3-0 NOV 2018

Sheet No. 1

Roll No.

B.Tech. (1st Semester) Examination,

Dec 2018

Subject: Mathematics-I (Code: MAT-105T/103T)

Time: 03 Hours Max. Marks: 50

Reappear (Old Scheme)

Note:

I. Answer Five questions: Selecting at least 1 question from each unit. Marks allotted for each question are shown on right hand margin.

II. The candidates, before starting to write the solutions, should please check the Question Paper for any discrepancy, and also ensure that they have been delivered the question paper of right course no. and right subject title.

Unit-1

1(a)	For the curve $y = \frac{ax}{a+x}$, if ρ is the radius of curvature at any point (x, y), show that $$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2.$$	[5]
1(b)	Find the center of curvature of the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ (or the astroid $x = acos^3\theta, y = asin^3\theta$)	[5]
2(a)	Find the asymptotes parallel to the axes for the curve $$x^2y^2 = a^2(x^2 + y^2)$$ and show that they form a square of sides $2a$.	[5]
2(b)	Trace the curve $r = acos2\theta$.	[5]
3(a)	Prove that $e^x cosx = 1 + x - \frac{2x^3}{3!} - \frac{2^2x^4}{4!} - \frac{2^2x^5}{5!} + \frac{2^3x^7}{7!} + \cdots$.	[5]
3(b)	Expand $log_e x$ in powers of $(x-1)$ and hence evaluate $log_e 1.1$correct to 4 decimal Places.	[5]

Unit-2

4(a)	If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$.	[5]
4(b)	If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.	[5]

5(a)	If $u = \sin^{-1}(x - y), x = 3t\ and\ y = 4t^3$, show that $\frac{du}{dt} = 3(1 - t^2)^{-1/2}$.	[5]
5(b)	If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.	[5]
6(a)	Prove that a rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.	[5]
6(b)	If $u = x + y + z, v = x^2 + y^2 + z^2, w = yz + zx + xy$, then show that $grad\ u, grad\ v, grad\ w$ are coplanar.	[5]

Unit-3

7(a)	Find eigen values and eigen vectors of $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.	[5]
7(b)	Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Hence compute A^{-1}.	[5]
8(a)	Find the latent roots, eigen vectors, the modal matrix of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$, hence reduce $x^2 + 3y^2 + 3z^2 - 2yz$ to canonical form.	[5]
8(b)	If $x = 2cos\alpha\ cosh\beta, y = 2sin\alpha\ sinh\beta$, prove that $\sec(\alpha + i\beta) + \sec(\alpha - i\beta) = \frac{4x}{x^2+y^2}$.	[5]
9(a)	If $\cosh(u + iv) = x + iy$, prove that $\frac{x^2}{cosh^2u} + \frac{y^2}{sinh^2u} = 1$ and $\frac{x^2}{cos^2v} - \frac{y^2}{sin^2v} = 1$.	[5]
9(b)	Find the sum of series $c\ sin\alpha + \frac{c^3}{3} sin3\alpha + \frac{c^5}{5} sin5\alpha + \cdots \infty$.	[5]