Central Limit Theorem Questions from N.P. Bali's Books:

Question 1:

A population has a mean $\mu = 50$ and standard deviation $\sigma = 10$. A random sample of size $n = 25$ is taken. Find the probability that the sample mean lies between 48 and 52.

Solution:

The standard error of the mean (SEM) is given by:

$$\text{SEM} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

Convert 48 and 52 to z-scores:

$$z_1 = \frac{48 - 50}{2} = -1, \quad z_2 = \frac{52 - 50}{2} = 1$$

Using the standard normal distribution table:

$$P(-1 \leq Z \leq 1) = P(Z \leq 1) - P(Z \leq -1) = 0.8413 - 0.1587 = 0.6826$$

Thus, the probability is **0.6826**.

Question 2:

The mean of a population is $\mu = 70$, and the standard deviation is $\sigma = 12$. A random sample of size $n = 36$ is drawn. Find the probability that the sample mean is less than 68.

Solution:

$$\text{SEM} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{36}} = 2$$

Calculate the z-score for 68:

$$z = \frac{68 - 70}{2} = -1$$

Using the standard normal distribution table:

$$P(Z \leq -1) = 0.1587$$

Thus, the probability is **0.1587**.

Question 3:

A population has $\mu = 500$ and $\sigma = 100$. A random sample of size $n = 49$ is taken. What is the probability that the sample mean lies between 490 and 510?

Solution:

$$\text{SEM} = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{49}} = 14.29$$

Convert 490 and 510 to z-scores:

$$z_1 = \frac{490 - 500}{14.29} \approx -0.70, \quad z_2 = \frac{510 - 500}{14.29} \approx 0.70$$

Using the standard normal distribution table:

$$P(-0.70 \leq Z \leq 0.70) = P(Z \leq 0.70) - P(Z \leq -0.70) = 0.7580 - 0.2420 = 0.516$$

Thus, the probability is **0.516**.

Question 4:

A population has $\mu = 1200$ and $\sigma = 200$. A sample of size $n = 64$ is selected. Find the probability that the sample mean is greater than 1250.

Solution:

$$\text{SEM} = \frac{\sigma}{\sqrt{n}} = \frac{200}{\sqrt{64}} = 25$$

Calculate the z-score for 1250:

$$z = \frac{1250 - 1200}{25} = 2$$

Using the standard normal distribution table:

$$P(Z \geq 2) = 1 - P(Z \leq 2) = 1 - 0.9772 = 0.0228$$

Thus, the probability is **0.0228**.

Question 5:

The average monthly expenditure of a family is $\mu = 15000$ with a standard deviation $\sigma = 3000$. A sample of size $n = 100$ is drawn. Find the probability that the sample mean lies between 14500 and 15500.

Solution:

$$\text{SEM} = \frac{\sigma}{\sqrt{n}} = \frac{3000}{\sqrt{100}} = 300$$

Convert 14500 and 15500 to z-scores:

$$z_1 = \frac{14500 - 15000}{300} = -1.67, \quad z_2 = \frac{15500 - 15000}{300} = 1.67$$

Using the standard normal distribution table:

$$P(-1.67 \leq Z \leq 1.67) = P(Z \leq 1.67) - P(Z \leq -1.67) = 0.9525 - 0.0475 = 0.905$$

Thus, the probability is **0.905**.

Chi-Square Test Questions from N.P. Bali's Books:

Question 1:

A coin is tossed 100 times, and the results are 55 heads and 45 tails. Test whether the coin is unbiased using the chi-square test at a 5% significance level.

Solution:

1. **Null Hypothesis** (H_0): The coin is unbiased.
   Expected frequencies: Heads = Tails = $\frac{100}{2} = 50$.
2. **Chi-Square Formula**:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

   Where O is the observed frequency, and E is the expected frequency.

3. Calculation:

$$\chi^2 = \frac{(55 - 50)^2}{50} + \frac{(45 - 50)^2}{50} = \frac{25}{50} + \frac{25}{50} = 1$$

4. **Degrees of Freedom**: $df = k - 1 = 2 - 1 = 1$.
5. **Critical Value** ($\chi^2_{0.05,1}$): From the chi-square table, $\chi^2_{0.05,1} = 3.841$.

Since $\chi^2 = 1 < 3.841$, we **fail to reject** H_0. The coin is unbiased.

