11 DEC 2017

Sheet No. 1/2

Roll No. ____________________

National Institute of Technology, Kurukshetra

B. Tech. (Semester I) ***End semester Examination*** **2017-18**

Subject: Mathematics- **I (Old Scheme, Reappear, *July 2012 onwards*)**
(Code: **MAT-103/MAT-105T**) Time: 3 hours Max. Marks: 50

Instructions:

I. Answer **FIVE** questions out of the following selecting at least **one** question from each unit. All questions carry equal marks.

II. **Do not** over attempt. Attempt the questions in **ascending serial order** only.

III. The Candidates, before starting to write the solutions, should please check the Question Paper for any discrepancy, and also ensure that they have been delivered the question paper of **right Code** and right subject title.

IV. Symbols have their usual meanings until and unless stated otherwise.

	Unit-I
1(a)	Derive a formula for radius of curvature of a polar curve. Hence find the radius of curvature at the point (r,θ) of the curve $r^n = a^n \cos n\theta$.
(b)	Find the centre of curvature for the cycloid $x = a(t - \sin t),\ y = a(1-\cos t)$.
2(a)	Given $\log_{10} 4 = 0.6021$, calculate approximately $\log_{10} 404$.
(b)	Write the expansion of $\frac{x}{2}\left(\frac{1+e^{-x}}{1-e^{-x}}\right)$ in ascending powers of x.
3(a)	Find all the asymptotes of the curve $4x^3 + 2x^2 - 3xy^2 - y^3 - xy - y^2 - 1 = 0$.
(b)	Trace the curve $x^2y^2 = a^2\left(y^2 - x^2\right)$.
	Unit-II
4(a)	Given $u = e^{r\cos\theta}\cos(r\sin\theta),\ v = e^{r\cos\theta}\sin(r\sin\theta)$, prove that $v_\theta = ru_r$ and $u_\theta = -rv_r$.
(b)	If $x^2 + y^2 + z^2 - 2xyz = 1$, show that $\frac{dx}{\sqrt{(1-x^2)}} + \frac{dy}{\sqrt{(1-y^2)}} + \frac{dz}{\sqrt{(1-z^2)}} = 0$.
5(a)	If f and $\overrightarrow{A}$ are point functions, prove that the components of $\overrightarrow{A}$ normal and tangential to the surface f=0 are $\frac{\left(\overrightarrow{A}.\overrightarrow{\nabla}f\right)\overrightarrow{\nabla}f}{\left(\overrightarrow{\nabla}f\right)^2}$ and $\frac{\overrightarrow{\nabla}f\times\left(\overrightarrow{A}\times\overrightarrow{\nabla}f\right)}{\left(\overrightarrow{\nabla}f\right)^2}$ respectively.

5(b)	Evaluate $\int_0^x \frac{\log(1+xy)}{1+y^2}dy$ by Leibnitz rule.
6(a)	What is the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the normal to the surface $x\log z - y^2 = -4$ at (-1, 2, 1)?
(b)	The acceleration of a particle at any time $t \geq 0$ is given by $(12\cos 2t)\hat{i} - (8\sin 2t)\hat{j} + (16t)\hat{k}$, the velocity and displacement are initially zero. Find the velocity and displacement at any time.
	Unit-III
7(a)	Evaluate A^{-1} for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using Cayley Hamilton theorem.
(b)	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ into canonical form by an orthogonal transformation and give the matrix of transformation. Also state the nature of the quadratic form.
8(a)	Prove that every Hermitian matrix can be written as A+iB, where A is real and symmetric and B is real and skew symmetric.
(b)	Prove that $\left(\frac{1+\tanh\theta}{1-\tanh\theta}\right)^3 = \cosh 6\theta + \sinh 6\theta$.
9(a)	Sum the series $\sin\alpha - \frac{\sin(\alpha+2\beta)}{2!} + \frac{\sin(\alpha+4\beta)}{4!} - \ldots + \infty$.
(b)	Prove that $i\log\left(\frac{a-ib}{a+ib}\right) = \tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)$.

------------------*The End*------------------