**1. Introduction**

**1.1 Problem Statement**

Elective care in the National Health Service (NHS) is under significant strain, with waiting lists reaching historic highs in the aftermath of the COVID-19 pandemic. Cataract surgery, one of the highest-volume elective procedures in the UK, has been particularly affected. A key operational challenge is the fragmentation of **Patient Treatment Lists (PTLs)**: each NHS trust and independent sector provider maintains its own queue. Patients are typically referred by optometrists to a single provider, after which they remain on that provider’s waiting list regardless of alternative capacity elsewhere.

This siloed system creates **inequities and inefficiencies**. For instance, NHS hospitals often report backlogs of around six months, while independent sector providers have average waits of around four months. As a result, a patient’s access to surgery is dictated more by referral pathways and local list management than by medical urgency or regional system capacity. Fragmentation prevents load balancing, leading to underutilisation in some facilities and overburden in others.

Cataract care provides an ideal case study. It is a relatively standardised procedure, clinically significant for patient quality of life, and accounts for a substantial proportion of elective activity. Addressing PTL fragmentation in cataract surgery therefore offers both **clinical impact**—by improving patient outcomes—and **system impact**—by demonstrating how unified elective lists can increase efficiency [1]–[3].

**1.2 Significance and Context**

The strategic importance of PTL reform is evident in national policy. The NHS *Long Term Plan* [4] emphasises reducing unwarranted variation and improving equity of access, while the NHS *Operational Plan 2025/26* [5] highlights elective recovery as a national priority. Surgical care accounts for 10–30% of hospital expenditure [6], making efficiency gains in operating theatre use particularly consequential.

At the patient level, prolonged waits for cataract surgery lead to impaired mobility, reduced independence, and poorer quality of life. Timely intervention restores vision and functionality, reducing downstream social and healthcare costs. Addressing waiting times is therefore not simply an operational challenge but also a **population health imperative**.

The relevance of queueing theory and simulation extends beyond healthcare. Similar problems have been addressed in **call centres** [7], **airports** [8], **air traffic flow** [9], and **road networks** [10]. In each case, pooling and intelligent scheduling reduce congestion and variability. Applying such methods to NHS PTLs represents a translation of proven operational research into healthcare delivery.

**1.3 Motivation and Objectives**

The motivation for this study stems from the combination of a pressing operational challenge and the availability of methods capable of addressing it. The COVID-19 pandemic exacerbated waiting list pressures, exposing the limitations of fragmented PTLs and static planning. Evidence from operations research indicates that pooling resources can significantly reduce delays [6], [11]. Simulation studies in oncology [12] and emergency care [13] show that reconfiguring patient flows yields measurable improvements.

The objectives of this study are:

1. To **mathematically model** separate versus consolidated PTLs using queueing theory (M/G/c).
2. To **simulate** patient flows via discrete-event simulation (DES), incorporating real-world complexities such as priorities, cancellations, and triage pathways.
3. To **optimise** patient allocation using reinforcement learning (RL), comparing heuristic and adaptive policies.

By combining these methods, the project provides both theoretical insights and applied recommendations. Expected outcomes include reductions in average waiting times and improvements in utilisation, aligning with NHS strategic goals for elective recovery.

**1.4 Thesis Overview**

The dissertation proceeds as follows. **Chapter 2 (Background)** reviews the literature and foundations of queueing, simulation, and reinforcement learning. **Chapter 3 (Methodology)** describes model construction. **Chapter 4 (Results)** compares outcomes across scenarios. **Chapter 5 (Discussion)** interprets findings in light of NHS policy. **Chapter 6 (Conclusion)** summarises contributions and outlines future work.

**2. Background**

**2.1 Problem-Specific Literature Review**

Healthcare queueing has been widely studied, particularly in emergency and surgical contexts. Green et al. [13] demonstrated how queueing models explain emergency department crowding, while Cochran and Roche [14] extended multi-class networks to ambulatory care. Cardoen et al. [6] reviewed decades of operating theatre scheduling, showing efficiency gains through queue pooling and better allocation. Beliën and Demeulemeester [15] applied mixed-integer programming to surgical scheduling, illustrating capacity bottlenecks.

DES has been successfully applied in oncology. Santibañez et al. [12] redesigned cancer pathways, reducing delays by centralising decision points. Harper and Shahani [24] applied simulation to NHS hospitals, finding that bottlenecks at triage points created unnecessary waiting times. More recent reviews confirm the enduring importance of simulation [11], [16].

Evidence from **ophthalmology** further supports simulation methods. Hussain et al. (2019) used DES to predict demand for cataract services, demonstrating how stochastic modelling can anticipate surges in referrals. Similar work in **orthopaedics** shows that waiting times can be cut by redesigning scheduling policies [31].

Pooling benefits are well established across industries. Larson [9] studied **air traffic flow**, demonstrating how centralised slot allocation reduced delays. Helbing [10] modelled **road traffic**, showing how self-organisation improved flow stability. Wu and Mengersen [8] simulated **airport security**, illustrating variance reduction via pooling. Koole [7] applied similar methods to **call centres**, showing centralised routing increases efficiency. These analogies highlight how fragmented NHS PTLs parallel problems long solved in other domains.

Despite these successes, PTL consolidation remains under-researched in healthcare. Most studies address local optimisation [6], [15] rather than system-wide pooling. Moreover, few combine queueing, DES, and RL into an integrated solution. This gap motivates the present study.

**2.2 Queueing Theory Foundations**

Queueing theory provides the mathematical language to analyse waiting systems [17], [18]. Systems are denoted by **Kendall’s notation A/B/c**, where *A* specifies the arrival distribution, *B* the service-time distribution, and *c* the number of servers. For example, M/M/1 denotes Poisson arrivals, exponential service times, and one server. Little’s Law [19] provides the relation $L=\lambda W$, linking arrivals, workload, and system length.

