Department of Mathematics

National Institute of Technology Kurukshetra

B.Tech. (Ist Semester) MID TERM 1

Jan -2021

Subject: Differential Calculus and Differential Equation

Code: MAIR 11 **Branch-PI,EC,ME**

Time: 40 mins **Max. Marks: 15**

Timings: 8. 30a.m-9.10 a.m.

Note: 1. All questions are compulsory.

2.This question paper consists of two parts. Part A has 10 objective questions of one mark each and Part B has 2 questions of 2.5 mark each.

PART A

1.For a given matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3. The other two eigen values are

(a) $7, -5$ (b)$3, -5$ (c)2 ,5 (d) 3 ,5

2.The eigen vector of the matrix $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ corresponding to $\lambda = 2$ is

(a)$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. The nature of the Quadratic form associated with the matrix is

$$\begin{bmatrix} 112 & 690 & -367 \\ 0 & 0 & 467 \\ 0 & 0 & 891 \end{bmatrix}$$

(a) Indefinite (b) Negative Semi Definite (c) Positive Definite

(d) Positive Semi Definite

4. The matrix Q = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies the characteristic equation

(a)$\lambda^3 + 6\lambda^2 + 9\lambda - 4 = 0$ (b) $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

(c) $\lambda^3 - 6\lambda^2 - 9\lambda - 4 = 0$ (d) $\lambda^3 - 6\lambda^2 + 9\lambda + 4 = 0$

5. The eigen values of a skew-symmetric matrix are

(a) Unit Modulus (b)Real

(c)Purely imaginary or zero (d) None of the above

6. A 4×4 matrix P have eigen values $1, -1, 2, -2$ then the eigen values of the matrix $Q=2P + P^{-1} - I$ are

(a) $2, -4, 7/2, -11/2$ (b) $2, -4, 1, -1$

(c) $-1, -4, 2, 7$ (d) $2, -2, -1, 1$

7. If $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & x \neq 0, y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$

At the origin the function F(x,y) is

(a)Continuous (b)Discontinuous (c)Not Defined (d)None of the above

8. The total differential of the function $\psi = e^{\xi} cos\eta$

(a) $d\psi = e^{\xi}\, cos\eta\, d\xi + e^{\xi}\, sin\,\eta\, d\eta$ (b) $d\psi = -e^{\xi}\, cos\eta\, d\xi - e^{\xi}\, sin\,\eta\, d\xi$

(c) $d\psi = e^{\xi}\, cos\eta\, d\xi - e^{\xi}\, sin\,\eta\, d\eta$ (d) $d\psi = e^{\xi}\, cos\eta\, d\eta - e^{\xi}\, sin\,\eta\, d\xi$

9. At point (1, 2/3) the function F(x,y)=$4x^2 + 9y^2 - 8x - 12y + 4$ has

(a) Local minimum value -4 (b) Local maximum value -4

(c) No extreme Value (d) Case is doubtful

10. Which of the following is not true

(a)Every Square matrix satisfies its own characteristic equation.

(b)If X is an eigen vector of A corresponding to the eigen value λ , then $P^{-1}X$ is an eigen vector of B corresponding to the eigen value λ , where P is the similarity matrix.

(c)If two matrices are similar , then they have same characteristic equation.

(d)Two matrices having same eigen values are always similar.

PART B

11. Determine the modal matrix for the matrix Q=$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and check pairwise orthogonality of the eigen vectors.

12. Find the value of n so that the equation $v = r^n(3cos^2\theta - 1)$ satisfies the relation

$\frac{\partial}{\partial r}\left(r^2 \frac{\partial v}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\ \frac{\partial v}{\partial \theta}\right) = 0$