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Sheet No. 1

Roll No. _________

# B.Tech. (1st Semester) Examination,

**November-December 2017**

**Subject: Mathematics-I** **(Code: MAIR-11)**

**Time: 03 Hours** **Max. Marks: 50**

**Note:**

I. Answer any Five questions. All questions carry equal marks.

II. The candidates, before starting to write the solutions, should please check the Question Paper for any discrepancy, and also ensure that they have been delivered the question paper of right course no. and right subject title.

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**1(a)** Reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to canonical form by orthogonal transformation. Determine index, signature and nature of quadratic form. [5]

**1(b)** Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

and hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. [5]

**2(a)** Diagonalize the matrix

$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Hence find modal matrix P. Then obtain the matrix $B=A^2+5A+3I$. [5]

**2(b)** If $f(x,y)=\tan^{-1}(xy)$, find an approximate value of f(1.1, 0.8) using Taylor's series upto quadratic approximation [take $\tan^{-1}(1)=0.7854$ ]. [5]

**3(a)** If $V=f(2x-3y, 3y-4z, 4z-2x)$. Prove that $6V_x+ 4V_y +3V_z=0$. [5]

**3(b)** If $x = r\sin\theta\cos\varphi$, $z = r\cos\theta$, show that $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2\sin\theta$. [5]

**4(a)** Solve $(D^2+4)\, y = x^2 \sin 2x$, Using method of Undetermined coefficient. [5]

**4(b)** Solve $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \sin(\log x^2)$. [5]

**5(a)** Solve $(D^2+2D+5)\, y= e^{-x}\sec 2x$, by using variation of parameter method. [5]

**5(b)** Find Laplace inverse of $\dfrac{1}{(s^2+a^2)^2}$ using convolution theorem. [5]

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| 6(a) | Solve the differential equation using Laplace transform $(D^2+9)\, y = \cos 2t$, if $y(0)=1$, $y\left(\frac{\pi}{2}\right) = -1$. | [5] |
| 6(b) | Show that $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{p}}e^{-1/4p}$ | [5] |
| 7(a) | An uncharged condenser of capacity C is charged by applying an e.m.f. $E\sin\frac{t}{\sqrt{LC}}$ through leads of self inductance L and negligible resistance. Prove that at any time t, the charge on one of the plate is $\frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right]$. | [5] |
| 7 (b) | A pot is baked in a kitchen and emerges at a temp. of 300 °C into a workshop that is constantly at temp. of 30 °C. After 1 hour temp. of pot is 100 °C. Assume Newton's law of cooling. (i) What will be temp. of pot after 2 hours ? (ii) How long will it take until the pot cools to 32 °C ? | [5] |