Multiple queue types exist:

* **M/M/1**: exponential interarrival and service, one server.
* **M/D/1**: exponential interarrival, deterministic service.
* **M/G/1**: exponential arrivals, general service distribution.
* **G/G/c**: general arrivals, general service times, *c* servers.

In healthcare, variability in arrivals and service makes **M/G/c models** most relevant. The **Pollaczek–Khinchine formula** extends M/G/1 analysis to waiting times, while **Kingman’s approximation** estimates delay in G/G/1 queues. These tools highlight how variability amplifies congestion.

In cataract surgery, variability is substantial. Consultant-led cases average 19 minutes, trainee-led cases extend to 30–40 minutes, and complex HRG-coded procedures may last over an hour [20], [21]. This leads to coefficients of variation ≥0.8 in NHS settings, versus ~0.6 in independent sector clinics.

Operational assumptions:

* **NHS throughput**: ~16 cases/day.
* **Independent throughput**: up to 30/day.
* **Backlog**: ~6 months (NHS) vs 4 months (independents).
* **Priority classes**: Urgent, TWW, Routine.

Analogies illustrate PTL dynamics. **Airline overbooking** uses stochastic models to balance capacity and cancellations. **Telecommunications** applies Erlang models to call routing. **Traffic networks** apply risk pooling to avoid local congestion [9], [10]. Similarly, PTL consolidation pools queues, reducing variance and improving fairness.

**2.3 Discrete-Event Simulation Foundations**

DES models systems as entities (patients) progressing through events (referral, surgery) over time [22]. Unlike system dynamics (aggregate-level) or agent-based modelling (individual-level interactions), DES captures stochastic processes at the patient level. It is especially suited to healthcare, where arrivals, cancellations, and resource use vary.

Brailsford et al. [22] reviewed DES in healthcare, emphasising its ability to reflect real-world uncertainty. Jun et al. [23] demonstrated its use for hospital planning, while Harper and Shahani [24] identified bottlenecks using DES in NHS contexts. Günal and Pidd [11] noted its suitability for policy testing, and Landa et al. [16] reviewed applications across elective care.

For this project, DES was implemented in Python (SimPy). Features included:

* **Priority classes**: Urgent, TWW, Routine.
* **Separate vs consolidated PTLs**.
* **Triage bias**: optometrist-driven vs NHS centralised triage.
* **Capacity realism**: NHS ~16 vs independent ~30/day.
* **Cancellations/rescheduling**: utilisation capped at 95%.

Replication strategies ensured robust results: 10+ runs per scenario, warm-up periods to remove initialisation bias, and validation against M/G/c approximations via Little’s Law.

DES is particularly valuable because it provides a **digital twin** of the cataract pathway. This enables safe experimentation with alternative designs and supports RL training.

**2.4 Reinforcement Learning Foundations**

Reinforcement learning formalises sequential decision-making as a **Markov Decision Process (MDP)** [25]. States represent the system (queue lengths, utilisation), actions select providers, and rewards encode performance. The Bellman equation governs value updates. RL agents learn by balancing exploration of new actions against exploitation of known good ones.

In this project, two algorithms were tested:

* **Cross-Entropy Method (CEM):** a stochastic optimiser that iteratively refines random policy distributions. It served as a baseline.
* **Deep Q-Network (DQN):** the main algorithm. Advantages include:
  + **Discrete action suitability:** 6 providers.
  + **Neural approximation:** handles 27D state.
  + **Experience replay and target networks:** stabilise learning.
  + **Healthcare precedent:** proven in similar scheduling tasks [26].

Alternatives were considered but excluded:

* Q-learning, SARSA: infeasible for large state space.
* A3C/A2C, PPO: high variance or computationally heavy.
* DDPG: designed for continuous actions.

Training occurred within the DES digital twin. This allowed safe experimentation without patient risk. Over episodes, DQN learned adaptive policies outperforming static heuristics such as “always send to shortest queue.” Policies sometimes diverted patients proactively to balance future load.

RL has been applied to treatment policy learning [27], ICU sepsis management [32], and radiotherapy scheduling [26]. However, integration with DES for PTL consolidation remains novel.

**2.5 Forecasting and RL Side Track**

While RL primarily supported allocation, it also has potential for **demand forecasting**. Classical methods such as ARIMA [28] and Prophet [29] model trends and seasonality. Hybrid approaches combining statistical and ML models outperform classical methods in predicting healthcare arrivals [30].

In this project, forecasting was treated as a complementary strand. Forecasted arrivals could feed into queueing models and DES to project future backlogs. RL could then adapt to these scenarios by simulating demand surges. For instance, winter pressures could be modelled, and RL trained to allocate dynamically under peak load.

Geographic extensions are also possible. Incorporating **LSOA-level data** could balance waiting time with travel distance, aligning allocation with equity as well as efficiency.

Although forecasting was not central to the waiting-time analysis, its integration illustrates how RL and DES can support **strategic as well as operational planning**.

**2.6 Synthesis and Research Gaps**

The review suggests:

* **Queueing theory** provides analytic insight but oversimplifies variability.
* **DES** captures complexity but relies on fixed policies.
* **RL** enables adaptive allocation but requires simulation.

Few studies integrate all three in the NHS context. This project contributes by:

* Modelling PTLs using M/G/c and DES.
* Incorporating NHS-specific details: HRG complexity, triage bias, throughput assumptions.
* Training RL agents in a digital twin for adaptive allocation.
* Exploring RL/forecasting as tools for demand prediction.

This integrated framework fills a gap in healthcare operations research and provides evidence for NHS policy debates on elective recovery.

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