ASSIGNMENT- Parameter Estimation

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(1) $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$

By taking ln of likelihood function we get,

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

Partial derivatives w.r.t θ_1, θ_2

(a)
$$\frac{\partial(\ln L(\theta_1, \theta_2))}{\partial\theta_1} = \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(b)
$$\frac{\partial}{\partial\theta_2}(\ln L(\theta_1, \theta_2)) = \sum_{i=1}^{n} \left(-\frac{(x_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{(x_i-\theta_1)^2}{\theta_2}\right) - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\theta_2 = \underbrace{\frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2}_{\text{corresponds}}$$

(2) MLE for binomial distribution $B(m, \theta)$

m = +ve integer

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

By natural log, we get :

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m - x_i) \ln(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

solving for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i (1-\theta) = \sum_{i=1}^{n} (m - x_i)\theta$$

$$\theta \left(\sum_{i=1}^{n} x_i \right) = m \sum_{i=1}^{n} \theta$$

$$\underbrace{\theta = \frac{1}{m} \sum_{i=1}^{n} x_i}_{\text{corresponds to sample mean}}$$