ARJUNA 2.0

-JEE 2024 Exam-

(12 OCT 2023)

MATHEMATICS

CHAPTER-08

COMPLEX NUMBER - I

Lecture – 07

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Today's Targets

1. Cube Root of Unity
2. nth Root of Unity
3. Question Practice
4.

LAST CLASS

Khatarnaak Result:

$$\boxed{\begin{aligned} 1+e^{i\theta} &= 2\cos\frac{\theta}{2}\, e^{i\frac{\theta}{2}} \\ 1-e^{i\theta} &= 2\sin\frac{\theta}{2}\, e^{i\left(\frac{\theta}{2}-\frac{\pi}{2}\right)} \end{aligned}}$$

Cube Root of Unity:

$\boxed{z^3=1}$ or $\boxed{z=(1)^{\frac{1}{3}}}$

$\longrightarrow 1$

$\longrightarrow -\frac{1}{2}+\frac{\sqrt{3}}{2}i=\omega$

$\longrightarrow -\frac{1}{2}-\frac{\sqrt{3}}{2}i=\omega^2$

Demoivre's Theorem:

$$(\cos\theta + i\sin\theta)^n$$

 # RESULT – 1 : *When power 'n' is an integer*

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)}$$

 # RESULT – 2 : When power is "rational" ($\frac{p}{q}$, where p, q ∈ Z)

$$(\cos\theta + i\sin\theta)^{p/q} \longrightarrow q \text{ values}$$

$$\cos\left((2n\pi+\theta)\frac{p}{q}\right) + i\sin\left((2n\pi+\theta)\frac{p}{q}\right)$$

$\boxed{n=0,1,2,3,\ldots,q-1}$

PROPERTIES OF CUBE ROOT OF UNITY

1. Product of cube roots of unity = $\boxed{\omega^3 = 1}$
   $1 \times \omega \times \omega^2$
   (any higher power can be reduced to small power)

   Example : $\omega^{1992} = (\omega^3)^{664} = (1)^{664} = 1$

   $\omega^{71} = \omega^{69} \cdot \omega^2 = (\omega^3)^{23} \cdot \omega^2 = 1 \times \omega^2 = \omega^2$

   $\frac{1}{\omega^{328}} = \frac{1}{\underbrace{\omega^{327}}_{1}\omega^1} = \frac{1}{\omega} \times \frac{\omega^2}{\omega^2} = \omega^2$

2. $\boxed{\omega \text{ \& } \omega^2 \text{ are conjugate \& reciprocal of each other.}}$
3. Sum of cube roots of unity = $\boxed{1 + \omega + \omega^2 = 0}$

$\omega^{\boxed{\text{3 ka multiple}}} = 1$

$\omega^3 = \omega^2 \cdot \omega = 1$

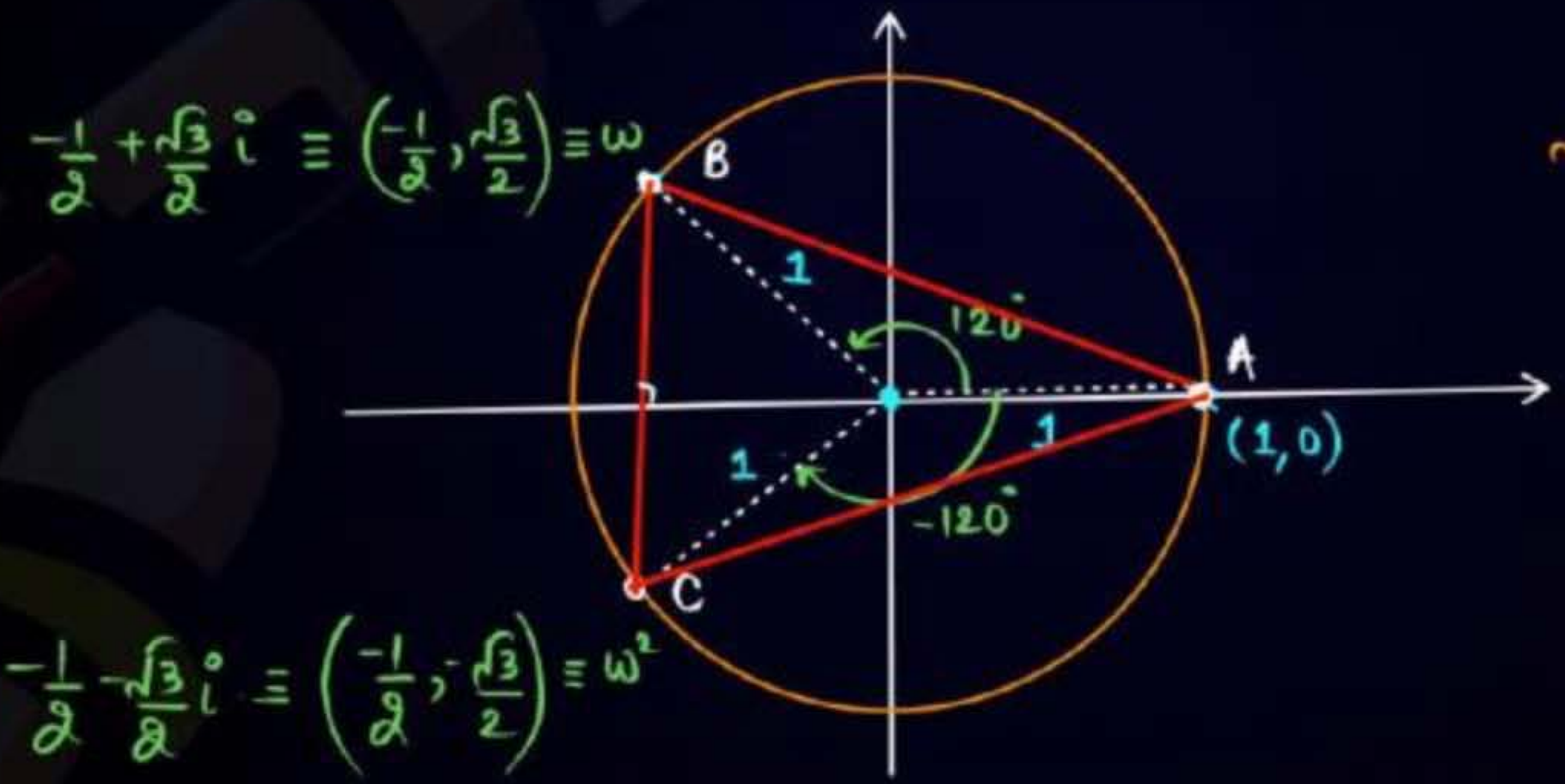
$\omega^2 = \frac{1}{\omega}$ or $\frac{1}{\omega^2} = \omega$

