

30 NOV 2018

Sheet No. 1/2

Roll No.

NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

B. Tech. (Semester – I) Examination, Nov.-Dec. 2018

Subject: Differential Calculus and Differential Equations

Paper Code: MAIR 11

Time: 03 Hours

Max. Marks: 50

Note:

- I. Answer FIVE questions out of the following SEVEN questions. Marks allotted for each question is shown on the right hand margin.
- II. The candidates, before starting to write the solutions, should please check the question paper for any discrepancy, and also ensure that they have been delivered the question paper of **right course number** and the **right subject title**.
- III. Unless stated otherwise, the symbols have their usual meanings in context with the subject.

1(a)	If $u^3 + v + w = x + y^2 + z^2$; $u + v^3 + w = x^2 + y + z^2$; $u + v + w^3 = x^2 + y^2 + z$, then Prove that- $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$	[5]
(b)	Find the maximum value of $u = x^p y^q z^r$ when the variables x, y, z are subject to the condition $ax + by + cz = p + q + r$.	[5]
2(a)	If $x^x y^y z^z = c$, then show that - at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log x)^{-1}$.	[5]
(b)	Diagonalize the following matrix- $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$	[5]
3(a)	Find the characteristic equation of the following matrix- $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Hence find A^{-1} and A^4 .	[5]

(b)	Reduce the following Quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a Canonical form by Orthogonal transformation and discuss its nature. Also find the Modal Matrix	[5]
4(a)	Solve the following ODE by Variation of Parameter method- $(D^2 - 1)y = 2(1 - e^{-2x})^{-1/2}$	[5]
(b)	Solve the following ODE - $\{(1 + 2x)^2 D^2 - 6(1 + 2x)D + 16\}y = 8(1 + 2x)^2$ Given that $y(0) = 0, y'(0) = 2$	[5]
5(a)	Solve the following ODE by Method of Undetermined Coefficient- $(D^2 - 6D + 9)y = x^2 e^{3x}$	[5]
(b)	Express the following function into Unit Step Function- $f(t) = \begin{cases} t - 1 & \text{for } 1 < t < 2 \\ 3 - t & \text{for } 2 < t < 3 \end{cases}$ Then find its Laplace Transform.	[5]
6(a)	Using Laplace Transform, Solve the following ODE - $y''' - 3y'' + 3y' - y = t^2 e^t$ Given that $y(0) = 1, y'(0) = 0, y''(0) = -2$.	[5]
(b)	Using Convolution Theorem, find the inverse Laplace Transform of the following function- $\frac{16}{(s-2)(s+2)^2}$	[5]
7(a)	Derive the formula of Half-Life of a radioactive element. Then solve the following Problem- If the half-life of a particular radioactive substance is known to be 10 days and there are 25 mg. initially, how much is present after 8 days.	[5]
(b)	Write the Differential equations of free undamped, free damped and forced oscillation of a spring. Also find the solution of equation in case of free undamped oscillations of a spring.	[5]