Department of Mathematics

National Institute of Technology Kurukshetra

B.Tech. (Ist Semester) MID TERM 1

Jan -2021

Subject: Differential Calculus and Differential Equation

Code: MAIR 11 **Branch-IT,EE,CS,CE**

Time : 40 mins **Max. Marks: 15**

Timings: 10.20a.m-11.00 a.m.

Note: 1. All questions are compulsory.

2.This question paper consists of two parts. Part A has 10 objective questions of one mark each and Part B has 2 questions of 2.5 mark each.

PART A

1.For a given matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, one of the eigen value is 6. The other two eigen values are

(a) $-2, -3$ (b)$3, -3$ (c)$2 , -2$ (d) 2, 3

2.The eigen vector of the matrix $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ corresponding to $\lambda = 5$ is

(a)$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 9 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. The nature of the Quadratic form associated with the matrix is

$$\begin{bmatrix} -2 & 1 & 2 \\ 0 & 0 & 7 \\ 0 & 0 & -5 \end{bmatrix}$$

(a) Indefinite (b) Negative Semi Definite (c) Positive Definite

(d) Positive Semi Definite

4. The matrix Q = $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ satisfies the characteristic equation

(a)$\lambda^3 + 6\lambda^2 + 11\lambda - 6 = 0$ (b) $\lambda^3 - 6\lambda^2 + 11\lambda + 6 = 0$

(c) $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ (d) $\lambda^3 - 6\lambda^2 + 11\lambda + 4 = 0$

5. The eigen values of an orthogonal matrix are

(a) Unit Modulus (b)Real

(c)Pure imaginary or zero (d) None of the above

6. A 3×3 matrix P have eigen values $1, 2, -1$ then the eigen values of the matrix $Q = P - P^{-1} + P^2$ are

(a) 1, 11/2, 1 (b) 2,-4, 1

(c) 1, -4, 2 (d) 2,-1,1

7. If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & x \neq 0, y \neq 0 \\ 0 & x = 0, y = 0 \end{cases}$

At the origin the function $f(x,y)$ is

(a) Continuous (b) Discontinuous (c)Not Defined (d)None of the above

8. The total differential of the function $z = tan^{-1}(\frac{y}{x})$

(a) $dz = \frac{1}{x^2+y^2}(-ydx + xdy)$ (c) $dz = \frac{1}{x^2+y^2}(ydx + xdy)$

(c) $dz = \frac{1}{x^2+y^2}(-ydx - xdy)$ (d) $dz = \frac{1}{x^2+y^2}(ydx - xdy)$

9. At point (1/6, 0) the function F$(x,y) = 3x^2 + y^2 - x$ has

(a) Local minimum value $\frac{-1}{12}$ (b) Local maximum value $\frac{-1}{12}$

(c) No extreme Value (d) Case is doubtful

10. Which of the following is not true

(a) Eigen vectors corresponding to distinct eigen values are linearly independent.

(b) A square matrix B of order 3 is diagonalizable if and only if it has 3 linearly independent eigen vectors.

(c) Determinant of a skew symmetric matrix of odd order is zero.

(d) The matrix A^TA has imaginary eigen values if A be a real square matrix.

PART B

11. Determine the modal matrix for the matrix Q=$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ and check the pairwise orthogonality of the eigen vectors.

12. If $x^x y^y z^z = c$, show that at x = y = z,

$\frac{\partial^2 z}{\partial x \partial y} = -(xlogex)^{-1}$