4. *Geometrical Plotting :*

Modulus of all 3 roots $(1, \omega, \omega^2)$ is unity $\Rightarrow$ $\boxed{1 = |\omega| = |\omega^2|}$

$\boxed{\text{All 3 roots lies on circle with centre (0, 0) \& radius = 1}}$

ΔABC in an equilateral Δ with side length = $\sqrt{3}$ units



area $= \frac{\sqrt{3}}{4}(\sqrt{3})^2$

ar(ΔABC) $= \frac{3\sqrt{3}}{4}$

Bumper Question

If ω is imaginary cube root of unity $(\omega \neq 1)$, then find value of:

(i) $(1+\omega^2-\omega)(1+\omega-\omega^2)$ = 4.

(ii) $(1+5\omega^5+\omega^4)(1+5\omega^4+\omega^2)(5\omega^3+\omega+\omega^2)$ = 64.

(iii) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$

(iv) $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^8+\omega^{16})$ ...... upto $2n$ terms.

(i) $(-\omega-\omega)\times(-\omega^2-\omega^2)$

$-2\omega \times -2\omega^2 = 4\omega^3 = 4$

(ii) $(1+5\omega^2+\omega)(1+5\omega+\omega^2)(5+\omega+\omega^2)$

$(-\omega^2+5\omega^2)(-\omega+5\omega)(5-1)$

$4\omega^2 \cdot 4\omega \cdot 4 = 64.$

(iii) $(1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$

$= \left((1-\omega)(1-\omega^2)\right)^2 = \left(1-(\omega^2+\omega)+\omega^3\right)^2$

$= \left(1-(-1)+1\right)^2 = 9.$

Bumper Question

If ω is imaginary cube root of unity ($\omega \neq 1$), then find value of:

(i) $(1 + \omega^2 - \omega)(1 + \omega - \omega^2)$

(ii) $(1 + 5\omega^5 + \omega^4)(1 + 5\omega^4 + \omega^2)(5\omega^3 + \omega + \omega^2)$

(iii) $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$ $\rightarrow (1-\omega^4+\omega^8)$

(iv) $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^8 + \omega^{16})$ ...... upto $2n$ terms. $\rightarrow \left(1-\omega^{(2^p)}+\omega^{(2^{p+1})}\right)$

$\omega^4 \to \omega$

$$\underbrace{(-\omega-\omega)\times(-\omega^2-\omega^2)}_{\Downarrow\ 4} \times \underbrace{(-\omega-\omega)\times(-\omega^2-\omega^2)}_{\Downarrow\ 4} \times \underbrace{\text{-----}}_{\Downarrow\ 4}\ \cdots$$

$4 \cdot 4 \cdot 4 \ldots$ n times $\Rightarrow$ fnf $\Rightarrow 4^n$ Ans.

5. ***Sum of p^{th} power of roots ($p \equiv$ integer):***

$$\boxed{\begin{aligned}(1)^p+(\omega)^p+(\omega^2)^p &= 0 \quad , \ p \text{ is not a multiple of } 3.\\ &= 3 \quad , \ p \text{ is multiple of } 3.\end{aligned}}$$

Ex. 7^{th} power $\Rightarrow 1+\omega^7+\omega^{14} =$ $1+\omega+\omega^2 = 0$.

$(1)^7+(\omega)^7+(\omega^2)^7$

12^{th} power $\Rightarrow 1+\omega^{12}+\omega^{24} =$ 3.

