Roll No.....

NATIONAL INSTITUTE OF TECHNOLOGY, KURUKSHETRA

B. Tech. (Semester – I) Theory Examination, December 2019

Subject: Differential Calculus and Differential Equations

Paper Code: MAIR 11

Time: 03 Hours **Max. Marks: 50**

Note:

I. Answer all questions. Marks allotted for each question is shown on the right hand margin.

II. The candidates, before starting to write the solutions, should please check the question paper for any discrepancy, and also ensure that they have been delivered the question paper of **right course number** and the **right subject title**.

III. Unless stated otherwise, the symbols have their usual meanings in context with the subject.

1	**(a)** Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	[5]
	(b) State and prove Cayley-Hamilton theorem.	[5]
2	**(a)** If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, then find $\frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}$.	[5]
	(b) Find the minimum distance from the point (1, 2, 0) to the cone $z^2 = x^2 + y^2$, by Lagrange's method of undetermined multipliers.	[5]
	OR	
	(a) If $x + y = 2\,e^{\theta}\cos\phi$ *and* $x - y = 2\,i\,e^{\theta}\sin\phi$, prove that $\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial\phi^2} = 4xy\,\frac{\partial^2 u}{\partial x\partial y}$.	[5]
	(b) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	[5]
3	**(a)** Solve the differential equation: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10\,y + 37\sin 3x = 0$, and hence find the value of y when $x = \frac{\pi}{2}$, given that $y = 3$, $\frac{dy}{dx} = 0$, *when* $x = 0$.	[6]
	(b) Solve the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$, by the method of undetermined coefficients.	[4]

	OR **(a)** Solve the differential equation: $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10\,y = e^{2x}\sin x.$ **(b)** Use the variation of parameters method to solve the differential equation: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x.$	[5] [5]
4	**(a)** The equation of an electromotive force in terms of current i for an electrical circuit having resistance R and a Condenser of Capacity C in series is $E = Ri + \int \frac{i}{C}\,dt.$ Find the current i at any time t, when $E = E_0 \sin wt.$ **(b)** A body executes damped forced vibrations given by the equation $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin wt,$ solve the equation for both the cases when $w^2 \neq b^2 - k^2$ and when $w^2 = b^2 - k^2.$	[5] [5]
5	**(a) (i).** Find the Laplace transform of $\left[\frac{e^{2t} - \cos 3t}{t}\right].$ **(ii).** Find the inverse Laplace transform of $\log\left(1 + \frac{1}{s^2}\right).$ **(b)** Solve the differential equation by using Laplace transform $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,$ where $x(0) = 0,\ x'(0) = 1.$	[3] [3] [4]

THE END