Roll No......................

# NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

## B. Tech. (Semester – II) Theory Examination, December, 2019

## Subject: Integral Calculus and Difference Equations

## Paper Code: MAIR 12

**Time: 03 Hours** **Max. Marks: 50**

Note:

I. Answer all questions, considering the internal choices in question no 2, 3 and 4. Marks allotted for each question is shown on the right hand margin.

II. The candidates, before starting to write the solutions, should please check the question paper for any discrepancy, and also ensure that they have been delivered the question paper of **right course number** and the **right subject title**.

| | | |
|---|---|---|
| 1 | (a) Find a power series solution in powers of $x$ for $x(1+x)y' - (2x+1)y = 0$. | [7] |
| | (b) Show that the function $f_1(x) = 1,\ f_2(x) = x$ are orthogonal on the interval $(-a, a)$ and determine the constants $A$ and $B$ so that the function $f_3(x) = 1 + Ax + Bx^2$ is orthogonal to both $f_1$ and $f_2$ on the interval $(-a, a)$. | [3] |
| 2 | (a) Change the order of integration in $\int_0^a \int_y^a \frac{x}{(x^2+y^2)}\,dxdy$ and hence evaluate the same. | [5] |
| | (b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{(1-x^2-y^2-z^2)}}\,dzdydx$. | [5] |
| | **OR** | |
| | (b) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ | [5] |
| 3 | (a) The integers 0,1,1,2,3,5,8,13,21, … … .. are said to form a Fibonacci sequence. Form the Fibonacci difference equation and solve it. | [5] |
| | (b) Using $Z$ transform, solve the difference equation $u_{n+2} + 5u_{n+1} + 4u_n = 2^n$, given that $u_0 = 1, u_1 = -4$ | [5] |
| | **OR** | |
| | (b) Find the inverse $Z$ transform of $\frac{4z^2-2z}{z^3-5z^2+8z-4}$. | [5] |

| | | |
|---|---|---|
| 4 | (a) A vector field is given by $\vec{A} = (x^2 + xy^2)i + (y^2 + x^2y)j$. Show that the field is irrotational and find the scalar potential | [5] |
| | (b) Verify Green's theorem in the plane for $\oint_C [(3x^2 - 8y^2)\,dx + (4y - 6xy)dy]$, where $C$ is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. | [5] |
| | **OR** | |
| | (b) Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 2z^2$ at the point $P\ (1,2,3)$ in the direction of the line $PQ$, where $Q$ is the point $(5,0,4)$. In what direction, it will be maximum. ? Find also the magnitude of this maximum. | [5] |
| 5 | (a) Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right)$ in the interval $(0,2\pi)$ and hence deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots$ | [5] |
| | (b) Represent $f(x) = \sin\frac{\pi x}{L}$ in $0 < x < L$ by a Fourier cosine series. | [5] |

THE END