$(1)^{12}+(\omega)^{12}+(\omega^2)^{12}$

$1 \qquad 1.$

JEE Mains-2020

The sum of 162$^{\text{th}}$ power of the roots of the equation $\boxed{x^3 - 2x^2 + 2x - 1 = 0}$ is

$$\boxed{(x-1)(x^2-x+1) = 0}$$

$\rightarrow 1.$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

$\frac{1}{2} + \frac{\sqrt{3}}{2}i = -\omega^2$

$\frac{1}{2} - \frac{\sqrt{3}}{2}i = -\omega.$

$$(1)^{162} + (-\omega^2)^{162} + (-\omega)^{162} = 3$$

Ans.

as you know:

$$x^2+x+1=0 \begin{cases} \omega \\ \omega^2 \end{cases}$$

$$\boxed{x^2+x+1 = (x-\omega)(x-\omega^2)} \longrightarrow \text{identity}$$

Put: $x = \frac{a}{b}$

$$\frac{a^2}{b^2}+\frac{a}{b}+1 = \left(\frac{a}{b}-\omega\right)\left(\frac{a}{b}-\omega^2\right)$$

$$\frac{a^2+ab+b^2}{\cancel{b^2}} = \frac{(a-\omega b)(a-\omega^2 b)}{\cancel{b^2}}$$

$$\boxed{a^2+ab+b^2 = (a-b\omega)(a-b\omega^2)}$$

$b \to -b$

$$\boxed{a^2-ab+b^2 = (a+b\omega)(a+b\omega^2)}$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$\boxed{a^3+b^3 = (a+b)(a+b\omega)(a+b\omega^2)}$$

6. *Important factorisation :*

(i) $\boxed{a^2 + ab + b^2 = (a - \omega b)(a - \omega^2 b)}$ $= (b - a\omega)(b - a\omega^2)$

(ii) $a^2 - ab + b^2 = (a + \omega b)(a + \omega^2 b)$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2) = (a + b)(a + \omega b)(a + \omega^2 b)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2) = (a - b)(a - \omega b)(a - \omega^2 b)$

(v) $\boxed{a^2 + b^2 + c^2 - ab - bc - ca = \frac{(a-b)^2+(b-c)^2+(c-a)^2}{2} = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)}$

Basic Maths.

$$RHS = (a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

$$= a^2 + ab\omega + ca\omega^2 + ab\omega^2 + b^2\omega^3 + bc\omega^4 + ac\omega + bc\omega^2 + c^2\omega^3$$

$\omega^3 \to 1$, $\omega^4 \to \omega$

$$= a^2 + b^2 + c^2 - ac - ab - bc.$$

aab - yaad kre :

$$a^2+b^2+c^2-ab-bc-ca = (a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

$$= (b+a\omega+c\omega^2)(b+a\omega^2+c\omega)$$

$$= (c+a\omega+b\omega^2)(c+a\omega^2+b\omega)$$

7. *Introduction of 'ω':*

(i) $\boxed{\text{'}\omega\text{' is non real complex cube root of unity.}}$

(ii) Roots of equation

$$\boxed{x^2 + x + 1 = 0} \text{ or } \boxed{x + \frac{1}{x} = -1} \to \omega,\ \omega^2$$

$$-\frac{1}{2} + \frac{\sqrt{3}}{2}i = \omega$$

$$-1 + \sqrt{3}i = 2\omega$$

$$1 - \sqrt{3}i = -2\omega$$

$$\boxed{x^2 - x + 1 = 0} \text{ or } \boxed{x + \frac{1}{x} = 1} \to -\omega,\ -\omega^2$$

(iii) $(1 + \sqrt{3}i) = -2\omega^2$

$(1 - \sqrt{3}i) = -2\omega$

$\to 1, \sqrt{3}, i, 2 \to$ vo-bhi aaya hua h.

ω

Complex Cube Root of unity

Bumper Question

(i) If α & β are roots of $\boxed{x^2 - x + 1 = 0}$, then $\alpha^{2009} + \beta^{2009} =$ 1 [AIEEE-2010]

$-\omega \quad -\omega^2$

(ii) Let α & β are roots of $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15} =$ -256 [JEE Mains-2019]

(iii) Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then $a + b =$

[JEE Mains-2020]

(i) $(-\omega)^{2009} + (-\omega^2)^{2009}$

$-(\omega)^2 - \omega = -(\omega^2+\omega)$

$= -(-1)$

$= 1.$

(ii) $x^2 + 2x + 2 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4\times 2}}{2} = \frac{-2 \pm 2i}{2}$

$-1+i = \sqrt{2}\, e^{i\frac{3\pi}{4}}$

$-1-i = \sqrt{2}\cdot e^{-i\frac{3\pi}{4}}$

$\alpha^{15} = \left(\sqrt{2}\, e^{i\frac{3\pi}{4}}\right)^{15} = (\sqrt{2})^{15}\, e^{i\frac{45\pi}{4}}$

$\beta^{15} = \left(\sqrt{2}\, e^{-i\frac{3\pi}{4}}\right)^{15} = (\sqrt{2})^{15}\, e^{-i\frac{45\pi}{4}}$

$\Big\} = 2\cdot(\sqrt{2})^{15}\cdot \cos\frac{45\pi}{4}$

$= 2\cdot(\sqrt{2})^{15}\left(-\frac{1}{\sqrt{2}}\right)$

$-2\cdot(\sqrt{2})^{14}$

$-2\cdot 2^7 = -2^8 = -256.$

Bumper Question

(i) If α & β are roots of $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ 1 [AIEEE-2010]

$-\omega \quad -\omega^2$

(ii) Let α & β are roots of $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15} =$ -256 [JEE Mains-2019]

(iii) Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then $a + b = 1 + 101$ $= 102$

[JEE Mains-2020]