Question 2:

A teacher claims that the performance of students in three categories (excellent, good, average) is equally distributed. A random sample of 90 students gives the following data:

- Excellent: 30, Good: 40, Average: 20.
  Test the teacher's claim at a 5% significance level.

Solution:

1. **Null Hypothesis** (H_0): The distribution is equal.
   Expected frequencies:
   $E = \frac{90}{3} = 30$ for each category.
2. **Chi-Square Calculation**:

$$\chi^2 = \frac{(30-30)^2}{30} + \frac{(40-30)^2}{30} + \frac{(20-30)^2}{30}$$

$$\chi^2 = 0 + \frac{100}{30} + \frac{100}{30} = \frac{200}{30} = 6.67$$

3. **Degrees of Freedom**: $df = k - 1 = 3 - 1 = 2$.
4. **Critical Value** ($\chi^2_{0.05,2}$): From the table, $\chi^2_{0.05,2} = 5.991$.

Since $\chi^2 = 6.67 > 5.991$, we **reject** H_0. The performance is not equally distributed.

Question 3:

The following data shows the observed frequencies of colors of balls drawn from a bag: Red = 40, Blue = 30, Green = 50. Test whether the colors are uniformly distributed at a 5% significance level.

Solution:

1. **Null Hypothesis (H_0)**: The colors are uniformly distributed.
   Expected frequencies:
   $E = \frac{40+30+50}{3} = 40.$
2. **Chi-Square Calculation**:
$$\chi^2 = \frac{(40-40)^2}{40} + \frac{(30-40)^2}{40} + \frac{(50-40)^2}{40}$$
$$\chi^2 = 0 + \frac{100}{40} + \frac{100}{40} = 5$$
3. **Degrees of Freedom**: $df = k - 1 = 3 - 1 = 2.$
4. **Critical Value ($\chi^2_{0.05,2}$)**: $\chi^2_{0.05,2} = 5.991.$

Since $\chi^2 = 5 < 5.991$, we **fail to reject** H_0. The colors are uniformly distributed.

Question 4:

A die is rolled 120 times with the following results:

1 = 15, 2 = 20, 3 = 25, 4 = 30, 5 = 20, 6 = 10. Test whether the die is fair at a 5% significance level.

Solution:

1. **Null Hypothesis** (H_0): The die is fair.

   Expected frequency for each face:

   $E = \frac{120}{6} = 20.$

2. **Chi-Square Calculation**:

   $$\chi^2 = \frac{(15-20)^2}{20} + \frac{(20-20)^2}{20} + \frac{(25-20)^2}{20} + \frac{(30-20)^2}{20} + \frac{(20-20)^2}{20} + \frac{(10-20)^2}{20}$$

   $$\chi^2 = \frac{25}{20} + 0 + \frac{25}{20} + \frac{100}{20} + 0 + \frac{100}{20} = 12.5$$

3. **Degrees of Freedom**: $df = k - 1 = 6 - 1 = 5.$

4. **Critical Value** ($\chi^2_{0.05,5}$): $\chi^2_{0.05,5} = 11.070.$

Since $\chi^2 = 12.5 > 11.070$, we **reject** H_0. The die is not fair.

Question 5:

A survey of 200 people shows the following preferences for three brands:
Brand A = 80, Brand B = 50, Brand C = 70. Test whether the preferences are in the ratio 2:1:1 at a 5% significance level.

Solution:

1. **Null Hypothesis** (H_0): The preferences follow the ratio 2:1:1.
   Total Ratio: $2 + 1 + 1 = 4$.
   Expected frequencies:
   Brand A = $\frac{2}{4} \times 200 = 100$,
   Brand B = $\frac{1}{4} \times 200 = 50$,
   Brand C = $\frac{1}{4} \times 200 = 50$.
2. **Chi-Square Calculation**:

$$\chi^2 = \frac{(80 - 100)^2}{100} + \frac{(50 - 50)^2}{50} + \frac{(70 - 50)^2}{50}$$

$$\chi^2 = \frac{400}{100} + 0 + \frac{400}{50} = 4 + 0 + 8 = 12$$

3. **Degrees of Freedom**: $df = k - 1 = 3 - 1 = 2$.
4. **Critical Value** ($\chi^2_{0.05,2}$): $\chi^2_{0.05,2} = 5.991$.

Since $\chi^2 = 12 > 5.991$, we **reject** H_0. The preferences do not follow the given ratio.