Department of Mathematics
National Institute of Technology Kurukshetra
B.Tech. (II Semester) END SEMESTER Exam, June-2021

Subject: Integral Calculus and Difference Equations **Max. Marks: 50**
Code: MAIR 12
Branch: CE, CS, EC, EE, IT, ME, PI
Timings: 11:00 a.m.-01:00 p.m.
Note:

1. All questions are compulsory.
2. This question paper consists of 23 MCQs. It has 6 MCQs (Q.No. 1-6) of one mark each, 7 MCQs (Q.No. 7-13) of 2 marks each and 10 MCQs (Q.No. 14-23) of 3 marks each.

1. If $f(x) = \sinh x$ in $-\pi < x < \pi$, then the value of a_n is –

   A. $\frac{2}{n}$ B. $\frac{2}{1-n^2}$ C. 0 D. $\frac{4}{n\pi}$

2. The Z-transform of $\frac{(\log c)^n}{n!}$ is –

   A. $e^{c/z}$ B. $c^{1/z}$ C. $e^{1/z \log c}$ D. $(\log c)^z$

3. The differential equation $x^2y'' + x\left(x - \dfrac{1}{2}\right)y' + \frac{1}{2}y = 0$ has ...... point at $x = 0$.

   A. Regular singular B. Irregular singular C. Ordinary D. Cannot be predicted

4. If $f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 50, & \pi < x < 2\pi \end{cases}$ and $f(x) = f(x + 2\pi)\ \forall x$, then the sum of the Fourier series of $f(x)$ at $x = \pi$ is –

   A. $\frac{49}{2}$ B. $\frac{50}{3}$ C. -1 D. Doesn't exist

5. On changing the order of integration, the integral $\int_0^1 \int_x^{\sqrt{2-x^2}} f(x, y)\, dxdy$ becomes –

   A. $\int_0^{\sqrt{2}} \int_0^y f(x, y)\, dxdy$ B. $\int_0^1 \int_0^y f(x, y)\, dxdy + \int_1^2 \int_0^{-\sqrt{2-y^2}} f(x, y)\, dxdy$

   C. $\int_0^{\sqrt{2}} \int_y^{\sqrt{2-y^2}} f(x, y)\, dxdy$ D. $\int_0^1 \int_0^y f(x, y)\, dxdy + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-y^2}} f(x, y)\, dxdy$

6. The maximum rate of change of $f(x, y, z) = x^2 - 2y^2 + z^2$ at the point $(1, 1, 0)$ is –

   A. 20 B. $\sqrt{2}$ C. $2\sqrt{5}$ D. -1

7. The complete solution for the difference equation, $y_{n+2} + 2y_{n+1} - 3y_n = 4 \cdot 2^n$ is –

   A. $A+B\cdot(3)^n+\frac{1}{5}\cdot 2^n$ B. $A+B\cdot(-3)^n+\frac{4}{5}\cdot 2^n$ C. $A+B\cdot(-3)^n-\frac{4}{3}\cdot 2^n$ D. $(A+nB)(3)^n+\frac{2}{5}\cdot 2^n$

8. Area outside the circle $r = 2$ and inside $r = 4\sin\theta$ is –

A. $\frac{4\pi}{3} + 2\sqrt{3}$ B. $\frac{-2\pi}{3} + 4$ C. $\frac{\pi}{6} + \sqrt{3}$ D. None of these

9. For the Sturm-Liouville problem, $y'' + \lambda y = 0,\ y'(0) = y'(\pi) = 0$, one eigen function is $y = \cos 4x$, then the corresponding eigen value is –

A. 0 B. 2 C. 4 D. 16

10. By using beta and gamma functions, evaluation of the integral, $\int_0^3 \dfrac{dx}{\sqrt{3x - x^2}}$ is –

A. $1/2$ B. $\pi/4$ C. π D. None of these

11. A difference equation generated by $y_n = A \cdot 2^n + B \cdot 3^n + \frac{1}{2}$, is –

A. $y_{n+2} - 5y_{n+1} + 6y_n = 1$ B. $3y_{n+2} + 5y_{n+1} + 6y_n = 0$
C. $3y_{n+2} + 5y_{n+1} + 6y_n = 2$ D. $y_{n+2} - 5y_{n+1} + 6y_n = 0$