ω

$$a = (1+\omega)\sum_{k=0}^{100}(\omega^{2k})$$

$$a = (1+\omega)\left\{1 + \omega^2 + \omega^4 + \omega^6 + \omega^8 \ldots + \omega^{200} + \omega^{202} - \omega^{202}\right\}$$

$$a = (1+\omega)\left\{(1+\omega^2+\omega) + (1+\omega^2+\omega) + (\ldots) + \ldots (\;) - \underbrace{\omega^{202}}_{-\omega}\right\}$$

$$a = (1+\omega)(-\omega)$$

$$a = -\omega^2 \times -\omega = \omega^3 = 1.$$

$$b = \sum_{k=0}^{100}(\omega)^{3k}$$

$$= 1 + \omega^3 + \omega^6 + \omega^9 + \ldots + \omega^{300}$$

$$= 101$$

Fourth Root of Unity:

$$x^4 = 1 \quad \text{or} \quad x = (1)^{\frac{1}{4}}$$

$$x^4 - 1^4 = 0$$

$$\underbrace{(x^2-1)}(x^2+1) = 0$$

$$x^2 - 1 = 0 \Rightarrow \boxed{x = \pm 1}$$

$$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow \boxed{x = \pm i}$$

$$z = (1)^{\frac{1}{4}} \to 1,\ -1,\ i,\ -i$$

OR

$$(1)^{\frac{1}{4}} = (\cos 0 + i \sin 0)^{\frac{1}{4}}$$

$$= \cos\frac{2n\pi}{4} + i \sin\frac{2n\pi}{4}$$

$n = 0, 1, 2, 3$.

$n = 0 \Rightarrow 1$

$n = 1 \Rightarrow i$

$n = 2 \Rightarrow -1$

$n = 3 \Rightarrow -i$

$\boxed{(1)^{\frac{1}{3}} \rightarrow \begin{cases} 1 \\ \omega \\ \omega^2 \end{cases}}$

$\boxed{(1)^{\frac{1}{4}} \rightarrow \begin{cases} 1 \\ -1 \\ i \\ -i \end{cases}}$

Note : Cube root of any number

Example :

(i) Solve: $\boxed{z^3 + 8 = 0}$ $\Rightarrow z^3 = -8 \Rightarrow z = (-8)^{\frac{1}{3}} = \left((-2)^3 \times 1\right)^{\frac{1}{3}} = -2\,(1)^{\frac{1}{3}} \Rightarrow -2, -2\omega, -2\omega^2$

$(1)^{\frac{1}{3}} \to 1, \omega, \omega^2$

(ii) Solve : $z^3 + 3z^2 + 3z - 26 = 0 \longrightarrow \boxed{z^3 + 3z^2 + 3z + 1} - 27 = 0$

$(z+1)^3 = 27 \Rightarrow (z+1) = (27)^{\frac{1}{3}} = 3.(1)^{\frac{1}{3}}$

$(1)^{\frac{1}{3}} \to 1, \omega, \omega^2$

$z + 1 = 3, 3\omega, 3\omega^2$

$\boxed{z = 2, 3\omega - 1, 3\omega^2 - 1}$

(iii) Solve : $z^3 + 16 = 0 \longrightarrow z^3 = -16$

$z = (-16)^{\frac{1}{3}} = -(16)^{\frac{1}{3}}(1)^{\frac{1}{3}}$

$(1)^{\frac{1}{3}} \to 1, \omega, \omega^2$

$= -(16)^{\frac{1}{3}}, -(16)^{\frac{1}{3}}\omega, -(16)^{\frac{1}{3}}\omega^2$

(iv) Solve : $z^4 - 16 = 0$

$z^4 = 2^4$

$z = 2(1)^{\frac{1}{4}}$ — $1, -1, i, -i$

$\boxed{2, -2, 2i, -2i}$

(v) Solve : $(z - 3)^3 = 125$

QIBY!

Let $\boxed{\omega \neq 1}$ be a $\boxed{\text{cube root of unity}}$. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2 : a, b, c$ distinct non-zero integers$\}$ equals 3

$\bar{\omega} = \omega^2$

$\bar{\omega}^2 = \omega$.

$$\# \quad |\underbrace{a + b\omega + c\omega^2}_{\text{complex no.}}|^2 = (a + b\omega + c\omega^2)(a + b\bar{\omega} + c\bar{\omega}^2)$$

$$= (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$$

$$= a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2}$$

min.

$\boxed{a = 1, b = 2, c = 3.}$

$$\frac{(1-2)^2 + (2-3)^2 + (3-1)^2}{2} = 3$$

min.

$\boxed{a = -1, b = 1, c = 2}$

$$\frac{(-1-1)^2 + (1-2)^2 + (2-(-1))^2}{2} = \frac{4+1+9}{2} = 7.$$

JEE MAINS 2022

Let α be root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

HW.

A 1

B α

C $1 + \alpha$

D $1 + 2\alpha$

JEE Mains-2021

Let $z = \frac{1 - i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of

$$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$$ is

H.W.

n^{th} Root of unity:

$$\boxed{z^n = 1} \rightarrow \boxed{n\text{-Roots}}$$

OR

$$z = (1)^{\frac{1}{n}} = (\cos 0 + i\sin 0)^{\frac{1}{n}}$$

$$z = (1)^{\frac{1}{n}} = \left(\cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}\right)$$

$$\boxed{m = 0, 1, 2, \dots (n-1)}$$

$$\alpha = e^{i\left(\frac{2\pi}{n}\right)}$$

$\boxed{m=0}$ $z = \cos 0 + i\sin 0 = 1$

$\boxed{m=1}$ $z = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n} = \boxed{e^{i\left(\frac{2\pi}{n}\right)} = \alpha}$ say

