paths, trees, cycles).
- Understood the importance of building a strong foundation before diving into advanced topics.
- Realized that basic clarity leads to better problem-solving in complex topics.

2. **Research Paper Exploration**

- Studied the paper "Conflict and Fairness in Resource Allocation".
- Learned how resource allocation problems can be modeled using conflict graphs.
- Gained insight into parameterized complexity and its real significance in NP-hard problems.
- Got exposure to the thought process involved in academic research.

3. **Analyzing Algorithms and Their Efficiency**

- Understanding algorithmic time and space complexity, using Big-O notation,
- Classifying problems into P, NP, NP-Complete, and NP-Hard categories,
- Exploring greedy algorithms, divide-and-conquer, dynamic programming, and backtracking approaches for graph-based problems (like shortest paths, spanning trees, and coloring).

A major highlight of the internship was the study of the research paper titled *"Conflict and Fairness in Resource Allocation"*. In the paper, a conflict graph is used to represent incompatibility between resources, and the challenge is to allocate disjoint sets of compatible resources to agents such that minimum satisfaction (utility) across all agents is maximized. Through this, I was introduced to deeper algorithmic ideas such as NP-hardness proofs, structural parameterization, kernelization, and FPT algorithms based on parameters like graph sparsity, treewidth, and neighborhood diversity. Moreover, the paper connected this technical foundation with the idea of algorithmic fairness, which is increasingly relevant in real-world applications ensuring that algorithmic decisions do not lead to biased or unequal outcomes.In summary, this internship has laid a strong academic and intellectual foundation in graph theory and algorithm design, while also offering a first-hand experience of engaging with ongoing research.

LITERATURE REVIEW

We review two recent research works that contributed significantly to the understanding developed during this internship, particularly in connecting graph-theoretic and algorithmic techniques to real-world fairness challenges.

1. **Conflict and Fairness in Resource Allocation** ***(Bodlaender et al., 2024)***

This paper models resource allocation under fairness constraints using conflict graphs, where each agent requests a bundle of resources and conflicts arise when resources are incompatible (represented by edges in the graph). The central goal is to allocate resources such that every agent receives a conflict-free set while maximizing the minimum utility among all agents, a fairness measure. The problem is proven to be NP-hard in general, but the authors explore tractable cases using parameterized complexity.

Key algorithmic contributions include:

- Fixed-parameter tractable (FPT) results under parameters like bundle size, treewidth, and neighborhood diversity of the conflict graph.
- Kernelization techniques and W[1]-hardness proofs for various parameter combinations.
- A broader link between graph structure and feasibility of fair allocation.

This work serves as a foundational reference for connecting algorithmic fairness with classical graph theory, particularly in modeling compatibility and conflict.

2. **Fairness in Competitive Ecosystems** ***(Gradwohl et al., 2025)***

This paper examines algorithmic fairness in environments where multiple decision-makers (classifiers) interact, such as competing companies using independent ML models. A central insight is that even if each classifier is individually fair (e.g., satisfies equal opportunity), their combined interaction may lead to systemic unfairness.

Important findings include:

- The formalization of ecosystem-level fairness loss due to interaction effects.
- The paradox where improving the fairness of one classifier may reduce overall fairness.
- Theoretical and empirical analysis demonstrating how fairness in isolation differs from fairness in interconnected systems.

This work highlights the importance of contextual and systemic perspectives when designing fair algorithms, a point we refer to in later sections discussing fairness-aware graph algorithms.To support our understanding of the above research, we relied heavily on the textbook *"Graph Theory" by E. Balagurusam*y. This book offered clear and accessible explanations of core topics.The book helped clarify essential definitions and theorems, and served as a reliable reference while navigating advanced topics like Menger's Theorem, Euler and Hamiltonian paths, and the use of adjacency matrices in algorithm design. It created a strong theoretical base that allowed for deeper engagement with complex research problems.

CONCLUSION

This internship has been a deeply enriching and transformative academic experience that enabled me to explore the foundational and advanced aspects of Graph Theory and the Analysis of Algorithms. By approaching the subjects from the ground up, I was able to strengthen my core understanding of key concepts such as connectivity, paths, trees, cycles, and graph-based optimization, which are vital for modeling a wide range of real-world systems. Learning how to analyze the efficiency of algorithms, understand computational complexity, and evaluate problem tractability through modern frameworks like parameterized complexity has added immense value to my perspective

A highlight of the internship was the opportunity to study and engage with recent research papers that connected theoretical knowledge to real-world fairness challenges. The paper "Conflict and Fairness in Resource Allocation" introduced an elegant way to model incompatibilities using conflict graphs and explore fair allocation through algorithmic and graph-theoretic techniques. In parallel, the study of "Fairness in Competitive Ecosystems" offered a critical lens on fairness in distributed decision systems, showing that fairness must be evaluated not only within individual components but also at the system-wide level. Together, these works sparked a deeper curiosity about how mathematical tools and ethical considerations combine in algorithm design. I must also acknowledge the role of the book "Graph Theory" by E. Balagurusamy, which served as an excellent guide throughout this learning journey, helping clarify complex ideas with simplicity and precision.

I would like to take this opportunity to express my sincere thanks to **Dr. Vikash Tripathi**, IIT Mandi, for his constant support, valuable feedback, and inspiring mentorship. His guidance helped me not only approach problems methodically but also encouraged me to develop a genuine interest in the subject A heartfelt vote of thanks to the authors of the academic resources and papers I referred to, their work formed the intellectual foundation of this report and greatly enriched my understanding.

In conclusion, this internship has not only expanded my technical knowledge but has also given me the confidence to engage with academic research, approach problems analytically, and explore the intersection of theory and real-world impact. It marks the beginning of a more mature and thoughtful academic journey, and I look forward to continuing this exploration in the field of mathematics, theoretical computer science and beyond.