

Department of Mathematics
National Institute of Technology Kurukshetra
B.Tech. (I Semester) MID TERM-1 Exam, 17 January-2022

Subject: Differential Calculus and Differential Equations

Code: MAIR 11

Max. Marks: 15

Branch: CE, CS, EC, EE, IT, ME, PI

Timings: 9: 30a.m-10:10 a.m.

Note: a) All questions are compulsory.

b) The question paper consists of 11 objective questions. First seven questions are of one mark each and rest are of two mark each.

1. Find the eigen values of A where

$$A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

A. 0, 0, 9

B. 3, 3, 3

C. 0, 3, 6

D. All the above.

2. If $A = [a_{ij}]_{n \times n}$ and $B = [b_{ij}]_{n \times n}$ are two matrices such that ' B ' is similar to ' A '. Then which of the following statement is not always true

A. $|A| = |B|$

B. Trace of A = Trace of B

C. A and B have the same eigen values.

D. Eigen vectors of ' A ' and ' B ' with corresponding eigen values are same.

3. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Then which of the followings are eigen vectors of A corresponding to eigen values $\lambda = 2$

A. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

D. None of the above.

4. If $f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

At the origin the function $f(x, y)$ is

- A. Continuous
- B. Discontinuous
- C. Not Defined
- D. None of the above

5. Let $A = [a_{ij}]_{3 \times 3}$ be a skew-symmetric matrix such that ' $2i$ ' is its one eigen value then other two eigen values are,

- A. $-2i, 0$.
- B. $\frac{1}{2i}, 0$.
- C. $-\frac{1}{2i}, 0$
- D. None of above.

6. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 - y^2}$ is equal to

- A. 0
- B. Does not exist
- C. 1
- D. -1

7. If $z = g(x^a y^b)$ satisfies the equation $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$ then

- A. $3b^2 = 4a^2$
- B. $4b^2 = 3a^2$
- C. $4b^2 = 9a^2$
- D. $9b^2 = 4a^2$

8. Let $A = [a_{ij}]_{2 \times 2}$ such that trace of A is 1 and $|A| = 1$, then find ' A^3 '

- A. I
- B. A
- C. A^2
- D. $-I$

9. What is the nature of the quadratic form $2x_1 x_2 + 2x_1 x_3 - 2x_2 x_3$

- A. Indefinite
- B. Positive Definite
- C. Positive Semi-definite
- D. None of the above.

10.If the matrix $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Then which of the following is true

- A. A is diagonalizable
- B. A is similar to diagonal matrix
- C. 0, 0 are the eigen values of A
- D. All the above

11.If $z = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$, then $\frac{\partial^2 z}{\partial y \partial x}$ is

- A. $\frac{x^2 - y^2}{x^2 + y^2}$
- B. $\frac{x^2 + y^2}{x^2 - y^2}$
- C. $\frac{x^2 y^2}{x^2 - y^2}$
- D. $\frac{x^2 y^2}{x^2 + y^2}$