$\boxed{m=2}$ $z = \cos\frac{4\pi}{n} + i\sin\frac{4\pi}{n} = e^{i\left(\frac{4\pi}{n}\right)} = \alpha^2$

$= \alpha^3$

$= \alpha^4$

$\boxed{m=n-1}$ $z = \cos\frac{2(n-1)\pi}{n} + i\sin\frac{2(n-1)\pi}{n} = \left(e^{i\frac{2\pi}{n}}\right)^{n-1}$

$= \alpha^{n-1}$

PROPERTIES OF nth ROOT OF UNITY

(1) If $\alpha = e^{i\left(\frac{2\pi}{n}\right)}$ then $\boxed{\alpha^n = 1}$.

ex:- $e^{i\frac{3\pi}{4}} = \left(\underbrace{e^{i\frac{2\pi}{8}}}_{8^{th}\text{ root of } 1.}\right)^3 = \left(8^{th}\text{ root of unity}\right)^3$.

Ex. : $e^{i\left(\frac{\pi}{7}\right)}$ is $\underline{\ 14^{th}\ }$ root of unity.

$e^{i\frac{2\pi}{14}} \leftarrow$

(2) All n-roots $(1, \alpha, \alpha^2, \ldots\ldots \alpha^{n-1})$ are in G.P. with common ratio $= \alpha = e^{i\frac{2\pi}{n}}$.

(3) Sum of all n-roots of unity $= \boxed{1 + \alpha + \alpha^2 + \alpha^3 + \ldots\ldots\ldots + \alpha^{n-1} = \text{zero}.}$

$z^n = 1.$

$z^n - 1 = 0 \to z^n + \underset{\sim}{0} z^{n-1} + 0 z^{n-2} + \ldots + (-1) = 0 \to$ S.O.R $= -\frac{0}{1} = 0.$

$$\# \quad \boxed{\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \cos\frac{6\pi}{n} + \cdots + \cos\frac{2(n-1)\pi}{n} = -1}$$

Trigo Series with angles in A.P.

Trigo ✓

$$\overset{1}{1} + \overset{\alpha}{\left(\cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)} + \overset{\alpha^2}{\left(\cos\frac{4\pi}{n} + i\sin\frac{4\pi}{n}\right)} + \cdots + \overset{\alpha^{n-1}}{\left(\cos\frac{2(n-1)\pi}{n} + i\sin\frac{2(n-1)\pi}{n}\right)} = 0 + 0i$$

$$1 + \cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \cdots + \cos\frac{2(n-1)\pi}{n} = 0.$$

$$\sin\frac{2\pi}{n} + \sin\frac{4\pi}{n} + \cdots + \sin\frac{2(n-1)\pi}{n} = 0.$$

$\#\quad x^3-1=0 \quad \Rightarrow \quad x^3+0x^2+0x-1=0 \quad \longrightarrow P=1.$

$\#\quad x^4-1=0. \quad \Rightarrow \quad x^4+0x^3+0x^2+0x-1=0. \Rightarrow P=-1.$

$\#\quad x^5-1=0. \quad \Rightarrow \quad x^5+0x^4+0x^3+0x^2+0x-1=0. \Rightarrow P=1$

$\#\quad x^6-1=0. \quad \Rightarrow \quad x^6+0x^5+0x^4+0x^3+0x^2+0x-1=0. \Rightarrow P=-1.$

$\vdots$

$$\#\quad x^n-1=0. \longrightarrow P=(-1)^{n-1} \begin{cases} =1, & n\equiv \text{odd} \\ =-1, & n\equiv \text{even} \end{cases}$$

(4) **Product of all roots of unity =** $\boxed{1 \cdot \alpha \cdot \alpha^2 \cdot \alpha^3 \ldots\ldots\ldots \alpha^{n-1} = (-1)^{n-1}}$

(5) **All roots are unimodular** $\Rightarrow \boxed{1 = |\alpha| = |\alpha^2| = \ldots\ldots\ldots\ldots |\alpha^{n-1}|}$ → unimodular.

$\alpha\bar{\alpha} = 1$

$\alpha = \frac{1}{\bar{\alpha}}$

"**Conjugate Pairs**" :-

$1, \boxed{\alpha, \alpha^2, \alpha^3, \ldots\ldots\ldots, \alpha^{n-2}, \alpha^{n-1}}$

(α, α^{n-1})

$(\alpha^2, \alpha^{n-2}) \ldots\ldots$

\# $\alpha^n = 1$.

\# $\alpha^{n-1} \cdot \alpha^1 = 1$.