12. Circulation of the vector field $F = (x-y)\,\hat{\imath} + x\,\hat{\jmath}$ around the circle $R(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath},\ 0 \le t \le 2\pi$, is –

A. 0 B. π C. 2π D. 4π

13. By using convolution theorem, the inverse Z-transform of $\dfrac{z^2}{(z-1)^3}$, is –

A. n^2 B. $\frac{n(n-1)}{2}$ C. $\frac{n(n+1)}{2}$ D. $\frac{n}{2}$

14. Volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the surface of the paraboloid $x^2 + y^2 = 3z$, is –

A. $\frac{3\pi}{2}$ B. $\frac{19\pi}{6}$ C. $\frac{14\pi}{3}$ D. None of these

15. The Fourier series of the function $f(x) = \begin{cases} -\frac{\pi}{2} - \frac{x}{2}, & -\pi \le x < 0, \\ \frac{\pi}{2} - \frac{x}{2}, & 0 \le x \le \pi, \end{cases}$ is –

A. $\frac{\pi^3}{4} + 6\pi \sum_{n=1}^{\infty} (-1)^n \dfrac{\cos nx}{n^2}$ B. $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{2\sin nx}{n}$ C. $\sum_{n=1}^{\infty} \dfrac{\sin nx}{n}$ D. None of these

16. By using the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}, w = \frac{z}{3}$, evaluation of the integral

$$\int_0^3 \int_0^4 \int_{y/2}^{y/2+1} \left(\frac{2x-y}{2} + \frac{z}{3}\right) dxdydz, \text{ is–}$$

A. 6 B. 12 C. 21 D. 30

17. The indicial roots of the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ about point $x = 0$ are –

A. Real and equal
B. Distinct and do not differ by an integer
C. Distinct and differ by an integer
D. None of these

18. Using the Z-transform, solution of the difference equation $y_{n+2} - 6y_{n+1} + 9y_n = 0,\ \ y_0 = 6, y_1 = 27,$ is –

A. $6 \cdot 3^n + 3n$ B. $6 \cdot 3^n + 9 \cdot (-3)^n$ C. $6 \cdot 3^n + 9n^2$ D. $(6 + 3n)3^n$

19. If $F = xz\,\hat{i} + xy\,\hat{j} + 3xz\,\hat{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, then by using Stoke's theorem, line integral $\int_C F \cdot dR$ is equal to–

A. $\sqrt{6}$ B. -1 C. 1 D. None of these

20. For the Sturm-Liouville problem, $y'' + \lambda y = 0,\ y(0) = y'(\pi) = 0$, the eigen values are–

A. $\lambda_k = \frac{(2k-1)^2}{4}$, k a positive integer
B. $\lambda_k = \frac{(2k-1)\pi}{2}$, k a positive integer
C. $\lambda_k = \frac{k\pi}{2}$, k an odd integer and also $k = 0$
D. $\lambda_k = \frac{k^2\pi^2}{4}$, k a positive integer

21. By divergence theorem, the surface integral $\int_S (lx + my + nz^2)\, dS$, where S denotes the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$; and l, m, n are the direction cosines of the external normal to S, is equal to–

A. $\frac{3\pi}{7}$ B. $\frac{7\pi}{6}$ C. $\frac{4\pi}{3}$ D. None of these

22. The half range sine series for $f(x) = \cos x$ in $0 \le x \le \pi$ is–

A. $\frac{4}{\pi}\sum_{n=1}^{\infty} \frac{n}{n^2-1} \sin nx$ B. $\frac{8}{\pi}\sum_{n=1}^{\infty} \frac{n}{2n^2-1} \sin 2nx$ C. $\frac{8}{\pi}\sum_{n=1}^{\infty} \frac{n}{4n^2-1} \sin 2nx$ D. $\frac{2}{\pi}\sum_{n=1}^{\infty} \frac{n}{n-1} \sin nx$

23. Evaluation of the integral $\iint_R \sqrt{|y - x^2|}\, dxdy$, where R is the rectangle $-1 \le x \le 1, 0 \le y \le 2$, is–

A. $\frac{\pi}{2}$ B. $\frac{\pi}{2} + \frac{4}{3}$ C. $\frac{4\pi}{3} + \frac{1}{2}$ D. None of these