$\alpha^{n-1} \cdot \frac{1}{\bar{\alpha}} = 1$

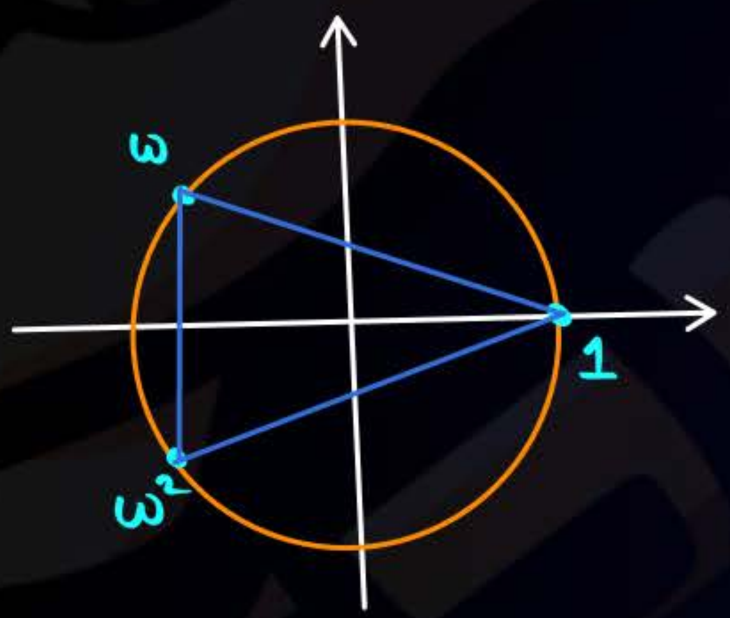
$\boxed{\alpha^{n-1} = \bar{\alpha}}$

(6) ***Sum of p^{th} powers of all roots of unity :***

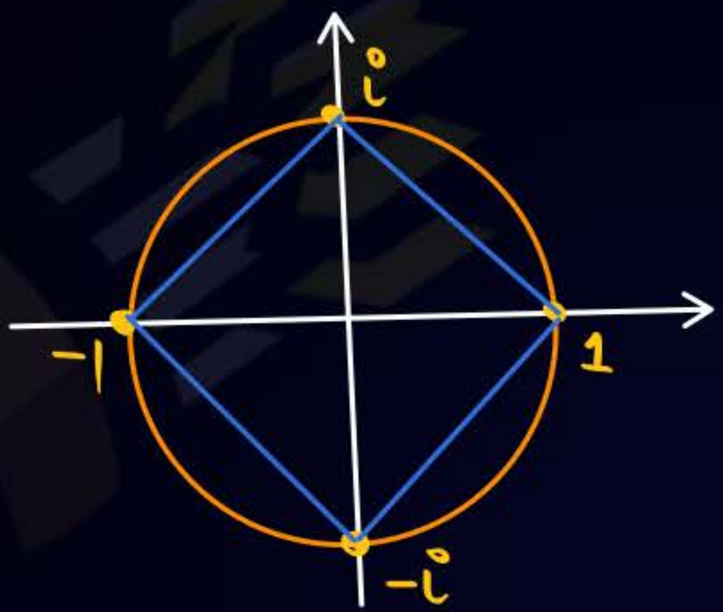
$1 + \alpha^p + \alpha^{2p} + \alpha^{3p} + \ldots\ldots\ldots + (\alpha^{n-1})^p = 0$, **If 'p' is not a multiple of n**

$= n$, **If 'p' is multiple of n.**

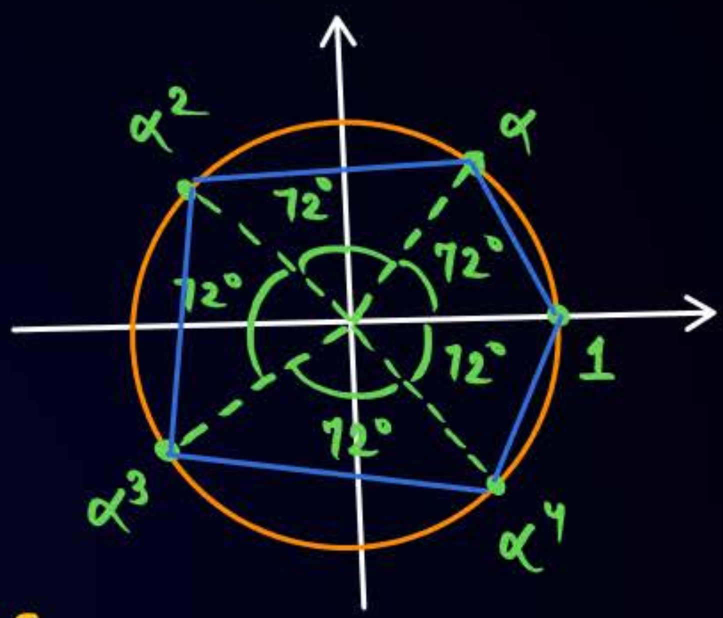
$z^3 = 1$



$z^4 = 1$

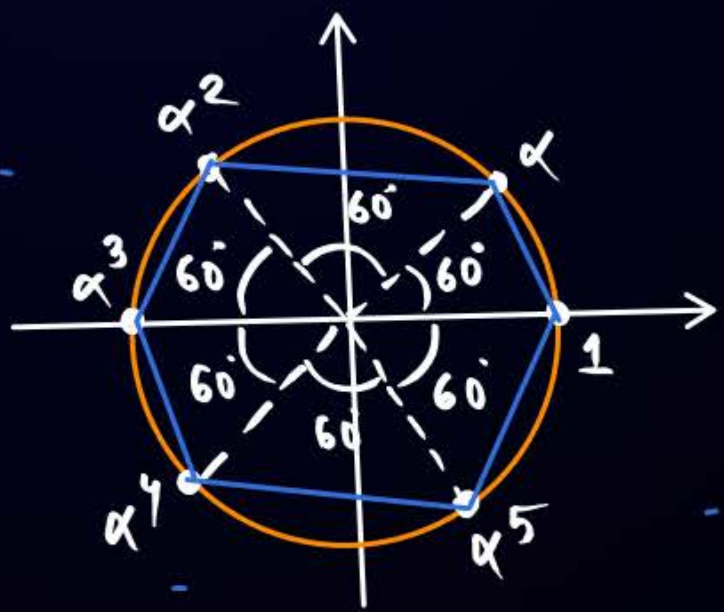


$z^5 = 1$



$$\alpha = e^{i\frac{2\pi}{5}}$$

$z^6 = 1$



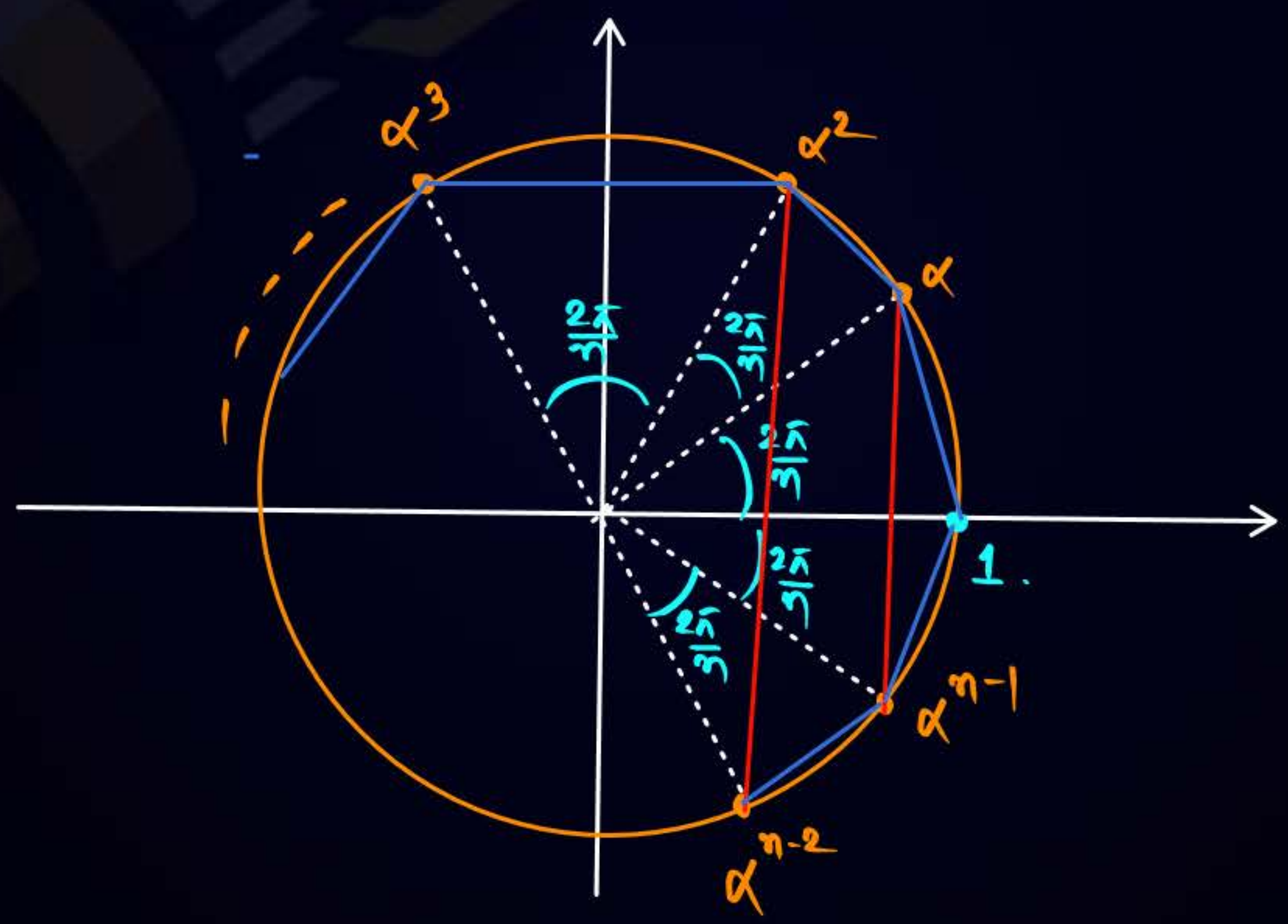
$$\alpha = e^{i\frac{2\pi}{6}}$$

(7) **Geometrical Plotting :**

All of them lies on Circle with centre (0, 0) & Radius = 1

Note : All n-roots divide circle in n-equal parts.

All n-roots are vertices of n-sided regular polygon.



Example

If $\alpha = e^{i\left(\frac{2\pi}{7}\right)}$ then form a quadratic equation whose roots are A & B where $A = \alpha + \alpha^2 + \alpha^4$ & $B = \alpha^3 + \alpha^5 + \alpha^6$.

$$\alpha = e^{i\frac{2\pi}{7}}$$

7^{th} root of unity

$$\alpha^7 = 1.$$

$$\boxed{1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6 = 0}$$

$$\alpha^9 = \underbrace{\alpha^7}_{1}\cdot\alpha^2 = \alpha^2.$$

$$A + B = \alpha+\alpha^2+\alpha^4+\alpha^3+\alpha^5+\alpha^6 = -1. \qquad \alpha^8 = \alpha.$$

$$A \cdot B = (\alpha+\alpha^2+\alpha^4)(\alpha^3+\alpha^5+\alpha^6)$$

$$= \alpha^4+\alpha^6+\alpha^7+\alpha^5+\alpha^7+\alpha^8+\alpha^7+\alpha^9+\alpha^{10}.$$

$$= \underbrace{\alpha^4+\alpha^6+\alpha^5+\alpha^1+\alpha^2+\alpha^3}_{-1} + 3\alpha^7$$

$$-1+3(1) = 2.$$

$$x^2-(-1)x+(2) = 0.$$

$$x^2+x+2 = 0 \quad \begin{cases} A \\ B \end{cases}$$

JEE (Adv.)-2013

Evaluate : $\sum_{\lambda=1}^{12}\left(\sin\frac{2\lambda\pi}{13}-i\cos\frac{2\lambda\pi}{13}\right)$

H.W.

DO IT BY YOURSELF (DIBY)

COMPLEX NUMBER - I

DIBY-01

Select the true statement from the following.

A $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}=0$

B $1+i^{14}+i^{18}+i^{22}$ is a real number

C $i^{3k}+i^{3k+1}+i^{3k+2}+i^{3k+3}=0$

D $6i^{54}+5i^{37}-i^{11}+6i^{68}=7i$

[Ans. : (A, B, C)]

DIBY-02

Find the value of $\sum_{k=1}^{k=4m+1}\left(i^k + i^{2k} + i^{3k} + i^{4k}\right)$ [Ans. : 4m where m ∈ I]

DIBY-03

Find the value of $\left(\frac{1+i}{1-i}\right)^{10001} + \left(\frac{1-i}{1+i}\right)^{10001}$. [Ans. : Zero]

DIBY-04

If $\frac{x-3}{3+i} + \frac{y-3}{3-i} = i$, where $x, y \in R$, then find (x, y). [Ans. : $y = 8, x = -2$]

DIBY-05

Find the imaginary part of $\frac{(1+2i)^2-(1-i)^3}{(3+2i)^3-(2+i)^2}$ [Ans. : $\frac{-15}{954}$]

DIBY-06

Find the reciprocal of $3+\sqrt{7}i$ [Ans. : $\frac{3}{16}-\frac{\sqrt{7}}{16}i$]

DIBY-07

Find the square root of $3 - 4i$

A $\pm(2 - i)$

B $\pm(2 + i)$

C $\pm\left(\sqrt{3} - 2i\right)$

D $\pm\left(\sqrt{3} + 2i\right)$

[Ans. : (A)]

JEE Mains 2016 # DIBY-08

A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is

A $\frac{\pi}{6}$

B $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

C $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D $\frac{\pi}{3}$

IIT-JEE 2012 (Paper-1) # DIBY-09

Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then a cannot take the value

A -1

B $1/3$

C $1/2$

D $3/4$

DIBY-10

Solve $z^2 + |z| = 0$ [Ans. : $(0,0)(0,1)(0,-1)$]

DIBY-11

If $|z - i| = 1$ and $Arg(z) = \frac{\pi}{2}$, then find z. [Ans. : $2i$]

DIBY-12

Write the complex number $\frac{\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)(\sqrt{3}+i)}{i-1}$ in [Ans. : $\sqrt{2}\text{cis}\left(\frac{-11\pi}{12}\right)$]

DIBY-13

If $z = \frac{1+i\sqrt{3}}{2i\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$, find $|z|$ and $\text{Arg}(z)$. [Ans. : 1 & $-\frac{\pi}{2}$]

DIBY-14

Simplify $\left(\dfrac{1+i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}{1-i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}\right)^{8}$ [Ans. : -1]

DIBY-15

If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then : [JEE (Main)-2019]

(A) $z\bar{w} = \frac{1-i}{\sqrt{2}}$

(B) $\bar{z}w = i$

(C) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

(D) $\bar{z}w = -i$

DIBY-16

The imaginary part of $(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$ can be [JEE (Main)-2020]

(A) $\sqrt{6}$

(B) $-\sqrt{6}$

(C) $-2\sqrt{6}$

(D) 6

DIBY-17

The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is [JEE (Main)-2020]

(A) $-2^{15}i$ (B) -2^{15} (C) $2^{15}i$ (D) 65

DIBY-18

If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a+b$ is equal to [JEE (Main)–2020]

(A) 33 (B) 9 (C) 24 (D) 57

DIBY-19

If ω is imaginary cube root of unity, then evaluate $\frac{1}{1+2\omega} + \frac{1}{2+\omega} - \frac{1}{1+\omega}$.

[Ans. : zero]

DIBY-20

If ω is imaginary cube root of unity, then evaluate $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$.

[Ans. : -1]

DIBY-21

If ω is imaginary cube root of unity, then evaluate $(1-\omega+\omega^2)(1+\omega-\omega^2)(-1+\omega+\omega^2)$

[Ans. : -8]

DIBY-22

If α, β & γ are roots of $x^3-3x^2+3x+7=0$ and $\dfrac{\alpha-1}{\beta-1}+\dfrac{\beta-1}{\gamma-1}+\dfrac{\gamma-1}{\alpha-1}=W$, then find $|W|$.

[Ans. : 3]

DIBY-20

Let z_0 be roots of quadratic $x^2 + x + 1 = 0$ and $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then $\arg(z)$=

[JEE Mains-2019]

[Ans. : $pi/4$]

DAILY HOME WORK

All DPP's till date & Re-attempt all the Ques. of Today's Lecture.

Next chapter

$\boxed{P \& C}$

Linear Inequalities.

Recorded form.


#futureIITians
THANK YOU